HOMEWORK 8: DUE MONDAY DECEMBER 2

MATH 196, SECTION 57 (VIPUL NAIK)

1. Routine problems

Please write your solutions clearly, show relevant steps, but be concise. Underline, highlight, or box your final answers to make life easy for the grader.

(1) Exercise 3.4.1 (Page 159): Determine whether the vector $\vec{x}$ is in the span V of the vectors $\vec{v}_1, \dots, \vec{v}_m$ (proceed "by inspection" if possible, and use the reduced row-echelon form if necessary). If $\vec{x}$ is in V, find the coordinates of $\vec{x}$ with respect to the basis $\mathcal{B} = (\vec{v}_1, \dots, \vec{v}_m)$ of V, and write the coordinate vector $[\vec{x}]_{\mathcal{B}}$:

$$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}; \vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(2) Exercise 3.4.2 (Page 159; was 3.4.4 in the 4th Edition): Determine whether the vector $\vec{x}$ is in the span V of the vectors $\vec{v}_1, \dots, \vec{v}_m$ (proceed "by inspection" if possible, and use the reduced row-echelon form if necessary). If $\vec{x}$ is in V, find the coordinates of $\vec{x}$ with respect to the basis $\mathcal{B} = (\vec{v}_1, \dots, \vec{v}_m)$ of V, and write the coordinate vector $[\vec{x}]_{\mathcal{B}}$:

$$\vec{x} = \begin{bmatrix} 23 \\ 29 \end{bmatrix}; \vec{v}_1 = \begin{bmatrix} 46 \\ 58 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 61 \\ 67 \end{bmatrix}$$

(3) Exercise 3.4.18 (Page 159): Determine whether the vector $\vec{x}$ is in the span V of the vectors $\vec{v}_1, \dots, \vec{v}_m$ (proceed "by inspection" if possible, and use the reduced row-echelon form if necessary). If $\vec{x}$ is in V, find the coordinates of $\vec{x}$ with respect to the basis $\mathcal{B} = (\vec{v}_1, \dots, \vec{v}_m)$ of V, and write the coordinate vector $[\vec{x}]_{\mathcal{B}}$:

$$\vec{x} = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \end{bmatrix}; \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

(4) Exercise 3.4.25 (Page 160): Find the matrix B of the linear transformation $T(\vec{x}) = A\vec{x}$ with respect to the basis $\mathcal{B} = (\vec{v}_1, \dots, \vec{v}_m)$.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}; \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(5) Exercise 3.4.37 (Page 160): Find a basis $\mathcal{B}$ of $\mathbb{R}^n$ such that the $\mathcal{B}$-matrix B of the given linear transformation T is diagonal: Orthogonal projection T onto the line in $\mathbb{R}^2$ spanned by $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(6) Exercise 4.1.3 (Page 176): Is the subset of P_2 given here a subspace? If so, find a basis, where p' is the derivative:

$$\{p(t) : p'(1) = p(2)\}$$

(7) Exercise 4.1.20 (Page 176): Find a basis for the following space and determine its dimension: the space of all matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in $\mathbb{R}^{2\times 2}$ such that $a + d = 0$.

(8) Exercise 4.1.27 (Page 176): Find a basis for the following space and determine its dimension: the space of all 2×2 matrices A that commute with $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(9) Exercise 4.1.28 (Page 176): Find a basis for the following space and determine its dimension: the space of all 2×2 matrices A that commute with $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(10) Exercise 4.2.23 (Page 184): Find out if the transformation here is linear, and if so, determine whether it is an isomorphism:

$$T(f(t)) = f(7) \text{ from } P_2 \text{ to } \mathbb{R}$$

(11) Exercise 4.2.24 (Page 184): Find out if the transformation here is linear, and if so, determine whether it is an isomorphism:

$$T(f(t)) = f''(t)f(t) \text{ from } P_2 \text{ to } P_2$$

(12) Exercise 4.2.37 (Page 185): Find out if the transformation here is linear, and if so, determine whether it is an isomorphism:

$$T(f) = f + f' \text{ from } C^\infty \text{ to } C^\infty$$

(13) Exercise 4.2.43 (Page 185): Find out if the transformation here is linear, and if so, determine whether it is an isomorphism:

$$T(f(t)) = \begin{bmatrix} f(5) \\ f(7) \\ f(11) \end{bmatrix} \text{ from } P_2 \text{ to } \mathbb{R}^3$$

(14) Exercise 4.2.45 (Page 185): Find out if the transformation here is linear, and if so, determine whether it is an isomorphism:

$$T(f(t)) = t(f(t)) \text{ from } P \text{ to } P$$

Here, P is used for the vector space of all polynomials, with no specified bound on the degree.

(15) Exercise 4.2.49 (Page 185): Find out if the transformation here is linear, and if so, determine whether it is an isomorphism:

$$T(f(t)) = f(t^2) \text{ from } P \text{ to } P$$

Here, P is used for the vector space of all polynomials, with no specified bound on the degree.

2. Advanced problems

(1) Exercise 4.2.65 (Page 185): We will define a transformation T from $\mathbb{R}^{n \times m}$ to $F(\mathbb{R}^m, \mathbb{R}^n)$; recall that $F(\mathbb{R}^m, \mathbb{R}^n)$ is the space of all functions from $\mathbb{R}^m$ to $\mathbb{R}^n$. For a matrix A in $\mathbb{R}^{n \times m}$, the value $T(A)$ will be a function from $\mathbb{R}^m$ to $\mathbb{R}^n$; thus, we need to define $(T(A))(\vec{v})$ for a vector $\vec{v}$ in $\mathbb{R}^m$. We let

$$(T(A))(\vec{v}) = A\vec{v}$$

- (a) Show that T is a linear transformation.
- (b) Find the kernel of T.
- (c) Show that the image of T is the space $L(\mathbb{R}^m, \mathbb{R}^n)$ of all linear transformations from $\mathbb{R}^m$ to $\mathbb{R}^n$.
- (d) Find the dimension of $L(\mathbb{R}^m, \mathbb{R}^n)$.

(2) Exercise 4.2.66 (Page 185): Find the kernel and nullity of the linear transformation $T(f) = f - f'$ from C^∞ to C^∞.