Digital Logic Systems

Recitation 7: Foundations of combinational circuits & Quiz rehearsal

Guy Even Moti Medina

School of Electrical Engineering Tel-Aviv Univ.

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- Let $C = (G, \pi)$ denote a combinational circuit where $G = (V, E)$ is a directed graph and $\pi : V \to \Gamma \cup IO$ is a labeling.
- Let $c : \Gamma \cup IO \to \mathbb{R}^{\geq 0}$ denote a cost function. Usually, input-gates and output-gates have zero cost.

Definition

The cost of C is defined by

$$c(C) \triangleq \sum_{v \in V} c(\pi(v)).$$

Propagation delay

The propagation delays $t_{pd}(v)$ are computed by Algorithm $\mathrm{SIM}(C, \vec{x})$.

Definition

The propagation delay of C is defined by

$$t_{pd}(C) \triangleq \max_{v \in V} t_{pd}(v).$$

We often refer to the propagation delay of a combinational circuit as its depth or simply its delay.

Definition

The propagation delay of a path p in G is defined as

$$t_{pd}(p) \triangleq \sum_{v \in p} t_{pd}(\pi(v)).$$

Critical paths

Algorithm $\mathrm{SIM}(C, \vec{x})$ computes the largest delay of a path in G.

Claim (4)

$$t_{pd}(C) = \max\left\{t_{pd}(p) \mid p \textit{ is a path in } G\right\}$$

Definition

Let $G = (G, \pi)$ denote a combinational circuit. A path p in G is critical if $t_{pd}(p) = t_{pd}(C)$.

We focus on critical paths that are maximal (i.e., cannot be further augmented). This means that maximal critical paths begin in an input-gate and end in an output-gate.

Recall Claim 4

Claim (4)

$$t_{pd}(C) = \max\left\{t_{pd}(p) \mid p \textit{ is a path in } G\right\}$$

The number of paths can be exponential in n. Does this mean that we cannot compute the propagation delay of a combinational circuit in linear time?

Answer: No. There is no need to enumerate all the paths in order to find the longest one. We can compute the propagation delay of a combinational circuit $C = (G, \pi)$ in linear time (in the size of the DAG G), e.g., using the SIM algorithm.

The number of paths can be exponential in n.

Question

Describe a combinational circuit with n gates that has at least $2^{n/2-1}$ paths.

Proof.

On the whiteboard. $\square$

Associative Boolean functions

Definition

A Boolean function $f : \{0,1\}^2 \to \{0,1\}$ is *associative* if

$$f(f(\sigma_1, \sigma_2), \sigma_3) = f(\sigma_1, f(\sigma_2, \sigma_3)),$$

for every $\sigma_1, \sigma_2, \sigma_3 \in \{0,1\}$.

We "extend" $f : \{0,1\}^2 \to \{0,1\}$ to $f_n : \{0,1\}^n \to \{0,1\}$ as follows.

Definition

Let $f : \{0,1\}^2 \to \{0,1\}$ denote a Boolean function. The function $f_n : \{0,1\}^n \to \{0,1\}$, for $n \geq 1$, is defined recursively as follows.

1. If $n = 1$, then $f_1(x) = x$.
2. If $n = 2$, then $f_2 = f$.
3. If $n > 2$, then f_n is defined based on f_{n-1} as follows:

$$f_n(x_1, x_2, \ldots x_n) \triangleq f(f_{n-1}(x_1, \ldots, x_{n-1}), x_n).$$

Associative Boolean functions (cont.)

If $f(x_1, x_2)$ is an associative Boolean function, then one could define f_n in many equivalent ways, as summarized in the following claim.

Claim

If $f : \{0,1\}^2 \to \{0,1\}$ is an associative Boolean function, then

$$f_n(x_1, x_2, \ldots x_n) = f(f_{n-k}(x_1, \ldots, x_{n-k}), f_k(x_{n-k+1}, \ldots, x_n)),$$

for every $n \geq 2$ and $k \in [1, n-1]$.

Questions from Quiz(zes)

Calm down!