The following questions cover models that have been discussed in Chapter 9 and the type of programming that you will be doing in the practical exam.

Note that you must use "Assignment 7.Rmd" for your solutions so you can produce a pdf of your solutions (and associated R output). This is again good practice for the practical exam.

1. We have the following linear regression model:

$$\begin{aligned} Y_i &= \beta x_i + \epsilon_i \text{ for } i = 1, \ldots, n, \\ \pi(\beta) &= \beta^{-2} \text{ for } \beta \in (1, \infty), \\ \epsilon_i | \sigma^2 &\sim \mathsf{N}(0, \sigma^2) \text{ for } i = 1, \ldots, n, \\ \sigma^2 &\sim \mathsf{InvGamma}(0.1, 0.1). \end{aligned}$$

   1.1 Show that $\pi(\beta)$ is a probability density function.

   1.2 What are the prior modes for β and σ^2? (You can look up the mode for σ^2 on Wikipedia.)

   1.3 Write the model code needed for Stan.

   1.4 We make the following pairs of observations:

x	y
2.13	4.02
4.32	8.73
3.60	7.33
0.19	0.51
5.62	12.09
2.86	5.99
4.50	8.91
1.95	4.02

   Write out R code to sample from the posterior given these data, and produce posterior summaries for β and σ^2.

2. An expert posits the following model for some count data:

$$\begin{aligned} X | \lambda &\sim \mathsf{Poisson}(\lambda), \\ \lambda &\sim \mathsf{Fréchet}(2, 4). \end{aligned}$$

   2.1 Write some model code for Stan that will enable us to explore the preposterior distribution for X.

   2.2 Write out R code to sample from the preposterior for X for this model.

   2.3 The expert believes that there is very little chance of X being less than 4. How might the model be changed to accommodate this?