HOMEWORK 1: DUE MONDAY OCTOBER 7

MATH 196, SECTION 57 (VIPUL NAIK)

Please submit routine and advanced problems *separately* on the due date of the homework. A grader (not me!) is assigned to grade the routine problems. I'll be grading the advanced problems myself.

The bulk of your homework will be the routine problems and these should be your first priority. There may be weeks where the homework does not contain any advanced problems.

1. Routine problems

Please write your solutions clearly, show relevant steps, but be concise. Underline, highlight, or box your final answers to make life easy for the grader.

(1) Exercise 1.1.8 (Page 5): Find all solutions of the linear system using elimination as discussed in Section 1.1. Then check your solutions.

$$\begin{aligned} x + 2y + 3z &= 0 \\ 4x + 5y + 6z &= 0 \\ 7x + 8y + 10z &= 0 \end{aligned}$$

(2) Exercise 1.1.10 (Page 5): Find all solutions of the linear system using elimination as discussed in Section 1.1. Then check your solutions.

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 4y + 7z &= 2 \\ 3x + 7y + 11z &= 8 \end{aligned}$$

(3) Exercise 1.1.17 (Page 5): Find all solutions of the linear system:

$$\begin{aligned} x + 2y &= a \\ 3x + 5y &= b \end{aligned}$$

where a and b are arbitrary constants.

(4) Exercise 1.1.26 (Page 6) (was Exercise 1.1.22 in the 4th edition): Consider the differential equation

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - x = \cos(t)$$

The equation could described a forced damped oscillator, as we will see in Chapter 9 (well, we actually *won't* see it in the course, but you are welcome to read it up). We are told that the differential equation has a solution of the form

$$x(t) = a\sin(t) + b\cos(t)$$

Find a and b, and graph the solution.

(5) Exercise 1.1.20 (Page 5) (was Exercise 1.1.25 in the 4th Edition): Consider the linear system

$$\begin{aligned} x+y-z &= 2 \\ x+2y+z &= 3 \\ x+y+(k^2-5)z &= k \end{aligned}$$

where k is an arbitrary constant. For which value(s) of k does this system have a unique solution? For which value(s) of k does this system have infinitely many solutions? For which value(s) of k is the system inconsistent?

(6) Exercise 1.1.34 (Page 7) (was Exercise 1.1.32 in the 4th Edition): Find all the polynomials $f(t)$ of degree ≤ 2 (of the form $f(t) = a + bt + ct^2$) whose graph runs through the points $(1, 1)$ and $(2, 0)$ and such that $\int_1^2 f(t)\,dt = -1$.

(7) Exercise 1.1.40 (Page 7) (was Exercise 1.1.39 in the 4th Edition): Find the ellipse centered at the origin that runs through the points $(1, 2)$, $(2, 2)$, and $(3, 1)$. Write your equation in the form $ax^2 + bxy + cy^2 = 1$.

(8) Exercise 1.1.42a (Page 7) (was Exercise 1.1.40a in the 4th Edition): Solve the lower triangular system:

$$\begin{aligned} x_1 &= -3 \\ -3x_1+x_2 &= 14 \\ x_1+2x_2+x_3 &= 9 \\ -x_1+8x_2-5x_3+x_4 &= 33 \end{aligned}$$

by forward substituting, finding x_1 first, then x_2, then x_3, and finally x_4.

(9) Exercise 1.1.48 (Page 8): A hermit eats only two kinds of food: brown rice and yogurt. The rice contains 3 grams of protein and 30 grams of carbohydrate per serving, while the yogurt contains 12 grams of protein and 20 grams of carbohydrates (per serving).
   (a) If the hermit wants to take in 60 grams of protein and 300 grams of carbohydrates per day, how many servings of each item should he consume?
   (b) If the hermit wants to take in P grams of protein and C grams of carbohydrates per day, how many servings of each item should he consume?

(10) Exercise 1.1.49 (Page 8) (was Exercise 1.1.47 in the 4th edition): I have 32 bills in my wallet, in the denominations of US$ 1, 5, and 10, worth $ 100 in total. How many do I have of each denomination?

2. Advanced problems

(1) Exercise 1.1.24 (Page 6) (was Exercise 1.1.20 in the 4th Edition): The Russian-born U.S. economist and Nobel laureate Wassily Leontief (1906-1999) was interested in the following question: What output should each of the industries in an economy produce to satisfy the total demand for all products? Here, we consider a very simple example of input-output analysis, an economy with only two industries, A and B. Assume that the consumer demand for their products is, respectively, 1000 and 780, in millions of dollars per year.

What outputs a and b (in millions of dollars per year) should the two industries generate to satisfy the demand? One complication is that some of the output of each industry needs to be diverted to the other industry to help it produce stuff (e.g., A may be producing electricity). Suppose that industry B needs 10 cents worth of electrical power for each $ 1 of output that B produces and industry A needs 20 cents worth of B's outputs for each $ 1 of output A produces. Find the outputs a and b needed to satisfy both consumer and interindustry demand.

(The book has pictorial depictions that you may find useful).

You can read more at:

`http://en.wikipedia.org/wiki/Input-output_model`