CLASS QUIZ: OCTOBER 3: LIMITS AND CONTINUITY

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________________

You can access more limit-related quiz questions online at:

`http://calculus.subwiki.org/wiki/Quiz:Limit`

Starred and double starred questions are questions you should feel free to discuss with your colleagues, both before and during class.

(1) (**) We call a function f left continuous on an open interval I if, for all $a \in I$, $\lim_{x \to a^-} f(x) = f(a)$. Which of the following is an example of a function that is left continuous but not continuous on $(0, 1)$? *Previous year's performance*: 6/13 correct

(A) $f(x) := \begin{cases} x, & 0 < x \le 1/2 \\ 2x, & 1/2 < x < 1 \end{cases}$

(B) $f(x) := \begin{cases} x, & 0 < x < 1/2 \\ 2x, & 1/2 \le x < 1 \end{cases}$

(C) $f(x) := \begin{cases} x, & 0 < x \le 1/2 \\ 2x - (1/2), & 1/2 < x < 1 \end{cases}$

(D) $f(x) := \begin{cases} x, & 0 < x < 1/2 \\ 2x - (1/2), & 1/2 \le x < 1 \end{cases}$

(E) All of the above

Your answer: ____________________________

(2) (**) Suppose f and g are functions $(0, 1)$ to $(0, 1)$ that are both left continuous on $(0, 1)$. Which of the following is *not* guaranteed to be left continuous on $(0, 1)$? *Previous year's performance*: 4/13 correct

(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be left continuous functions

Your answer: ____________________________

(3) Which of these is the correct interpretation of $\lim_{x \to c} f(x) = L$ in terms of the definition of limit? *Previous year's performance:* 9/12 *correct*

(A) For every $\alpha > 0$, there exists $\beta > 0$ such that if $0 < |x - c| < \alpha$, then $|f(x) - L| < \beta$.
(B) There exists $\alpha > 0$ such that for every $\beta > 0$, and $0 < |x - c| < \alpha$, we have $|f(x) - L| < \beta$.
(C) For every $\alpha > 0$, there exists $\beta > 0$ such that if $0 < |x - c| < \beta$, then $|f(x) - L| < \alpha$.
(D) There exists $\alpha > 0$ such that for every $\beta > 0$ and $0 < |x - c| < \beta$, we have $|f(x) - L| < \alpha$.
(E) None of the above

Your answer: ____________________________

(4) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function. Which of the following says that f does not have a limit at any point in $\mathbb{R}$ (i.e., there is no point $c \in \mathbb{R}$ for which $\lim_{x \to c} f(x)$ exists)? *Previous year's performance:* 10/12 *correct*

(A) For every $c \in \mathbb{R}$, there exists $L \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - L| \geq \varepsilon$.
(B) There exists $c \in \mathbb{R}$ such that for every $L \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x satisfying $0 < |x - c| < \delta$ and $|f(x) - L| \geq \varepsilon$.
(C) For every $c \in \mathbb{R}$ and every $L \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x satisfying $0 < |x - c| < \delta$ and $|f(x) - L| \geq \varepsilon$.
(D) There exists $c \in \mathbb{R}$ and $L \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - L| \geq \varepsilon$.
(E) All of the above.

Your answer: ____________________________

(5) In the usual $\varepsilon - \delta$ definition of limit for a given limit $\lim_{x \to c} f(x) = L$, if a given value $\delta > 0$ works for a given value $\varepsilon > 0$, then which of the following is true? *Previous year's performance:* 17/26 *correct*

(A) Every smaller positive value of δ works for the same ε. Also, the given value of δ works for every smaller positive value of ε.
(B) Every smaller positive value of δ works for the same ε. Also, the given value of δ works for every larger value of ε.
(C) Every larger value of δ works for the same ε. Also, the given value of δ works for every smaller positive value of ε.
(D) Every larger value of δ works for the same ε. Also, the given value of δ works for every larger value of ε.
(E) None of the above statements need always be true.

Your answer: ____________________________

(6) Which of the following is a correct formulation of the statement $\lim_{x \to c} f(x) = L$, in a manner that avoids the use of εs and δs?

(A) For every open interval centered at c, there is an open interval centered at L such that the image under f of the open interval centered at c (excluding the point c itself) is contained in the open interval centered at L.
(B) For every open interval centered at c, there is an open interval centered at L such that the image under f of the open interval centered at c (excluding the point c itself) contains the open interval centered at L.

(C) For every open interval centered at L, there is an open interval centered at c such that the image under f of the open interval centered at c (excluding the point c itself) is contained in the open interval centered at L.
(D) For every open interval centered at L, there is an open interval centered at c such that the image under f of the open interval centered at c (excluding the point c itself) contains the open interval centered at L.
(E) None of the above.

Your answer: ____________________________

(7) (*) Consider the function:

$$f(x) := \begin{cases} x, & x \text{ rational} \\ 1/x, & x \text{ irrational} \end{cases}$$

What is the set of all points at which f is continuous? *Previous year's performance*: 5/13 correct
(A) $\{0, 1\}$
(B) $\{-1, 1\}$
(C) $\{-1, 0\}$
(D) $\{-1, 0, 1\}$
(E) f is continuous everywhere

Your answer: ____________________________