A method for stochastic multiple criteria decision making based on dominance degrees

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This paper proposes a method for solving the stochastic multiple criteria decision making (SMCDM) problem, where consequences of alternatives with respect to criteria are represented by random variables with probability distributions. Firstly, definitions and related analysis of dominance degree of one probability distribution over another are given. Then, by calculating the dominance degrees, the dominance degree matrix of alternative pairwise comparisons with respect to each criterion is built. Further, using PROMETHEE II method, an overall dominance degree matrix of alternative pairwise comparisons is constructed, and a net flow of each alternative is calculated. Based on the obtained net flows, a ranking of alternatives is determined. Finally, numerical examples for the three cases are given to illustrate the use of the proposed method.

Stochastic multiple criteria decision making (SMCDM) refers to the problem of selecting alternatives associated with multiple criteria, where consequences of alternatives with respect to criteria are in the form of random variables. SMCDM problems arise in many real-world situations [20,29,37]. For example, in the elevator group selection problem, consequences with respect to criteria such as the traffic component of passenger (incoming, outgoing or inter-floor) are random variables [29]. Research on SMCDM problems can be found in [1,4–7,11–25,28,29,32,34–38].

Keeney and Raiffa [12] initially proposed a method based on multiattribute utility theory (MAUT) to solve SMCDM problems. In their method, decision-maker's (DM's) multiattribute utility function is firstly estimated. Then, the utility value of each alternative is calculated using the multiattribute utility function, and a ranking of alternatives is determined by comparing the utility values. Thereafter, several outranking methods using confidence indices or preference indices have been proposed for solving SMCDM problems [1,11,20]. For example, in the method proposed by D'Avignon and Vincke [1], a distributive preference degree on alternative pairwise comparisons with respect to each criterion and a distributive outranking degree over all criteria are constructed. Further, an outranking approach is explored to obtain a ranking of alternatives. Besides, some methods using if-then rules can be found [30,31].

In recent years, some methods using stochastic dominance (SD) rules have been proposed to solve SMCDM problems [21–25,34–38]. These methods generally include two processes: comparison and selection. The former is to identify whether there exists a SD relation for comparison of any pair of alternatives using SD rules. The latter is to rank alternatives based on the determined SD relations using Rough Set Theory [35,36], outranking methods [23] or interactive procedures [24,25]. Besides, the method based on stochastic dominance degree (SDD) has been also proposed [38].

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Stochastic multiobjective acceptability analysis (SMAA) is another tool for supporting SMCDM or group decision making analysis, in which both criterion values and criterion weights are uncertain [14]. In SMAA, Monte Carlo simulation is used to generate random outcomes of criterion values and criterion weights. In each iteration, a ranking order of alternatives obtained based on the generated outcomes is recorded. After a large number of iterations, the rank acceptability index, holistic acceptability index, central weight and confidence factor of each alternative are respectively obtained, which are valuable data for the DM to identify desirable alternative(s). Recently, some extended SMAA methods have also been developed [4–6,15–19,28,29].

It can be seen that some scholars attempted to use the methods based on probability analysis to solve SMCDM problems. For example, in the method proposed by Zawisza and Trzpiot [37], probabilities on alternative pairwise comparisons and SD rules are used to determine dominance relations among alternatives. In the method proposed by Fan et al. [7], the ranking of alternatives can be determined on the basis of estimating superior, indifferent and inferior probabilities on alternative pairwise comparisons.

The above studies have made significant contributions to SMCDM analysis. However, there are still some limitations when the existing methods are used. For example, when the method based on MAUT is used, a multiattribute utility function is needed to be estimated beforehand, whereas determination of such a function is not easy [23]. Although the outranking method using confidence indices or preference indices can be used to obtain a ranking of alternatives, the confidence indices or preference indices may be not easily interpreted by DMs [27]. Particularly, when the methods using SD rules are applied, SD relations for comparisons of some pairs of alternatives can not be often determined. This would easily cause generation of the unclear ranking of alternatives, i.e., the ranking order of some alternatives cannot be distinguished (see [23,35,36]). Also, when the methods based on probability analysis are used, the thresholds for judging the results of alternative pairwise comparisons are needed to be defined beforehand (see [7,37]). Although indirect elicitation techniques can be used to determine the thresholds, it could be troublesome and time-consuming.

Based on the above analysis, it is necessary to make a further study for solving SMCDM problems. A key of the study is to overcome the shortages which occur either in the processes of determining SD relations or defining the thresholds for judging the results of alternative pairwise comparisons. In this paper, a method is proposed to solve the SMCDM problem. In the proposed method, firstly, definitions of dominance degree of one alternative over another are given. Then, the dominance degree matrix of alternative pairwise comparisons with respect to each criterion is constructed by calculating the dominance degrees. Further, using PROMETHEE II method [2], an overall dominance degree matrix of alternative pairwise comparisons is constructed, and a net flow of each alternative is calculated. Finally, a ranking of alternatives is determined based on the obtained net flows.

The rest of this paper is structured as follows: Section 2 gives concepts and calculation formulas on dominance degree, and conducts related theoretical analysis. In Section 3, the dominance degree matrix of alternative pairwise comparisons with respect to each criterion is constructed, and PROMETHEE II method is used to rank alternatives. In Section 4, numerical examples for the three cases are examined to illustrate the potential applications of the proposed method. And lastly, Section 5 summarizes and highlights the main features of this paper.

In decision analysis, it is usually assumed that the DM would select an alternative with the greatest utility value. However, it would be not easy for the DM to make selection if consequences of alternatives are in the form of random variables. Here, for simplicity, we consider a case of two alternatives. Let $U(x)$ be the DM's utility function and x be possible outcomes of random variable X. Suppose that consequences of two alternatives A_1 and A_2 are random and respectively represented by random variables X_1 and X_2 with probability distributions $f_1(x)$ and $f_2(x)$. Then it is difficult to select a better alternative between A_1 and A_2 since either of two cases $U(x_1)> U(x_2)$ and $U(x_1)< U(x_2)$ could occur. Hence, to select a better one between the two alternatives, it is necessary to assess probabilities of $U(x_1)> U(x_2)$ and $U(x_1)< U(x_2)$. To assess the probabilities, we make the minimal assumption that the DM's utility function, $U(x)$, increases with x, $U'(x)> 0$ [10]. Based on the assumption, we can know that the probabilities of $U(x_1)> U(x_2)$ and $U(x_1)< U(x_2)$ are equal to the probabilities of $x_1 > x_2$ and $x_1 < x_2$, respectively. Further, by calculating the probability that the possible outcome of one random variable is greater than another, we can measure the dominance degree of $f_1(x)$ over $f_2(x)$ (noted as $f_1(x) \succ f_2(x)$) or $f_2(x)$ over $f_1(x)$ ($f_2(x) \succ f_1(x)$) to compare the two alternatives.

Ordinarily, there are two kinds of random variables, i.e., continuous random variable with continuous probability distribution (probability density function) and discrete random variable with discrete probability distribution (probability mass function). Thus, there are three possible cases when we compare two probability distributions, i.e., (1) two continuous probability distributions, (2) two discrete probability distributions and (3) a continuous probability distribution and a discrete probability distribution. In the following, we will give definitions and related analysis of dominance degree for the three cases, respectively.

2.1. Comparing two continuous probability distributions

Let X_1 and X_2 be two independent continuous random variables with probability distributions $f_1(x)$ and $f_2(x)$, respectively, where $\int_{-\infty}^{+\infty} f_1(x)dx = 1$ and $\int_{-\infty}^{+\infty} f_2(x)dx = 1$. Let x_1 and x_2 be outcomes of X_1 and X_2, respectively. Let $p(x_1 > x_2)$, $p(x_1 = x_2)$ and

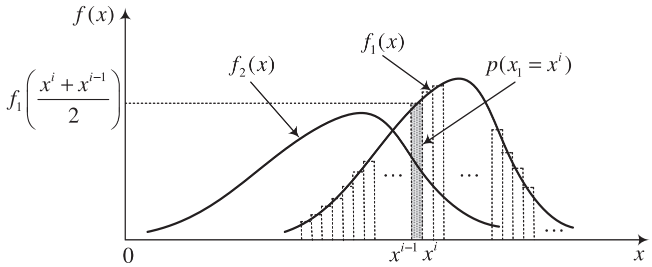

Fig. 1. Probability $p(x_1 = x^i)$.

$p(x_1 < x_2)$ denote probabilities of $x_1 > x_2$, $x_1 = x_2$ and $x_1 < x_2$, respectively. To calculate $p(x_1 > x_2)$, $p(x_1 = x_2)$ and $p(x_1 < x_2)$, we subdivide space $(-\infty,+\infty)$ into $q+1$ intervals $(-\infty,x^1]$, $(x^1,x^2],\ldots,$ $(x^{i-1},x^i],\ldots,(x^{q-1},x^q]$ and $(x^q,+\infty)$, $-\infty < x^1 < x^2 < \cdots < x^{i-1} < x^i < \cdots < x^q < +\infty$. As shown in Fig. 1, the probability that event $x_1 = x^i$ occurs can be approximately represented by

$$p(x_1 = x^i) \approx (x^i - x^{i-1})f_1\left(\frac{x^i + x^{i-1}}{2}\right). \tag{1}$$

In the situation of $x_1 = x^i$ (see Fig. 2), the probabilities that events $x_1 > x_2$, $x_1 = x_2$ and $x_1 < x_2$ occur can be respectively represented by

$$p(x_1 > x_2 | x_1 = x^i) \approx \sum_{j=1}^{i-2}(x^{j+1} - x^j)f_2\left(\frac{x^{j+1} + x^j}{2}\right), \tag{2}$$

$$p(x_1 = x_2 | x_1 = x^i) \approx (x^i - x^{i-1})f_2\left(\frac{x^i + x^{i-1}}{2}\right), \tag{3}$$

$$p(x_1 < x_2 | x_1 = x^i) \approx \sum_{j=i}^{q}(x^{j+1} - x^j)f_2\left(\frac{x^{j+1} + x^j}{2}\right). \tag{4}$$

Since X_1 and X_2 are two independent random variables, the probabilities that events $x_1 > x_2$, $x_1 = x_2$ and $x_1 < x_2$ occur can be respectively calculated by

$$\begin{aligned} p(x_1 > x_2) &= \sum_{i=1}^{q} p(x_1 = x^i)p(x_1 > x_2 | x_1 = x^i) \\ &\approx \sum_{i=2}^{q}\sum_{j=1}^{i-2}(x^i - x^{i-1})(x^{j+1} - x^j)f_1\left(\frac{x^i + x^{i-1}}{2}\right)f_2\left(\frac{x^{j+1} + x^j}{2}\right), \end{aligned} \tag{5}$$

$$\begin{aligned} p(x_1 = x_2) &= \sum_{i=1}^{q} p(x_1 = x^i)p(x_1 = x_2 | x_1 = x^i) \\ &\approx \sum_{i=2}^{q}(x^i - x^{i-1})^2 f_1\left(\frac{x^i + x^{i-1}}{2}\right)f_2\left(\frac{x^i + x^{i-1}}{2}\right), \end{aligned} \tag{6}$$

$$\begin{aligned} p(x_1 < x_2) &= \sum_{i=1}^{q} p(x_1 = x^i)p(x_1 < x_2 | x_1 = x^i) \\ &\approx \sum_{i=2}^{q}\sum_{j=i}^{q}(x^i - x^{i-1})(x^{j+1} - x^j)f_1\left(\frac{x^i + x^{i-1}}{2}\right)f_2\left(\frac{x^{j+1} + x^j}{2}\right). \end{aligned} \tag{7}$$

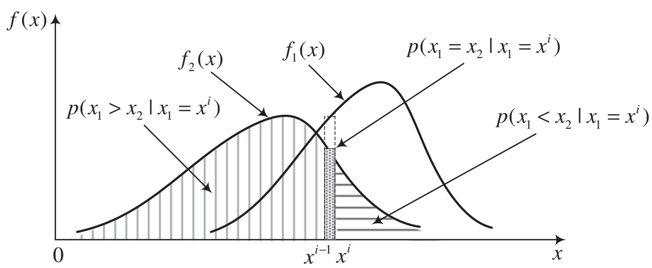

Fig. 2. Probabilities $p(x_1 > x_2 | x_1 = x^i)$, $p(x_1 = x_2 | x_1 = x^i)$ and $p(x_1 < x_2 | x_1 = x^i)$.

Further, if the number of intervals increases infinitely, i.e., $q + 1 \to +\infty$, then $(x^i - x^{i-1}) \to 0$, $i = 1,2,\ldots,q$. Thus, $p(x_1 > x_2)$, $p(x_1 = x_2)$ and $p(x_1 < x_2)$ can be respectively expressed in the following integral forms:

$$p(x_1 > x_2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{x_1} f_1(x_1) f_2(x_2) dx_2 dx_1, \tag{8}$$

$$p(x_1 = x_2) = \int_{-\infty}^{+\infty} \int_{x_1}^{x_1} f_1(x_1) f_2(x_2) dx_2 dx_1, \tag{9}$$

$$p(x_1 < x_2) = \int_{-\infty}^{+\infty} \int_{x_1}^{+\infty} f_1(x_1) f_2(x_2) dx_2 dx_1. \tag{10}$$

Obviously, $p(x_1 = x_2) = 0$.

Based on the above analysis, we give the following definition.

Definition 1. Let X_1 and X_2 be two independent continuous random variables with probability distributions $f_1(x)$ and $f_2(x)$, respectively, where $\int_{-\infty}^{+\infty} f_1(x)dx = 1$ and $\int_{-\infty}^{+\infty} f_2(x)dx = 1$. Then the dominance degree of $f_1(x)$ over $f_2(x)$ (noted as $D_{f_1 \succ f_2}$) is given by

$$D_{f_1 \succ f_2} = \int_{-\infty}^{+\infty} \int_{-\infty}^{x_1} f_1(x_1) f_2(x_2) dx_2 dx_1, \tag{11}$$

and accordingly, the dominance degree of $f_2(x)$ over $f_1(x)$ (noted as $D_{f_2 \succ f_1}$) is given by

$$D_{f_2 \succ f_1} = \int_{-\infty}^{+\infty} \int_{x_1}^{+\infty} f_1(x_1) f_2(x_2) dx_2 dx_1. \tag{12}$$

Property 1. $D_{f_1 \succ f_2} + D_{f_2 \succ f_1} = 1$.

Proof. By Eqs. (11) and (12), we have

$$\begin{aligned} D_{f_1 \succ f_2} + D_{f_2 \succ f_1} &= \int_{-\infty}^{+\infty} \int_{-\infty}^{x_1} f_1(x_1) f_2(x_2) dx_2 dx_1 + \int_{-\infty}^{+\infty} \int_{x_1}^{+\infty} f_1(x_1) f_2(x_2) dx_2 dx_1 \\ &= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{x_1} f_1(x_1) f_2(x_2) dx_2 + \int_{x_1}^{+\infty} f_1(x_1) f_2(x_2) dx_2 \right] dx_1 \quad = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_1(x_1) f_2(x_2) dx_2 dx_1 \\ &= \left[\int_{-\infty}^{+\infty} f_1(x_1) dx_1 \right] \left[\int_{-\infty}^{+\infty} f_2(x_2) dx_2 \right]. \end{aligned}$$

Since $\int_{-\infty}^{+\infty} f_1(x)dx = 1$ and $\int_{-\infty}^{+\infty} f_2(x)dx = 1$, we know $D_{f_1 \succ f_2} + D_{f_2 \succ f_1} = 1$. □

Property 2. $0 \leqslant D_{f_1 \succ f_2} \leqslant 1$ *and* $0 \leqslant D_{f_2 \succ f_1} \leqslant 1$.

Proof. Since probability distributions $f_1(x)$ and $f_2(x)$ are non-negative, we have $D_{f_1 \succ f_2} \geqslant 0$ and $D_{f_2 \succ f_1} \geqslant 0$ by Eqs. (11) and (12). Thus, we know $0 \leqslant D_{f_1 \succ f_2} \leqslant 1$ and $0 \leqslant D_{f_2 \succ f_1} \leqslant 1$ since $D_{f_1 \succ f_2} + D_{f_2 \succ f_1} = 1$. □

2.2. Comparing two discrete probability distributions

Let Y_1 and Y_2 be two independent discrete random variables with probability distributions $g_1(y)$ and $g_2(y)$, respectively, where $\sum_{y=-\infty}^{+\infty} g_1(y) = 1$ and $\sum_{y=-\infty}^{+\infty} g_2(y) = 1$. Let y_1 and y_2 be outcomes of Y_1 and Y_2, respectively. Let $p(y_1 > y_2)$, $p(y_1 = y_2)$ and $p(y_1 < y_2)$ denote probabilities of $y_1 > y_2$, $y_1 = y_2$ and $y_1 < y_2$, respectively. Similar to the analysis of Section 2.1, we have

$$p(y_1 > y_2) = \sum_{y_1=-\infty}^{+\infty} \sum_{y_2=-\infty}^{y_1} g_1(y_1) g_2(y_2) - \sum_{y_1=-\infty}^{+\infty} g_1(y_1) g_2(y_1), \tag{13}$$

$$p(y_1 = y_2) = \sum_{y_1=-\infty}^{+\infty} g_1(y_1) g_2(y_1), \tag{14}$$

$$p(y_1 < y_2) = \sum_{y_1=-\infty}^{+\infty} \sum_{y_2=y_1}^{+\infty} g_1(y_1) g_2(y_2) - \sum_{y_1=-\infty}^{+\infty} g_1(y_1) g_2(y_1). \tag{15}$$

Here, event $y_1 = y_2$ can be regarded as a situation where events $y_1 > y_2$ and $y_1 < y_2$ occur with the same probability simultaneously. Thus, in the situation of $y_1 = y_2$, the probabilities that events $y_1 > y_2$ and $y_1 < y_2$ occur are 0.5, i.e., $p(y_1 > y_2 | y_1 = y_2) = p(y_1 < y_2 | y_1 = y_2) = 0.5$. Based on the above analysis, we give Definition 2.

Definition 2. Let Y_1 and Y_2 be two independent discrete random variables with probability distributions $g_1(y)$ and $g_2(y)$, respectively, where $\sum_{y=-\infty}^{+\infty} g_1(y) = 1$ and $\sum_{y=-\infty}^{+\infty} g_2(y) = 1$. Then the dominance degree of $g_1(y)$ over $g_2(y)$ (noted as $D_{g_1 \succ g_2}$) is given by

$$D_{g_1 \succ g_2} = \sum_{y_1=-\infty}^{+\infty} \sum_{y_2=-\infty}^{y_1} g_1(y_1)g_2(y_2) - 0.5 \sum_{y_1=-\infty}^{+\infty} g_1(y_1)g_2(y_1), \tag{16}$$

and accordingly, the dominance degree of $g_2(y)$ over $g_1(y)$ (noted as $D_{g_2 \succ g_1}$) is given by

$$D_{g_2 \succ g_1} = \sum_{y_1=-\infty}^{+\infty} \sum_{y_2=y_1}^{+\infty} g_1(y_1)g_2(y_2) - 0.5 \sum_{y_1=-\infty}^{+\infty} g_1(y_1)g_2(y_1). \tag{17}$$

Property 3. $D_{g_1 \succ g_2} + D_{g_2 \succ g_1} = 1$.

Proof. By Eqs. (16) and (17), we have

$$\begin{aligned}
D_{g_1 \succ g_2} + D_{g_2 \succ g_1} &= \sum_{y_1=-\infty}^{+\infty} \sum_{y_2=-\infty}^{y_1} g_1(y_1)g_2(y_2) + \sum_{y_1=-\infty}^{+\infty} \sum_{y_2=y_1}^{+\infty} g_1(y_1)g_2(y_2) - \sum_{y_1=-\infty}^{+\infty} g_1(y_1)g_2(y_1) \\
&= \sum_{y_1=-\infty}^{+\infty} g_1(y_1)\left[\sum_{y_2=-\infty}^{y_1} g_2(y_2) + \sum_{y_2=y_1}^{+\infty} g_2(y_2)\right] - \sum_{y_1=-\infty}^{+\infty} g_1(y_1)g_2(y_1) \\
&= \sum_{y_1=-\infty}^{+\infty} g_1(y_1)\left[\sum_{y_2=-\infty}^{+\infty} g_2(y_2) + g_2(y_1)\right] - \sum_{y_1=-\infty}^{+\infty} g_1(y_1)g_2(y_1) \\
&= \sum_{y_1=-\infty}^{+\infty} g_1(y_1) \sum_{y_2=-\infty}^{+\infty} g_2(y_2) + \sum_{y_1=-\infty}^{+\infty} g_1(y_1)g_2(y_1) - \sum_{y_1=-\infty}^{+\infty} g_1(y_1)g_2(y_1) = \left[\sum_{y_1=-\infty}^{+\infty} g_1(y_1)\right]\left[\sum_{y_2=-\infty}^{+\infty} g_2(y_2)\right].
\end{aligned}$$

Since $\sum_{y=-\infty}^{+\infty} g_1(y) = 1$ and $\sum_{y=-\infty}^{+\infty} g_2(y) = 1$, we know $D_{g_1 \succ g_2} + D_{g_2 \succ g_1} = 1$. □

Property 4. $0 \leqslant D_{g_1 \succ g_2} \leqslant 1$ *and* $0 \leqslant D_{g_2 \succ g_1} \leqslant 1$.

The proof of Property 4 is similar to that of Property 2.

2.3. Comparing a continuous probability distribution with a discrete probability distribution

Let X be an independent continuous random variable with probability distribution $f(x)$, and Y be an independent discrete random variable with probability distribution $g(y)$, where $\int_{-\infty}^{+\infty} f(x)dx = 1$ and $\sum_{y=-\infty}^{+\infty} g(y) = 1$. Let x and y be outcomes of X and Y, respectively. Let $p(x > y)$, $p(x = y)$ and $p(x < y)$ denote probabilities of $x > y$, $x = y$ and $x < y$, respectively. Similar to the analysis of Section 2.1, we have

$$p(x > y) = \sum_{y=-\infty}^{+\infty} \left[g(y) \int_{y}^{+\infty} f(x)dx\right], \tag{18}$$

$$p(x = y) = \sum_{y=-\infty}^{+\infty} \left[g(y) \int_{y}^{y} f(x)dx\right], \tag{19}$$

$$p(x < y) = \sum_{y=-\infty}^{+\infty} \left[g(y) \int_{-\infty}^{y} f(x)dx\right]. \tag{20}$$

Obviously, $p(x = y) = 0$. Based on the above analysis, we give Definition 3.

Definition 3. Let X be an independent continuous random variable with probability distribution $f(x)$, and Y be an independent discrete random variable with probability distribution $g(y)$, where $\int_{-\infty}^{+\infty} f(x)dx = 1$ and $\sum_{y=-\infty}^{+\infty} g(y) = 1$. Then the dominance degree of $f(x)$ over $g(y)$ (noted as $D_{f \succ g}$) is given by

$$D_{f \succ g} = \sum_{y=-\infty}^{+\infty} \left[g(y) \int_{y}^{+\infty} f(x)dx\right], \tag{21}$$

and accordingly, the dominance degree of $g(y)$ over $f(x)$ (noted as $D_{g \succ f}$) is given by

$$D_{g \succ f} = \sum_{y=-\infty}^{+\infty} \left[g(y) \int_{-\infty}^{y} f(x)dx\right]. \tag{22}$$

Property 5. $D_{f\succ g} + D_{g\succ f} = 1$.

Proof. By Eqs. (21) and (22), we have

$$D_{f\succ g} + D_{g\succ f} = \sum_{y=-\infty}^{+\infty}\left[g(y)\int_{-\infty}^{y} f(x)dx\right] + \sum_{y=-\infty}^{+\infty}\left[g(y)\int_{y}^{+\infty} f(x)dx\right] = \sum_{y=-\infty}^{+\infty}\left[g(y)\int_{-\infty}^{y} f(x)dx + g(y)\int_{y}^{+\infty} f(x)dx\right]$$
$$= \sum_{y=-\infty}^{+\infty} g(y)\left[\int_{-\infty}^{y} f(x)dx + \int_{y}^{+\infty} f(x)dx\right] = \left[\sum_{y=-\infty}^{+\infty} g(y)\right]\left[\int_{-\infty}^{+\infty} f(x)dx\right].$$

Since $\int_{-\infty}^{+\infty} f(x)dx = 1$ and $\sum_{y=-\infty}^{+\infty} g(y) = 1$, we know $D_{f\succ g} + D_{g\succ f} = 1$. □

Property 6. $0 \leqslant D_{f\succ g} \leqslant 1$ and $0 \leqslant D_{g\succ f} \leqslant 1$.

The proof of Property 6 is similar to that of Property 2.

2.4. Comparing two normal probability distributions

The normal probability distribution is most widely used in many practical decision-making problems with random variables because it models well the additive effect of many independent factors [3,8,9,18,33]. In the following, we analyze a case that continuous probability distributions are normal ones.

Consider two independent normal random variables Z_1 and Z_2 with probability distributions $h_1(z) = \frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z-\mu_1)^2/2\sigma_1^2}$ and $h_2(z) = \frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z-\mu_2)^2/2\sigma_2^2}$, respectively, where μ_1 and μ_2 are means, and σ_1 and σ_2 are standard deviations. According to the operation rules of normal random variables [3,8], probability distribution of $Z = Z_1 - Z_2$ is also a normal one, i.e.,

$$h(z) = \int_{-\infty}^{+\infty}\left(\frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z_1-\mu_1)^2/2\sigma_1^2}\right)\left(\frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z+\mu_2)^2/2\sigma_2^2}\right)dz_1 = \frac{1}{\sigma\sqrt{2\pi}}e^{-(z-\mu)^2/2\sigma^2}, \tag{23}$$

where $\mu = \mu_1 - \mu_2$ and $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$. By Eq. (23), we have the following theorem.

Theorem 1. *Let Z_1 and Z_2 be two independent normal random variables with probability distributions $h_1(z) = \frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z-\mu_1)^2/2\sigma_1^2}$ and $h_2(z) = \frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z-\mu_2)^2/2\sigma_2^2}$, respectively, where μ_1 and μ_2 are means, and σ_1 and σ_2 are standard deviations. Then the dominance degree of $h_1(z)$ over $h_2(z)$ (noted as $D_{h_1\succ h_2}$) is given by*

$$D_{h_1\succ h_2} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\alpha} e^{-t^2/2}dt, \tag{24}$$

where $\alpha = \frac{\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}$.

Proof. By Eq. (11), we have $D_{h_1\succ h_2} = \int_{-\infty}^{+\infty}\int_{-\infty}^{z_1} h_1(z_1)h_2(z_2)dz_2dz_1$, i.e., $D_{h_1\succ h_2} = \int_{-\infty}^{+\infty}\int_{-\infty}^{z_1}\left(\frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z_1-\mu_1)^2/2\sigma_1^2}\right)\left(\frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z_2-\mu_2)^2/2\sigma_2^2}\right) dz_2dz_1$. Further, let $z = z_1 - z_2$ and substitute $z_1 - z$ for z_2, then we have

$$D_{h_1\succ h_2} = \int_{-\infty}^{+\infty}\int_{-\infty}^{z_1}\left(\frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z_1-\mu_1)^2/2\sigma_1^2}\right)\left(\frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z_1-z-\mu_2)^2/2\sigma_2^2}\right)d(z_1-z)dz_1$$
$$= \int_{0}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z_1-\mu_1)^2/2\sigma_1^2}\right)\left(\frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z+\mu_2)^2/2\sigma_2^2}\right)dz_1dz.$$

By Eq. (23), we know $\int_{-\infty}^{+\infty}\left(\frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z_1-\mu_1)^2/2\sigma_1^2}\right)\left(\frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z+\mu_2)^2/2\sigma_2^2}\right)dz_1 = \frac{1}{\sigma\sqrt{2\pi}}e^{-(z-\mu)^2/2\sigma^2}$, where $\mu = \mu_1 - \mu_2$ and $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$. Consequently, we have $D_{h_1\succ h_2} = \int_0^{+\infty}\frac{1}{\sigma\sqrt{2\pi}}e^{-(z-\mu)^2/2\sigma^2}dx$. Further, let $t = \frac{z-\mu}{\sigma}$ and substitute $(\sigma t + \mu)$ for z, then we have $D_{h_1\succ h_2} = \frac{1}{\sqrt{2\pi}}\int_{-\alpha}^{+\infty} e^{-t^2/2}dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\alpha} e^{-t^2/2}dt$, where $\alpha = \frac{\mu}{\sigma} = \frac{\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}$. □

Based on Theorem 1, we have the following conclusions.

Corollary 1. *Let Z_1 and Z_2 be two independent normal random variables with probability distributions $h_1(z) = \frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z-\mu_1)^2/2\sigma_1^2}$ and $h_2(z) = \frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z-\mu_2)^2/2\sigma_2^2}$, respectively. If and only if $\mu_1 \geqslant \mu_2$, then $D_{h_1\succ h_2} \geqslant 0.5$. Particularly, if and only if $\mu_1 = \mu_2$, then $D_{h_1\succ h_2} = 0.5$.*

Proof. If and only if $\mu_1 \geqslant \mu_2$, then $\alpha = \frac{\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}} \geqslant 0$. By Eq. (24), we have $D_{h_1\succ h_2} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\alpha} e^{-t^2/2}dt \geqslant \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} e^{-t^2/2}dt = 0.5$. Particularly, if and only if $\mu_1 = \mu_2$, $\alpha = \frac{\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}} = 0$. By Eq. (24), we have $D_{h_1\succ h_2} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\alpha} e^{-t^2/2}dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} e^{-t^2/2}dt = 0.5$. □

Theorem 2. *Let Z_1, Z_2 and Z_3 be three independent normal random variables with probability distributions $h_1(z)=\frac{1}{\sigma_1\sqrt{2\pi}}e^{-(z-\mu_1)^2/2\sigma_1^2}$, $h_2(z)=\frac{1}{\sigma_2\sqrt{2\pi}}e^{-(z-\mu_2)^2/2\sigma_2^2}$ and $h_3(z)=\frac{1}{\sigma_3\sqrt{2\pi}}e^{-(z-\mu_3)^2/2\sigma_3^2}$, respectively. If $D_{h_1\succ h_2}\geqslant 0.5$ and $D_{h_2\succ h_3}\geqslant 0.5$, then $D_{h_1\succ h_3}\geqslant 0.5$. Particularly, if $D_{h_1\succ h_2}=0.5$ and $D_{h_2\succ h_3}=0.5$, then $D_{h_1\succ h_3}=0.5$.*

Proof. If $D_{h_1\succ h_2}\geqslant 0.5$ and $D_{h_2\succ h_3}\geqslant 0.5$, according to Corollary 1, we have $\mu_1\geqslant\mu_2$ and $\mu_2\geqslant\mu_3$, respectively. Thus, we have $\mu_1\geqslant\mu_3$. Further, we have $D_{h_1\succ h_3}\geqslant 0.5$ by Corollary 1. Particularly, if $D_{h_1\succ h_2}=0.5$ and $D_{h_2\succ h_3}=0.5$, we have $\mu_1=\mu_2$ and $\mu_2=\mu_3$ by Corollary 1, respectively. Therefore, we have $\mu_1=\mu_3$. Further, by Corollary 1 we have $D_{h_1\succ h_3}=0.5$. □

As for a situation of uniform distributions [15], there are also conclusions similar to Corollary 1 and Theorem 2. The details are omitted to save the space of this paper.

Remark 1. Consider arbitrary two alternatives A_1 and A_2 in decision analysis. Let $f_1(x)$ and $f_2(x)$ be probability distributions on consequences of A_1 and A_2, respectively. $D_{f_1\succ f_2}$ can be also regarded as the dominance degree of A_1 over A_2 (noted as $D_{A_1\succ A_2}$), i.e., $D_{f_1\succ f_2}\Longleftrightarrow D_{A_1\succ A_2}$. The greater $D_{f_1\succ f_2}$ is, the greater the dominance degree of alternative A_1 over A_2 will be.

Consider a SMCDM problem. Let $A=\{A_1,A_2,\ldots,A_m\}(m\geqslant 2)$ be a finite set of m alternatives, where A_i denotes the ith alternative. Let $C=\{C_1,C_2,$
$,C_n\}(n\geqslant 2)$ be a finite set of n criteria, where C_j denotes the jth criterion. Let $w=(w_1,w_2,\ldots,w_n)^T$ be a vector of criterion weights, where w_j denotes the weight or the importance degree of criterion C_j, such that $\sum_{j=1}^{n}w_j=1$ and $0\leqslant w_j\leqslant 1$, $j=1,2,\ldots,n$. Usually, the vector of criterion weights can be obtained either directly from the DM or indirectly using existing procedures such as AHP [26]. Let $\overline{X}=[X_{ij}]_{m\times n}$ be a decision matrix, where X_{ij} denotes the consequence of alternative A_i with respect to criterion C_j. Here, X_{ij} is a random variable with probability distribution $f_{ij}(x)$. The problem concerned in this paper is how to rank alternatives or select the most desirable alternative(s) among the finite set A based on $\overline{X}$ and w.

To solve the SMCDM problem mentioned above, we propose a decision analysis method based on dominance degrees. A description of the method is given below.

By Eqs. (11), (12), (16), (17), (21), (22) and (24), the dominance degree matrix D_j of alternative pairwise comparisons with respect to criterion C_j can be constructed, i.e.,

$$D_j=[D_{ikj}]_{m\times m}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_m\end{array}\begin{array}{c}\begin{array}{cccc}A_1 & A_2 & \cdots & A_m\end{array}\\ \begin{bmatrix} D_{11j} & D_{12j} & \ldots & D_{1mj}\\ D_{21j} & D_{22j} & \ldots & D_{2mj}\\ \vdots & \vdots & & \vdots\\ D_{m1j} & D_{m2j} & \ldots & D_{mmj}\end{bmatrix}\end{array},\quad j=1,2,\ldots,n,$$

where D_{ikj} denotes the dominance degree of A_i over A_k with respect to criterion C_j, and $D_{ikj}+D_{kij}=1$.

Based on $D_1,D_2,\ldots,D_n$, PROMETHEE II method [2] is employed to rank alternatives or select the most desirable alternative(s). In the following, a computation procedure is briefly described.

First, overall dominance degree matrix D of alternative pairwise comparisons is constructed, i.e.,

$$D=[D_{ik}]_{m\times m}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_m\end{array}\begin{array}{c}\begin{array}{cccc}A_1 & A_2 & \cdots & A_m\end{array}\\ \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1m}\\ D_{21} & D_{22} & \cdots & D_{2m}\\ \vdots & \vdots & & \vdots\\ D_{m1} & D_{m2} & \cdots & D_{mm}\end{bmatrix}\end{array},$$

where D_{ik} denotes the overall dominance degree of A_i over A_k and is calculated by

$$D_{ik}=\sum_{j=1}^{n}w_jD_{ikj},\quad i,k=1,2,\ldots,m. \tag{25}$$

From Eq. (25), it can be seen that $D_{ik}+D_{ki}=1$ and $0\leqslant D_{ik}\leqslant 1$.

Then, based on matrix D, outgoing flow $\Phi^+(A_i)$, entering flow $\Phi^-(A_i)$ and net flow $\Phi(A_i)$ of each alternative can be calculated. $\Phi^+(A_i)$ denotes a measure of dominance degree of alternative A_i over all the other alternatives, which is given by

$$\Phi^+(A_i)=\frac{1}{m-1}\sum_{\substack{k=1\\k\neq i}}^{m}D_{ik},\quad i=1,2,\quad,m. \tag{26}$$

$\Phi^{-}(A_i)$ denotes a measure of dominance degree of all the other alternatives over alternative A_i, which is given by

$$\Phi^{-}(A_i) = \frac{1}{m-1}\sum_{\substack{k=1\\k\neq i}}^{m} D_{ki}, \quad i = 1,2,\ldots,m. \tag{27}$$

$\Phi(A_i)$ denotes a measure of the difference between $\Phi^{+}(A_i)$ and $\Phi^{-}(A_i)$, which is given by

$$\Phi(A_i) = \Phi^{+}(A_i) - \Phi^{-}(A_i), \quad i = 1,2,\ldots,m. \tag{28}$$

The greater $\Phi(A_i)$ is, the better alternative A_i will be. Therefore, according to net flows $\Phi(A_1), \Phi(A_2), \ldots, \Phi(A_m)$, we can determine a ranking order of all the alternatives or select the most desirable alternative(s).

In summary then, the proposed procedure for solving the SMCDM problem is:

Step 1. Set up the dominance degree matrices using Eqs. (11), (12), (16), (17), (21), (22) and (24), i.e., $D_j = [D_{ikj}]_{m\times m}$, $j = 1,2,\ldots,n$.
Step 2. Construct overall dominance degree matrix $D = [D_{ik}]_{m\times m}$ using Eq. (25).
Step 3. Calculate the outgoing flow, entering flow and net flow using Eqs. (26)–(28), i.e., $\Phi^{+}(A_i)$, $\Phi^{-}(A_i)$ and $\Phi(A_i)$, $i = 1,2,\ldots,m$.
Step 4. Determine a ranking order of alternatives according to the obtained net flows. Besides, the dominance degree of any two neighbor alternatives in the ranking order is marked according to overall dominance degree matrix D.

Table 1
The evaluations provided by experts.

Criteria	Values	Alternatives									
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
C_1	1	0	0	0	1/7	0	1/7	1/7	1/7	0	0
	2	3/7	1/7	0	0	0	0	0	2/7	0	1/7
	3	1/7	0	0	0	1/7	0	0	2/7	0	2/7
	4	0	2/7	0	0	0	0	0	1/7	0	2/7
	5	2/7	1/7	3/7	1/7	0	0	3/7	1/7	2/7	1/7
	6	0	2/7	1/7	0	2/7	0	1/7	0	1/7	0
	7	1/7	0	1/7	0	2/7	1/7	0	0	3/7	1/7
	8	0	1/7	2/7	1/7	0	3/7	1/7	0	1/7	0
	9	0	0	0	4/7	2/7	0	0	0	0	0
	10	0	0	0	0	0	2/7	1/7	0	0	0
C_2	1	0	1/7	1/7	0	0	0	1/7	3/7	0	0
	2	2/7	0	0	0	0	0	3/7	3/7	0	1/7
	3	1/7	0	0	1/7	0	4/7	1/7	0	1/7	0
	4	0	0	0	1/7	0	0	0	1/7	1/7	0
	5	2/7	0	0	0	1/7	0	1/7	0	0	0
	6	0	1/7	1/7	1/7	2/7	0	1/7	0	1/7	0
	7	0	1/7	0	0	1/7	1/7	0	0	4/7	2/7
	8	1/7	1/7	2/7	3/7	2/7	2/7	0	0	0	3/7
	9	1/7	3/7	1/7	1/7	1/7	0	0	0	0	0
	10	0	0	2/7	0	0	0	0	0	0	1/7
C_3	1	0	0	1/7	0	1/7	0	0	2/7	0	1/7
	2	0	0	0	0	0	0	3/7	1/7	0	2/7
	3	1/7	0	0	1/7	0	0	1/7	4/7	1/7	0
	4	3/7	0	0	0	0	1/7	1/7	0	2/7	0
	5	0	1/7	0	0	0	1/7	2/7	0	2/7	0
	6	1/7	0	0	0	0	0	0	0	0	2/7
	7	0	1/7	0	1/7	0	0	0	0	2/7	2/7
	8	1/7	2/7	0	2/7	3/7	2/7	0	0	0	0
	9	1/7	3/7	2/7	1/7	1/7	1/7	0	0	0	0
	10	0	0	4/7	2/7	2/7	2/7	0	0	0	0
C_4	1	0	1/7	0	1/7	0	0	0	2/7	0	0
	2	0	0	0	0	0	0	0	0	1/7	0
	3	3/7	0	0	0	0	0	1/7	0	0	0
	4	0	0	0	0	0	0	0	1/7	1/7	0
	5	2/7	0	0	0	0	1/7	1/7	2/7	0	0
	6	0	0	0	0	1/7	1/7	0	1/7	3/7	3/7
	7	0	0	1/7	0	1/7	1/7	0	0	0	1/7
	8	1/7	2/7	4/7	0	3/7	2/7	3/7	1/7	1/7	1/7
	9	0	2/7	0	1/7	1/7	1/7	1/7	0	0	1/7
	10	1/7	2/7	2/7	5/7	1/7	1/7	1/7	0	1/7	1/7

In this section, numerical examples for the three cases are examined to illustrate the potential application of the proposed method.

Example 1. We use the example investigated by Zaras and Martel [34] and Nowak [23]. Consider a problem of selecting the most desirable computer development project(s) from 10 alternative projects (A_1,A_2,...,A_{10}). When making a decision, the criteria considered include: personal resources effort (C_1), discounted profit (C_2), chances of success (C_3) and technological orientation (C_4). The vector of criterion weights provided by the DM is $w = (0.09,0.55,0.27,0.09)^T$. To solve the problem, the DM invites seven experts to participate in the decision analysis. The evaluations on the alternatives with respect to the criteria provided by the seven experts are expressed in the form of probability distributions, as shown in Table 1. For instance, three experts in the seven give their evaluations on project A_1 with respect to criterion C_1 using score 2, then the 'probability' that the evaluation on project A_1 is score 2 is regarded as 3/7 (see Table 1). To rank the projects, computation processes and results using the proposed method are summarized below.

Using data in Table 1, firstly, the probability distribution of random variable on each alternative with respect to each criterion is determined. Then, the dominance degree of one alternative over another with respect to each criterion can be obtained using Eqs. (16) and (17). Thus, four dominance degree matrices D_1, D_2, D_3 and D_4 are respectively constructed, i.e.,

$$D_1 = \begin{bmatrix} 0.5 & 0.3163 & 0.1531 & 0.1837 & 0.1327 & 0.1531 & 0.2857 & 0.6122 & 0.1327 & 0.4286 \\ 0.6837 & 0.5 & 0.2959 & 0.2245 & 0.2449 & 0.1939 & 0.4082 & 0.8265 & 0.2551 & 0.6735 \\ 0.8469 & 0.7041 & 0.5 & 0.2755 & 0.3878 & 0.2551 & 0.5714 & 0.9694 & 0.4694 & 0.8776 \\ 0.8163 & 0.7755 & 0.7245 & 0.5 & 0.6122 & 0.5102 & 0.6633 & 0.8571 & 0.7245 & 0.8265 \\ 0.8673 & 0.7551 & 0.6122 & 0.3878 & 0.5 & 0.3265 & 0.6531 & 0.9388 & 0.5714 & 0.8367 \\ 0.8469 & 0.8061 & 0.7449 & 0.4898 & 0.6735 & 0.5 & 0.7143 & 0.8673 & 0.7755 & 0.8469 \\ 0.7143 & 0.5918 & 0.4286 & 0.3367 & 0.3469 & 0.2857 & 0.5 & 0.8367 & 0.3878 & 0.7449 \\ 0.3878 & 0.1735 & 0.0306 & 0.1429 & 0.0612 & 0.1327 & 0.1633 & 0.5 & 0.0204 & 0.2959 \\ 0.8673 & 0.7449 & 0.5306 & 0.2755 & 0.4286 & 0.2245 & 0.6122 & 0.9796 & 0.5 & 0.8878 \\ 0.5714 & 0.3265 & 0.1224 & 0.1735 & 0.1633 & 0.1531 & 0.2551 & 0.7041 & 0.1122 & 0.5 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.5 & 0.2857 & 0.2551 & 0.3163 & 0.2551 & 0.4694 & 0.7041 & 0.8776 & 0.3776 & 0.2959 \\ 0.7143 & 0.5 & 0.4184 & 0.6020 & 0.6939 & 0.7245 & 0.8571 & 0.8878 & 0.7245 & 0.5204 \\ 0.7449 & 0.5816 & 0.5 & 0.6530 & 0.6020 & 0.7551 & 0.8571 & 0.8878 & 0.7653 & 0.5918 \\ 0.6837 & 0.3980 & 0.3470 & 0.5 & 0.4796 & 0.7143 & 0.8980 & 0.9694 & 0.6633 & 0.4592 \\ 0.7449 & 0.3061 & 0.3980 & 0.5204 & 0.5 & 0.7245 & 0.9490 & 1 & 0.6735 & 0.4082 \\ 0.5306 & 0.2755 & 0.2449 & 0.2857 & 0.2755 & 0.5 & 0.7959 & 0.9184 & 0.4286 & 0.3061 \\ 0.2959 & 0.1429 & 0.1429 & 0.1020 & 0.0510 & 0.2041 & 0.5 & 0.7143 & 0.1020 & 0.0918 \\ 0.1224 & 0.1122 & 0.1122 & 0.0306 & 0 & 0.0816 & 0.2857 & 0.5 & 0.0306 & 0.0510 \\ 0.6224 & 0.2755 & 0.2347 & 0.3367 & 0.3265 & 0.5714 & 0.8980 & 0.9694 & 0.5 & 0.2245 \\ 0.7041 & 0.4796 & 0.4082 & 0.5408 & 0.5918 & 0.6939 & 0.9082 & 0.9490 & 0.7755 & 0.5 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0.5 & 0.1939 & 0.1633 & 0.2449 & 0.2449 & 0.2245 & 0.7755 & 0.9592 & 0.5204 & 0.6122 \\ 0.8061 & 0.5 & 0.2041 & 0.4490 & 0.4184 & 0.4694 & 0.9796 & 1 & 0.9184 & 0.8980 \\ 0.8367 & 0.7959 & 0.5 & 0.6735 & 0.6837 & 0.6735 & 0.8571 & 0.8776 & 0.8571 & 0.8673 \\ 0.7551 & 0.5510 & 0.3265 & 0.5 & 0.4796 & 0.5 & 0.9286 & 0.9592 & 0.8469 & 0.8980 \\ 0.7551 & 0.5816 & 0.3163 & 0.5204 & 0.5 & 0.5204 & 0.8571 & 0.8776 & 0.8571 & 0.8673 \\ 0.7755 & 0.5306 & 0.3265 & 0.5 & 0.4796 & 0.5 & 0.9286 & 1 & 0.8367 & 0.8367 \\ 0.2245 & 0.0204 & 0.1429 & 0.0714 & 0.1429 & 0.0714 & 0.5 & 0.6837 & 0.2143 & 0.3673 \\ 0.0408 & 0 & 0.1224 & 0.0408 & 0.1224 & 0 & 0.3163 & 0.5 & 0.0408 & 0.3061 \\ 0.4796 & 0.0816 & 0.1429 & 0.1531 & 0.1429 & 0.1633 & 0.7857 & 0.9592 & 0.5 & 0.5510 \\ 0.3878 & 0.1020 & 0.1327 & 0.1020 & 0.1327 & 0.1633 & 0.6327 & 0.6939 & 0.4490 & 0.5 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 0.5 & 0.2653 & 0.1837 & 0.2143 & 0.2041 & 0.2347 & 0.2959 & 0.5612 & 0.3878 & 0.2245 \\ 0.7347 & 0.5 & 0.5714 & 0.2959 & 0.6327 & 0.6531 & 0.6327 & 0.8571 & 0.7347 & 0.6735 \\ 0.8163 & 0.4286 & 0.5 & 0.2857 & 0.5816 & 0.6429 & 0.5918 & 0.9388 & 0.8163 & 0.7041 \\ 0.7857 & 0.7041 & 0.7143 & 0.5 & 0.7755 & 0.7755 & 0.7755 & 0.8776 & 0.7857 & 0.7755 \\ 0.7959 & 0.3673 & 0.4184 & 0.2245 & 0.5 & 0.5714 & 0.5408 & 0.9184 & 0.7653 & 0.6224 \\ 0.7653 & 0.3469 & 0.3571 & 0.2245 & 0.4286 & 0.5 & 0.5 & 0.8673 & 0.6939 & 0.5306 \\ 0.7041 & 0.3673 & 0.4082 & 0.2245 & 0.4592 & 0.5 & 0.5 & 0.8061 & 0.6531 & 0.5204 \\ 0.4388 & 0.1429 & 0.0612 & 0.1224 & 0.0816 & 0.1327 & 0.1939 & 0.5 & 0.2959 & 0.1224 \\ 0.6122 & 0.2653 & 0.1837 & 0.2143 & 0.2347 & 0.3061 & 0.3469 & 0.7041 & 0.5 & 0.3163 \\ 0.7755 & 0.3265 & 0.2959 & 0.2245 & 0.3776 & 0.4694 & 0.4796 & 0.8776 & 0.6837 & 0.5 \end{bmatrix}.$$

Here, to save the space, we just give a demonstration on calculation of D_{341} in matrix D_1. From Table 1, probability distributions on alternatives A_3 and A_4 with respect to criterion C_1 can be determined, i.e.,

$$f_{31}(x) = \begin{cases} 0, & x = 1, \\ 0, & x = 2, \\ 0, & x = 3, \\ 0, & x = 4, \\ 3/7, & x = 5, \\ 1/7, & x = 6, \\ 1/7, & x = 7, \\ 2/7, & x = 8, \\ 0, & x = 9, \\ 0, & x = 10, \end{cases} \quad \text{and} \quad f_{41}(x) = \begin{cases} 1/7, & x = 1, \\ 0, & x = 2, \\ 0, & x = 3, \\ 0, & x = 4, \\ 1/7, & x = 5, \\ 0, & x = 6, \\ 0, & x = 7, \\ 1/7, & x = 8, \\ 4/7, & x = 9, \\ 0, & x = 10. \end{cases}$$

Using Eq. (16), we know $D_{341} = \frac{3}{7} \times \left(\frac{1}{7} + \frac{1}{7}\right) + \frac{1}{7} \times \left(\frac{1}{7} + \frac{1}{7}\right) + \frac{1}{7} \times \left(\frac{1}{7} + \frac{1}{7}\right) + \frac{2}{7} \times \left(\frac{1}{7} + \frac{1}{7} + \frac{1}{7}\right) - 0.5 \times \left(\frac{3}{7} \times \frac{1}{7} + \frac{2}{7} \times \frac{1}{7}\right) = \frac{27}{98} \approx 0.2755$.

Further, using Eq. (25), the overall dominance degree matrix of alternative pairwise comparisons can be obtained, i.e.,

$$D = \begin{bmatrix} 0.5 & 0.2618 & 0.2147 & 0.2759 & 0.2367 & 0.3537 & 0.6490 & 0.8473 & 0.3950 & 0.3868 \\ 0.7382 & 0.5 & 0.3633 & 0.4992 & 0.5736 & 0.6014 & 0.8296 & 0.9098 & 0.7355 & 0.6499 \\ 0.7853 & 0.6367 & 0.5 & 0.5915 & 0.6029 & 0.6780 & 0.8075 & 0.8970 & 0.7680 & 0.7020 \\ 0.7241 & 0.5008 & 0.4085 & 0.5 & 0.5182 & 0.6436 & 0.8741 & 0.9483 & 0.7294 & 0.6392 \\ 0.7633 & 0.4264 & 0.3970 & 0.4818 & 0.5 & 0.6198 & 0.8608 & 0.9541 & 0.7221 & 0.5900 \\ 0.6463 & 0.3986 & 0.3220 & 0.3564 & 0.3802 & 0.5 & 0.7978 & 0.9312 & 0.5939 & 0.5182 \\ 0.3510 & 0.1704 & 0.1925 & 0.1259 & 0.1392 & 0.2022 & 0.5 & 0.7253 & 0.2076 & 0.2635 \\ 0.1527 & 0.0902 & 0.1030 & 0.0517 & 0.0459 & 0.0688 & 0.2747 & 0.5 & 0.0563 & 0.1483 \\ 0.6050 & 0.2645 & 0.2320 & 0.2706 & 0.2779 & 0.4061 & 0.7924 & 0.9437 & 0.5 & 0.3806 \\ 0.6132 & 0.3501 & 0.2980 & 0.3608 & 0.4100 & 0.4818 & 0.7365 & 0.8517 & 0.6194 & 0.5 \end{bmatrix}.$$

In matrix D, for instance, the computation process of D_{34} is: $D_{34} = 0.2755 \times 0.09 + 0.6530 \times 0.55 + 0.6735 \times 0.27 + 0.2857 \times 0.09 = 0.5915$. Next, using Eq. (26), outgoing flows of the 10 alternatives can be obtained, i.e.,

$$\begin{aligned} &\Phi^+(A_1) = 0.4579, \quad \Phi^+(A_2) = 0.7112, \quad \Phi^+(A_3) = 0.7743, \quad \Phi^+(A_4) = 0.7207, \\ &\Phi^+(A_5) = 0.7017, \quad \Phi^+(A_6) = 0.6050, \quad \Phi^+(A_7) = 0.3197, \\ &\Phi^+(A_8) = 0.1657, \quad \Phi^+(A_9) = 0.5192, \quad \Phi^+(A_{10}) = 0.5801. \end{aligned}$$

Using Eq. (27), entering flows of the 10 alternatives can be obtained, i.e.,

$$\begin{aligned} &\Phi^-(A_1) = 0.6532, \quad \Phi^-(A_2) = 0.3999, \quad \Phi^-(A_3) = 0.3368, \quad \Phi^-(A_4) = 0.3904, \\ &\Phi^-(A_5) = 0.4094, \quad \Phi^-(A_6) = 0.5062, \quad \Phi^-(A_7) = 0.7914, \\ &\Phi^-(A_8) = 0.9454, \quad \Phi^-(A_9) = 0.5919, \quad \Phi^-(A_{10}) = 0.5310. \end{aligned}$$

Using Eq. (28), net flows of the 10 alternatives can be obtained, i.e.,

$$\begin{aligned} &\Phi(A_1) = -0.1953, \quad \Phi(A_2) = 0.3112, \quad \Phi(A_3) = 0.4376, \quad \Phi(A_4) = 0.3303, \\ &\Phi(A_5) = 0.2923, \quad \Phi(A_6) = 0.0988, \quad \Phi(A_7) = -0.4716, \\ &\Phi(A_8) = -0.7796, \quad \Phi(A_9) = -0.0728, \quad \Phi(A_{10}) = -0.0492. \end{aligned}$$

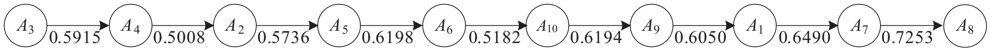

Fig. 3. The ranking of alternatives obtained by the proposed method.

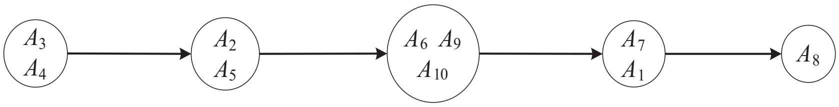

Fig. 4. The ranking of alternatives obtained by Zaras and Martel's method [34].

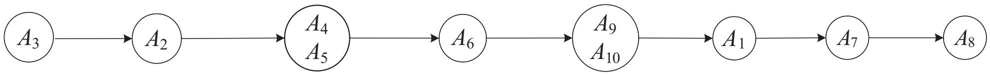

Fig. 5. The ranking of alternatives obtained by Nowak's method [23].

Finally, according to the obtained net flows and matrix D, a ranking order of the 10 alternatives with dominance degrees is shown in Fig. 3. In Fig. 3, the dominance degree of alternative A_3 over A_4 is 0.5915, and that of A_4 over A_2 is 0.5008, etc.

To compare the result obtained by the proposed method with those obtained by the existing methods, the results obtained by Zaras and Martel's method [34] and Nowak's method [23] are shown in Figs. 4 and 5, respectively. It can be seen from Figs. 3–5 that the ranking result obtained by the proposed method and those by the existing methods are slightly different. Using the proposed method, not only could it be easier to obtain a clear ranking order of alternatives but also the dominance degree between any two neighbor alternatives in the ranking order can be given.

Example 2. Consider a problem of selecting the most desirable strategy for an electricity retailer, which was investigated by Lahdelma et al. [18]. In the problem, there are nine alternatives ($A_1, A_2, \ldots, A_9$) and four criteria: long-term profit (C_1), short-term profit (C_2), market share (C_3) and green market share (C_4). Suppose that the consequence of alternative A_i with respect to criterion C_j is a random variable with normal probability distribution $f_{ij}(x) = \frac{1}{\sigma_{ij}\sqrt{2\pi}} e^{-(x-\mu_{ij})^2/2\sigma_{ij}^2}$, which is shown in Table 2. Suppose that the vector of criterion weights is $w = (0.33, 0.35, 0.15, 0.17)^T$ (see Table 5 in [18]). To rank the alternatives, computation processes and results using the proposed method are summarized below.

Firstly, using Eq. (24), four dominance degree matrices D_1, D_2, D_3 and D_4 are built as follows:

$$D_1 = \begin{bmatrix} 0.5 & 0.5273 & 0.8132 & 0.4895 & 0.1828 & 0.4791 & 0.4777 & 0.4624 & 0.4688 \\ 0.4727 & 0.5 & 0.8107 & 0.4594 & 0.1460 & 0.4484 & 0.4444 & 0.4307 & 0.4348 \\ 0.1868 & 0.1893 & 0.5 & 0.1640 & 0.0273 & 0.1582 & 0.1414 & 0.1480 & 0.1363 \\ 0.5105 & 0.5406 & 0.8360 & 0.5 & 0.1716 & 0.4888 & 0.4879 & 0.4708 & 0.4782 \\ 0.8172 & 0.8540 & 0.9728 & 0.8284 & 0.5 & 0.8198 & 0.8397 & 0.8072 & 0.8334 \\ 0.5209 & 0.5516 & 0.8418 & 0.5112 & 0.1802 & 0.5 & 0.5 & 0.4821 & 0.4903 \\ 0.5223 & 0.5556 & 0.8586 & 0.5121 & 0.1603 & 0.5 & 0.5 & 0.4807 & 0.4895 \\ 0.5376 & 0.5693 & 0.8520 & 0.5292 & 0.1928 & 0.5179 & 0.5193 & 0.5 & 0.5097 \\ 0.5312 & 0.5652 & 0.8637 & 0.5218 & 0.1666 & 0.5097 & 0.5105 & 0.4903 & 0.5 \end{bmatrix},$$

Table 2
Means and standard deviations for consequences of alternatives with respect to criteria.

Alternatives	(μ_{ij}, σ_{ij})			
	C_1	C_2	C_3	C_4
A_1	(439,143)	(163,36)	(12.1,0.5)	(9.3,6.5)
A_2	(426,125)	(159,31)	(12.1,0.5)	(14.8,6.5)
A_3	(264,135)	(104,32)	(13.1,0.5)	(9.3,6.5)
A_4	(444,125)	(163,31)	(12.1,0.5)	(9.3,6.5)
A_5	(605,115)	(220,31)	(11.0,0.5)	(9.3,6.5)
A_6	(449,126)	(166,32)	(12.1,0.5)	(4.3,6.5)
A_7	(449,107)	(164,27)	(12.1,0.5)	(9.3,6.5)
A_8	(457,126)	(165,32)	(12.1,0.5)	(9.3,6.5)
A_9	(453,107)	(163,27)	(12.1,0.5)	(14.8,6.5)

$$D_2 = \begin{bmatrix} 0.5 & 0.5336 & 0.8897 & 0.5 & 0.1151 & 0.4752 & 0.4911 & 0.4834 & 0.5 \\ 0.4665 & 0.5 & 0.8915 & 0.4637 & 0.0821 & 0.4376 & 0.4516 & 0.4464 & 0.4612 \\ 0.1103 & 0.1085 & 0.5 & 0.0927 & 0.0046 & 0.0853 & 0.0759 & 0.0888 & 0.0794 \\ 0.5 & 0.5363 & 0.9073 & 0.5 & 0.0968 & 0.4732 & 0.4903 & 0.4821 & 0.5 \\ 0.8849 & 0.9179 & 0.9954 & 0.9032 & 0.5 & 0.8873 & 0.9134 & 0.8915 & 0.9172 \\ 0.5248 & 0.5624 & 0.9147 & 0.5268 & 0.1128 & 0.5 & 0.5191 & 0.5088 & 0.5286 \\ 0.5089 & 0.5484 & 0.9241 & 0.5097 & 0.0866 & 0.4810 & 0.5 & 0.4905 & 0.5104 \\ 0.5166 & 0.5536 & 0.9112 & 0.5179 & 0.1085 & 0.4912 & 0.5095 & 0.5 & 0.5191 \\ 0.5 & 0.5388 & 0.9206 & 0.5 & 0.0828 & 0.4714 & 0.4896 & 0.4810 & 0.5 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0.5 & 0.5 & 0.0787 & 0.5 & 0.9401 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.0787 & 0.5 & 0.9401 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.9214 & 0.9214 & 0.5 & 0.9214 & 0.9985 & 0.9214 & 0.9214 & 0.9214 & 0.9214 \\ 0.5 & 0.5 & 0.0787 & 0.5 & 0.9401 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.0599 & 0.0599 & 0.0015 & 0.0599 & 0.5 & 0.0599 & 0.0599 & 0.0599 & 0.0599 \\ 0.5 & 0.5 & 0.0787 & 0.5 & 0.9401 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.0787 & 0.5 & 0.9401 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.0787 & 0.5 & 0.9401 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.0787 & 0.5 & 0.9401 & 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 0.5 & 0.2748 & 0.5 & 0.5 & 0.5 & 0.7068 & 0.5 & 0.5 & 0.2748 \\ 0.7252 & 0.5 & 0.7252 & 0.7252 & 0.7252 & 0.8733 & 0.7252 & 0.7252 & 0.5 \\ 0.5 & 0.2748 & 0.5 & 0.5 & 0.5 & 0.7068 & 0.5 & 0.5 & 0.2748 \\ 0.5 & 0.2748 & 0.5 & 0.5 & 0.5 & 0.7068 & 0.5 & 0.5 & 0.2748 \\ 0.5 & 0.2748 & 0.5 & 0.5 & 0.5 & 0.7068 & 0.5 & 0.5 & 0.2748 \\ 0.2932 & 0.1267 & 0.2932 & 0.2932 & 0.2932 & 0.5 & 0.2932 & 0.2932 & 0.1267 \\ 0.5 & 0.2748 & 0.5 & 0.5 & 0.5 & 0.7068 & 0.5 & 0.5 & 0.2748 \\ 0.5 & 0.2748 & 0.5 & 0.5 & 0.5 & 0.7068 & 0.5 & 0.5 & 0.2748 \\ 0.7252 & 0.5 & 0.7252 & 0.7252 & 0.7252 & 0.8733 & 0.7252 & 0.7252 & 0.5 \end{bmatrix}.$$

Then, using Eq. (25), overall dominance degree matrix D can be obtained, i.e.,

$$D = \begin{bmatrix} 0.5 & 0.4825 & 0.6766 & 0.4965 & 0.3266 & 0.5196 & 0.4895 & 0.4818 & 0.4514 \\ 0.5175 & 0.5 & 0.7146 & 0.5122 & 0.3412 & 0.5246 & 0.5030 & 0.5025 & 0.4709 \\ 0.3234 & 0.2854 & 0.5 & 0.3098 & 0.2454 & 0.3404 & 0.3964 & 0.3198 & 0.2764 \\ 0.5035 & 0.4878 & 0.6902 & 0.5 & 0.3165 & 0.5220 & 0.4926 & 0.4864 & 0.4564 \\ 0.6734 & 0.6588 & 0.7546 & 0.6835 & 0.5 & 0.7102 & 0.6908 & 0.6508 & 0.6266 \\ 0.4804 & 0.4754 & 0.6596 & 0.4780 & 0.2898 & 0.5 & 0.4715 & 0.4632 & 0.4441 \\ 0.5105 & 0.4970 & 0.7036 & 0.5074 & 0.3092 & 0.5285 & 0.5 & 0.4921 & 0.4630 \\ 0.5182 & 0.5033 & 0.6969 & 0.5159 & 0.3276 & 0.5380 & 0.5097 & 0.5 & 0.4709 \\ 0.5486 & 0.5351 & 0.7423 & 0.5455 & 0.3483 & 0.5567 & 0.5381 & 0.5293 & 0.5 \end{bmatrix}.$$

Further, using Eqs. (26)–(28), outgoing flows, entering flows and net flows of the nine alternatives can be obtained, respectively, i.e.,

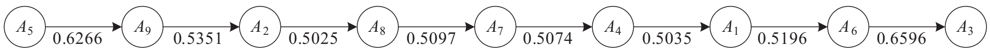

Fig. 6. The ranking of alternatives obtained by the proposed method.

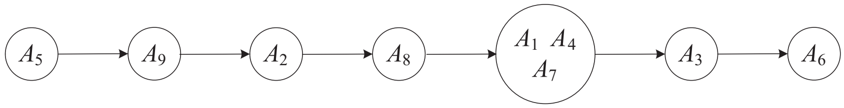

Fig. 7. The ranking of alternatives obtained by Lahdelma et al. [18].

$$\begin{aligned}
&\Phi^{+}(A_1) = 0.4906, \quad \Phi^{+}(A_2) = 0.5108, \quad \Phi^{+}(A_3) = 0.2996,\\
&\Phi^{+}(A_4) = 0.4944, \quad \Phi^{+}(A_5) = 0.6811, \quad \Phi^{+}(A_6) = 0.4702,\\
&\Phi^{+}(A_7) = 0.5013, \quad \Phi^{+}(A_8) = 0.5101, \quad \Phi^{+}(A_9) = 0.5430;\\
&\Phi^{-}(A_1) = 0.5094, \quad \Phi^{-}(A_2) = 0.4907, \quad \Phi^{-}(A_3) = 0.7048,\\
&\Phi^{-}(A_4) = 0.5061, \quad \Phi^{-}(A_5) = 0.3131, \quad \Phi^{-}(A_6) = 0.5300,\\
&\Phi^{-}(A_7) = 0.4990, \quad \Phi^{-}(A_8) = 0.4907, \quad \Phi^{-}(A_9) = 0.4575;\\
&\Phi(A_1) = -0.0189, \quad \Phi(A_2) = 0.0202, \quad \Phi^{-}(A_3) = -0.4052,\\
&\Phi(A_4) = -0.0116, \quad \Phi(A_5) = 0.3680, \quad \Phi^{-}(A_6) = -0.0598,\\
&\Phi(A_7) = 0.0024, \quad \Phi(A_8) = 0.0193, \quad \Phi(A_9) = 0.0855.
\end{aligned}$$

Finally, according to the obtained net flows and matrix D, a ranking order of the nine alternatives with dominance degrees is shown in Fig. 6.

Lahdelma et al. also investigated this example. In their study, rank acceptability index of each alternative with respect to each ranking position can be obtained (see Table 4 and Fig. 2 in [18]), and a ranking order of alternatives is determined according to holistic acceptability indices (see Fig. 7).

It can be seen from Figs. 6 and 7 that implications of the two results are different. Fig. 6 shows the ranking result of alternatives with dominance degrees, while the result in Fig. 7 is based on the analysis of holistic acceptability indices.

Example 3. Consider a problem of selecting the most desirable vendor(s). There are four candidates (A_1, A_2, A_3 and A_4) and four criteria: responsiveness to customer needs (C_1), price (C_2), on-time delivery (C_3) and product quality (C_4). The vector of criterion weights is $w = (0.25, 0.25, 0.25, 0.25)^T$, which is directly obtained from assignment of the DM. Evaluations of the candidates with respect to the criteria are shown in Table 3. In Table 3, some evaluations are determined using assessment information provided by the invited experts, the others are determined through the statistical analysis of historical data. To select the most desirable candidate(s), computation processes and results using the proposed method are summarized as follows.

Table 3
Discrete probability distributions and normal probability distributions for consequences of alternatives with respect to criteria.

Criteria	Alternatives	Scores										(μ_{ij}, σ_{ij})
		1	2	3	4	5	6	7	8	9	10	
C_1	A_1	0	0.1	0.1	0.3	0.2	0.2	0.1	0	0	0	
	A_2	0	0	0.1	0.3	0.2	0.2	0.1	0.1	0	0	
	A_3	0.1	0	0.2	0.2	0.3	0.1	0.1	0	0	0	
	A_4	0	0	0	0.2	0.3	0.4	0.1	0	0	0	
C_2	A_1											(5.5, 1.3)
	A_2											(5, 1.1)
	A_3											(6, 1.5)
	A_4											(4.5, 1)
C_3	A_1	0	0.1	0.1	0.2	0.1	0.3	0.1	0.1	0	0	
	A_2											(4.5, 1.2)
	A_3	0	0	0.2	0.3	0.2	0.2	0.1	0	0	0	
	A_4											(4.6, 1.1)
C_4	A_1											(6, 1.2)
	A_2	0	0	0	0.1	0.2	0.3	0.2	0.2	0	0	
	A_3											(6.3, 1)
	A_4	0	0	0	0	0.1	0.2	0.3	0.3	0.1	0	

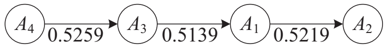

Fig. 8. The ranking of alternatives with dominance degrees.

Firstly, using Eqs. (16), (17), (21), (22) and (24), four dominance degree matrices D_1, D_2, D_3 and D_4 are built as follows:

$$D_1 = \begin{bmatrix} 0.5 & 0.405 & 0.545 & 0.335 \\ 0.595 & 0.5 & 0.635 & 0.435 \\ 0.455 & 0.365 & 0.5 & 0.29 \\ 0.665 & 0.565 & 0.71 & 0.5 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.5 & 0.6155 & 0.4006 & 0.7290 \\ 0.3845 & 0.5 & 0.2954 & 0.6317 \\ 0.5994 & 0.7046 & 0.5 & 0.7973 \\ 0.2710 & 0.3683 & 0.2027 & 0.5 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0.5 & 0.6125 & 0.575 & 0.5989 \\ 0.3875 & 0.5 & 0.4680 & 0.4755 \\ 0.425 & 0.5320 & 0.5 & 0.5090 \\ 0.4011 & 0.5245 & 0.4910 & 0.5 \end{bmatrix}, \quad D_4 = \begin{bmatrix} 0.5 & 0.4548 & 0.4238 & 0.2554 \\ 0.5452 & 0.5 & 0.4778 & 0.31 \\ 0.5762 & 0.5222 & 0.5 & 0.3002 \\ 0.7446 & 0.69 & 0.6998 & 0.5 \end{bmatrix}.$$

Then, using Eq. (25), overall dominance degree matrix D can be set up, i.e.,

$$D = \begin{bmatrix} 0.5 & 0.5219 & 0.4861 & 0.4796 \\ 0.4781 & 0.5 & 0.4691 & 0.4631 \\ 0.5139 & 0.5309 & 0.5 & 0.4741 \\ 0.5204 & 0.5369 & 0.5259 & 0.5 \end{bmatrix}.$$

Further, using Eqs. (26)–(28), outgoing flows, entering flows and net flows of the four candidates can be obtained, respectively, i.e.,

$$\begin{aligned} &\Phi^+(A_1) = 0.4959, \quad \Phi^+(A_2) = 0.4701, \quad \Phi^+(A_3) = 0.5063, \quad \Phi^+(A_4) = 0.5278, \\ &\Phi^-(A_1) = 0.5041, \quad \Phi^-(A_2) = 0.5299, \quad \Phi^-(A_3) = 0.4937, \quad \Phi^-(A_4) = 0.4723, \\ &\Phi(A_1) = -0.0082, \quad \Phi(A_2) = -0.0598, \quad \Phi(A_3) = 0.0126, \quad \Phi(A_4) = 0.0555. \end{aligned}$$

According to the obtained net flows and matrix D, a ranking order of the four candidates with dominance degrees is shown in Fig. 8.

This paper presents a method for solving the SMCDM problem. In the method, firstly, the dominance degree matrix of alternative pairwise comparisons with respect to each criterion is constructed through comparisons of probability distributions. Then, using PROMETHEE II method, an overall dominance degree matrix is built, and the net flow of each alternative is calculated. Further, a ranking order of alternatives is determined based on the obtained net flows. Compared with the existing methods, the proposed method has distinct characteristics as discussed below.

In the proposed method, definitions and calculation formulas of dominance degree of one probability distribution over another are given to measure the dominance degree for comparison of a pair of alternatives. It can be seen that the use of dominance degree overcomes the shortages which occur either in the processes of determining SD relations for comparisons among alternatives or defining the thresholds for judging the results of alternative pairwise comparisons.

Using the proposed method, not only could it be easier to obtain a clear ranking order of alternatives but also the dominance degree between any two neighbor alternatives in the ranking order can be given. This is important for solving the SMCDM problem and conductive to support the decision making of DMs.

The proposed method has a clear logic and a simple computation process. It is also a supplement or an extension of the existing methods and gives the DM one more choice for solving SMCDM problems.

In terms of future research, the proposed method can be extended to solve MCDM problems with multiple information forms such as crisp numbers, interval numbers, stochastic information and so on.

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