HYBRID TAKE-HOME PLUS CLASS QUIZ: MONDAY NOVEMBER 12: TAYLOR SERIES AND POWER SERIES

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ____________________________

PLEASE ATTEMPT THIS QUIZ BEFORE CLASS. I WILL GIVE YOU TIME TO REVIEW AND UPDATE YOUR SOLUTIONS IN CLASS, BUT THIS WILL NOT BE SUFFICIENT TO ATTEMPT ALL QUESTIONS FROM SCRATCH.

FEEL FREE TO DISCUSS ALL QUESTIONS, BUT ONLY ENTER ANSWER CHOICES THAT YOU PERSONALLY ENDORSE.

For these questions, we denote by $C^\infty(\mathbb{R})$ the space of functions from $\mathbb{R}$ to $\mathbb{R}$ that are *infinitely* differentiable *everywhere* in $\mathbb{R}$.

We denote by $C^k(\mathbb{R})$ the space of functions from $\mathbb{R}$ to $\mathbb{R}$ that are at least k times continuously differentiable on all of $\mathbb{R}$. Note that for $k \ge l$, $C^k(\mathbb{R})$ is a subspace of $C^l(\mathbb{R})$. Further, $C^\infty(\mathbb{R})$ is the intersection of $C^k(\mathbb{R})$ for all k.

We say that a function f is analytic about c if the Taylor series of f about c converges to f on some open interval about c. We say that f is *globally analytic* if the Taylor series of f about 0 converges to f everywhere on $\mathbb{R}$.

It turns out that if a function is globally analytic, then it is analytic not only about 0 but about any other point. In particular, globally analytic functions are in $C^\infty(\mathbb{R})$.

(1) Which of the following functions is in $C^\infty(\mathbb{R})$ but is *not* analytic about 0? *Two years ago:* 3/26 *correct*
(A) $f_1(x) := \begin{cases} (\sin x)/x, & x \neq 0 \\ 1, & x = 0 \end{cases}$
(B) $f_2(x) := \begin{cases} e^{-1/x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
(C) $f_3(x) := \begin{cases} e^{-1/x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
(D) $f_4(x) := \begin{cases} \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$
(E) All of the above.

Your answer: ____________________

(2) Which of the following functions is in $C^\infty(\mathbb{R})$ and is analytic about 0 but is not globally analytic? *Two years ago:* 7/26 *correct*
(A) $x \mapsto \ln(1 + x^2)$
(B) $x \mapsto \ln(1 + x)$
(C) $x \mapsto \ln(1 - x)$
(D) $x \mapsto \exp(1 + x)$
(E) $x \mapsto \exp(1 - x)$

Your answer: ____________________

(3) Suppose f and g are globally analytic functions and g is nowhere zero. Which of the following is *not necessarily* globally analytic?
(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$

(C) fg, i.e., the function $x \mapsto f(x)g(x)$
(D) f/g, i.e., the function $x \mapsto f(x)/g(x)$
(E) $f \circ g$, i.e., the function $x \mapsto f(g(x))$

Your answer: ____________________________

(4) Which of the following is an example of a globally analytic function whose reciprocal is in $C^\infty(\mathbb{R})$ but is not globally analytic? *Two years ago:* 10/26 *correct*
(A) x
(B) x^2
(C) $x+1$
(D) x^2+1
(E) e^x

Your answer: ____________________________

(5) Consider the rational function $1/\prod_{i=1}^{n}(x-\alpha_i)$, where the α_i are all distinct real numbers. This rational function is analytic about any point other than the α_is, and in particular its Taylor series converges to it on the interval of convergence. What is the radius of convergence for the Taylor series of the rational function about a point c not equal to any of the α_is? *Two years ago:* 10/26 *correct*
(A) It is the minimum of the distances from c to the α_is.
(B) It is the second smallest of the distances from c to the α_is.
(C) It is the arithmetic mean of the distances from c to the α_is.
(D) It is the second largest of the distances from c to the α_is.
(E) It is the maximum of the distances from c to the α_is.

Your answer: ____________________________

(6) What is the interval of convergence of the Taylor series for arctan about 0? *Two years ago:* 11/26 *correct*
(A) $(-1,1)$
(B) $[-1,1)$
(C) $(-1,1]$
(D) $[-1,1]$
(E) All of $\mathbb{R}$

Your answer: ____________________________

(7) What is the radius of convergence of the power series $\sum_{k=0}^{\infty} 2^{\sqrt{k}}x^k$? Please keep in mind the square root in the exponent.
(A) 0
(B) $1/2$
(C) $1/\sqrt{2}$
(D) 1
(E) infinite

Your answer: ____________________________