The following questions cover ideas that have been discussed in Chapter 11 and the type of programming that you will be doing in the practical exam.

Note that you must use "Assignment 8.Rmd" for your solutions so you can produce a pdf of your solutions (and associated R output). This is again good practice for the practical exam.

1. We are trying to decide between three competing models:

$$\begin{aligned}
\mathcal{M}_1 &: H|\lambda \sim \mathsf{Po}(\lambda), & \lambda \sim \mathsf{Gamma}(2,2);\\
\mathcal{M}_2 &: H|\lambda \sim \mathsf{Bin}(100, \lambda/100), & \lambda \sim \mathsf{Gamma}(2,2);\\
\mathcal{M}_3 &: H|\lambda \sim \mathsf{N}(\lambda, \lambda(100-\lambda)/100), & \lambda \sim \mathsf{Gamma}(2,2).
\end{aligned}$$

   We receive the following data:

$$\underline{h} = \{1,0,0,0,0,2,0,0,0,0,0,0,0,1,1,1,0,0,1,0\}.$$

   1.1 Write some model code for Stan that covers each of the three models (with three separate Stan files).

   1.2 Write out R code to sample from the posterior for each model.

   1.3 Use the `loo` package to find pseudo-BMA weights for each of the models. Do the results match your expectations?

2. We are again trying to decide between three competing data-generating models:

$$\begin{aligned}
\mathcal{M}_1 &: Y|\alpha \sim \mathsf{Exp}(\alpha),\\
\mathcal{M}_2 &: Y|\nu \sim \chi^2(\nu),\\
\mathcal{M}_3 &: \sqrt{Y}\Big|\beta \sim \text{Maxwell–Boltzmann}(\beta).
\end{aligned}$$

   2.1 Find a single model for Y that would accommodate all of these models as special cases.

   2.2 If we expect Y to be about 2 on average and to be less than 4 about 95% of the time, set up a prior for the parameters of the model that roughly matches these beliefs. (Hint: you will need to set up the preposterior distribution and try different prior options.)