THE LIMITS OF PROPAGANDA WITH STRATEGIC COMMUNICATION

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MOTIVATION

- Autocratic regimes tend to repress the citizens that provide **evidence of misinformation** in the news

  E.g., new laws introducing war censorship in Russia:

  - criminal cases on charges of “false information” or “discreditation” of Russian armed forces

- Citizens are not homogeneous in their beliefs about the bias in the news

- **Sharing information might be a strategic tool** that agents can use to increase the awareness of citizens

How does communication affect the optimal intensity of propaganda?

MOTIVATION

- A **skeptical** receiver knows the bias introduced by the government
- A **credulous** receiver takes the meaning of a message as given
- **Communication is costly**: opportunity cost of time

Then, higher bias in the news..

- increases the impact on credulous citizens
- increases incentives of skeptical citizens to share information about the bias

The goal is to analyze **the limits on propaganda** that arise from **costly communication** between skeptical and credulous citizens

RELATED LITERATURE

- **Bayesian Persuasion**

  (Kamenica and Gentzkow, 2011; Bergemann and Morris, 2019)

- **Propaganda**

  (Gehlbach and Sonin, 2014; Chen, Lu and Suen, 2016; Egorov and Sonin, 2020)

  Persuasion on Networks (Egorov and Sonin, 2020)

  - A single sender who maximizes the expected amount of action by agents connected in a network
  - Receivers compare the cost of getting this information with the value that information provides to them

ENVIRONMENT

- Two states: $\omega \in \Omega = \{0, 1\}$, $P(\omega = 1) = \mu \in \left(0, \frac{1}{2}\right)$
- Economy is populated with N agents: $i \in \{1, .., N\}$
- Each agent i chooses an action: $y_i \in \{0, 1\}$
- The final choice profile of society: $y \in \Omega^N$
- The government wants to maximize expected amount of action 1
- State-dependent payoffs of citizens are:

$$u_i(y, \omega) = \alpha 1\{y_i = \omega\} + (1 - \alpha) \frac{1}{N - 1} \sum_{j \neq i} 1\{y_j = \omega\}$$

PROPAGANDA

- Each agent i observes a free public signal $\hat{\omega}$ provided by the media. The signal is structured so that:

$$P(\hat{\omega} = 1|\omega = 0) = \beta, \text{ and } P(\hat{\omega} = 1|\omega = 1) = 1$$

- Each agent is skeptical with probability q and credulous with $(1 - q)$
  - skeptical agents know the true value of β
  - credulous agents believe that $\beta = 0$

The posterior probabilities after observing $\hat{\omega}$ are:

- skeptical: $P(\omega = 1|\hat{\omega} = 1) = \dfrac{\mu}{\mu + (1 - \mu)\beta}$, $P(\omega = 0|\hat{\omega} = 0) = 1$
- credulous: $P(\omega = 0|\hat{\omega} = 0) = P(\omega = 1|\hat{\omega} = 1) = 1$

COMMUNICATION

Credulous citizens can learn the value of β indirectly from other agents.

- Agent i is connected to agent j with probability $p_i \rightarrow$ there is outgoing link from i to j
- $p_i \sim F(\cdot)$, where $F(\cdot)$ is absolutely continuous and has finite L1-norm
- Each agent i knows p_i and the distribution $F(\cdot)$

Each agent i chooses $m_{ij} \in \{0, 1\}$:

- $m_{ij} = 1$: agent j learns β with p_i, agent i pays a cost $c_m \in \left(0, \frac{(1-\alpha)(1-q)}{N-1}\right)$
- $m_{ij} = 0$: remain silent

TIMELINE OF THE GAME

1. The government chooses the editorial policy $\beta \in [0, 1]$ and commits to it. Skeptical citizens observe the editorial policy.
2. The state of the world $\omega \in \{0, 1\}$ is realized. Signal $\hat{\omega} \in \{0, 1\}$ is generated according to the editorial policy.
3. **Communication Stage:** Each citizen chooses whether to send costly messages.
4. All citizens observe signals (if any), and each agent i chooses action $y_i \in \{0, 1\}$.
5. Payoffs are realized.

Equilibrium strategy profile: $\left\{ \beta^*, (m_i^*)_{i=1}^N, (y_i^*)_{i=1}^N \right\}$

LAST STAGE

Note: the decision of each agent depends only on the posterior

1. If $\hat{\omega} = 0$: $y_i = 0$ for all i as $P(\omega = 0|\hat{\omega} = 0) = 1$

   $\Rightarrow$ **no communication**

2. If $\hat{\omega} = 1$: decision depends on the belief about β

   - uninformed: $y_i = 1$
   - informed: $y_i = 1$ if $P(\omega = 1|\hat{\omega} = 1) \geq \frac{1}{2}$ and $y_i = 0$ otherwise

   $\Rightarrow$ **no communication** if $P(\omega = 1|\hat{\omega} = 1) \geq \frac{1}{2}$

Skeptical citizens have incentives to send messages only if:

$$\hat{\omega} = 1 \text{ and } P(\omega = 1|\hat{\omega} = 1) = \frac{\mu}{\mu + (1-\mu)\beta} < \frac{1}{2}.$$

EXPECTED GAIN FROM COMMUNICATION

What are the incentives of agent i to send a message to agent k?

Probability that agent k is not informed if $m_{ik} = 0$:

$$(1-q)\prod_{\substack{j\neq i\\ j\neq k}}(1-p_j q m_{jk})$$

Then, the expected gain from $m_{ik} = 1$ when $\omega = 0$:

$$\frac{1-\alpha}{N-1}(1-q)\mathbb{E}\left[\prod_{\substack{j\neq i\\ j\neq k}}(1-p_j q m_{jk})\right] p_i = \frac{1-\alpha}{N-1}(1-q)\,p_i\mathbb{E}\left[(1-p_j q m_{jk})\right]^{N-2}$$

Cutoff Strategy

For any level of bias $\beta \in [0, 1]$, there is a unique cutoff $p^* \in [0, 1]$ such that:

$$m_i(p_i) = \begin{cases} 1 \text{ if } p_i \geq p^*, \\ 0 \text{ otherwise.} \end{cases}$$

Then, the indifference condition is:

$$\underbrace{p^* \left(1 - q \int_{p^*}^{1} p dF(p)\right)^{N-2}}_{L(p^*)} = c_m K(\beta), \tag{1}$$

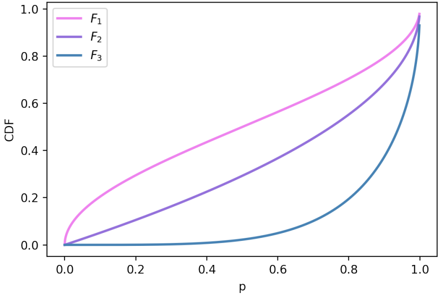
where $K(\beta) = \frac{N-1}{(1-\alpha)(1-q)} \frac{\mu+(1-\mu)\beta}{(1-\mu)\beta-\mu}$.

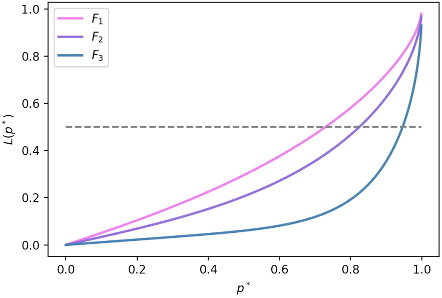
Note: $c_m K(\beta) \leq 1$ when $\beta \geq \frac{\mu}{1-\mu}\left(1 + \frac{2c_m(N-1)}{(1-\alpha)(1-q)-c_m(N-1)}\right) \equiv \bar{\beta}$.

COMPARATIVE STATICS

First Order Dominance

If $F_1(\cdot)$ first-order stochastically dominates $F_2(\cdot)$ then $p_1^*(\beta) \leq p_2^*(\beta)$ for any $\beta \in [0, 1]$.

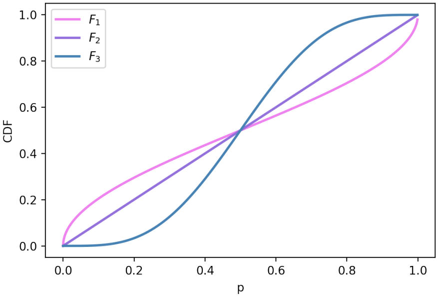


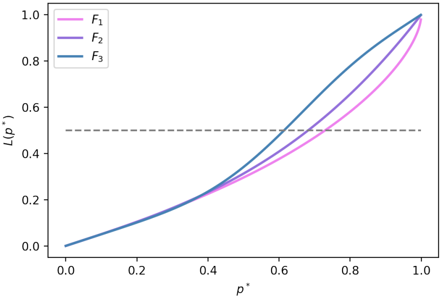


COMPARATIVE STATICS

Mean Preserving Spread

If $p_1 \sim F_1(\cdot)$ is a mean preserving spread of $p_2 \sim F_2(\cdot)$ then there exists $\hat{\beta} \in (\bar{\beta}, 1)$ such that $p_1^*(\beta) \geq p_2^*(\beta)$ for $\beta \leq \hat{\beta}$.





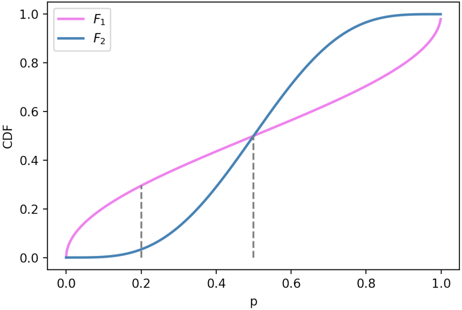
COMPARATIVE STATICS: DETAILS

$$\int_{p^*}^{1} p dF_1(p) \geq \int_{p^*}^{1} p dF_2(p) \Rightarrow L_1(p^*) \leq L_2(p^*)$$

$$\int_{p^*}^{1} p dF(p) = E[p] - p^* F(p^*) + \int_{0}^{p^*} F(p) d p$$

Mean preserving spread:

$$\int_{0}^{p^*} (F_1(p) - F_2(p)) d p \leq p^*(F_1(p^*) - F_2(p^*)) \Rightarrow L_1(p^*) \geq L_2(p^*)$$



COMPARATIVE STATICS: INTUITION

Note: If $p_1^* < p_2^*$, then

$$\left(1 - q \int_{p_1^*}^{1} p dF_1(p)\right)^{N-2} > \left(1 - q \int_{p_2^*}^{1} p dF_2(p)\right)^{N-2}.$$

$$P_{\text{inf}} = 1 - (1 - q)\left(1 - q \int_{p^*}^{1} p dF(p)\right)^{N-1}$$

- **First Order Stochastic Dominance**

  higher average p leads to lower average awareness

- **Mean preserving spread**

  higher inequality in the level of influence will lead to higher awareness of citizens when the level of propaganda is low enough

PROBLEM OF THE GOVERNMENT

From indifference condition (1) we find $p^*(\beta)$

Note: $p^*(\beta)$ is differentiable and $\frac{\partial p^*(\beta)}{\partial \beta} < 0$ for $\beta \in \left[\bar{\beta}, 1\right]$

Problem of the government:

$$(\mu + (1 - \mu)\beta)\,(1 - q)\left(1 - q\int_{p^*(\beta)}^{1} p dF(p)\right)^{N-1} \to \max_{\beta\in[\bar{\beta},1]} \tag{2}$$

First order condition of (2) can be written as:

$$\frac{1 - \mu}{\mu + \beta(1 - \mu)}\left(1 - q\int_{p^*(\beta)}^{1} p dF(p)\right) + (N - 1)q\,p^*(\beta)\,f(p^*(\beta))\frac{\partial p^*(\beta)}{\partial \beta} = 0 \tag{3}$$

OPTIMAL LEVEL OF PROPAGANDA

Instead of $p^*(\beta)$ we use $\beta(p^*)$:

$$\beta(p^*) = \frac{\mu}{1-\mu}\frac{(1-\alpha)(1-q)L(p^*) + c_m(N-1)}{(1-\alpha)(1-q)L(p^*) - c_m(N-1)},\ L(p^*) \geq \frac{(N-1)c_m}{(1-\alpha)(1-q)(1-2\mu)} \tag{4}$$

Then, **the problem of the government** is:

$$\big(\mu + (1-\mu)\beta(p^*)\big)(1-q)\left(1 - q\int_{p^*}^{1} p dF(p)\right)^{N-1} \rightarrow \max_{p^*\in[\bar{p},1]} \tag{5}$$

And equilibrium can be described by:

$$p^{*2} q f(p^*)\big((1-\alpha)(1-q)L(p^*) - c_m(2N-3)\big) - c_m\left(1 - q\int_{p^*}^{1} p dF(p)\right) = 0 \tag{6}$$

CONCLUSION

The trade-off of the government:

An increase in β..

- reduces the likelihood of signal $\hat{\omega} = 0$ ($\uparrow$)
- increases the expected gain from sending a message when $\hat{\omega} = 1$ ($\downarrow$)

Communication Structure:

- **First Order Stochastic Dominance**

  cannot lead to higher awareness and less aggressive propaganda
- **Mean Preserving Spread**

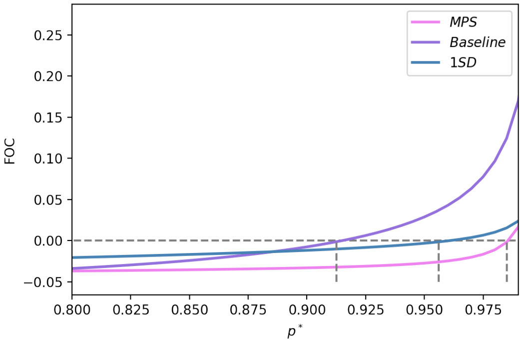
  (may lead to) higher awareness and/or less aggressive propaganda

NUMERICAL EXAMPLE

$$\beta^{\mathrm{MPS}} > \beta^{\mathrm{B}} > \beta^{\mathrm{1SD}}$$

but

$$P_{\mathrm{inf}}^{\mathrm{MPS}} > P_{\mathrm{inf}}^{\mathrm{B}} > P_{\mathrm{inf}}^{\mathrm{1SD}}$$



Thank you for your attention!