Practical 1 - Bayesian Term 2

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My main section

Here's some maths:

$$\log(x^3) \neq \frac{\exp(3x)}{x}.$$

Align subsection

Adding maths in an "align" environment makes it easier to line things up.

$$\begin{aligned} \mu &\sim \mathrm{N}(0,1), \\ X_i|\mu &\sim \mathrm{N}(\mu,1), \quad i = 1, \ldots, n. \end{aligned}$$

R section

Here's some very simple R code.

```r
x <- rnorm(100)
y <- runif(100) + x
```

3. First Stan Models

```r
library(rstan)
```

```
## Loading required package: StanHeaders
```

```
##
## rstan version 2.26.13 (Stan version 2.26.1)
```

```
## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)
## For within-chain threading using 'reduce_sum()' or 'map_rect()' Stan functions,
## change 'threads_per_chain' option:
## rstan_options(threads_per_chain = 1)
```

```
## Do not specify '-march=native' in 'LOCAL_CPPFLAGS' or a Makevars file

setwd('C:/Users/guyro/OneDrive/Y3/bayesian-modelling/practicals')

our_model <- stan_model('conjugate_1.stan')

our_sample <- sampling(our_model,
                       data = list(n = 7, X = c(1.20, 1.21, 3.06, 7.89, 5.67, 6.10, 3.90)))
```

```
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 2.8e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.28 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.019 seconds (Warm-up)
## Chain 1:                0.014 seconds (Sampling)
## Chain 1:                0.033 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 7e-06 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
```

```
## Chain 2:
## Chain 2:  Elapsed Time: 0.015 seconds (Warm-up)
## Chain 2:                 0.016 seconds (Sampling)
## Chain 2:                 0.031 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 5e-06 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.05 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.016 seconds (Warm-up)
## Chain 3:                 0.016 seconds (Sampling)
## Chain 3:                 0.032 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 5e-06 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.05 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.016 seconds (Warm-up)
## Chain 4:                 0.017 seconds (Sampling)
## Chain 4:                 0.033 seconds (Total)
```

```
## Chain 4:
```

```
summary(our_sample)
```

```
## $summary
##            mean     se_mean        sd        2.5%        25%        50%
## mu     3.615481 0.008685492 0.3582970   2.915165   3.377522   3.614112
## lp__ -27.387127 0.018972895 0.7388802 -29.371906 -27.541106 -27.106709
##            75%       97.5%     n_eff      Rhat
## mu     3.861638    4.318968 1701.757 1.000521
## lp__ -26.926326 -26.873518 1516.634 1.000022
##
## $c_summary
## , , chains = chain:1
##
##          stats
## parameter        mean         sd        2.5%        25%        50%        75%
##       mu     3.604697 0.3705729    2.880928   3.358923   3.589009   3.849454
##       lp__ -27.424106 0.7733782 -29.702169 -27.600906 -27.131080 -26.922081
##          stats
## parameter      97.5%
##       mu     4.349045
##       lp__ -26.874105
##
## , , chains = chain:2
##
##          stats
## parameter        mean         sd        2.5%        25%        50%        75%
##       mu     3.629614 0.3473248    2.952619   3.393848   3.620164   3.862268
##       lp__ -27.355102 0.6951193 -29.323610 -27.456845 -27.093160 -26.930751
##          stats
## parameter      97.5%
##       mu     4.315921
##       lp__ -26.873340
##
## , , chains = chain:3
##
##          stats
## parameter        mean         sd        2.5%        25%        50%        75%
##       mu     3.606118 0.3544005    2.932119   3.368342   3.613595   3.860344
##       lp__ -27.376989 0.6526949 -29.148714 -27.567946 -27.124294 -26.926887
##          stats
## parameter      97.5%
##       mu     4.264902
##       lp__ -26.873599
##
## , , chains = chain:4
##
##          stats
## parameter        mean         sd        2.5%        25%        50%        75%
##       mu     3.621493 0.3604078    2.883477   3.409294   3.620448   3.868008
##       lp__ -27.392310 0.8219337 -29.522814 -27.503389 -27.086413 -26.926796
##          stats
## parameter      97.5%
```

```
##      mu      4.341628
##      lp__ -26.873385
```

3.2 A Tricky Conjugate Analysis

```r
our_data <- list(n = 5,
                 y = c(15,10, 6, 8, 17))

our_model = stan_model('conjugate_2.stan')

our_sample = sampling(our_model, data=our_data)
```

```
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
## Chain 1: Rejecting initial value:
## Chain 1:    Error evaluating the log probability at the initial value.
## Chain 1: Exception: beta_lpdf: Random variable is 5.08211, but must be in the interval [0, 1] (in 's
## Chain 1:
## Chain 1: Gradient evaluation took 2.6e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.024 seconds (Warm-up)
## Chain 1:                0.017 seconds (Sampling)
## Chain 1:                0.041 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
## Chain 2: Rejecting initial value:
## Chain 2:    Error evaluating the log probability at the initial value.
## Chain 2: Exception: beta_lpdf: Random variable is 3.7669, but must be in the interval [0, 1] (in 'st
## Chain 2:
## Chain 2: Gradient evaluation took 6e-06 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.06 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
```

```
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.032 seconds (Warm-up)
## Chain 2:                0.025 seconds (Sampling)
## Chain 2:                0.057 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 7e-06 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.034 seconds (Warm-up)
## Chain 3:                0.03 seconds (Sampling)
## Chain 3:                0.064 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 8e-06 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.08 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
```

```
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.029 seconds (Warm-up)
## Chain 4:                0.024 seconds (Sampling)
## Chain 4:                0.053 seconds (Total)
## Chain 4:
```

```
summary(our_sample)
```

```
## $summary
##                mean      se_mean        sd        2.5%         25%         50%
## theta     0.2992017 0.001248048 0.0473426   0.2089462   0.2666868   0.2986447
## lp__    -55.1116034 0.017805104 0.7060301 -57.1825932 -55.2636946 -54.8426620
##                75%       97.5%     n_eff      Rhat
## theta    0.3304395   0.3962024 1438.938 1.005073
## lp__   -54.6651848 -54.6191555 1572.380 1.003533
##
## $c_summary
## , , chains = chain:1
##
##          stats
## parameter        mean          sd        2.5%          25%          50%
##     theta    0.2970602 0.04875704   0.2118245   0.2622185   0.2953694
##     lp__   -55.1494248 0.72205102 -57.1849058 -55.3213512 -54.8908830
##          stats
## parameter       75%        97.5%
##     theta    0.3307511   0.3981604
##     lp__   -54.6815735 -54.6191765
##
## , , chains = chain:2
##
##          stats
## parameter        mean          sd        2.5%         25%          50%          75%
##     theta    0.3013398 0.04634943   0.2172269    0.270068    0.2978345    0.3302126
##     lp__   -55.0782388 0.67732918 -56.8286039  -55.223076  -54.8296516  -54.6598232
##          stats
## parameter       97.5%
##     theta    0.3991576
##     lp__   -54.6198022
##
## , , chains = chain:3
##
##          stats
## parameter        mean          sd        2.5%          25%          50%          75%
##     theta    0.3000848 0.04938163   0.201346    0.2687104    0.3023064    0.3335028
##     lp__   -55.1574609 0.76437192 -57.544705  -55.3004980  -54.8542164  -54.6709126
##          stats
## parameter       97.5%
```

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```
##      theta    0.3969636
##      lp__   -54.6190061
##
## , , chains = chain:4
##
##          stats
## parameter      mean          sd        2.5%          25%          50%          75%
##      theta    0.298322 0.04469041    0.2108997    0.2685856    0.2987727    0.3267194
##      lp__   -55.061289 0.65102120  -57.1168032  -55.1824162  -54.8089453  -54.6533808
##          stats
## parameter      97.5%
##      theta    0.3870173
##      lp__   -54.6190529
```

3.3 Exploring example 8.3.1 (from notes)

```r
data <- list(N = 3 + 10,
             J = 2,
             group = c(rep(1,3),
                       rep(2,10)),
             X = c(0,1,0,
                   2,0,0,1,2,0,0,1,1,0))

our_model = stan_model('stan_3.stan')

fit = sampling(our_model, data=data)
```

```
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 2.1e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.21 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.048 seconds (Warm-up)
## Chain 1:                0.042 seconds (Sampling)
## Chain 1:                0.09 seconds (Total)
## Chain 1:
```

```
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 5e-06 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.05 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.078 seconds (Warm-up)
## Chain 2:                0.057 seconds (Sampling)
## Chain 2:                0.135 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 7e-06 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.081 seconds (Warm-up)
## Chain 3:                0.065 seconds (Sampling)
## Chain 3:                0.146 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'anon_model' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 7e-06 seconds
```

```
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.068 seconds (Warm-up)
## Chain 4:                0.059 seconds (Sampling)
## Chain 4:                0.127 seconds (Total)
## Chain 4:
```

```
summary(fit)
```

```
## $summary
##                 mean     se_mean           sd         2.5%         25%         50%
## beta        1.001547  0.02577484     1.043279   0.02460425   0.2763697   0.6793347
## lambda[1]  13.210192  5.02890893   166.805205   0.01985648   0.3125798   1.0337656
## lambda[2]  49.002284 40.24803056  1208.155170   0.03099554   0.3496148   1.0040398
## lp__       -4.802664  0.04573600     1.478810  -8.59349863  -5.4757186  -4.4114142
##                 75%        97.5%       n_eff       Rhat
## beta        1.414148   3.700666  1638.3585  1.001187
## lambda[1]   3.064183  47.277275  1100.2000  1.005160
## lambda[2]   3.057664  41.323120   901.0651  1.004164
## lp__       -3.714844  -3.111357  1045.4618  1.004201
##
## $c_summary
## , , chains = chain:1
##
##           stats
## parameter       mean         sd         2.5%         25%         50%        75%
##    beta      0.9754311  0.9647098   0.03168704   0.3005369   0.6738632   1.348395
##    lambda[1] 13.9242730 168.5071017  0.01975428   0.3133878   0.9495200   2.652733
##    lambda[2]  8.0094225  57.6479954  0.03908864   0.3738263   0.9694987   2.997383
##    lp__      -4.7377725   1.4435056  -8.50421219  -5.3612671  -4.3340772  -3.686648
##           stats
## parameter       97.5%
##    beta       3.702084
##    lambda[1] 40.790116
##    lambda[2] 36.705922
##    lp__      -3.102770
##
## , , chains = chain:2
##
```

```
##           stats
## parameter      mean       sd       2.5%        25%        50%        75%
##    beta      1.005232  1.122940  0.02827112  0.2631341  0.6568571  1.393425
##    lambda[1] 4.911897 14.392758  0.02241485  0.3063760  1.0761374  3.228670
##    lambda[2] 4.802460 14.182725  0.04019270  0.3296527  0.9817095  3.044323
##    lp__     -4.800914  1.346169 -7.92370458 -5.5967464 -4.4790986 -3.781065
##           stats
## parameter      97.5%
##    beta       3.856415
##    lambda[1] 34.112681
##    lambda[2] 36.203788
##    lp__      -3.133874
##
## , , chains = chain:3
##
##           stats
## parameter      mean       sd       2.5%        25%        50%        75%
##    beta      1.013449  1.020902  0.02879108  0.2752897  0.7168009  1.456866
##    lambda[1] 4.669405 23.425071  0.02523615  0.3053974  0.9690350  2.887870
##    lambda[2] 5.276758 23.045634  0.02489143  0.3481052  0.9712044  2.853128
##    lp__     -4.789135  1.433923 -8.48150510 -5.4730156 -4.4479281 -3.697601
##           stats
## parameter      97.5%
##    beta       3.603678
##    lambda[1] 27.843749
##    lambda[2] 27.906167
##    lp__      -3.104637
##
## , , chains = chain:4
##
##           stats
## parameter       mean          sd        2.5%        25%        50%        75%
##    beta       1.012076    1.059312  0.01124433  0.2606006  0.670974  1.457716
##    lambda[1] 29.335191  286.052787  0.01441152  0.3321787  1.127875  3.497357
##    lambda[2] 177.920496 2411.782611 0.03019328  0.3474776  1.103181  3.342690
##    lp__      -4.882836    1.670698 -9.27559582 -5.5328718 -4.394362 -3.733466
##           stats
## parameter      97.5%
##    beta        3.568424
##    lambda[1]  95.316027
##    lambda[2] 103.488383
##    lp__       -3.092083
```

```
# samples <- extract(fit, pars = c('lambda[1]','lambda[2]'))
# print(samples)
# hist(samples)
```