Goals as Reward-Generating Programs Domain Specific Language

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BODY

1 Modal Definitions in Linear Temporal Logic

1.1 Linear Temporal Logic definitions

We offer a mapping between the temporal sequence functions defined in **??** (**??**) and linear temporal logic (LTL) operators. As we were creating this DSL, we found that the syntax of the $\langle then \rangle$ operator felt more convenient than directly writing down LTL, but we hope the mapping helps reason about how we see our temporal operators functioning. LTL offers the following operators, using φ and ψ as the symbols (in our case, predicates). Assume the following formulas operate sequence of states $S_0, S_1, \cdots, S_n$:

- **Next**, $X\psi$: at the next timestep, ψ will be true. If we are at timestep i, then $S_{i+1} \vdash \psi$
- **Finally**, $F\psi$: at some future timestep, ψ will be true. If we are at timestep i, then $\exists j > i : S_j \vdash \psi$
- **Globally**, $G\psi$: from this timestep on, ψ will be true. If we are at timestep i, then $\forall j : j \geq i : S_j \vdash \psi$
- **Until**, $\psi U \varphi$: ψ will be true from the current timestep until a timestep at which φ is true. If we are at timestep i, then $\exists j > i : \forall k : i \leq k < j : S_k \vdash \psi$, and $S_j \vdash \varphi$.
- **Strong release**, $\psi M \varphi$: the same as until, but demanding that both ψ and φ are true simultaneously: If we are at timestep i, then $\exists j > i : \forall k : i \leq k \leq j : S_k \vdash \psi$, and $S_j \vdash \varphi$.

  Aside: there's also a **weak until**, $\psi W \varphi$, which allows for the case where the second is never true, in which case the first must hold for the rest of the sequence. Formally, if we are at timestep i, *if* $\exists j > i : \forall k : i \leq k < j : S_k \vdash \psi$, and $S_j \vdash \varphi$, and otherwise, $\forall k \geq i : S_k \vdash \psi$. Similarly there's **release**, which is the similar variant of strong release. We're leaving those two as an aside since we don't know we'll need them.

1.2 Satisfying a $\langle then \rangle$ operator

Formally, to satisfy a preference using a $\langle then \rangle$ operator, we're looking to find a sub-sequence of $S_0, S_1, \cdots, S_n$ that satisfies the formula we translate to. We translate a $\langle then \rangle$ operator by translating the constituent sequence-functions ($\langle once \rangle$, $\langle hold \rangle$, $\langle while\text{-}hold \rangle$)[1] to LTL. Since the translation of each individual sequence function leaves the last operand empty, we append a 'true' ($\top$) as the final operand, since we don't care what happens in the state after the sequence is complete.

(once ψ) $:= \psi X \cdots$

(hold ψ) $:= \psi U \cdots$

(hold-while $\psi\ \alpha\ \beta \cdots \nu$) $:= (\psi M \alpha) X (\psi M \beta) X \cdots X (\psi M \nu) X \psi U \cdots$ where the last $\psi U \cdots$ allows for additional states satisfying ψ until the next modal is satisfied.

For example, a sequence such as the following, which signifies a throw attempt:

```
(then
    (once (agent_holds ?b))
    (hold (and (not (agent_holds ?b)) (in_motion ?b)))
    (once (not (in_motion ?b)))
)
```

Can be translated to LTL using $\psi :=$ (agent_holds ?b), $\varphi :=$ (in_motion ?b) as:

$\psi X (\neg\psi \wedge \varphi) U (\neg\varphi) X \top$

Here's another example:

[1] These are the ones we've used so far in the interactive experiment dataset, even if we previously defined other ones, too.

```
(then
    (once (agent_holds ?b))    α
    (hold-while
        (and (not (agent_holds ?b)) (in_motion ?b))  β
        (touch ?b ?r)  γ
    )
    (once  (and (in ?h ?b) (not (in_motion ?b))))  δ
)
```

If we translate each predicate to the letter appearing in blue at the end of the line, this translates to:
$\alpha X(\beta M\gamma)X\beta U\delta X\top$