Chaotic Spectral Encryption (CSE) with Quantum Fourier Transform (QFT) and Recursive Lambda Updates: A Post-Quantum Security Approach

Abstract

This paper introduces a novel encryption framework, **Chaotic Spectral Encryption (CSE)**, which leverages adaptive spectral transformations, recursive lambda updates, and dynamically evolving spectral targets to create a secure, post-quantum cryptographic scheme. By integrating **Quantum Fourier Transform (QFT)** alongside classical **Fast Fourier Transform (FFT)** techniques, this framework enhances spectral obfuscation and ensures resilience against quantum cryptanalysis techniques such as **Grover's algorithm** and **Shor's algorithm**. Furthermore, the inclusion of **Lyapunov stability analysis** and **generalized update mappings** provides formal mathematical guarantees for system robustness. This paper explores the theoretical foundations, implementation strategy, and security implications of this hybrid encryption approach.

1. Introduction

1.1 Background and Motivation

The rapid advancement of quantum computing poses a significant threat to conventional cryptographic schemes. Algorithms such as **Shor's factorization algorithm** and **Grover's quantum search algorithm** can efficiently break RSA, ECC, and other cryptographic systems. Existing post-quantum cryptographic methods, such as **lattice-based cryptography**, provide resistance but often lack adaptivity. This paper introduces a **Chaotic Spectral Encryption (CSE) model** that combines classical chaos-based cryptographic principles with quantum computational techniques to ensure high security and adaptiveness.

1.2 Objectives

1. Develop an encryption scheme that **dynamically adjusts transformation rules** over time.
2. Implement a **chaotic spectral feedback mechanism** using FFT- and QFT-based adaptive encryption.
3. Design **recursive lambda updates** to introduce non-linearity and time-dependent diffusion properties.
4. Analyze security, efficiency, and quantum resistance of CSE.
5. Evaluate feasibility of implementation on **classical computing platforms (GPUs/FPGA)** and quantum-enabled hardware.

2. Methodology

2.1 Adaptive Dynamic Balance Function

The encryption system is governed by a **balance function** that dynamically modifies encryption parameters based on spectral feedback:

$$F(P) = \sum_{i=1}^{N} \left(\mathbf{w}_i^\top \boldsymbol{\lambda} - \boldsymbol{\beta}^\top \mathbf{S}_i \right) A_i,$$

where ($\boldsymbol{\lambda} = (\lambda_C, \lambda_S, \lambda_E)$) represents dynamic key parameters and ($\mathbf{S}_i$) contains spectral metrics.

2.2 Spectral Feedback and QFT Integration

The encryption scheme updates dynamically based on both **FFT** and **QFT**:

$$\mathcal{S}_{\text{dynamic}}(t+1) = (1-\tau)\mathcal{S}_{\text{dynamic}}(t) + \tau \text{FFT}(\lambda(t)).$$

A hybrid quantum approach is introduced via:

$$|\tilde{\psi}\rangle = \text{QFT}(|\psi\rangle),$$

where ($|\psi\rangle$) encodes spectral components and adaptive states.

2.3 Recursive Lambda Updates with FFT/QFT Error Correction

To ensure that encryption states do not repeat or become predictable, we use **recursive lambda updates**:

$$\lambda^{(t+1)} = \lambda^{(t)} - \eta \nabla_\lambda L(\lambda^{(t)}) - \gamma \mathcal{E}_{FFT/QFT}(\lambda^{(t)}),$$

where the error correction term is:

$$\mathcal{E}_{FFT/QFT}(\lambda) = \left\| \text{FFT/QFT}(\lambda) - \mathcal{S}_{\text{target}} \right\|.$$

This approach leverages both classical and quantum spectral corrections for robust cryptographic state evolution.

2.4 Convergence and Stability Guarantees

A generalized update mapping ensures **contractive behavior**:

$$\mathcal{T}(\lambda) = \lambda - \eta \nabla L(\lambda) - \gamma H_f(\lambda) + \delta D_f(\lambda),$$

and a Lyapunov function guarantees stability:

$$V(\lambda) = L(\lambda) + \kappa E(\lambda),$$

where

$$E(\lambda) = \| \text{FFT/QFT}(\lambda) - \mathcal{S}_{\text{target}} \|^2$$

3. Security and Performance Evaluation

3.1 Cryptanalysis and Quantum Resistance

The proposed scheme is evaluated against:

- **Grover's Algorithm:** Expanding the effective key space mitigates Grover's quadratic speedup.
- **Shor's Algorithm:** The absence of periodicity prevents exploitation of structured factorization.
- **Adaptive Cryptanalysis:** Continuous evolution ensures state unpredictability against

side-channel attacks.

3.2 Computational Complexity and Hardware Feasibility

- **Parallelization on GPUs/FPGA**: FFT-based spectral feedback is computationally efficient.
- **Quantum Compatibility**: QFT-based corrections can be implemented on near-term quantum processors for hybrid cryptographic use cases.

4. Conclusion

This research introduces **Chaotic Spectral Encryption (CSE)** as a post-quantum cryptographic approach integrating classical chaos theory, adaptive spectral transformations, and quantum spectral processing. By leveraging **QFT-enhanced error correction**, **recursive lambda updates**, and **dynamic spectral feedback**, this model provides a strong foundation for resilient encryption mechanisms in a quantum-threatened era. Future work includes further experimental validation on classical and quantum hardware platforms.

5. References

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