CLASS QUIZ: OCTOBER 15: ORDER OF ZERO, L'HOPITAL'S RULE

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

For these questions, keep in mind that the *order of a zero* for a function f at a point c in its domain (where it's continuous) such that $f(c) = 0$ is defined as the lub of the set $\{\beta \geq 0 \mid \lim_{x \to c} |f(x)|/|x - c|^\beta = 0\}$.

If f is an infinitely differentiable function at c, then the order, if finite, must be a positive integer. If the order is a positive integer r, then the first $r - 1$ derivatives of f at c equal zero and the r^{th} derivative at c is nonzero (assuming f to be infinitely differentiable).

For convenience, we take $c = 0$ in the next three questions, i.e., all limits are being taken as $x \to 0$.

(1) If f has a zero of order 2 and g has a zero of order 3 at the point 0, what is the order of zero for the pointwise sum $f + g$ at zero? (Example: $f(x) = \sin^2 x$ and $g(x) = \ln(1 + x^3)$).
(A) 1
(B) 2
(C) 3
(D) 5
(E) 6

Your answer: ______________________

(2) If f has a zero of order 2 and g has a zero of order 3 at the point 0, what is the order of zero for the pointwise product fg at zero? (Example: $f(x) = \sin^2 x$ and $g(x) = \ln(1 + x^3)$).
(A) 1
(B) 2
(C) 3
(D) 5
(E) 6

Your answer: ______________________

(3) If f has a zero of order 2 and g has a zero of order 3 at the point 0, what is the order of zero for the composite function $f \circ g$ at zero? (Example: $f(x) = \sin^2 x$ and $g(x) = \ln(1 + x^3)$).
(A) 1
(B) 2
(C) 3
(D) 5
(E) 6

Your answer: ______________________

(4) L'Hopital's rule can be related with order of zero in the following manner: Every time the rule is applied to a $(\to 0)/(\to 0)$ form, the order of zero of the numerator and denominator go *down by one*. Repeated application hopefully yields a situation where either the numerator or the denominator has a nonzero limiting value.

Assume that we start with a limit $\lim_{x \to c} f(x)/g(x)$ where both f and g are infinitely differentiable at c, and further, that $f(c) = g(c) = 0$. If the order of zero of f is d_f and the order of zero of g is d_g, which of the following is true?

(A) If $d_f = d_g$, then we need to apply the LH rule d_f times and we will then get a nonzero numerator and nonzero denominator, that we can evaluate to get the limit. If $d_f < d_g$, then we apply the LH rule d_f times to get a nonzero numerator and zero denominator, so the limit is undefined. If $d_g < d_f$, then we apply the LH rule d_g times to get a zero numerator and nonzero denominator, so the limit is zero.

(B) If $d_f = d_g$, then we need to apply the LH rule d_f times and we will then get a nonzero numerator and nonzero denominator, that we can evaluate to get the limit. If $d_f < d_g$, then we apply the LH rule d_f times to get a zero numerator and nonzero denominator, so the limit is undefined. If $d_g < d_f$, then we apply the LH rule d_g times to get a nonzero numerator and zero denominator, so the limit is zero.

(C) If $d_f = d_g$, then we need to apply the LH rule d_f times and we will then get a nonzero numerator and nonzero denominator, that we can evaluate to get the limit. If $d_f < d_g$, then we apply the LH rule d_f times to get a nonzero numerator and zero denominator, so the limit is zero. If $d_g < d_f$, then we apply the LH rule d_g times to get a zero numerator and nonzero denominator, so the limit is undefined.

(D) If $d_f = d_g$, then we need to apply the LH rule d_f times and we will then get a nonzero numerator and nonzero denominator, that we can evaluate to get the limit. If $d_f < d_g$, then we apply the LH rule d_f times to get a zero numerator and nonzero denominator, so the limit is zero. If $d_g < d_f$, then we apply the LH rule d_g times to get a nonzero numerator and zero denominator, so the limit is undefined.

(E) In all cases, we perform the LH rule $\min\{d_f, d_g\}$ times and obtain a nonzero numerator and nonzero denominator.

Your answer: ____________________________