Divide and Conquer

A divide and conquer algorithm recursively breaks down a problem into two or more subproblems until these have a simple direct solution. The solutions of the subproblems are then combined into a solution to the original problem.

- Efficient recursive algorithms for many problems
  - Base case = Direct solution
  - Recursive call = Split problem, solve both, recombine
- Foundation of two famous sorting algorithms
  - Merge Sort
  - Quick Sort

Merge Sort

- Base case
  - A single-element list is sorted
- Recursive call
  - Split the list into two halves, and merge-sort both independently
- Recombination
  - Combine the two sorted halves while maintaining (sorted) order


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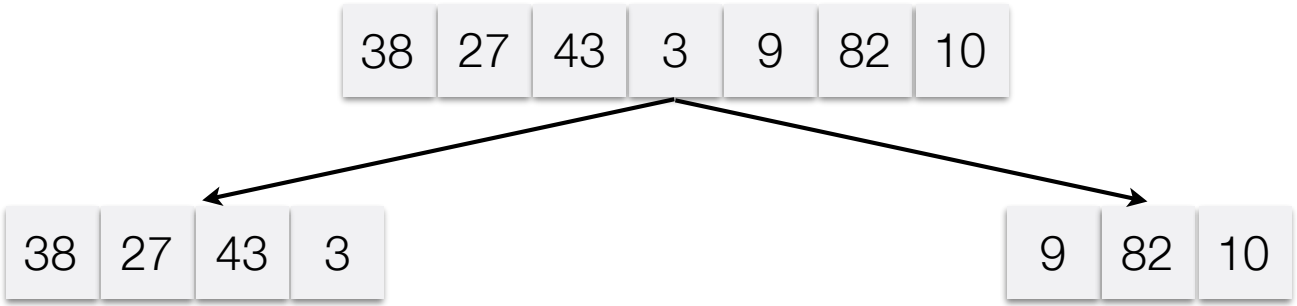
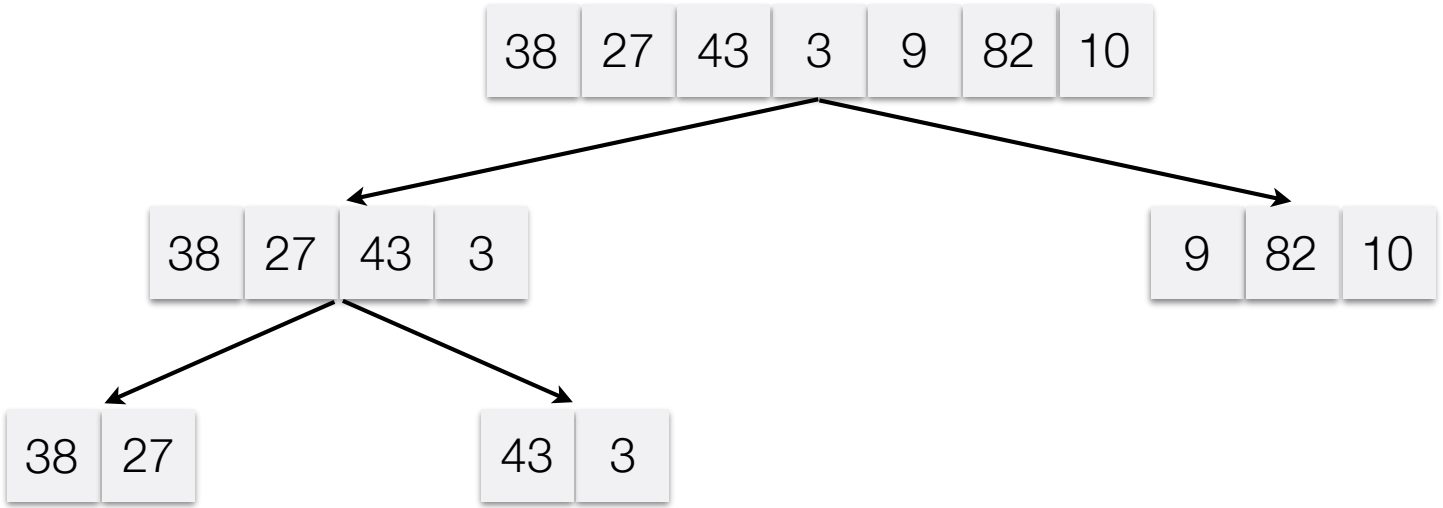


Step 1



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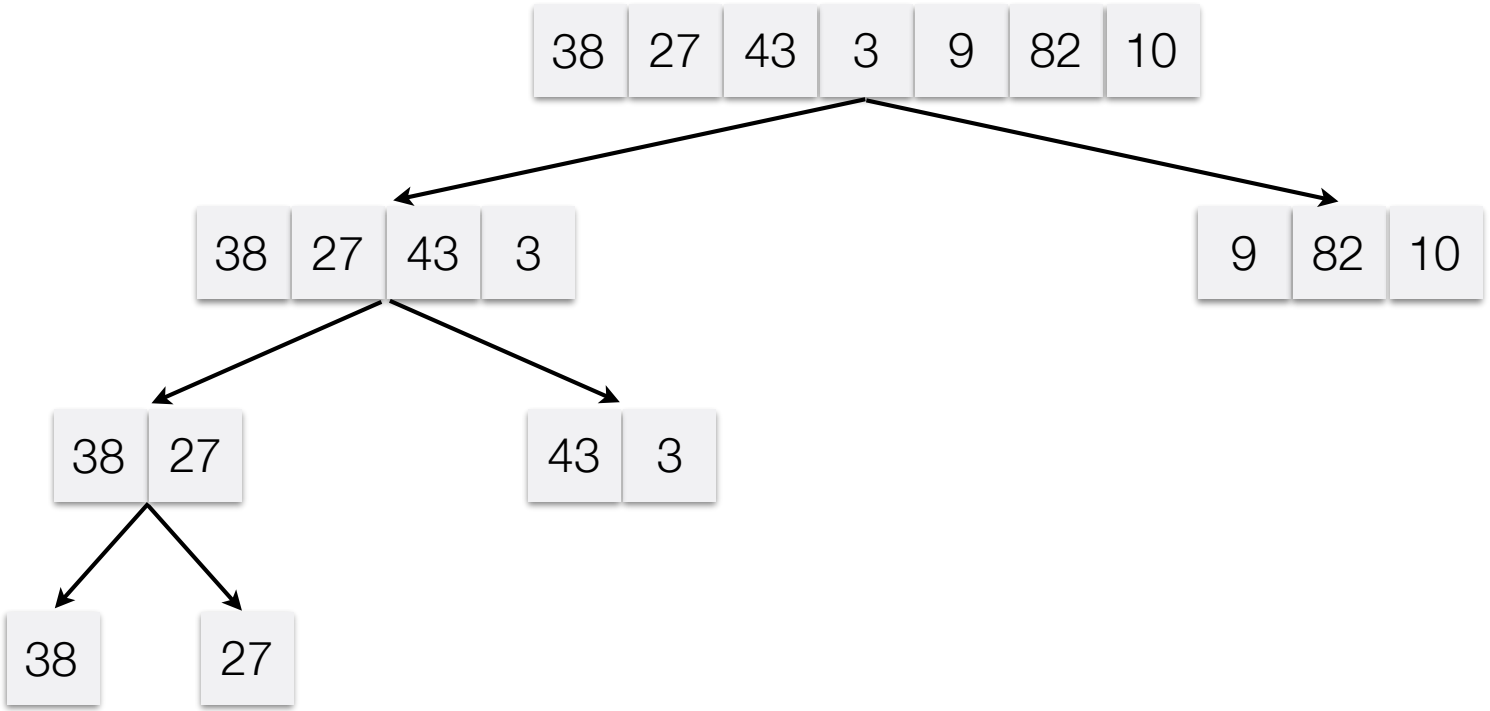

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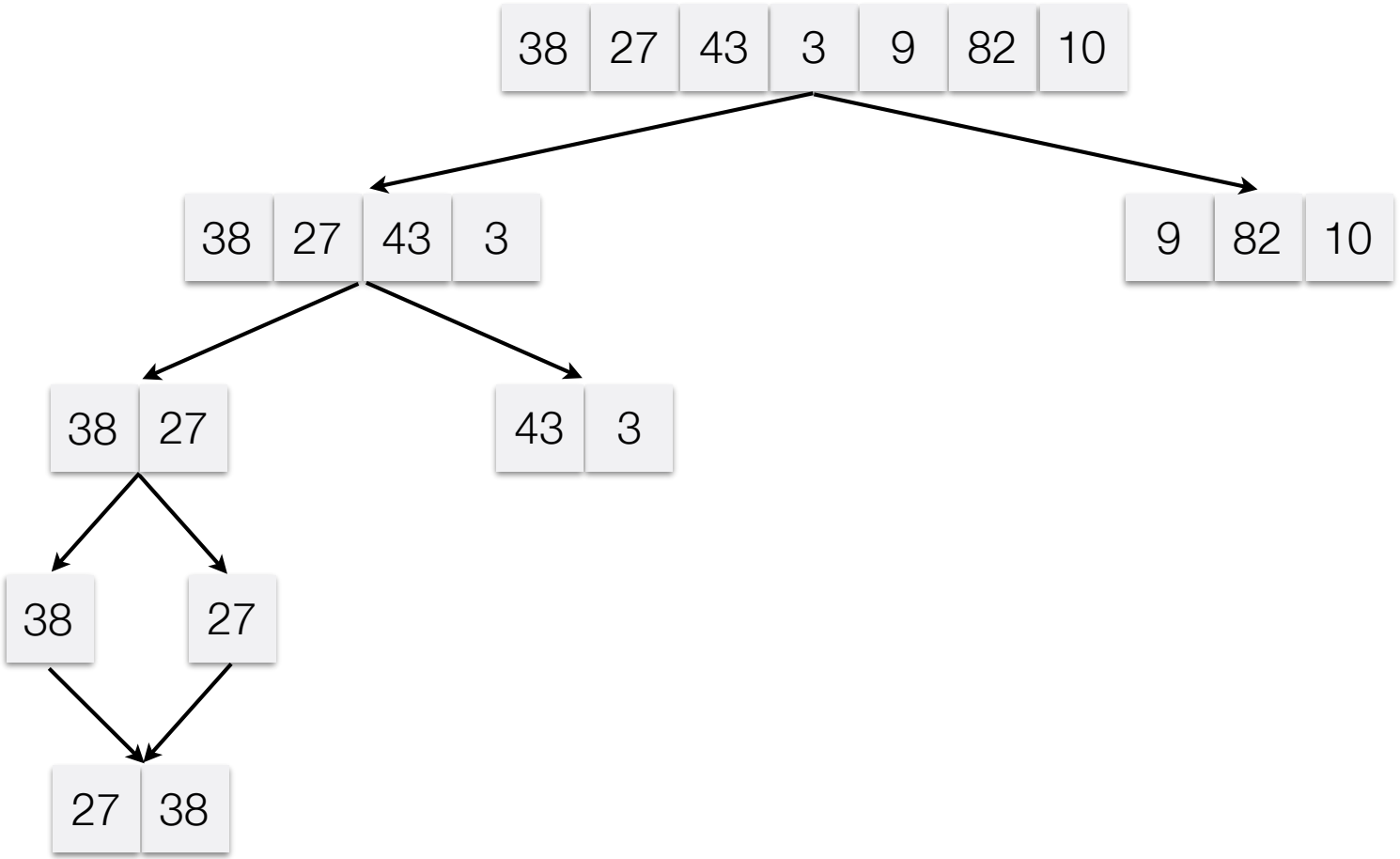

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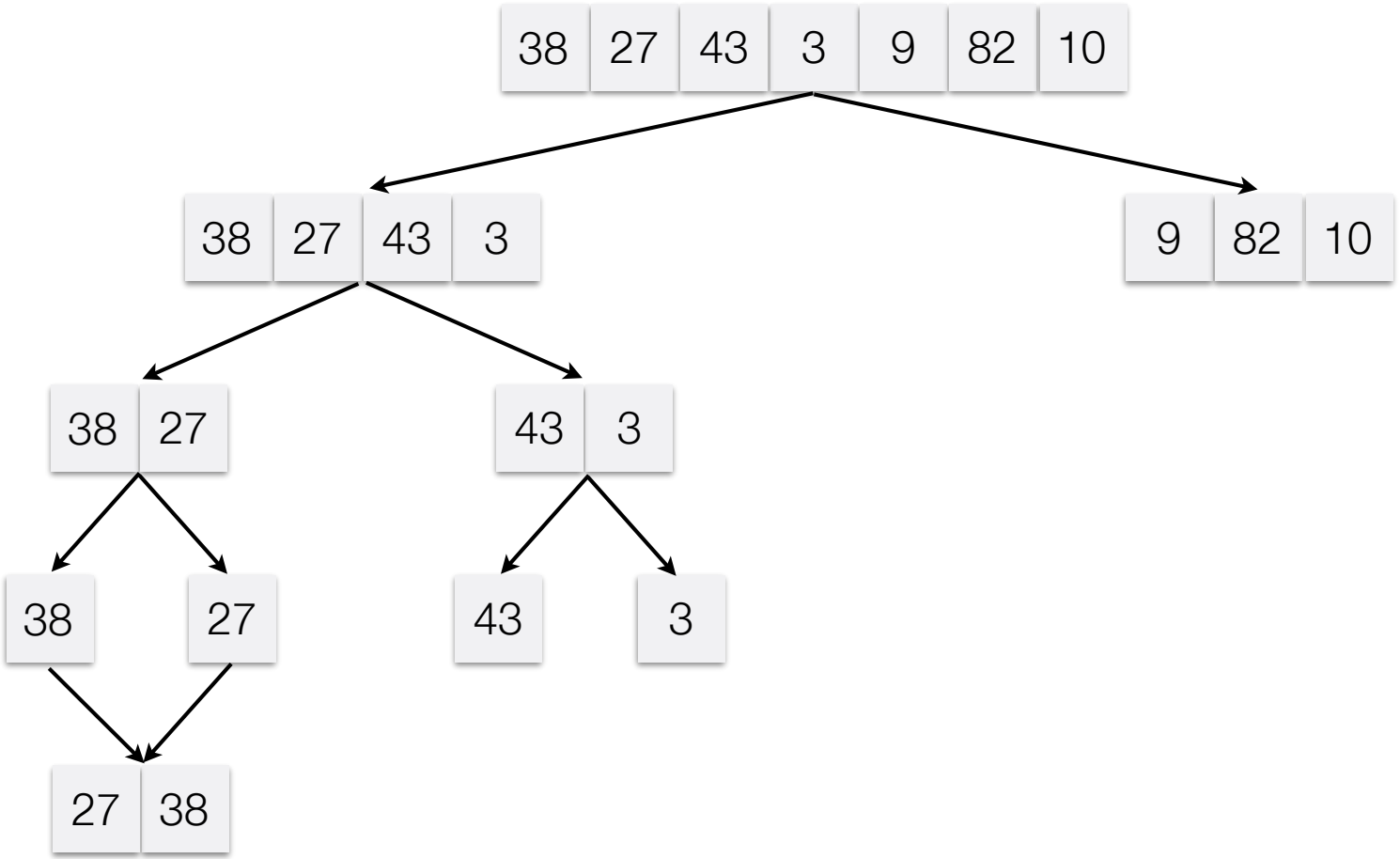

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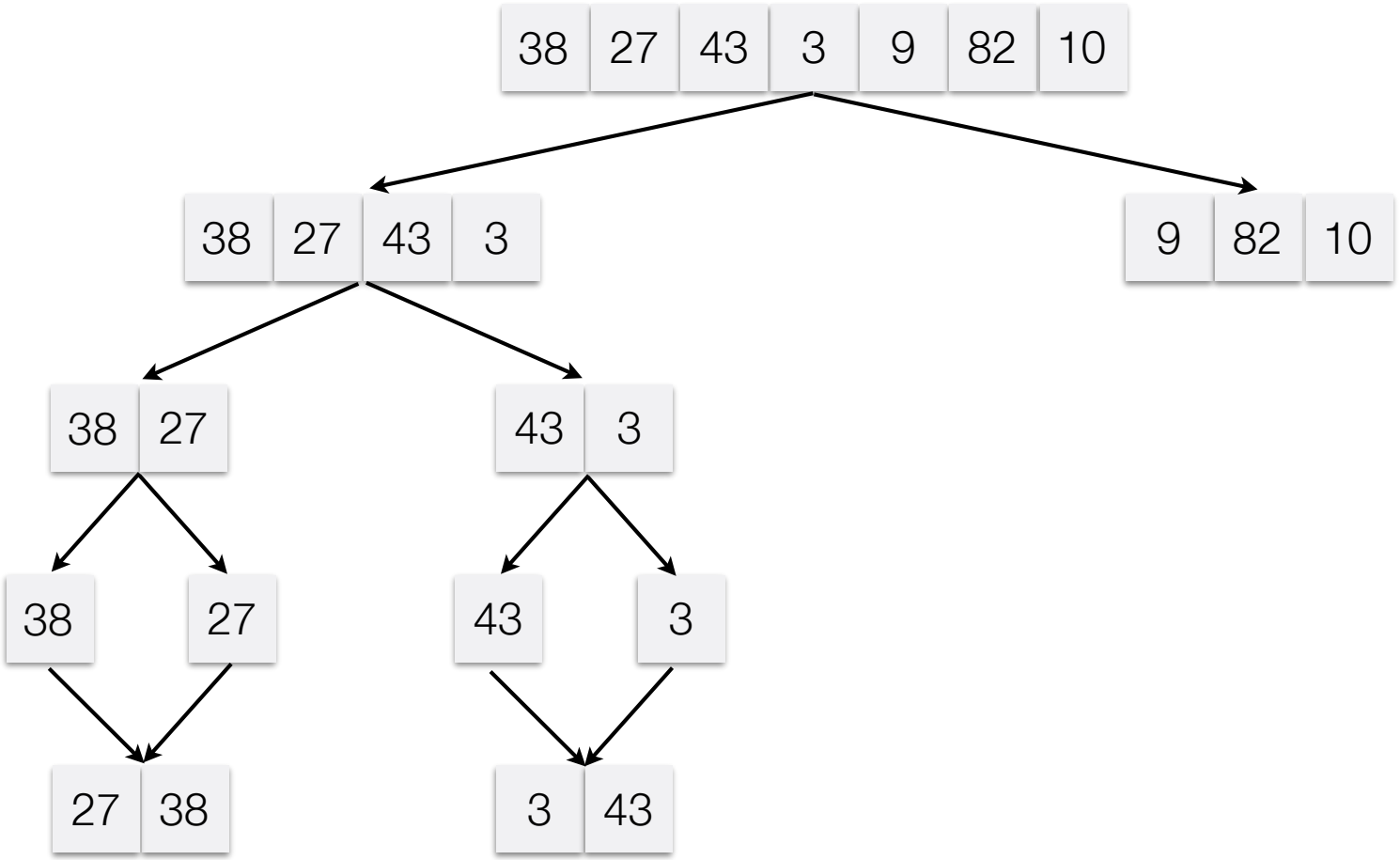

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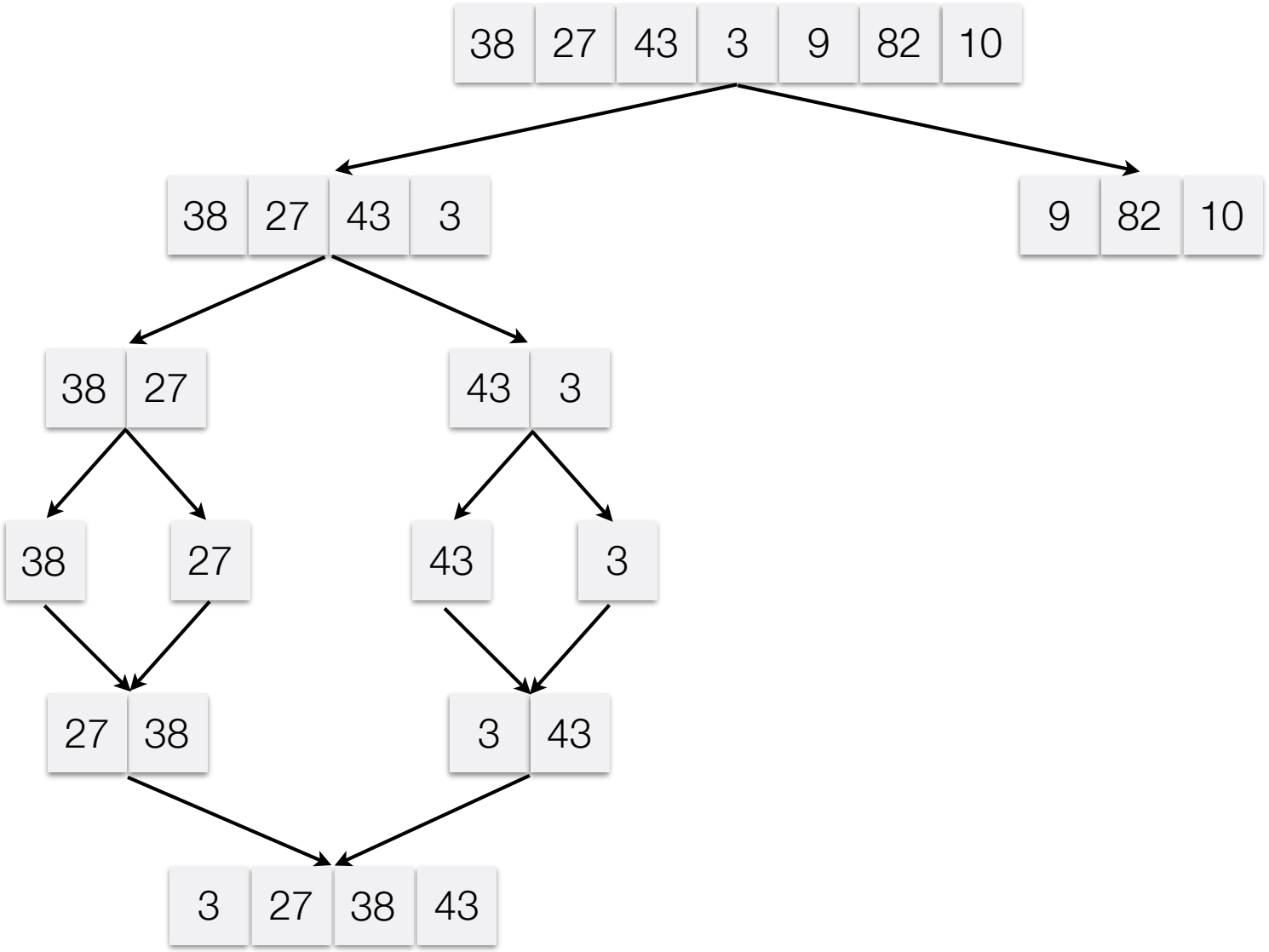
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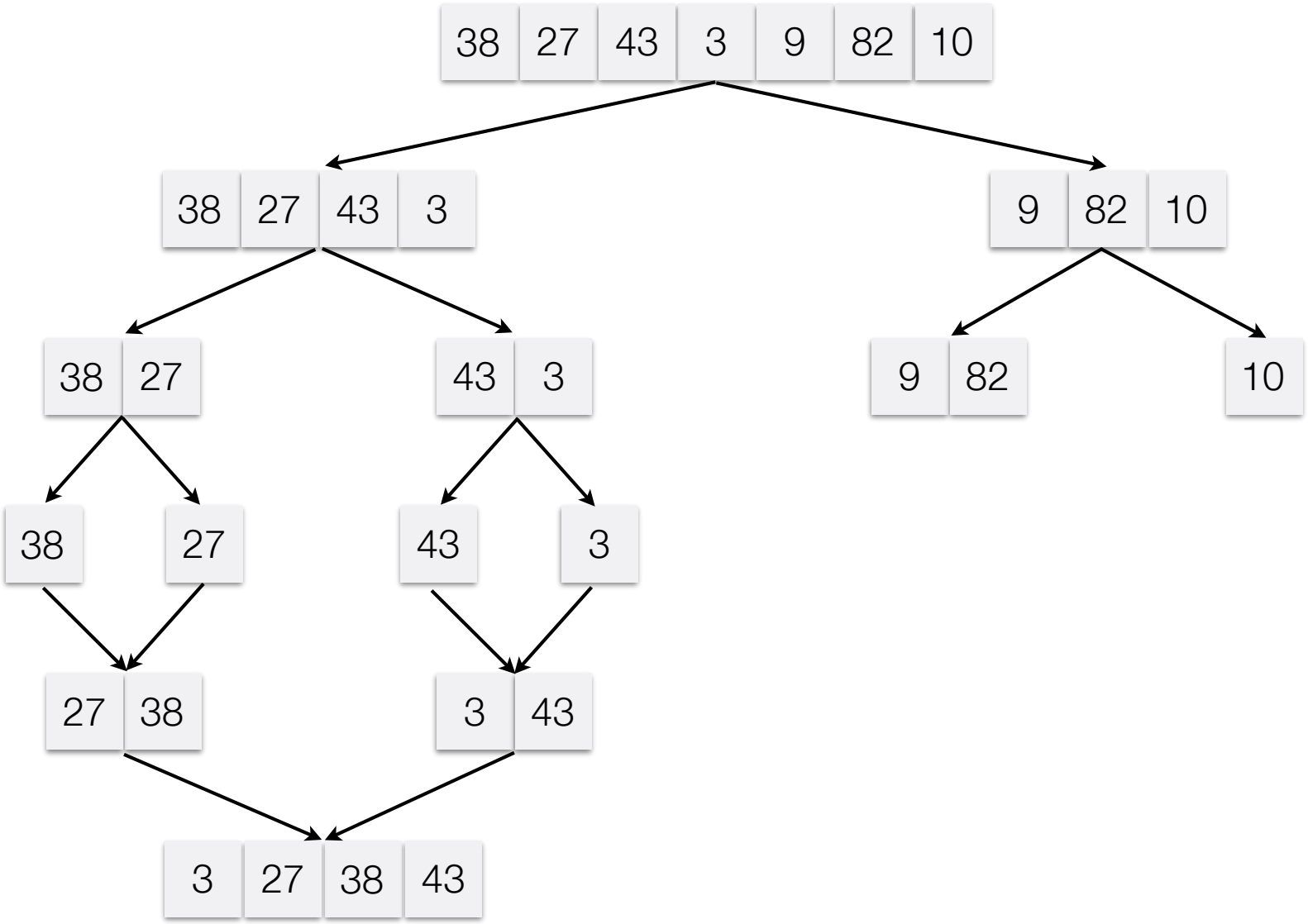
Step 7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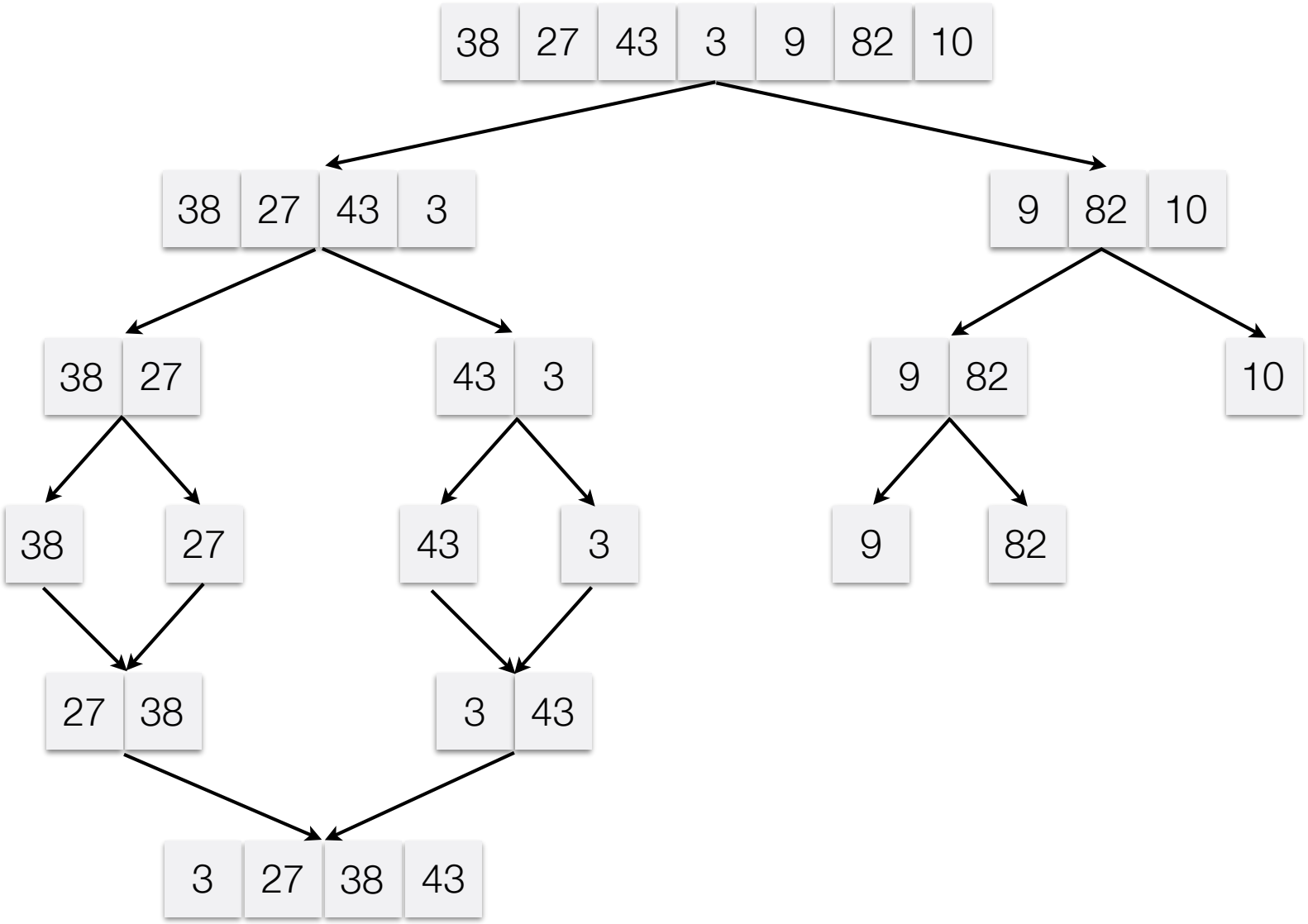
Step 8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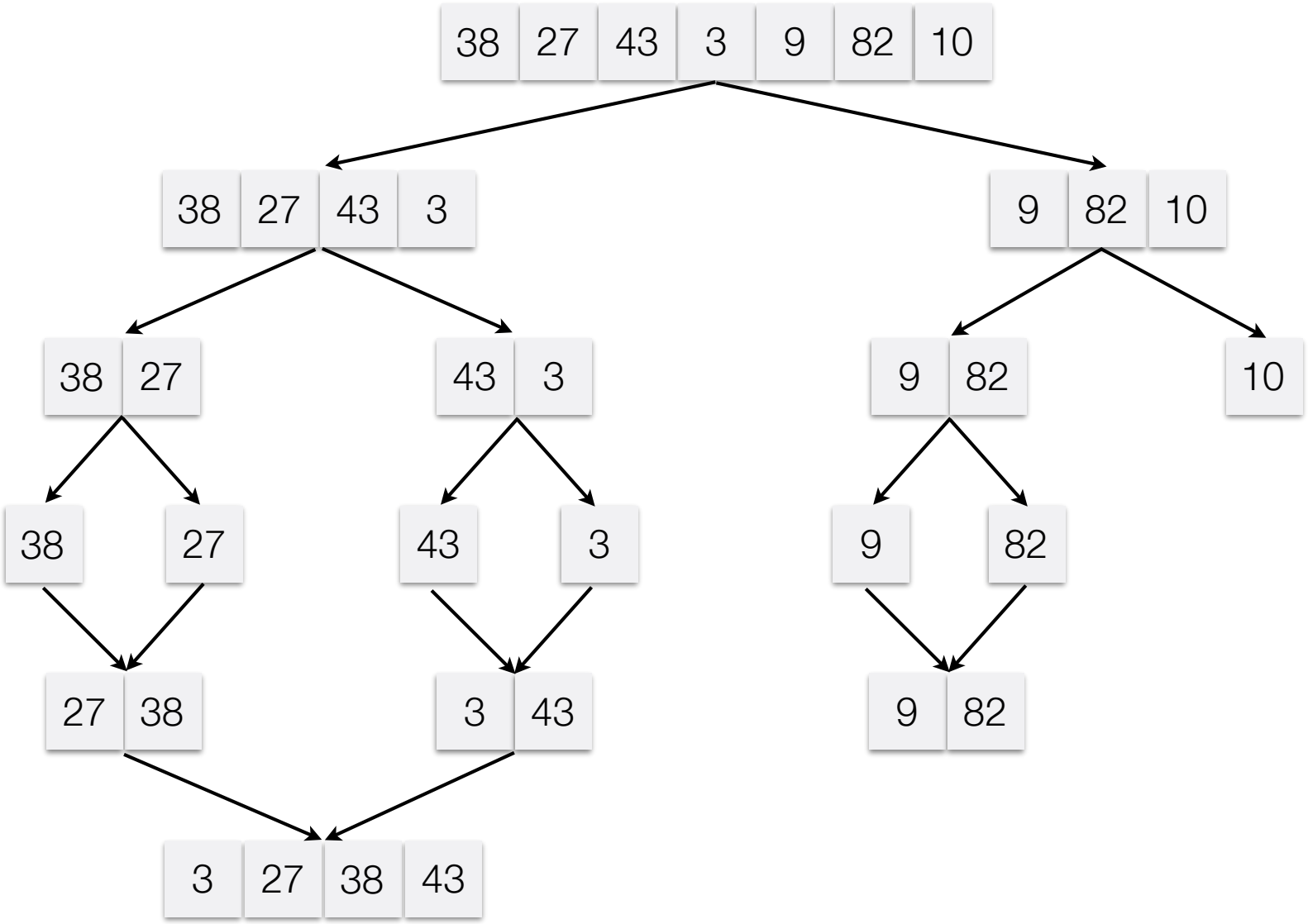
Step 9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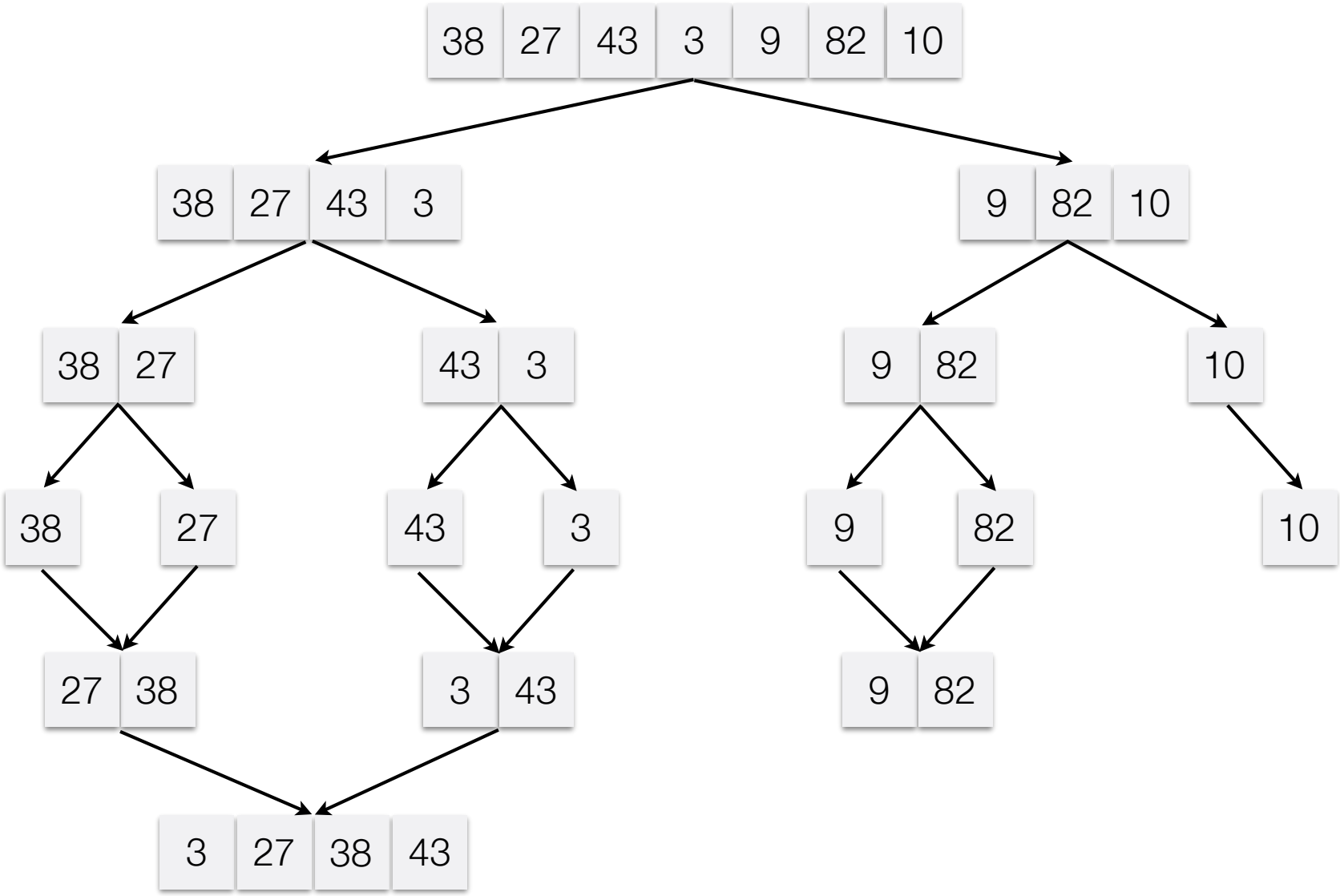
Step 10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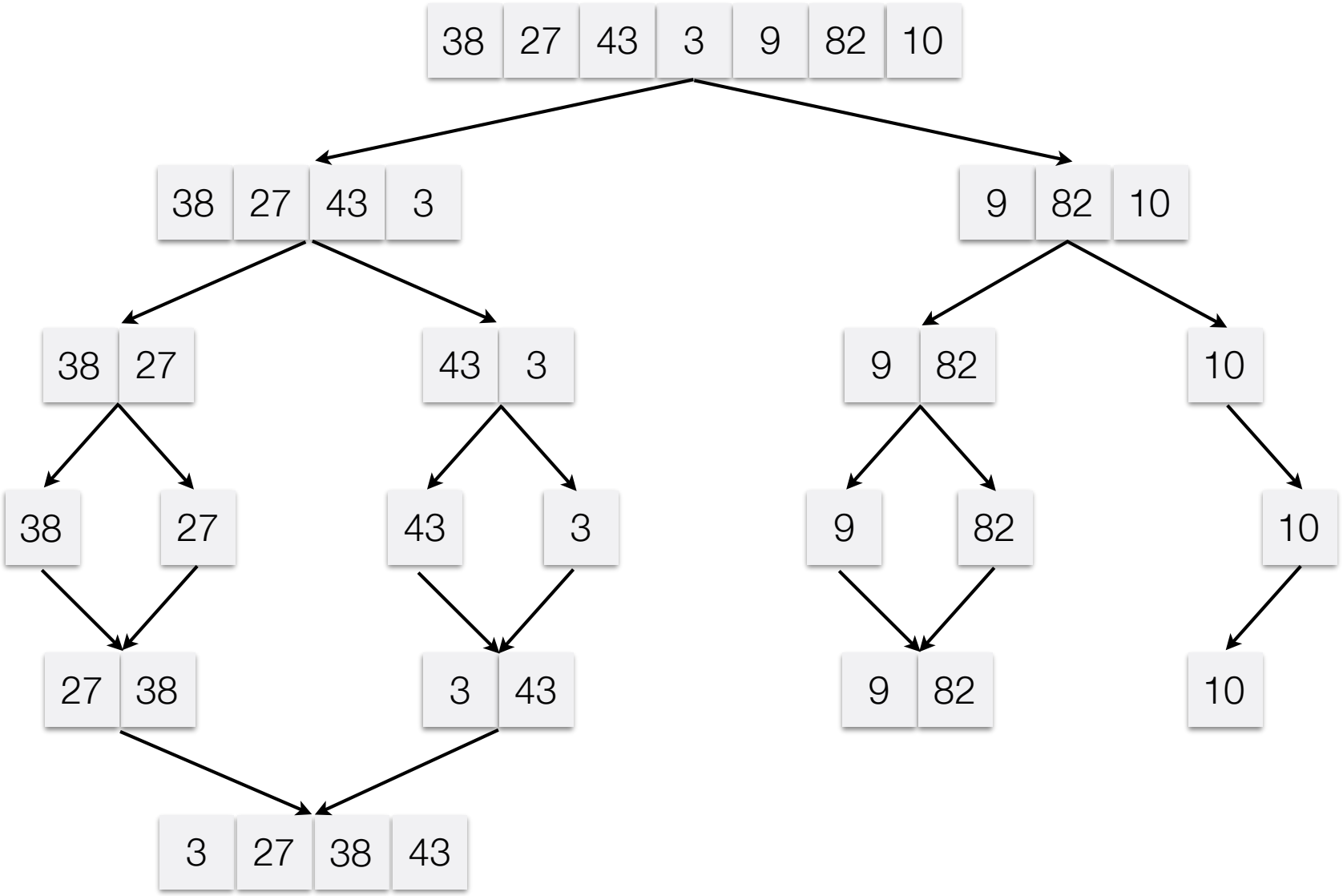
Step 11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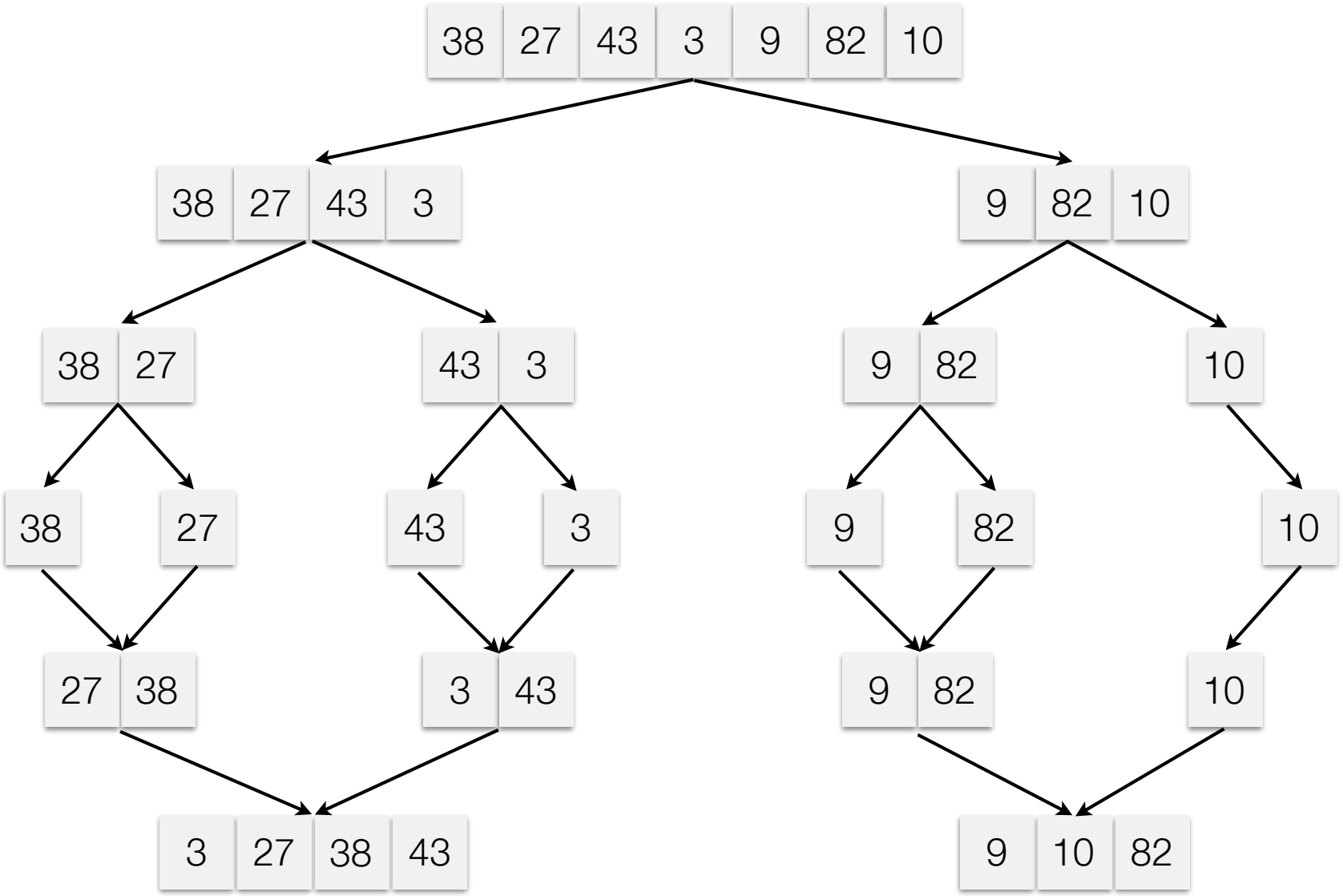
Step 12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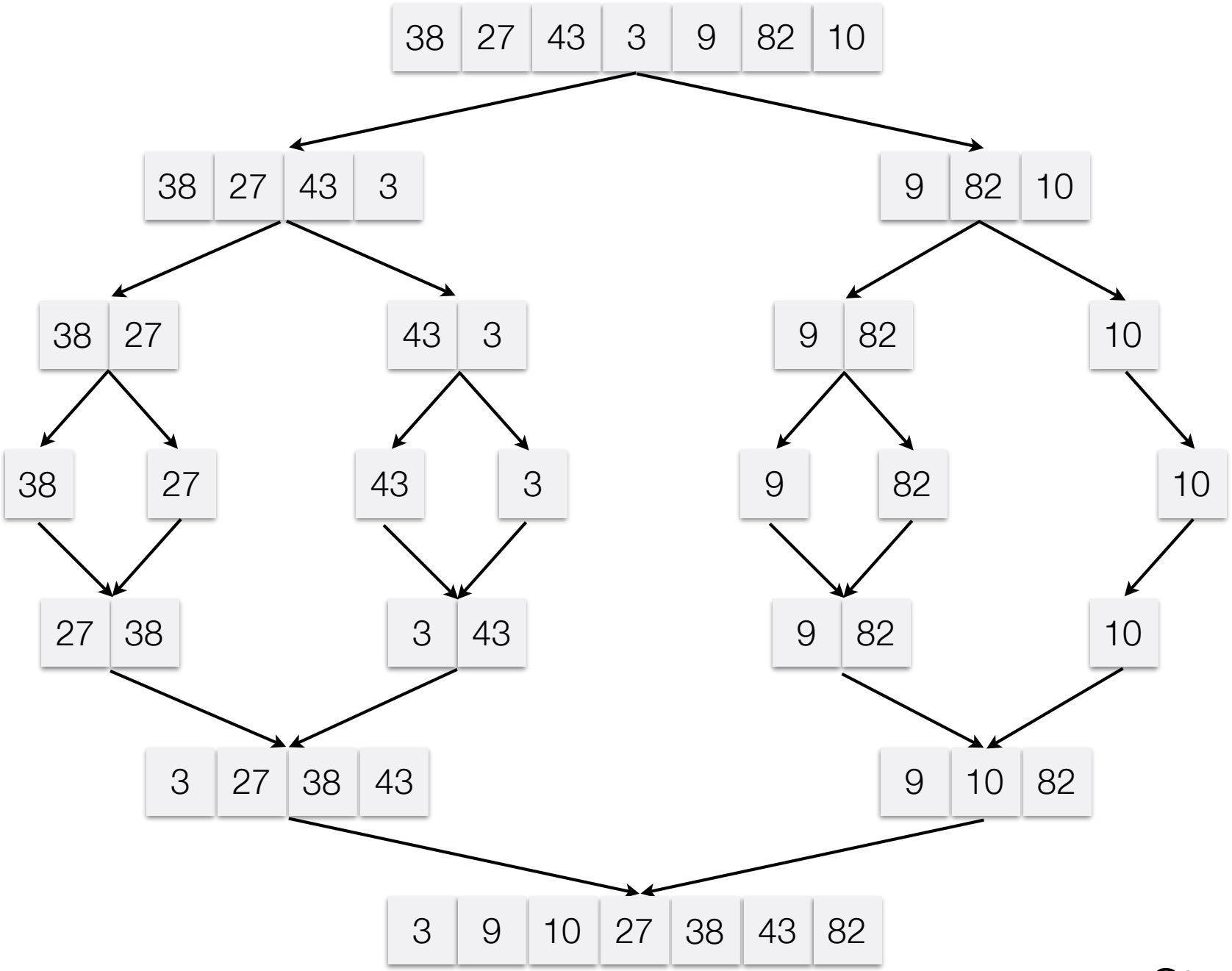
Step 13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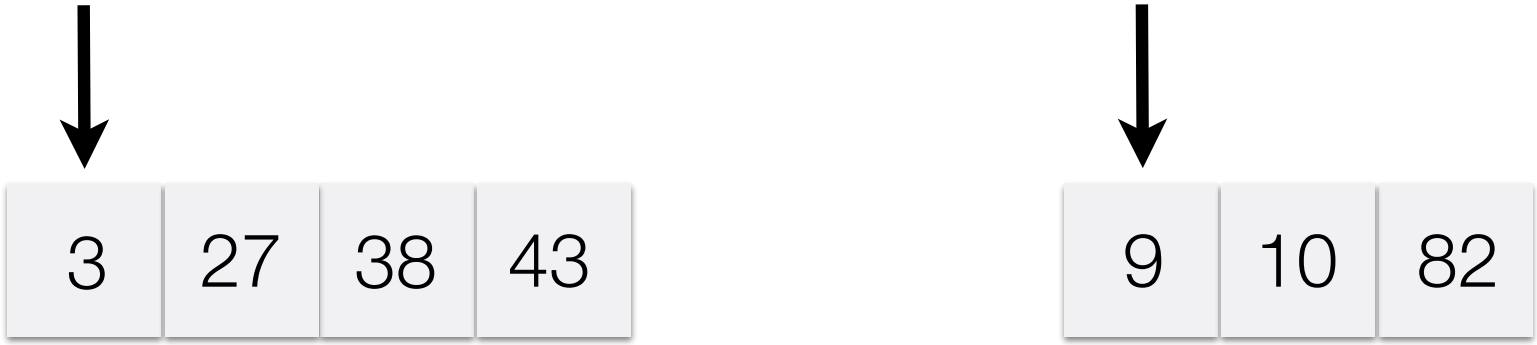
Step 13

Merging Lists

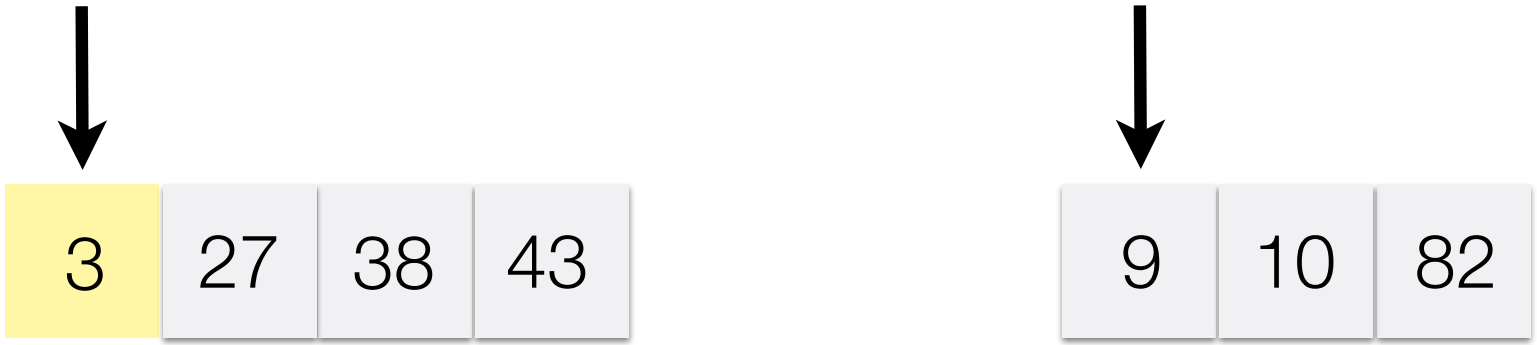
The core work of Merge sort is the merging of sublists

- Make a new list to store merged list
- Keep pointers into both source lists
    - While both pointers are not at the end of the lists
        - Compare current elements
        - Put least one in the destination list
        - Increase appropriate pointer into source lists

Merging Lists



Merging Lists



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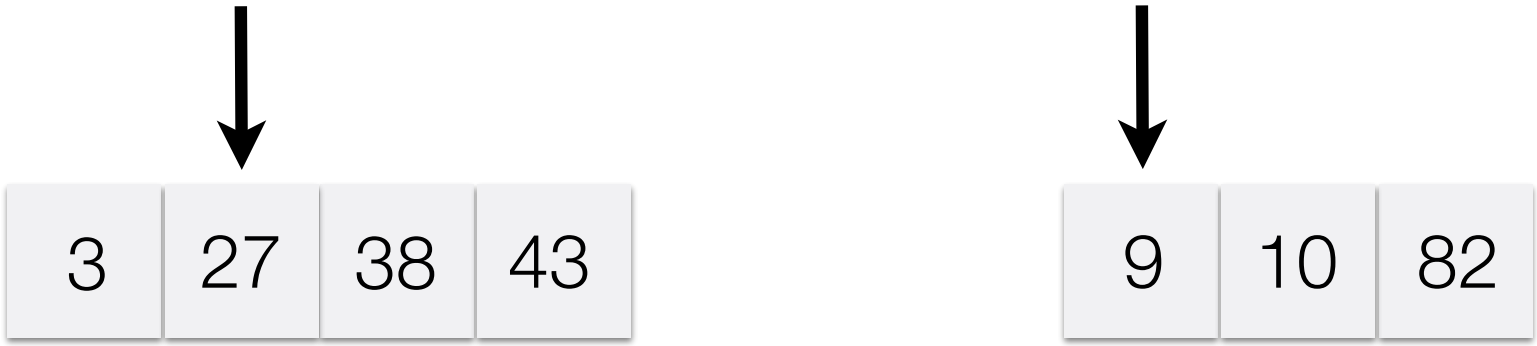
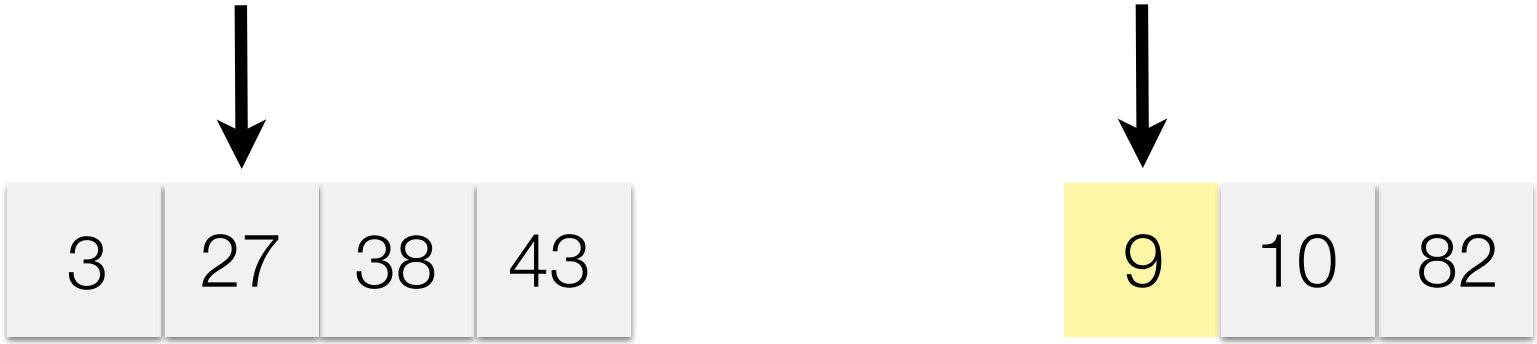


Merging Lists



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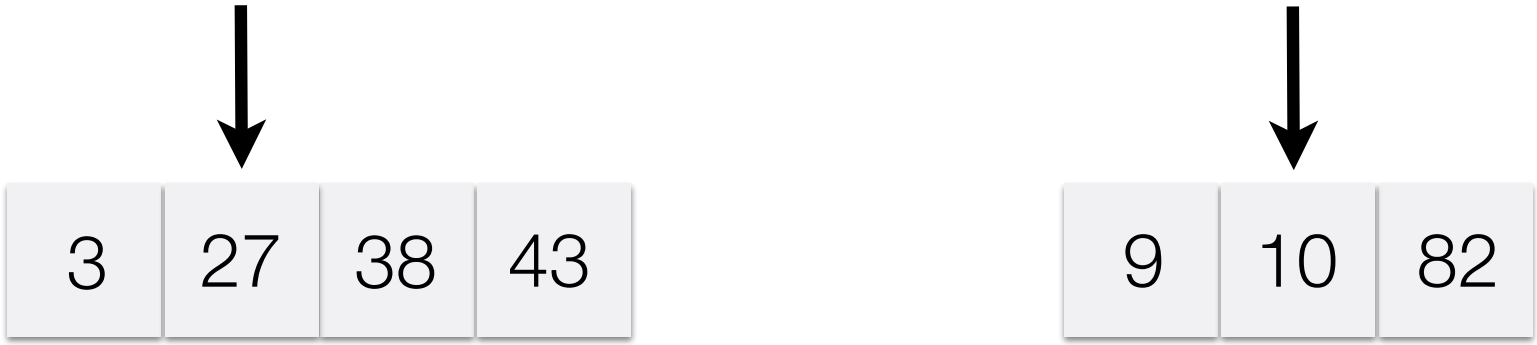

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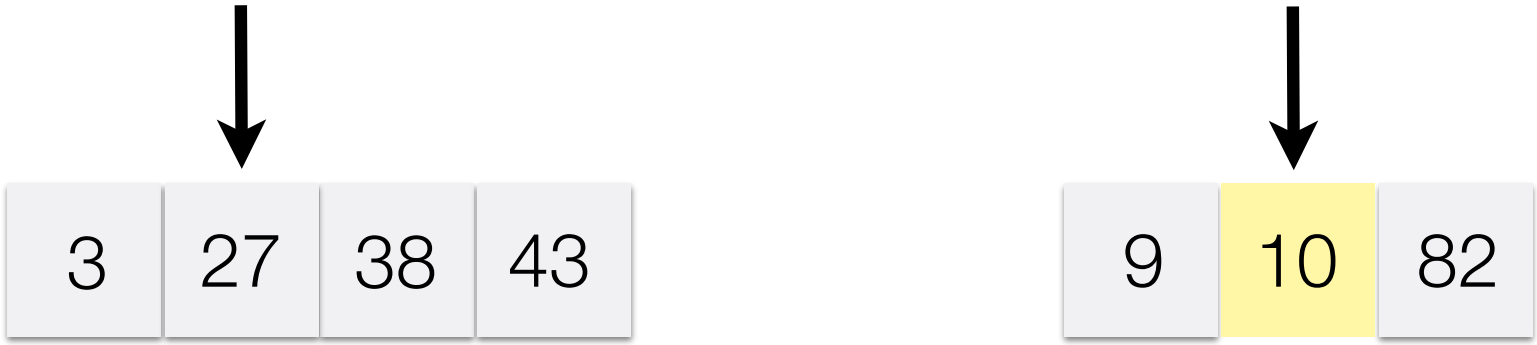

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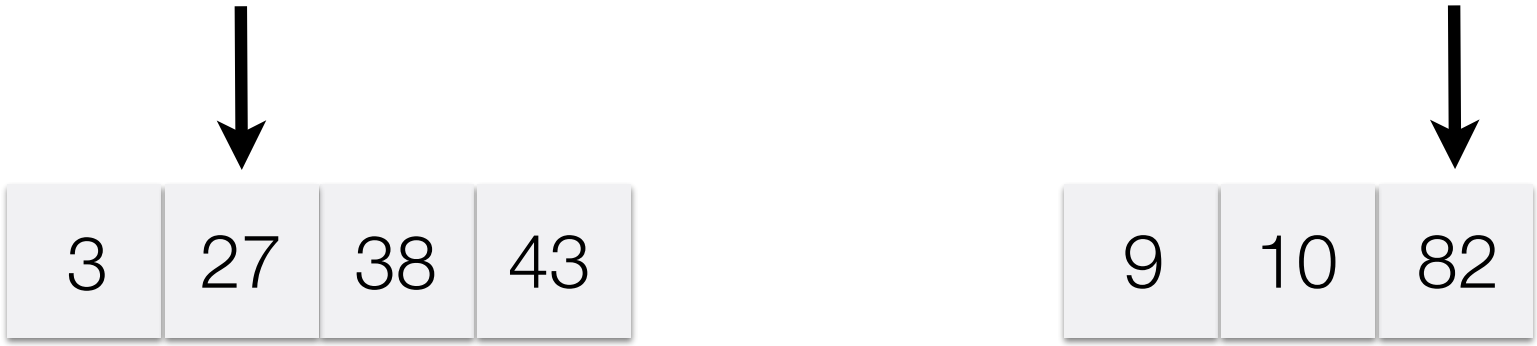

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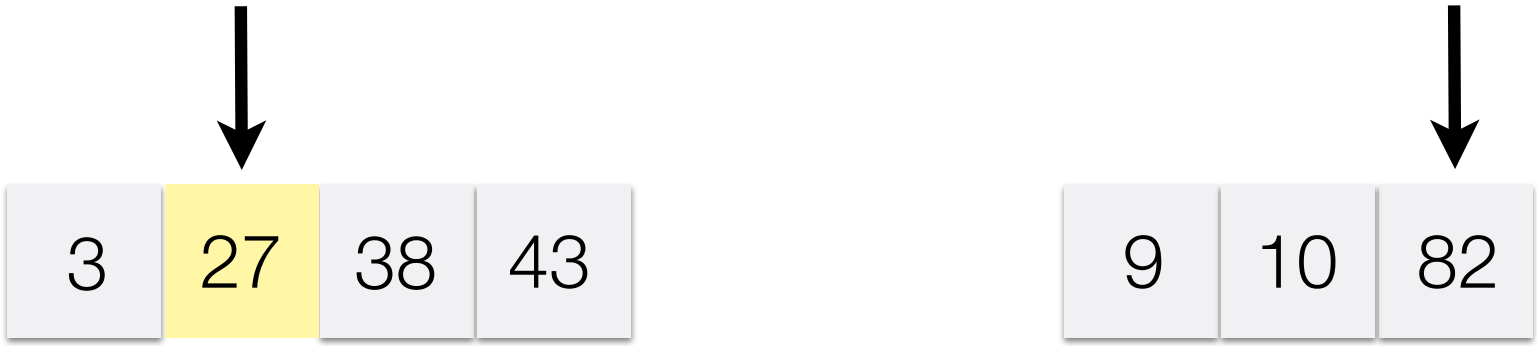

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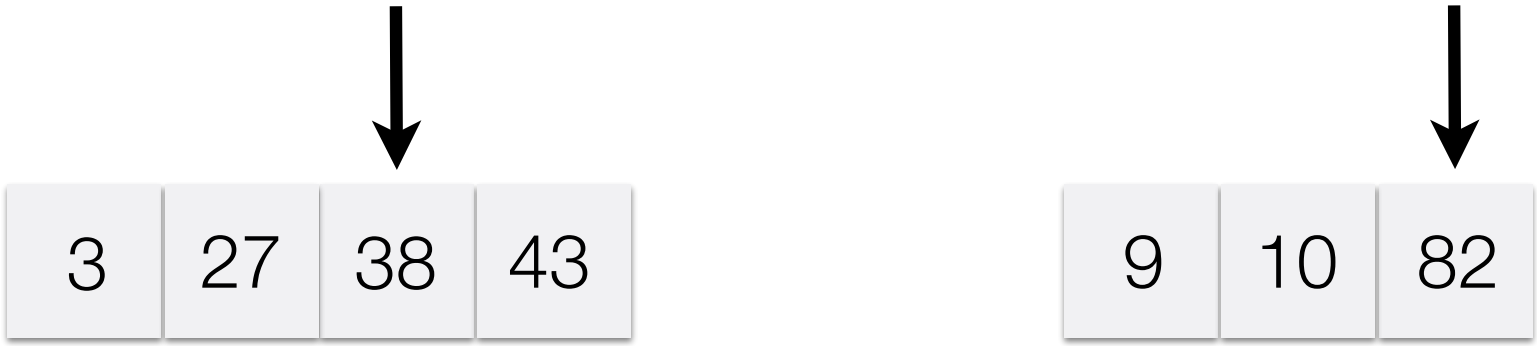

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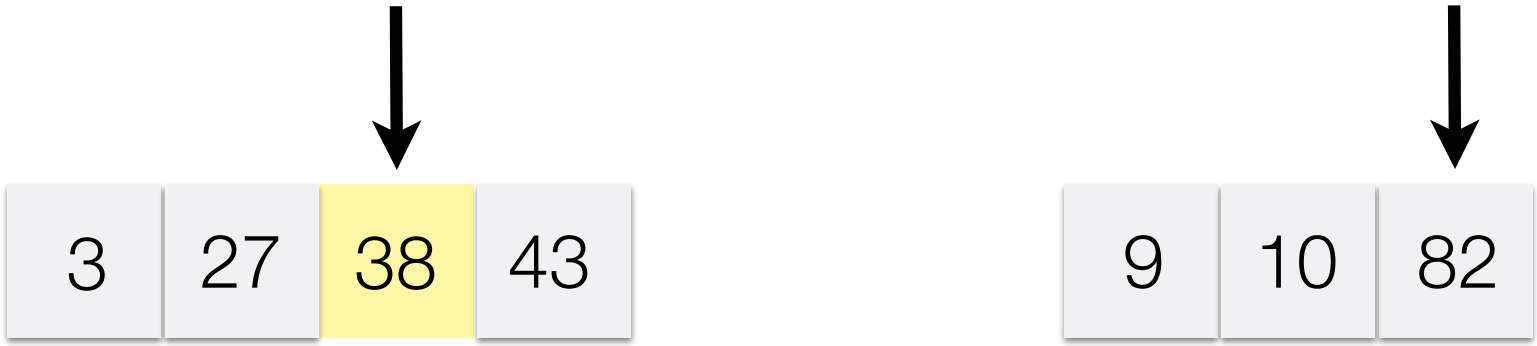

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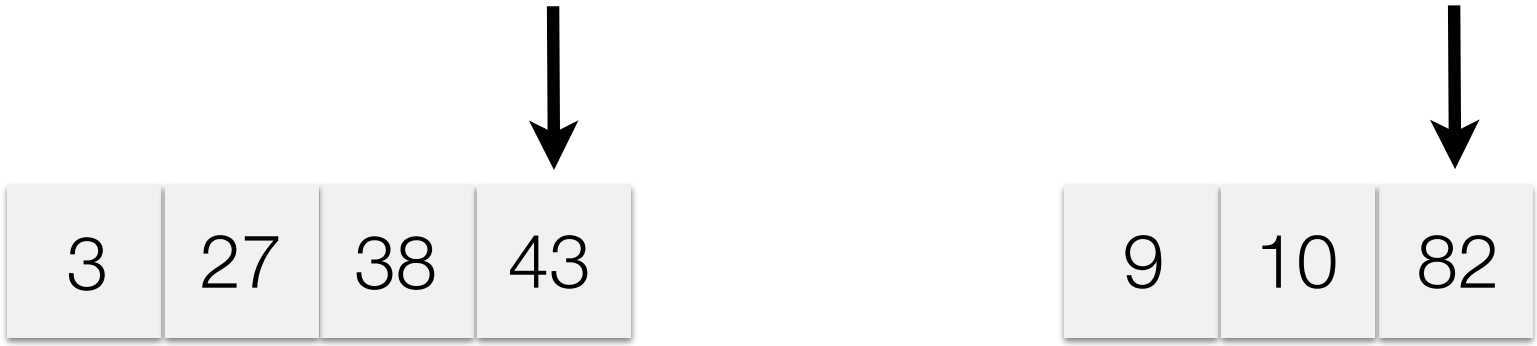

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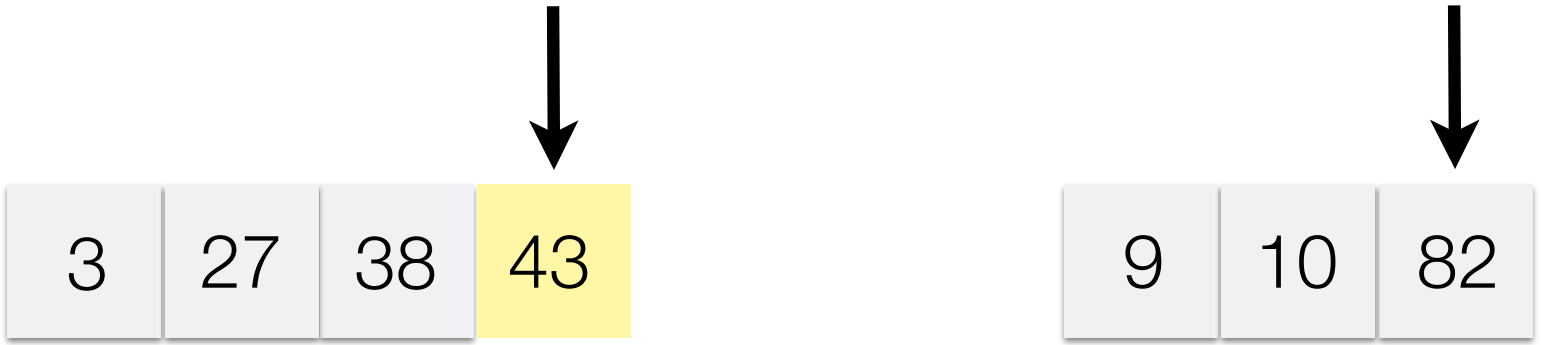

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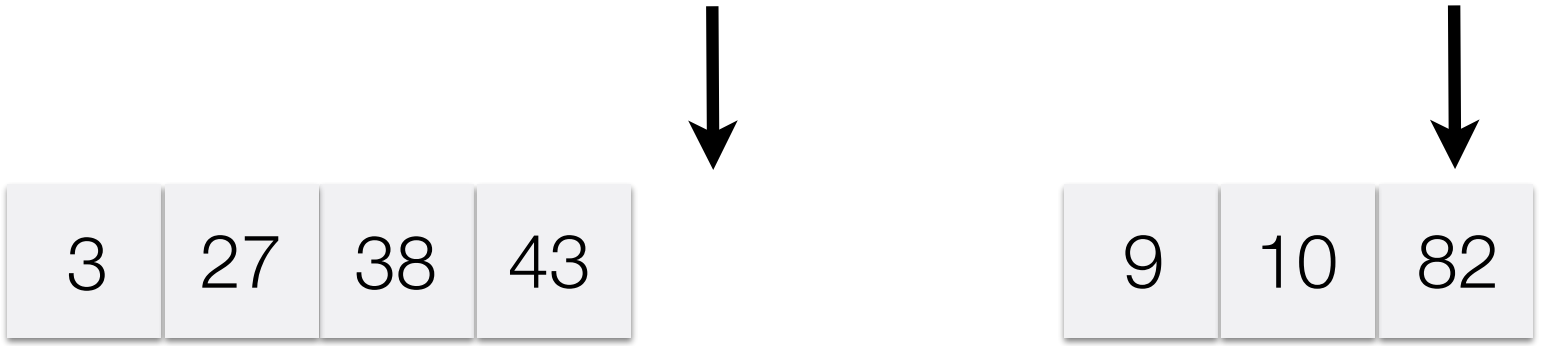

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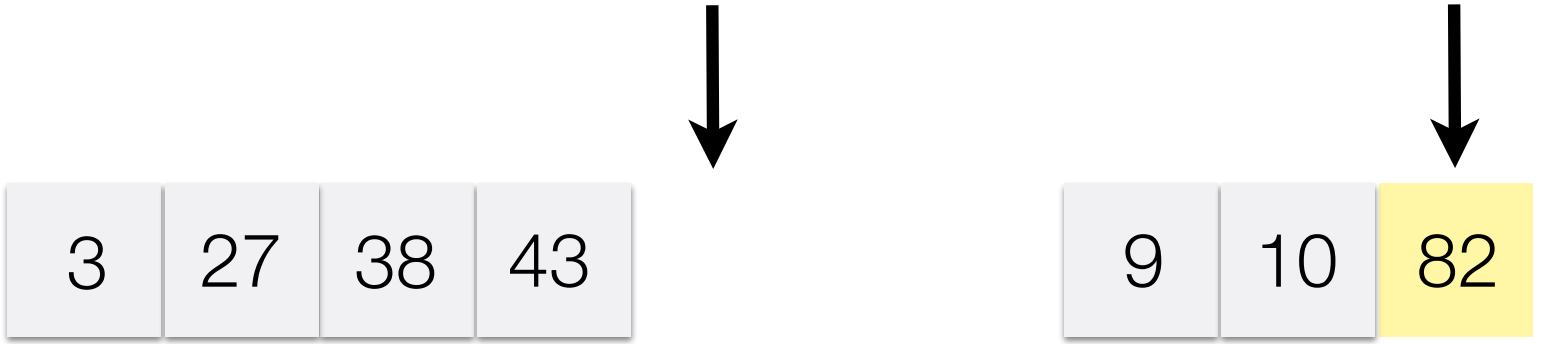

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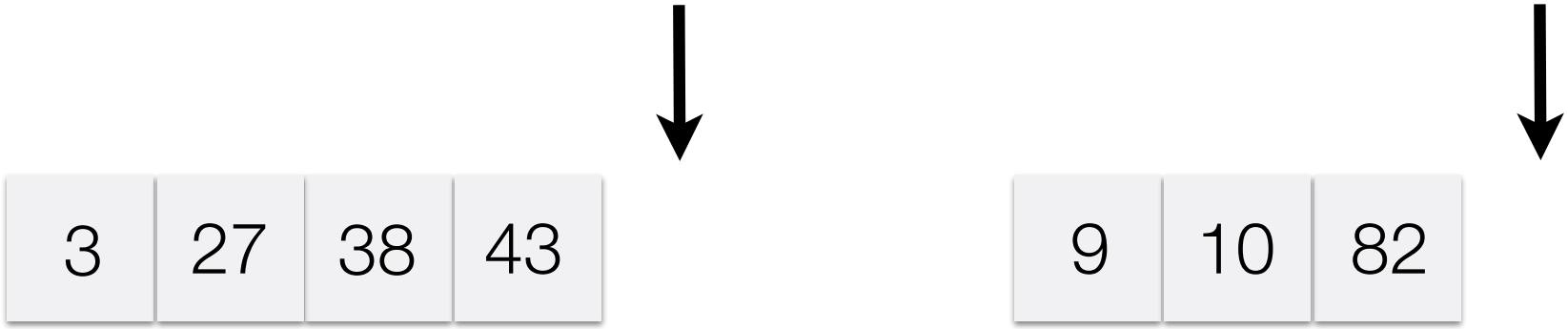

Merging Lists

```
public static List<Integer> merge(List<Integer> a, List<Integer> b) {
  List<Integer> result = new ArrayList<Integer>();
  int i = 0, j = 0;
  while (i < a.size() && j < b.size()) {
    if (a.get(i) < b.get(j)) {
      result.add(a.get(i));
      i++;
    } else {
      result.add(b.get(j));
      j++;
    }
  }
  while (i < a.size()) {
    result.add(a.get(i));
    i++;
  }
  while (j < b.size()) {
    result.add(b.get(j));
    j++;
  }
  return result;
}
```

Merge Sort

- Base case
  - A single-element list is sorted
- Recursive call
  - Split the list into two halves, and merge-sort both independently
- Recombination
  - Combine the two sorted halves while maintaining (sorted) order

Merge Sort

```
public static List<Integer> mergeSort(List<Integer> list) {
  // Base case
  if (list.size() <= 1) {
    return list;
  }

  // Middle position in list
  int mid = list.size() / 2;

  // Recursive calls to sort left and right sublists
  // subList(s,t) is a sublist from s (inclusive) to t (exclusive)
  List<Integer> left = mergeSort(list.subList(0, mid));
  List<Integer> right = mergeSort(list.subList(mid, list.size()));

  // Merge the sorted sublists
  return merge(left, right);
}
```