TAKE-HOME CLASS QUIZ: DUE FRIDAY FEBRUARY 1: LIMITS

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

This is based on the concept of limits you have seen in single variable calculus. You can view video playlists of the material here:

http://www.youtube.com/playlist?list=PL8483BCA409563C88

http://www.youtube.com/playlist?list=PLC0bHnWu122lmsGOHv39OSaNwD8MXvlTH

http://www.youtube.com/playlist?list=PLC0bHnWu122lZYgoCzVmWtnMAmUO6RDXC

PLEASE FEEL FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE ONLY ENTER FINAL ANSWER OPTIONS THAT YOU PERSONALLY CONSIDER MOST LIKELY TO BE CORRECT – DO NOT ENGAGE IN GROUPTHINK.

(1) We call a function f left continuous on an open interval I if, for all $a \in I$, $\lim_{x \to a^-} f(x) = f(a)$. Which of the following is an example of a function that is left continuous but not continuous on $(0,1)$? If all are examples, please select Option (E).
(A) $f(x) := \begin{cases} x, & 0 < x \le 1/2 \\ 2x, & 1/2 < x < 1 \end{cases}$
(B) $f(x) := \begin{cases} x, & 0 < x < 1/2 \\ 2x, & 1/2 \le x < 1 \end{cases}$
(C) $f(x) := \begin{cases} x, & 0 < x \le 1/2 \\ 2x - (1/2), & 1/2 < x < 1 \end{cases}$
(D) $f(x) := \begin{cases} x, & 0 < x < 1/2 \\ 2x - (1/2), & 1/2 \le x < 1 \end{cases}$
(E) All of the above

Your answer: ____________________________

(2) Suppose f and g are functions $(0,1)$ to $(0,1)$ that are both left continuous on $(0,1)$. Which of the following is *not* guaranteed to be left continuous on $(0,1)$? Please see Option (E) before answering.
(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be left continuous functions

Your answer: ____________________________

(3) Which of these is the correct interpretation of $\lim_{x \to c} f(x) = L$ in terms of the definition of limit? Please see Option (E) before answering.
(A) For every $\alpha > 0$, there exists $\beta > 0$ such that if $0 < |x - c| < \alpha$, then $|f(x) - L| < \beta$.
(B) There exists $\alpha > 0$ such that for every $\beta > 0$, and $0 < |x - c| < \alpha$, we have $|f(x) - L| < \beta$.
(C) For every $\alpha > 0$, there exists $\beta > 0$ such that if $0 < |x - c| < \beta$, then $|f(x) - L| < \alpha$.
(D) There exists $\alpha > 0$ such that for every $\beta > 0$ and $0 < |x - c| < \beta$, we have $|f(x) - L| < \alpha$.
(E) None of the above

Your answer: ____________________________

(4) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function. Which of the following says that f does not have a limit at any point in $\mathbb{R}$ (i.e., there is no point $c \in \mathbb{R}$ for which $\lim_{x \to c} f(x)$ exists)? If all, please select Option (E).

(A) For every $c \in \mathbb{R}$, there exists $L \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - L| \geq \varepsilon$.

(B) There exists $c \in \mathbb{R}$ such that for every $L \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x satisfying $0 < |x - c| < \delta$ and $|f(x) - L| \geq \varepsilon$.

(C) For every $c \in \mathbb{R}$ and every $L \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x satisfying $0 < |x - c| < \delta$ and $|f(x) - L| \geq \varepsilon$.

(D) There exists $c \in \mathbb{R}$ and $L \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - L| \geq \varepsilon$.

(E) All of the above.

Your answer: ____________________________

(5) In the usual $\varepsilon - \delta$ definition of limit for a given limit $\lim_{x \to c} f(x) = L$, if a given value $\delta > 0$ works for a given value $\varepsilon > 0$, then which of the following is true? Please see Option (E) before answering.

(A) Every smaller positive value of δ works for the same ε. Also, the given value of δ works for every smaller positive value of ε.

(B) Every smaller positive value of δ works for the same ε. Also, the given value of δ works for every larger value of ε.

(C) Every larger value of δ works for the same ε. Also, the given value of δ works for every smaller positive value of ε.

(D) Every larger value of δ works for the same ε. Also, the given value of δ works for every larger value of ε.

(E) None of the above statements need always be true.

Your answer: ____________________________

(6) Which of the following is a correct formulation of the statement $\lim_{x \to c} f(x) = L$, in a manner that avoids the use of εs and δs? Please see Option (E) before answering.

(A) For every open interval centered at c, there is an open interval centered at L such that the image under f of the open interval centered at c (excluding the point c itself) is contained in the open interval centered at L.

(B) For every open interval centered at c, there is an open interval centered at L such that the image under f of the open interval centered at c (excluding the point c itself) contains the open interval centered at L.

(C) For every open interval centered at L, there is an open interval centered at c such that the image under f of the open interval centered at c (excluding the point c itself) is contained in the open interval centered at L.

(D) For every open interval centered at L, there is an open interval centered at c such that the image under f of the open interval centered at c (excluding the point c itself) contains the open interval centered at L.

(E) None of the above.

Your answer: ____________________________

(7) Consider the function:

$$f(x) := \begin{cases} x, & x \text{ rational} \\ 1/x, & x \text{ irrational} \end{cases}$$

What is the set of all points at which f is continuous?

(A) $\{0, 1\}$
(B) $\{-1, 1\}$
(C) $\{-1, 0\}$
(D) $\{-1, 0, 1\}$
(E) f is continuous everywhere

Your answer: ____________________________

(8) The graph $y = f(x)$ of a function f defined on all reals has a horizontal asymptote $y = c$ as x approaches $+\infty$. Which of the following is the correct definition of this?

(A) For every $a \in \mathbb{R}$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $f(x) > a$.
(B) For every $a \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for all x satisfying $x > a$, we have $|f(x) - c| < \varepsilon$.
(C) For every $\varepsilon > 0$, there exists $a \in \mathbb{R}$ such that for all x satisfying $x > a$, we have $|f(x) - c| < \varepsilon$.
(D) For every $\delta > 0$, there exists $a \in \mathbb{R}$ such that for all x satisfying $0 < |x - c| < \delta$, we have $f(x) > a$.
(E) For every $\varepsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - c| < \varepsilon$.

Your answer: ____________________________

(9) Which of the following is the correct definition of $\lim_{x \to c^-} f(x) = -\infty$ (in words: the left hand limit of f at c is $-\infty$)?

(A) For every $a \in \mathbb{R}$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $f(x) > a$.
(B) For every $a \in \mathbb{R}$, there exists $\delta > 0$ such that for all x satisfying $0 < x - c < \delta$, we have $f(x) > a$.
(C) For every $a \in \mathbb{R}$, there exists $\delta > 0$ such that for all x satisfying $0 < x - c < \delta$, we have $f(x) < a$.
(D) For every $a \in \mathbb{R}$, there exists $\delta > 0$ such that for all x satisfying $0 < c - x < \delta$, we have $f(x) > a$.
(E) For every $a \in \mathbb{R}$, there exists $\delta > 0$ such that for all x satisfying $0 < c - x < \delta$, we have $f(x) < a$.

Your answer: ____________________________

(10) Suppose f is a function defined on all of $\mathbb{R}$ and $c \in \mathbb{R}$. Which of the following is the correct $\varepsilon - \delta$ definition for the statement "f is differentiable at c"?

(A) For every $L \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x such that $0 < |x - c| < \delta$ and $|f(x) - f(c) - L(x - c)| \geq |x - c|\varepsilon$.
(B) For every $L \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x such that $0 < |x - c| < \delta$ and $|f(x) - f(c) - L(x - c)| < |x - c|\varepsilon$.
(C) There exists $L \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every x satisfying $0 < |x - c| < \delta$, we have $|f(x) - f(c) - L(x - c)| \geq |x - c|\varepsilon$
(D) There exists $L \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every x satisfying $0 < |x - c| < \delta$, we have $|f(x) - f(c) - L(x - c)| < |x - c|\varepsilon$
(E) There exists $L \in \mathbb{R}$ such that there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x such that $0 < |x - c| < \delta$ and $|f(x) - f(c) - L(x - c)| < |x - c|\varepsilon$.

Your answer: ____________________________

(11) Suppose f is a function defined on all of $\mathbb{R}$ and $c \in \mathbb{R}$. Which of the following is the correct $\varepsilon - \delta$ definition for the statement "f is not differentiable at c"?

(A) For every $L \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x such that $0 < |x - c| < \delta$ and $|f(x) - f(c) - L(x - c)| \geq |x - c|\varepsilon$.

(B) For every $L \in \mathbb{R}$, there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x such that $0 < |x - c| < \delta$ and $|f(x) - f(c) - L(x - c)| < |x - c|\varepsilon$.
(C) There exists $L \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every x satisfying $0 < |x - c| < \delta$, we have $|f(x) - f(c) - L(x - c)| \geq |x - c|\varepsilon$
(D) There exists $L \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every x satisfying $0 < |x - c| < \delta$, we have $|f(x) - f(c) - L(x - c)| < |x - c|\varepsilon$
(E) There exists $L \in \mathbb{R}$ such that there exists $\varepsilon > 0$ such that for every $\delta > 0$, there exists x such that $0 < |x - c| < \delta$ and $|f(x) - f(c) - L(x - c)| < |x - c|\varepsilon$.

Your answer: ____________________________

(12) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function. Identify which of these definitions is *not* correct for $\lim_{x\to c} f(x) = L$, where c and L are both finite real numbers. If all are correct, please select Option (E).
(A) For every $\varepsilon > 0$, there exists $\delta > 0$ such that if $x \in (c-\delta, c+\delta) \setminus \{c\}$, then $f(x) \in (L-\varepsilon, L+\varepsilon)$.
(B) For every $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, there exist $\delta_1 > 0$ and $\delta_2 > 0$ such that if $x \in (c - \delta_1, c + \delta_2) \setminus \{c\}$, then $f(x) \in (L - \varepsilon_1, L + \varepsilon_2)$.
(C) For every $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, there exists $\delta > 0$ such that if $x \in (c - \delta, c + \delta) \setminus \{c\}$, then $f(x) \in (L - \varepsilon_1, L + \varepsilon_2)$.
(D) For every $\varepsilon > 0$, there exist $\delta_1 > 0$ and $\delta_2 > 0$ such that if $x \in (c - \delta_1, c + \delta_2) \setminus \{c\}$, then $f(x) \in (L - \varepsilon, L + \varepsilon)$.
(E) None of these, i.e., all definitions are correct.

Your answer: ____________________________

(13) In the usual $\varepsilon - \delta$ definition of limit, we find that the value $\delta = 0.2$ for $\varepsilon = 0.7$ for a function f at 0, and the value $\delta = 0.5$ works for $\varepsilon = 1.6$ for a function g at 0. What value of δ *definitely* works for $\varepsilon = 2.3$ for the function $f + g$ at 0?
(A) 0.2
(B) 0.3
(C) 0.5
(D) 0.7
(E) 0.9

Your answer: ____________________________

(14) The sum of limits theorem states that $\lim_{x\to c}[f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$ *if* the right side is defined. One of the choices below gives an example where the left side of the equality is defined and finite but the right side makes no sense. Identify the correct choice.
(A) $f(x) := 1/x$, $g(x) := -1/(x + 1)$, $c = 0$.
(B) $f(x) := 1/x$, $g(x) := (x - 1)/x$, $c = 0$.
(C) $f(x) := \arcsin x$, $g(x) := \arccos x$, $c = 1/2$.
(D) $f(x) := 1/x$, $g(x) = x$, $c = 0$.
(E) $f(x) := \tan x$, $g(x) := \cot x$, $c = 0$.

Your answer: ____________________________