TAKE-HOME CLASS QUIZ: DUE FRIDAY DECEMBER 7: DIFFERENTIAL EQUATIONS (SEE NOTE)

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

YOU ARE FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE MAKE SURE TO ONLY ENTER ANSWER CHOICES THAT YOU PERSONALLY ENDORSE

Note: Technically, I'm not supposed to have any materials due on Friday, but I think it's best you have a little extra time after absorbing the class material on Wednesday rather than doing the quiz in haste. However, if this date is not convenient for you, feel free to turn the quiz in during class Wednesday, or request for an extension to Saturday/Sunday.

(1) It takes time T for $1/10$ of a radioactive substance to decay. How much does it take for $3/10$ of the same substance to decay? *Two years ago:* 22/26 *correct*
(A) Between T and $2T$
(B) Between $2T$ and $3T$
(C) Exactly $3T$
(D) Between $3T$ and $4T$
(E) Between $4T$ and $5T$

Your answer: ______________________

(2) Suppose a function f satisfies the differential equation $f''(x) = 0$ for all $x \in \mathbb{R}$. Which of the following is true about $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$? *Two years ago:* 13/26 *correct*
(A) If either limit is finite, then both are finite and they are equal. Otherwise, both the limits are infinities of opposite signs.
(B) If either limit is finite, then both are finite and they are equal. Otherwise, both the limits are infinities of the same sign.
(C) One of the limits is finite and the other is infinite.
(D) Both the limits are finite and unequal.
(E) Both the limits are infinite but they may be of the same or of opposite signs.

Your answer: ______________________

(3) For y a function of x, consider the differential equation $(y')^2 - 3yy' + 2y^2 = 0$. What is the description of the **general solution** to this differential equation? *Two years ago:* 12/26 *correct*
(A) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are arbitrary real numbers.
(B) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are real numbers satisfying $C_1C_2 = 0$ (i.e., at least one of them is zero)
(C) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are real numbers satisfying $C_1 + C_2 = 0$.
(D) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are real numbers satisfying $C_1C_2 = 1$.
(E) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are real numbers satisfying $C_1 + C_2 = 1$.

Your answer: ______________________

(4) Suppose $F(t)$ represents the number of gigabytes of disk space that can be purchased with one dollar at time t in commercially available disk drive formats (not adjusted for inflation). Empirical observation shows that $F(1980) \approx 5 * 10^{-6}$, $F(1990) \approx 10^{-4}$, $F(2000) \approx 10^{-1}$, and $F(2010) \approx 10$. From these data, what is a good estimate for the "doubling time" of F, i.e., the time it takes for the number of gigabytes purchaseable with a dollar to double? *Two years ago:* 10/26 *correct*

(A) Between 6 months and 1 year.
(B) Between 1 year and 2 years.
(C) Between 2 years and 4 years.
(D) Between 4 years and 5 years.
(E) Between 5 years and 6 years.

Your answer: ____________________________

(5) The size S of an online social network satisfies the differential equation $S'(t) = kS(t)(1-(S(t))/(W(t)))$ where $W(t)$ is the world population at time t. Suppose $W(t)$ itself satisfies the differential equation $W'(t) = k_0 W(t)$ where k_0 is positive but much smaller than k. How would we expect S to behave, assuming that initially, $S(t)$ is positive but much smaller than $W(t)$? *Two years ago:* 11/26 *correct*
(A) It initially appears like an exponential function with exponential growth rate k, but over time, it slows down to (roughly) an exponential function with exponential growth rate k_0.
(B) It initially appears like an exponential function with exponential growth rate k_0, but over time, it speeds up to (roughly) an exponential function with exponential growth rate k.
(C) It behaves roughly like an exponential function with growth rate k_0 for all time.
(D) It behaves roughly like an exponential function with growth rate k for all time.
(E) It initially behaves like an exponential function with exponential growth rate k but then it starts declining.

Your answer: ____________________________

(6) Suppose the growth of a population P with time is described by the equation $dP/dt = aP^{1-\beta}$ with $a > 0$ and $0 < \beta < 1$. What can we say about the nature of the population as a function of t, assuming that the population at time 0 is positive? *Two years ago:* 8/26 *correct*
(A) The population grows as a sub-linear power function of t, i.e., roughly like t^γ where $0 < \gamma < 1$.
(B) The population grows as a linear power function of t, i.e., roughly like t.
(C) The population grows as a superlinear power function of t, i.e., roughly like t^γ where $\gamma > 1$.
(D) The population grows like an exponential function of t, i.e., roughly like e^{kt} for some $k > 0$.
(E) The population grows super-exponentially, i.e., it eventually surpasses any exponential function.

Your answer: ____________________________

(7) Suppose the growth of a population P with time is described by the equation $dP/dt = aP^{1+\theta}$ with $0 < \theta$ *and* $a > 0$. What can we say about the nature of the population as a function of t, assuming that the population at time 0 is positive? *Two years ago:* 3/26 *correct*
(A) The population approaches infinity in finite time, and the differential equation makes no sense beyond that.
(B) The population increases at a decreasing rate and approaches a horizontal asymptote, i.e., it proceeds to a finite limit as time approaches infinity.
(C) The population grows linearly.
(D) The population grows super-linearly but sub-exponentially.
(E) The population grows exponentially.

Your answer: ____________________________

(8) Let $r(t)$ denote the fractional growth rate per annum in per capita income, which we denote by $I(t)$. In other words, $r(t) = I'(t)/I(t)$, measured in units of (per year). It is observed that, over a certain time period, $r'(t) = kr(t)$ for a positive constant k. Assuming that the initial values of $I(t)$ and $r(t)$ are positive, what best describes the nature of the function $I(t)$? *Two years ago:* 2/26 *correct*
(A) $I(t)$ is a linear function of t, i.e., per capita income is getting incremented by a constant *amount* (rather than a constant proportion).
(B) $I(t)$ is a super-linear but sub-exponential function of t, i.e., per capita income is rising, but less than exponentially.

(C) $I(t)$ is an exponential function of t, i.e., per capita income is rising by a constant proportion per year.
(D) $I(t)$ is a super-exponential function of t but slower than a doubly exponential function of t.
(E) $I(t)$ is a doubly exponential function of t.

Your answer: ____________________________

(9) Suppose a function P of time t has the property that $P(0) > 1$, and $dP/dt = P \ln P$ for all $t \geq 0$. Which of the following best describes P as a function of t, for $t \geq 0$?
(A) P grows logarithmically in t.
(B) P grows linearly in t.
(C) P grows super-linearly but sub-exponentially in t.
(D) P grows exponentially in t.
(E) P grows super-exponentially in t.

Your answer: ____________________________

(10) An irreversible chemical reaction with reactants A and B and product C begins at time $t = 0$ with the quantity of C being 0 and with finite positive masses of A and B. The rate of reaction at any time t is proportional to the product of quantities (masses) of A and B at that time. By the law of conservation of mass, the total mass of the system is constant. What can we say about the **quantity (mass)** of C as a **function of time** t?
(A) It is increasing, concave up, and has a vertical asymptote.
(B) It is increasing and concave up till a finite time after which it becomes constant.
(C) It is increasing, concave down, and has a horizontal asymptote.
(D) It is concave down, initially increasing and later decreasing after reaching a local maximum.
(E) It is increasing, initially concave up and later concave down.

Your answer: ____________________________

(11) Consider the differential equation $(y' - 2x)(y' - 3x^2) = 0$ with independent variable x and dependent variable y. We are interested in the *global* solutions to this differential equation, i.e., the solutions to this differential equation for y as a continuously differentiable function of x defined on all $\mathbb{R}$ and satisfying this condition globally. What can we say about such solutions?
(A) Any function that is a global solution to this differential equation is infinitely differentiable on all of $\mathbb{R}$.
(B) Any function that is a global solution to this differential equation is twice differentiable on all of $\mathbb{R}$, but there exist solutions that are not thrice differentiable *anywhere* on $\mathbb{R}$.
(C) Any function that is a global solution to this differential equation is twice differentiable on all of $\mathbb{R}$, but there exist solutions that are not thrice differentiable at some isolated points in $\mathbb{R}$ (but are thrice differentiable elsewhere).
(D) Any function that is a global solution to this differential equation is once differentiable on all of $\mathbb{R}$, but there exist solutions that are not twice differentiable *anywhere* on $\mathbb{R}$.
(E) Any function that is a global solution to this differential equation is once differentiable on all of $\mathbb{R}$, but there exist solutions that are not twice differentiable at some isolated points in $\mathbb{R}$ (but are twice differentiable elsewhere).

Your answer: ____________________________