CLASS QUIZ: JANUARY 31: PARTIAL FRACTIONS AND RADICALS

MATH 153, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Which of these functions of x is *not* elementarily integrable?
(A) $x\sqrt{1+x^2}$
(B) $x^2\sqrt{1+x^2}$
(C) $x(1+x^2)^{1/3}$
(D) $x\sqrt{1+x^3}$
(E) $x^2\sqrt{1+x^3}$

Your answer: ____________________________

(2) Consider the function $f(k) := \int_1^2 \frac{dx}{\sqrt{x^2+k}}$. f is defined for $k \in (-1, \infty)$. What can we say about the nature of f within this interval?
(A) f is increasing on the interval $(-1, \infty)$.
(B) f is decreasing on the interval $(-1, \infty)$.
(C) f is increasing on $(-1, 0)$ and decreasing on $(0, \infty)$.
(D) f is decreasing on $(-1, 0)$ and increasing on $(0, \infty)$.
(E) f is increasing on $(-1, 0)$, decreasing on $(0, 2)$, and increasing again on $(2, \infty)$.

Your answer: ____________________________

(3) For which of these functions of x does the antiderivative necessarily involve *both* arctan *and* ln?
(A) $1/(x+1)$
(B) $1/(x^2+1)$
(C) $x/(x^2+1)$
(D) $x/(x^3+1)$
(E) $x^2/(x^3+1)$

Your answer: ____________________________

(4) Suppose F is a (not known) function defined on $\mathbb{R} \setminus \{-1, 0, 1\}$, differentiable everywhere on its domain, such that $F'(x) = 1/(x^3 - x)$ everywhere on $\mathbb{R} \setminus \{-1, 0, 1\}$. For which of the following sets of points is it true that knowing the value of F at these points **uniquely** determines F?
(A) $\{-\pi, -e, 1/e, 1/\pi\}$
(B) $\{-\pi/2, -\sqrt{3}/2, 11/17, \pi^2/6\}$
(C) $\{-\pi^3/7, -\pi^2/6, \sqrt{13}, 11/2\}$
(D) Knowing F at any of the above determines the value of F uniquely.
(E) None of the above works to uniquely determine the value of F.

Your answer: ____________________________

(5) Consider a rational function $f(x) := p(x)/q(x)$ where p and q are nonzero polynomials and the degree of p is strictly less than the degree of q. Suppose $q(x)$ is monic of degree n and has n distinct real roots $a_1, a_2, \dots, a_n$, so $q(x) = \prod_{i=1}^{n}(x - a_i)$. Then, we can write:

$$f(x) = \frac{c_1}{x - a_1} + \frac{c_2}{x - a_2} + \cdots + \frac{c_n}{x - a_n}$$

for suitable constants $c_i \in \mathbb{R}$. What can we say about the sum $\sum_{i=1}^{n} c_i$?
(A) The sum is always 0.
(B) The sum equals the leading coefficient of p.
(C) The sum is 0 if p has degree $n - 1$. If the degree of p is smaller, the sum equals the leading coefficient of p.
(D) The sum is 0 if p has degree smaller than $n - 1$. If p has degree equal to $n - 1$, the sum is the leading coefficient of p.
(E) The sum is 0 if p is a constant polynomial. Otherwise, it equals the leading coefficient of p.

Your answer: ____________________________

(6) *Hard right now, will become easier later*: Suppose F is a continuously differentiable function whose domain contains (a, ∞) for some $a \in \mathbb{R}$, and $F'(x)$ is a rational function $p(x)/q(x)$ on the domain of F. Further, suppose that p and q are nonzero polynomials. Denote by d_p the degree of p and by d_q the degree of q. Which of the following is a **necessary and sufficient condition** to ensure that $\lim_{x\to\infty} F(x)$ is finite?
(A) $d_p - d_q \geq 2$
(B) $d_p - d_q \geq 1$
(C) $d_p = d_q$
(D) $d_q - d_p \geq 1$
(E) $d_q - d_p \geq 2$

Your answer: ____________________________

For the remaining questions, we build on the observation: For any nonconstant monic polynomial $q(x)$, there exists a finite collection of transcendental functions $f_1, f_2, \dots, f_r$ such that the antiderivative of any rational function $p(x)/q(x)$, on an open interval where it is defined and continuous, can be expressed as $g_0 + f_1 g_1 + f_2 g_2 + \cdots + f_r g_r$ where $g_0, g_1, \dots, g_r$ are rational functions.

(7) For the polynomial $q(x) = 1 + x^2$, what collection of f_is works (all are written as functions of x)?
(A) $\arctan x$ and $\ln|x|$
(B) $\arctan x$ and $\arctan(1 + x^2)$
(C) $\ln|x|$ and $\ln(1 + x^2)$
(D) $\arctan x$ and $\ln(1 + x^2)$
(E) $\ln|x|$ and $\arctan(1 + x^2)$

Your answer: ____________________________

(8) For the polynomial $q(x) := 1 + x^2 + x^4$, what is the size of the smallest collection of f_is that works?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ____________________________