Graph Metrics in Neuroscience Network

A brief summary of relevant metrics, and how to calculate them.

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1. **Vertex degree / Degree centrality**

   The total number of vertices connected to the vertex. Vertex with a higher degree centrality is more central in a network.

   $C_d(v) = \sum_{i \neq j} W_{ij}$

   Where W_{ij} is the weight of the edge linking vertices i and j.

2. **Path length**

   The path with the least number of edges between two vertices.

3. **The clustering coefficient**

   Measure the tendency for any two vertices to cluster together.

   $C(i) = \frac{2}{s_i(s_i-1)} \sum_{j,h} (\widehat{w}_{ij} \widehat{w}_{jh} \widehat{w}_{hi})^{\frac{1}{3}}$

   Or

   $C(i) = \frac{2N_i}{K_i(K_i-1)}$, $N_i = number\ of\ links\ between\ the\ neighbers\ of\ i.$

   Centralities

4. **Eigenvector (degree-based) centrality**

   Measure a vertex importance in a network based on the centrality of his neighbors.

   $C_E(i) = \frac{1}{\lambda_i} \sum_{j=1}^{N} A_{ij} x_j$, $Ax = \lambda x$

   Where A is the adjencency matrix, x is an eigenvector of A with eigenvalue of λ.

5. **Closeness (shortest path based) centrality**

   Measures how quickly information can spread from a vertex to all other vertices in the network.

   $C_c(i) = \frac{N-1}{\sum_{i=1}^{N-1} d(i,j)}$

Where $d(i,j)$ is the shortest-path distance between I and j, N is the number of vertices in the graph.

6. **Betweenness (shortest path base) centrality**

   Measures the number of shortest paths that pass through a vertex, indicating its role in connecting different parts of the network.

   $$C_B(v) = \sum_{i \neq h \neq j \in V} \frac{\sigma_{ij}(h)}{\sigma_{ij}}$$

   Where V is a set of vertices, σ_{ij} is the total number of shortest paths between i and j, $\sigma_{ij}(h)$ is the number of those paths that pass through h.

7. **The minimum spanning tree**

   A subgraph that is a tree and connects all the vertices together with the minimum possible sum of edge weights.

   Metrics that related to it: **the total weight of the minimum spanning tree**, or **the specific edges included in the minimum spanning tree**.

8. **Modularity**

   State how divisible a network is into different modules (communities).

   A higher modularity score indicates a better-defined community structure, with more connections within communities than expected by random chance.

   $$Q = \frac{1}{m}\sum_{ij}\left[a_{ij} - \frac{k_i^{in}k_j^{out}}{m}\right]\delta_{c(i),c(j)}$$

   where m is total number of edges in the network, a_{ij} is element of adjacency matrix, k_i^{in} is in-degree of node i, k_j^{out} is out-degree of node j, $\delta_{c(i),c(j)}$ is Kronecker delta (only one if nodes i and j are in the same module and zero otherwise).

Global scale — aggregate measures

9. **The edge density**

   the proportion of connections that exists relative to the number of potential connections of a network. It provides insight into how connected or dense a network is. Often expressed as a ratio between the number of observed edges and the total number of possible edges in the graph.

   $D = \frac{2E}{V(V-1)}$, (undirected graphs)

10. Average path length

The average path length for all possible pairs of vertices in a network. Global measure information transport efficiency and integration in a network.

$$ASP = \sum_{i,j \in V} \frac{d(i,j)}{N(N-1)}$$

Where V is a set of vertices, $d(i,j)$ is the shortest path between vertices i and j, and N is the number of vertices in the network.

11. Global efficiency

Quantifies how efficiently information can be exchanged across the entire network.

$$E_{global} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d(i,j)}$$

where $d(i,j)$ is the length of the shortest path between nodes i and j and N is the number of nodes.

12. Global clustering coefficient

Quantifies the overall level of clustering or transitivity in a network.

$$C_{global} = \frac{1}{N} \sum c(i)$$

Bibliography

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