$D = \{x_1 \dots x_n\}$ taken from $x_1 \dots x_n \sim \mathrm{Pois}(\lambda)$

PMF for Pois $\Pr[X=k] = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$

1. Assuming $x_1 \dots x_n$ are independent

$\mathcal{L}(\lambda | D) =$

$$\Pr[X=x_1] \cdot \Pr[X=x_2] \dots$$

$$\prod_i \Pr[X=x_i] = e^{-n\lambda} \cdot \frac{\lambda^{\sum x_i}}{\prod x_i!}$$

Applying natural logs

$$\boxed{-n\lambda + \ln(\lambda) \sum x_i - \ln\left(\prod x_i!\right)}$$

2) for finding MLE $\hat{\lambda}$

$$\frac{d\ell}{d\lambda}\left(-n\lambda + \ln(\lambda)\cdot\sum x_i - \ln\left(\prod x_i!\right)\right) =$$

$$-n + \frac{\sum x_i}{\lambda} = 0 \Rightarrow \boxed{\hat{\lambda} = \frac{\sum x_i}{n}}$$

3) $$\mathbb{E}[\hat{\lambda}] = \frac{\sum E[x_i]}{n} = \frac{n\cdot\lambda}{n} = \boxed{\lambda}$$

$$Var[\hat{\lambda}] = Var\left[\frac{1}{n}\sum x_i\right] = \frac{1}{n^2}\sum Var[x_i] = \frac{n\lambda}{n^2} = \boxed{\frac{\lambda}{n}}$$

4) $\Phi^{-1}(0.975) = -\Phi^{-1}(0.025)$ so

$$\boxed{\hat{\lambda} \pm \Phi^{-1}(0.975)\sqrt{\frac{\hat{\lambda}}{n}}}$$