TAKE-HOME CLASS QUIZ: DUE OCTOBER 8: INTERPLAY OF CONTINUOUS AND DISCRETE

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

YOU ARE FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE MAKE SURE TO ONLY ENTER ANSWER CHOICES THAT YOU PERSONALLY ENDORSE

(1) Consider a function f defined on all real numbers. Consider also the sequence $a_n = f(n)$ defined for n a natural number. Which of the following is true?
(A) $\lim_{x \to \infty} f(x)$ is finite if and only if $\lim_{n \to \infty} a_n$ is finite, and if so, both limits are equal.
(B) $\lim_{x \to \infty} f(x)$ is finite if and only if $\lim_{n \to \infty} a_n$ is finite, but the limits need not be equal.
(C) If $\lim_{x \to \infty} f(x)$ is finite, then $\lim_{n \to \infty} a_n$ is finite, but the converse is not true. Moreover, if both limits are finite, they must be equal.
(D) If $\lim_{n \to \infty} a_n$ is finite, then $\lim_{x \to \infty} f(x)$ is finite, but the converse is not true. Moreover, if both limits are finite, they must be equal.
(E) It is possible for either of the limits $\lim_{x \to \infty} f(x)$ and $\lim_{n \to \infty} a_n$ to be finite, but for the other one not to be finite. Moreover, even if both limits exist, they need not be equal.

Your answer: ____________________________

(2) Consider a function $f : \mathbb{R} \to \mathbb{R}$. Restricting the domain of f to the natural numbers, obtain a sequence whose n^{th} member a_n is defined as $f(n)$. Which of the following statements is **false** about the relationship between f and the sequence (a_n)?
(A) If f is an increasing function, then (a_n) form an increasing sequence.
(B) If f is a decreasing function, then (a_n) form a decreasing sequence.
(C) If f is a bounded function, (i.e., its range is a bounded set) then (a_n) form a bounded sequence.
(D) If f is a periodic function, then (a_n) form a periodic sequence.
(E) If f has a limit at infinity, then (a_n) is a convergent sequence.

Your answer: ____________________________

(3) We are given a sequence $a_1, a_2, \dots, a_n, \dots$ of real numbers. The goal is to find a *continuous* function f on all of $\mathbb{R}$ such that $f(n) = a_n$ for all $n \in \mathbb{N}$. Which of the following is true?
(A) There is a unique choice of f that works.
(B) There exist infinitely many different choices of f that work.
(C) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be zero, one, or infinite.
(D) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be zero or one. It can never be infinite.
(E) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be one or infinite. It can never be zero.

Your answer: ____________________________

(4) We are given a sequence $a_1, a_2, \dots, a_n, \dots$ of real numbers. The goal is to find an *infinitely differentiable* function f on all of $\mathbb{R}$ such that $f(n) = a_n$ for all $n \in \mathbb{N}$. Which of the following is true?
(A) There is a unique choice of f that works.

(B) There exist infinitely many different choices of f that work.
(C) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be zero, one, or infinite.
(D) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be zero or one. It can never be infinite.
(E) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be one or infinite. It can never be zero.

Your answer: ____________________________

(5) We are given a sequence $a_1, a_2, \ldots, a_n, \ldots$ of real numbers. The goal is to find a *polynomial* function f on all of $\mathbb{R}$ such that $f(n) = a_n$ for all $n \in \mathbb{N}$. Which of the following is true?
(A) There is a unique choice of f that works.
(B) There exist infinitely many different choices of f that work.
(C) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be zero, one, or infinite.
(D) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be zero or one. It can never be infinite.
(E) The number of possible choices of f depends on the sequence. Depending on the sequence, the number of possible choices of f may be one or infinite. It can never be zero.

Your answer: ____________________________

For the remaining questions: For a function $f : \mathbb{N} \to \mathbb{R}$, define Δf as the function $n \mapsto f(n+1) - f(n)$. Denote by $\Delta^k f$ the function obtained by applying Δ k times to f.

(6) If $f(n) = n^2$, what is $(\Delta f)(n)$?
(A) 1
(B) n
(C) $2n - 1$
(D) $2n$
(E) $2n + 1$

Your answer: ____________________________

(7) If f is expressible as a polynomial function of degree $d > 0$, what is the smallest k for which $\Delta^k f$ is identically the zero function? *Hint: Think of the analogous question using continuous derivatives. Although Δ differs from the continuous derivative, much of the qualitative behavior is the same.*
(A) $d - 2$
(B) $d - 1$
(C) d
(D) $d + 1$
(E) $d + 2$

Your answer: ____________________________

(8) If f is a function such that $\Delta f = af$ for some positive constant a, and $f(1)$ is positive, which of the following best describes the nature of growth of f? *Hint: This is qualitatively similar to the analogous differential equation using continuous derivatives.*
(A) f grows like a sublinear function of n.
(B) f grows like a linear function of n.
(C) f grows like a superlinear but subexponential function of n.
(D) f grows like an exponential function of n.
(E) f grows like a superexponential function of n.

Your answer: ____________________________