CLASS QUIZ (TAKE-HOME): MARCH 2: LOOSE ENDS

MATH 153, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

Please attempt these quiz questions prior to class and turn them in during class on Wednesday March 2.

(1) Which of the following is the correct definition of $\lim_{x\to\infty} f(x) = L$ for L a finite number?
  (A) For every $\epsilon > 0$ there exists $a \in \mathbb{R}$ such that if $0 < |x - L| < \epsilon$ then $f(x) > a$.
  (B) For every $\epsilon > 0$ there exists $a \in \mathbb{R}$ such that if $x > a$ then $|f(x) - L| < \epsilon$.
  (C) For every $a \in \mathbb{R}$ there exists $\epsilon > 0$ such that if $x > a$ then $|f(x) - L| < \epsilon$.
  (D) For every $a \in \mathbb{R}$ there exists $\epsilon > 0$ such that if $0 < |x - L| < \epsilon$ then $f(x) > a$.
  (E) There exists $a \in \mathbb{R}$ and $\epsilon > 0$ such that if $x > a$ then $|f(x) - L| < \epsilon$.

Your answer: ______________________

(2) Suppose $\lim_{x\to\infty} f'(x)$ is finite. Which of the following is true (be careful about f versus f' when reading the choices)?
  (A) If $\lim_{x\to\infty} f'(x)$ is zero, then $\lim_{x\to\infty} f(x)$ is finite.
  (B) If $\lim_{x\to\infty} f(x)$ is finite, then $\lim_{x\to\infty} f'(x)$ is zero.
  (C) If $\lim_{x\to\infty} f(x)$ is finite, then $\lim_{x\to\infty} f(x)$ is zero.
  (D) All of the above.
  (E) None of the above.

Your answer: ______________________

(3) Suppose a function y of time t satisfies the differential equation $y' = f(y)$ for all time t, where f is a continuous function on $\mathbb{R}$. Further, suppose we know that $\lim_{t\to\infty} y = L$ for some finite L. What can we conclude is true about L?
  (A) $f(L) = L$
  (B) $f(L) = 0$
  (C) $f'(L) = L$
  (D) $f'(L) = 0$
  (E) $f''(L) = 0$

Your answer: ______________________

(4) A sequence a_n is found to satisfy the recurrence $a_{n+1} = 2a_n(1 - a_n)$. Assume that a_1 is strictly between 0 and 1. What can we say about the sequence (a_n)?
  (a) It is monotonic non-increasing, and its limit is 0.
  (b) It is monotonic non-decreasing, and its limit is 1.
  (c) From a_2 onward, it is monotonic non-decreasing, and its limit is 1/2.
  (d) From a_2 onward, it is monotonic non-increasing, and its limit is 1/2.
  (e) It is either monotonic non-decreasing or monotonic non-increasing everywhere, and its limit is 1/2.

Your answer: ______________________

(5) Suppose f is a continuous function on $\mathbb{R}$ and (a_n) is a sequence satisfying the recurrence $f(a_n) = a_{n+1}$ for all n. Further, suppose the limit of the a_ns for odd n is L and the limit of the a_ns for even n is M. What can we say about L and M?

(A) $f(L) = L$ and $f(M) = M$
(B) $f(L) = M$ and $f(M) = L$
(C) $f(L) = f(M) = 0$
(D) $f'(L) = f'(M) = 0$
(E) $f'(L) = M$ and $f'(M) = L$

Your answer: ____________________________

(6) Consider a function f on the natural numbers defined as follows: $f(m) = m/2$ if m is even, and $f(m) = 3m + 1$ if m is odd. Consider a sequence where a_1 is a natural number and we define $a_n := f(a_{n-1})$. It is conjectured (see *Collatz conjecture*) that (a_n) is eventually periodic, regardless of the starting point, and that there is only one possibility for the eventual periodic fragment. Which of the following can be the eventual periodic fragment?
(A) $(1, 2, 3)$
(B) $(1, 3, 2)$
(C) $(1, 2, 4)$
(D) $(1, 4, 2)$
(E) $(1, 3, 4)$

Your answer: ____________________________

(7) For which of the following properties p of sequences of real numbers does p equal *eventually* p?
(A) Monotonicity
(B) Periodicity
(C) Being a polynomial sequence (i.e., given by a polynomial function)
(D) Being a constant sequence
(E) Boundedness

Your answer: ____________________________

(8) Which of the following series converges? Assume for all series that the startin point of summation is large enough that the terms are well defined.
(A) $\sum 1/(k \ln(\ln k))$
(B) $\sum 1/(k \ln k)$
(C) $\sum 1/(k(\ln(\ln k))^2)$
(D) $\sum 1/(k(\ln k)(\ln(\ln k)))$
(E) $\sum 1/(k(\ln k)(\ln(\ln k))^2)$

Your answer: ____________________________

(9) Which of the following series converges?
(A) $\sum \frac{k+\sin k}{k^2+1}$
(B) $\sum \frac{k+\cos k}{k^3+1}$
(C) $\sum \frac{k^2-\sin k}{k+1}$
(D) $\sum \frac{k^3+\cos k}{k^2+1}$
(E) $\sum \frac{k}{\sin(k^3+1)}$

Your answer: ____________________________

Suppose F is a function of two real variables, say x and t, so $F(x, t)$ is a real number for x and t restricted to suitable open intervals in the real number. Suppose, further, that F is jointly continuous (whatever that means) in x and t.

Define $f(t) := \int_0^\infty F(x, t)\, dx$. Here, while doing the integration, t is treated as a constant. x, the variable of integration, is being integrated on $[0, \infty)$.

Suppose further that f is defined and continuous for t in $(0, \infty)$. *Note that similar computations we did in the midterm review session involved integration from $-\infty$ to ∞.*

In the next few questions, you are asked to compute the function f explicitly given the function F, for $t \in (0, \infty)$.

(10) $F(x,t) := e^{-tx}$. Find f.
(A) $f(t) = e^{-t}/t$
(B) $f(t) = e^{t}/t$
(C) $f(t) = 1/t$
(D) $f(t) = -1/t$
(E) $f(t) = -t$

Your answer: ____________________________

(11) $F(x,t) := 1/(t^2 + x^2)$. Find f.
(A) $f(t) = \pi/(2t)$
(B) $f(t) = \pi/t$
(C) $f(t) = 2\pi/t$
(D) $f(t) = \pi t$
(E) $f(t) = 2\pi t$

Your answer: ____________________________

(12) $F(x,t) := 1/(t^2 + x^2)^2$. Find f.
(A) $f(t) = \pi/t^3$
(B) $f(t) = \pi/(2t^3)$
(C) $f(t) = \pi/(4t^3)$
(D) $f(t) = \pi/(8t^3)$
(E) $f(t) = 3\pi/(8t^3)$

Your answer: ____________________________

(13) $F(x,t) = \exp(-(tx)^2)$. Use that $\int_0^\infty \exp(-x^2)\,dx = \sqrt{\pi}/2$. Find f.
(A) $f(t) = t^2\sqrt{\pi}/2$
(B) $f(t) = t\sqrt{\pi}/2$
(C) $f(t) = \sqrt{\pi}/2$
(D) $f(t) = \sqrt{\pi}/(2t)$
(E) $f(t) = \sqrt{\pi}/(2t^2)$

Your answer: ____________________________

(14) In the same general setup as above (but with none of these specific Fs), which of the following is a *sufficient* condition for f to be an increasing function of t?
(A) $t \mapsto F(x_0, t)$ is an increasing function of t for every choice of $x_0 \geq 0$.
(B) $x \mapsto F(x, t_0)$ is an increasing function of x for every choice of $t_0 \in (0, \infty)$.
(C) $t \mapsto F(x_0, t)$ is a decreasing function of t for every choice of $x_0 \geq 0$.
(D) $x \mapsto F(x, t_0)$ is a decreasing function of x for every choice of $t_0 \in (0, \infty)$.
(E) None of the above.

Your answer: ____________________________