Symbolic disintegration with a variety of base measures

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Chung-chieh Shan

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Exact calculation

for approximate computation

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for approximate computation

Automatic differentiation
for gradient descent

Automatic disintegration
for inference and sampling

Automatic simplification
for Rao-Blackwellization

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for inference and sampling

A variety of base measures

I’d also like to address this concept of being “fake” or “calculating.”

If being “fake” means not thinking or feeling the same way in one moment than you thought or felt in a different moment, then lord help us all.

If being “calculating” is thinking through your words and actions and modeling the behavior you would like to see in the world, even when it is difficult, then I hope more of you will become calculating.

—BenDeLaCreme

Exact calculation
for approximate computation

Automatic differentiation
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Automatic disintegration
for inference and sampling

A variety of base measures

Disintegration

$$(t, p) \leftarrow\!\!\sim \mu \quad = \quad \begin{array}{l} t \leftarrow\!\!\sim \beta \\ p \leftarrow\!\!\sim \kappa(t) \end{array} \quad : \mathbb{M}\,(T \times P)$$

Disintegration relates a joint measure $\mu : \mathbb{M}(T \times P)$

$$(t, p) \leftarrow\!\!\sim \mu \quad = \quad \begin{array}{l} t \leftarrow\!\!\sim \beta \\ p \leftarrow\!\!\sim \kappa(t) \end{array} \quad : \mathbb{M}(T \times P)$$

Disintegration relates a joint measure $\mu : \mathbb{M}(T \times P)$ with a base measure $\beta : \mathbb{M}\,T$

$$(t,p) \leftharpoonup\!\!\sim \mu \quad = \quad \begin{array}{l} t \leftharpoonup\!\!\sim \beta \\ p \leftharpoonup\!\!\sim \kappa(t) \end{array} \quad : \mathbb{M}(T \times P)$$

Disintegration relates a joint measure $\mu : \mathbb{M}\,(T \times P)$ with a base measure $\beta : \mathbb{M}\,T$ and a kernel $\kappa : T \to \mathbb{M}\,P$

by the equation

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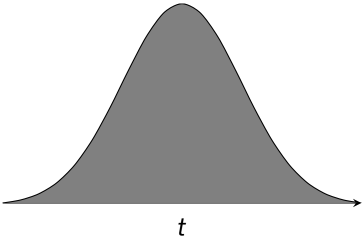
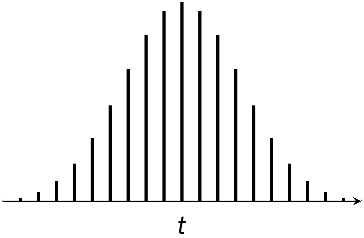
Generalizes density:

$$\begin{array}{l} t \leftsquigarrow \textbf{normal} \\ p = () \end{array} \quad = \quad \begin{array}{l} t \leftsquigarrow \textbf{lebesgue} \\ p \leftsquigarrow \textbf{factor}\ (\textbf{dnorm}\ t) \end{array} \quad : \mathbb{M}\,(\mathbb{R} \times \mathbb{1})$$





Disintegration relates **a joint measure** $\mu : \mathbb{M}(T \times P)$ with **a base measure** $\beta : \mathbb{M}\,T$ and **a kernel** $\kappa : T \to \mathbb{M}\,P$

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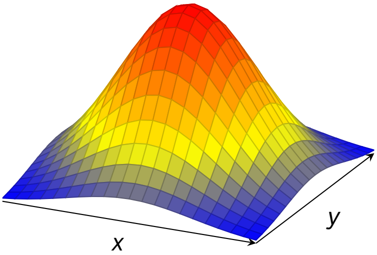
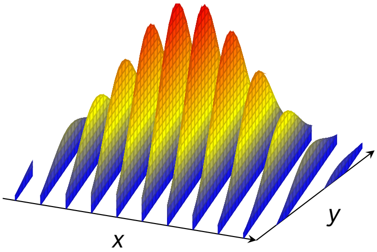
Generalizes conditioning:

$$\begin{array}{l} x \leftsquigarrow \textbf{normal} \\ y \leftsquigarrow \textbf{normal} \\ t \leftsquigarrow 5 \cdot x + 0.1 \cdot y \\ p = (x,y) \end{array} \quad = \quad \begin{array}{l} t \leftsquigarrow \textbf{lebesgue} \\ p \leftsquigarrow \ldots \Pr(p,t) \ldots \end{array} \quad : \mathbb{M}(\mathbb{R} \times \mathbb{R}^2)$$

"unnormalized conditioning"





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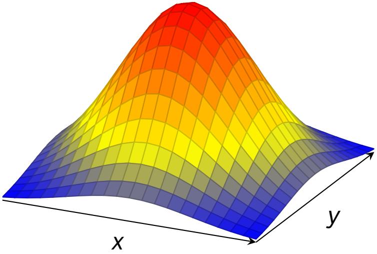

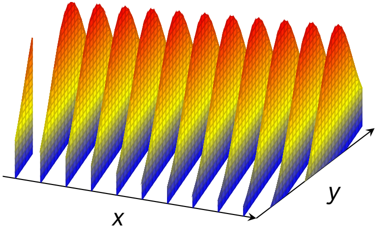

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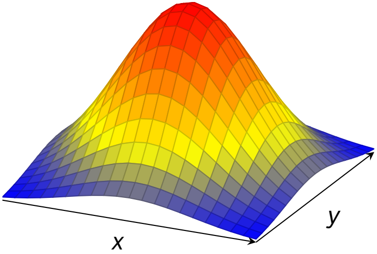

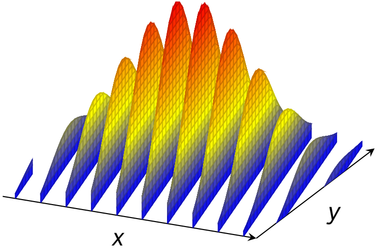

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Generalizes density and conditioning

Can be thought of as unnormalized conditioning

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A semantics-preserving transformation on probabilistic programs

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random choice (**normal**), scoring (**factor**)

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A semantics-preserving transformation on probabilistic programs

s-finite measures/kernels

random choice (**normal**), scoring (**factor**)

sequence of **operations**, not just a primitive

Derived by equational reasoning (hence proven sound by construction)

= This talk: equational reasoning on example programs

+ Established PL technology: lazy and partial evaluation (traversing computation graph)

LAFI 2019: Languages for Inference (formerly PPS)

Tue 15 Jan 2019
Lisbon Portugal
(Co-located with POPL)
https://popl19.sigplan.org/track/lafi-2019

Keywords

- ProbProg systems
- generative modelling
- inference
- autodiff
- semantics
- applications

Important Dates AoE (UTC-12h)

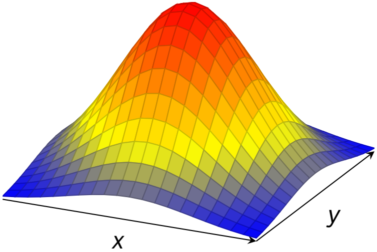
Submission deadline	Thu 1 Nov 2018
Author notification	Mon 3 Dec 2018

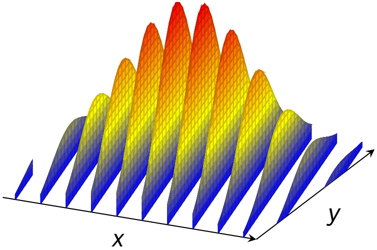
Fixed-base disintegration

$x \leftsquigarrow$ **normal**
$y \leftsquigarrow$ **normal**
$t = 5 \cdot x + 0.1 \cdot y$
$p = (x, y)$

$=$

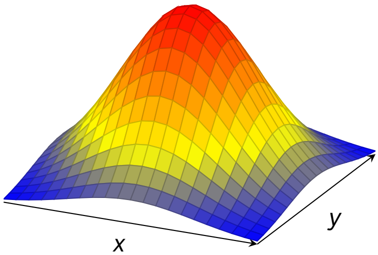
$t \leftsquigarrow$ **lebesgue**
$p \leftsquigarrow$ **???**

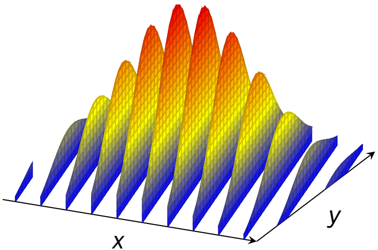




Fixed-base disintegration

$$\begin{array}{l} x \leftsquigarrow \textbf{normal} \\ y \leftsquigarrow \textbf{normal} \\ t = 5 \cdot x + 0.1 \cdot y \\ p = (x, y) \end{array} \quad = \quad \begin{array}{l} x \leftsquigarrow \textbf{normal} \\ y \leftsquigarrow \textbf{lebesgue} \\ t = 5 \cdot x + 0.1 \cdot y \\ \textbf{factor}\ (\textbf{dnorm}\ y) \\ p = (x, y) \end{array} \quad = \quad \begin{array}{l} t \leftsquigarrow \textbf{lebesgue} \\ x \leftsquigarrow \textbf{normal} \\ y = 10 \cdot (t - 5 \cdot x) \\ \textbf{factor}\ 10 \\ \textbf{factor}\ (\textbf{dnorm}\ y) \\ p = (x, y) \end{array}$$





Fixed-base disintegration

$x \leftsquigarrow \textbf{normal}$
$y \leftsquigarrow \textbf{normal}$
$t = 5 \cdot x + 0.1 \cdot y$
$p = (x, y)$

density

$x \leftsquigarrow \textbf{normal}$
$y \leftsquigarrow \textbf{lebesgue}$
$t = 5 \cdot x + 0.1 \cdot y$
$\textbf{factor}\ (\textbf{dnorm}\ y)$
$p = (x, y)$

reparameterize

$t \leftsquigarrow \textbf{lebesgue}$
$x \leftsquigarrow \textbf{normal}$
$y = 10 \cdot (t - 5 \cdot x)$
$\textbf{factor}\ 10$
$\textbf{factor}\ (\textbf{dnorm}\ y)$
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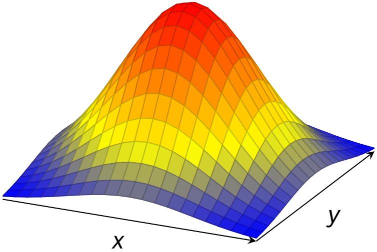

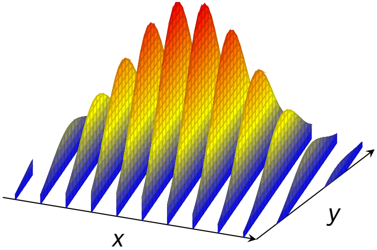

A growing variety of base measures

Bhat et al. POPL 2012,
Shan & Ramsey POPL 2017

$$\beta ::= \textbf{lebesgue} \mid \textbf{counting} \mid \beta \otimes \beta$$

A growing variety of base measures

$$\beta ::= \overbrace{\beta \otimes \cdots \otimes \beta}^{n}$$

Narayanan & Shan
ICFP 2017

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$$\beta ::= \textbf{lebesgue} \mid \textbf{counting} \mid \beta \otimes \beta$$

Narayanan & Shan

$$\beta ::= \textbf{ret}\ e \obar \beta \mid \beta \oplus \beta \mid (x \leftsquigarrow \beta) \otimes \beta$$

Discrete-continuous mixtures
Disjoint sums
Dependent products

A growing variety of base measures

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Applications:

- clamped model/observation
  (GPA, Tobit, camera)
  - likelihood ratio, importance sampling, mutual information
  - belief update, Gibbs sampling
- Metropolis-Hastings sampling
  - single site
  - reversible jump, light transport

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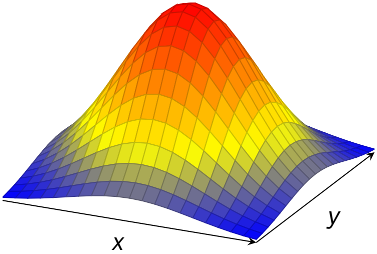
  - single site
  - reversible jump, light transport

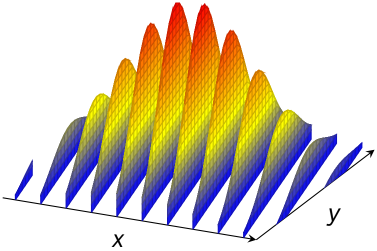
Clamped observation requires mixed base

$x \leftsquigarrow$ **normal**
$y \leftsquigarrow$ **normal**
$t = 5 \cdot x + 0.1 \cdot y$
$t' = \max\{-3, \min\{+3, t\}\}$
$p = (x, y)$

$\neq$

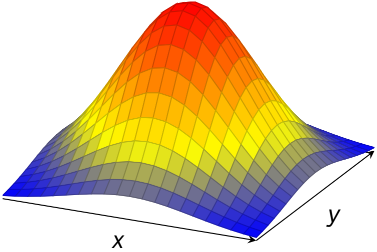
$t' \leftsquigarrow$ **lebesgue**
$p \leftsquigarrow$ **???**

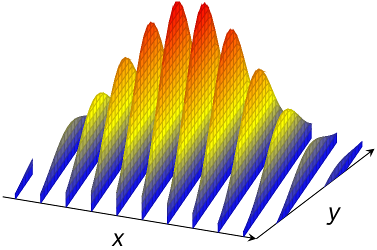




Clamped observation requires mixed base

$$
\begin{aligned}
&x \leftsquigarrow \textbf{normal}\\
&y \leftsquigarrow \textbf{normal}\\
&t = 5 \cdot x + 0.1 \cdot y\\
&t' \leftsquigarrow (\textbf{factor}\ \langle t < -3\rangle ; \textbf{ret}\ {-3}) \oplus\\
&\qquad (\textbf{factor}\ \langle t > +3\rangle ; \textbf{ret}\ {+3}) \oplus\\
&\qquad (\textbf{factor}\ \langle |t| \leq 3\rangle\ ; \textbf{ret}\ t \quad)\\
&p = (x, y)
\end{aligned}
\quad = \quad
\begin{aligned}
&t' \leftsquigarrow \textbf{ret}\ {-3} \oplus \textbf{ret}\ {+3} \oplus \textbf{lebesgue}\\
&p \leftsquigarrow \textbf{???}
\end{aligned}
$$





Base-checking disintegration

$$
\begin{array}{lcl}
x \leftsquigarrow \textbf{normal} & = & t' \leftsquigarrow \textbf{ret}\ {-3} \oplus \textbf{ret}\ {+3} \oplus \textbf{lebesgue} \\
y \leftsquigarrow \textbf{normal} & & p \leftsquigarrow \textbf{???} \\
t = 5 \cdot x + 0.1 \cdot y & & \\
\textbf{factor}\ \langle t < -3 \rangle & & \\
t' = -3 & & \\
p = (x, y) & &
\end{array}
$$

$$
\begin{array}{lcl}
x \leftsquigarrow \textbf{normal} & = & t \leftsquigarrow \textbf{ret}\ {-3} \oplus \textbf{ret}\ {+3} \oplus \textbf{lebesgue} \\
y \leftsquigarrow \textbf{normal} & & p \leftsquigarrow \textbf{???} \\
t = 5 \cdot x + 0.1 \cdot y & & \\
\textbf{factor}\ \langle |t| \leq 3 \rangle & & \\
p = (x, y) & &
\end{array}
$$

Base-checking disintegration

$$
\begin{aligned}
&x \leftsquigarrow \mathbf{normal}\\
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&t = 5 \cdot x + 0.1 \cdot y\\
&\mathbf{factor}\ \langle t < -3 \rangle\\
&t' = -3\\
&p = (x, y)
\end{aligned}
$$

density

$$
\begin{aligned}
&t' \leftsquigarrow \mathbf{ret}\ {-3} \obar \mathbf{ret}\ {+3} \obar \mathbf{lebesgue}\\
&\mathbf{factor}\ \langle t' = -3 \rangle\\
&x \leftsquigarrow \mathbf{normal}\\
&y \leftsquigarrow \mathbf{normal}\\
&t = 5 \cdot x + 0.1 \cdot y\\
&\mathbf{factor}\ \langle t < -3 \rangle\\
&p = (x, y)
\end{aligned}
$$

$$
\begin{aligned}
&x \leftsquigarrow \mathbf{normal}\\
&y \leftsquigarrow \mathbf{normal}\\
&t = 5 \cdot x + 0.1 \cdot y\\
&\mathbf{factor}\ \langle |t| \leq 3 \rangle\\
&p = (x, y)
\end{aligned}
$$

density

$$
\begin{aligned}
&x \leftsquigarrow \mathbf{normal}\\
&y \leftsquigarrow \mathbf{ret}\ (10 \cdot (-3 - 5 \cdot x)) \obar\\
&\qquad \mathbf{ret}\ (10 \cdot (+3 - 5 \cdot x)) \obar\\
&\qquad \mathbf{lebesgue}\\
&t = 5 \cdot x + 0.1 \cdot y\\
&\mathbf{factor}\ \langle t \notin \{\pm 3\} \rangle\\
&\mathbf{factor}\ (\mathbf{dnorm}\ y)\\
&\mathbf{factor}\ \langle |t| \leq 3 \rangle\\
&p = (x, y)
\end{aligned}
$$

reparameterize

$$
\begin{aligned}
&t \leftsquigarrow \mathbf{ret}\ {-3} \obar \mathbf{ret}\ {+3} \obar \mathbf{lebesgue}\\
&x \leftsquigarrow \mathbf{normal}\\
&y = 10 \cdot (t - 5 \cdot x)\\
&\mathbf{factor}\ 10\\
&\mathbf{factor}\ \langle t \notin \{\pm 3\} \rangle\\
&\mathbf{factor}\ (\mathbf{dnorm}\ y)\\
&\mathbf{factor}\ \langle |t| \leq 3 \rangle\\
&p = (x, y)
\end{aligned}
$$

Base-inferring disintegration

$x \leftsquigarrow$ **normal** $y \leftsquigarrow$ **normal** $t = 5 \cdot x + 0.1 \cdot y$ **factor** $\langle t < -3 \rangle$ $t' = -3$ $p = (x, y)$	$=$	$t' \leftsquigarrow \mathbb{B}$ $p \leftsquigarrow$ **???**
$x \leftsquigarrow$ **normal** $y \leftsquigarrow$ **normal** $t = 5 \cdot x + 0.1 \cdot y$ **factor** $\langle \lvert t \rvert \leq 3 \rangle$ $p = (x, y)$	$=$	$t \leftsquigarrow \mathbb{B}$ $p \leftsquigarrow$ **???**

Base-inferring disintegration

$$\begin{aligned}
&x \leftsquigarrow \textbf{normal}\\
&y \leftsquigarrow \textbf{normal}\\
&t = 5 \cdot x + 0.1 \cdot y\\
&\textbf{factor}\ \langle t < -3 \rangle\\
&t' = -3\\
&p = (x, y)
\end{aligned}$$

density

$$\begin{aligned}
&t' \leftsquigarrow \mathbb{B}\\
&\textbf{divide}\ (\textbf{ret}\ {-3})\ \mathbb{B}\ t'\\
&x \leftsquigarrow \textbf{normal}\\
&y \leftsquigarrow \textbf{normal}\\
&t = 5 \cdot x + 0.1 \cdot y\\
&\textbf{factor}\ \langle t < -3 \rangle\\
&p = (x, y)
\end{aligned}$$

$$\begin{aligned}
&x \leftsquigarrow \textbf{normal}\\
&y \leftsquigarrow \textbf{normal}\\
&t = 5 \cdot x + 0.1 \cdot y\\
&\textbf{factor}\ \langle |t| \le 3 \rangle\\
&p = (x, y)
\end{aligned}$$

density

$$\begin{aligned}
&x \leftsquigarrow \textbf{normal}\\
&y \leftsquigarrow \textbf{reparam}\ (y \mapsto 5 \cdot x + 0.1 \cdot y)\ \mathbb{B}\\
&t = 5 \cdot x + 0.1 \cdot y\\
&\textbf{factor}\ (10\ /\ \textbf{jacobian}\ (y \mapsto \cdots)\ \mathbb{B}\ t)\\
&\textbf{divide lebesgue}\ \mathbb{B}\ t\\
&\textbf{factor}\ (\textbf{dnorm}\ y)\\
&\textbf{factor}\ \langle |t| \le 3 \rangle\\
&p = (x, y)
\end{aligned}$$

reparameterize

$$\begin{aligned}
&t \leftsquigarrow \mathbb{B}\\
&x \leftsquigarrow \textbf{normal}\\
&y = 10 \cdot (t - 5 \cdot x)\\
&\textbf{factor}\ 10\\
&\textbf{divide lebesgue}\ \mathbb{B}\ t\\
&\textbf{factor}\ (\textbf{dnorm}\ y)\\
&\textbf{factor}\ \langle |t| \le 3 \rangle\\
&p = (x, y)
\end{aligned}$$

Base-inferring disintegration

$x \leftsquigarrow \textbf{normal}$
$y \leftsquigarrow \textbf{normal}$
$t = 5 \cdot x + 0.1 \cdot y$
$\textbf{factor}\ \langle t < -3 \rangle$
$t' = -3$
$p = (x, y)$

density

$t' \leftsquigarrow \mathtt{B}$
$\textbf{divide}\ (\textbf{ret}\ {-3})\ \mathtt{B}\ t'$
$x \leftsquigarrow \textbf{normal}$
$y \leftsquigarrow \textbf{normal}$
$t = 5 \cdot x + 0.1 \cdot y$
$\textbf{factor}\ \langle t < -3 \rangle$
$p = (x, y)$

$x \leftsquigarrow \textbf{normal}$
$y \leftsquigarrow \textbf{normal}$
$t = 5 \cdot x + 0.1 \cdot y$
$\textbf{factor}\ \langle |t| \leq 3 \rangle$
$p = (x, y)$

density

$x \leftsquigarrow \textbf{normal}$
$y \leftsquigarrow \textbf{reparam}\ (y \mapsto 5 \cdot x + 0.1 \cdot y)\ \mathtt{B}$
$t = 5 \cdot x + 0.1 \cdot y$
$\textbf{factor}\ (10\ /\ \textbf{jacobian}\ (y \mapsto \cdots)\ \mathtt{B}\ t)$
$\textbf{divide lebesgue}\ \mathtt{B}\ t$
$\textbf{factor}\ (\textbf{dnorm}\ y)$
$\textbf{factor}\ \langle |t| \leq 3 \rangle$
$p = (x, y)$

reparameterize

$t \leftsquigarrow \mathtt{B}$
$x \leftsquigarrow \textbf{normal}$
$y = 10 \cdot (t - 5 \cdot x)$
$\textbf{factor}\ 10$
$\textbf{divide lebesgue}\ \mathtt{B}\ t$
$\textbf{factor}\ (\textbf{dnorm}\ y)$
$\textbf{factor}\ \langle |t| \leq 3 \rangle$
$p = (x, y)$

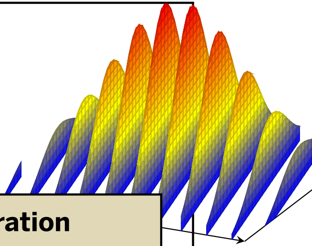
Base-inferring disintegration

Infer principal base measure

$$\mathbf{divide}\ \begin{pmatrix} \mathbf{ret}\ {-3} & \oplus \\ \mathbf{ret}\ {+3} & \oplus \\ \mathbf{lebesgue} & \end{pmatrix}\ \mathtt{B} \quad \begin{cases} \mathbf{divide}\ (\mathbf{ret}\ {-3})\ \mathtt{B}\ t' \\ \\ \mathbf{divide}\ (\mathbf{ret}\ {+3})\ \mathtt{B}\ t' \\ \\ \mathbf{divide}\ \mathbf{lebesgue}\ \mathtt{B}\ t \end{cases}$$

Exact calculation

for approximate computation



Automatic differentiation

for gradient descent

Automatic simplification

for Rao-Blackwellization

Automatic disintegration

for inference and sampling

Generalizes density, conditioning

Derived by equational reasoning

A variety of base measures

Mixtures, disjoint sums,
dependent products

Infer principal base measure

Can your computer do this?

GI on Kantor dwarfs any on any given world. To walk in the weak gravity by the great aluminum and ceramic banks in hot and cold storage is to walk past macro-encyclopedias—encyclopedias of encyclopedias! I recall my first time through, when I stood on a plane of scarlet glass under an array of floating light tubes and thought out: "What is the exact human population of the universe?" and was informed, for answer: "In a universe of c. six thousand two hundred inhabited worlds with human populations over two hundred and under five billion, 'population' itself becomes a fuzzy-edged concept. Over any moment there is a birth/death pulse of almost a billion. Those worlds on which humans have the legal status of the native population and little distinction is made among all these women present statistical problems from several points of view. Thus 'exactness' below five billion is not to be forthcoming. Here are some informative programs you may pursue that will allow you to ask your question in more meaningful terms ..."

—Samuel R. Delany,
Stars in my pocket like grains of sand