Subject of the lesson:

Sorting and Binary Search

Relevant STL Functions

- Sort: sort a range of values
- Searching a sorted range of values:
  - Binary search: Return true/false if the value exists
  - lower_bound(x): Return a pointer to the first element which is greater or equal to x.
  - upper_bound(x): Return a pointer to the first element which is strictly greater than x.
- For example, when searching for 3 in a sorted array:
  - 1 2 3 3 3 3 4 6
  - 1 2 4 6

When do we need to use binary search?

- If
  - We look for a minimal (or maximal) value k for which some property holds.
  - Given such k, it is relatively easy to check if the property holds.
  - The property is **monotonous**. If the property holds for k, then it holds for all j>k.
- Then we can use binary search to find the minimal k for which the property holds.

(Hypothetical) Example

- A construction company considers the placement of gas stations along a given road system. Construction is very expensive, and therefore the company wants to construct the least amount of gas stations that will cover the road.
- The only information we have is a “test function”:
  - Given k, the function returns true iff we can cover the road system using k gas stations or less.
- Clearly, if we can consturct k gas stations to cover the road, then we can also do it for every j>k.
- Therefore, we can use binary search to find the minimal possible value of k.
- In the problem we will encounter, you should look for a relatively simple “test function”.

Binary Search - Pseudocode

```
low = minimum possible index
high = maximum possible index
while (low < high)
       mid = (low + high)/2
       if check(mid)
              high = mid
       else
              low = mid + 1
return low
```

Sorting

- Sorting is a useful tool for a wide range of prolems.
- After sorting it is easier to:
  - Quickly search for elements
  - Go over the elements in order
  - Find identical elements
- Example from the first lesson - Lawn Mower problem (4954)
- In every problem where the input is not ordered, try to think if ordering the elements might help after some manipulations.

Sorting – Example

- In a given street there are n houses, and each house has a number n_i
- The post office wants to re-number the houses. They suggest the tenants to choose a number k, so the new number of house i will be $k \cdot i$.
- We need to find the k such which minimizes the amount of houses that need to change their number.

Solution

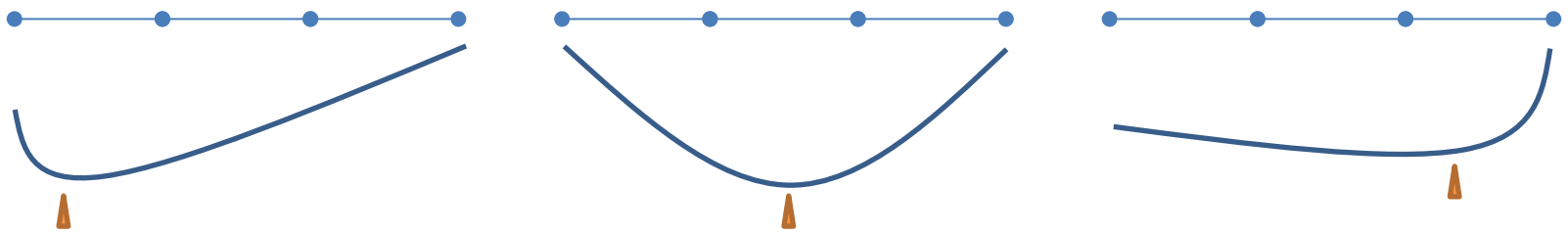
- Define $m_i = \frac{n_i}{i}$
- For a given k, every house i for which $k = m_i$ will not change its number (since $k \cdot i = m_i \cdot i = n_i$)
- Therefore, the most frequent value in the new array $(m_1, \ldots, m_n)$ will give us the k we are looking for.
- This value is easy to find after sorting the array.

Convex Functions

- Equivalent definitions
  - Line segment between any two point on the graph of the function lies above or on the graph
  - Epigraph (the set of points lying on or above its graph) is a convex set
  - Second derivative greater or equal to 0 for entire domain
- Examples:
  - Quadratic function x^2
  - Exponential function e^x
- If $f(x)$, $g(x)$ are convex functions, then $h(x) = \max\{f(x), g(x)\}$ is also a convex function.

Ternary Search

- Search for a maximum (or minimum) of a convex function.
- Logarithmic time complexity



Ternary Search – Pseudocode

```
left = minimum possible value
right = maximum possible value
while (right – left  > epsilon)
        mid_left = (2*left + right)/3
        mid_right = (left + 2*right)/3
        if check(mid_left) < check(mid_right)
                right = mid_right
        else
                left = mid_left
return left
```