HOMEWORK 4: DUE MONDAY OCTOBER 28

MATH 196, SECTION 57 (VIPUL NAIK)

1. Routine problems

Please write your solutions clearly, show relevant steps, but be concise. Underline, highlight, or box your final answers to make life easy for the grader.

The computational problem numbers have moved a bit since the 4th Edition; if using the 4th Edition, please check the correct problem number before comparing your answer with that at the back of the book (if you're doing so) for odd-numbered problems.

(1) Exercise 2.3.3 (Page 85): Compute the matrix product by hand:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(2) Exercise 2.3.6 (Page 85): Compute the matrix product by hand:

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

(3) Exercise 2.3.7 (Page 85): Compute the matrix product by hand:

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

(4) Exercise 2.3.8 (Page 85): Compute the matrix product by hand:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(5) Exercise 2.3.10 (Page 85): Compute the matrix product by hand:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

(6) Exercise 2.3.11 (Page 85): Compte the matrix product by hand:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

(7) Exercise 2.3.13 (Page 85): Compute the matrix product by hand:

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(8) Exercise 2.4.2 (Page 97): Determine whether the matrix is invertible. If it is, find the inverse. Do the computations with paper and pencil.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

(9) Exercise 2.4.8 (Page 97) (was 2.4.10 in the 4th Edition): Determine whether the matrix is invertible. If it is, find the inverse. Do the computations with paper and pencil.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$$

(10) Exercise 2.4.10 (Page 97): Determine whether the matrix is invertible. If it is, find the inverse. Do the computations with paper and pencil.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(11) Exercise 2.4.12 (Page 97) (was 2.4.14 in the 4th Edition) : Determine whether the matrix is invertible. If it is, find the inverse. Do the computations with paper and pencil.

$$\begin{bmatrix} 2 & 5 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}$$

(12) Exercise 2.4.15 (Page 97): Determine whether the matrix is invertible. If it is, find the inverse. Do the computations with paper and pencil.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 7 & 11 \\ 3 & 7 & 14 & 25 \\ 4 & 11 & 25 & 50 \end{bmatrix}$$

(13) Exercise 2.4.25 (Page 97): Is this nonlinear transformation invertible? If so, find the inverse.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1^3 \\ x_2 \end{bmatrix}$$

(14) Exercise 2.4.26 (Page 97): Is this nonlinear transformation invertible? If so, find the inverse.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1^3 + x_2 \end{bmatrix}$$

(15) Exercise 2.4.27 (Page 97): Is this nonlinear transformation invertible? If so, find the inverse.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 x_2 \end{bmatrix}$$

(16) Exercise 2.4.29 (Page 97): For which values of the constant k is the following matrix invertible?

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$$

(17) Exercise 2.4.30 (Page 98): For which values of the constants b and c is the following matrix invertible?

$$\begin{bmatrix} 0 & 1 & b \\ -1 & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

2. Problems for your own review, not for submission

(1) Exercise 2.4.34 (Page 98): Consider the diagonal matrix

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

   (a) For which values of a, b, and c is A invertible? If it is invertible, what is A^{-1}?
   (b) For which values of the diagonal elements is a diagonal matrix (of arbitrary size) invertible?

(2) Exercise 2.4.35 (Page 98): Consider the upper triangular 3×3 matrix

$$A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

   (a) For which values of a, b, c, d, e, and f is A invertible?
   (b) More generally, when is an upper triangular matrix (of arbitrary size) invertible?
   (c) If an upper triangular matrix is invertible, is its inverse an upper triangular matrix as well?
   (d) When is a lower triangular matrix invertible?

(3) Exercise 2.4.67 (Page 100): For two invertible $n \times n$ matrices A and B, determine whether the formula is true:

$$(A + B)^2 = A^2 + 2AB + B^2$$

(4) Exercise 2.4.70 (Page 100): For an invertible $n \times n$ matrix A, determine whether A^2 is invertible and $(A^2)^{-1} = (A^{-1})^2$.

(5) Exercise 2.4.68 (Page 100): For two invertible $n \times n$ matrices A and B, determine whether the formula is true:

$$(A - B)(A + B) = A^2 - B^2$$