CLASS QUIZ SOLUTIONS: SEPTEMBER 26; TOPIC: FUNCTIONS

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1. Performance review

10 people took this 5-question quiz. The performance was as follows:

(1) (A): Everybody got this correct.
(2) (E): Everybody got this correct.
(3) (B): Everybody got this correct.
(4) (B): Everybody got this correct.
(5) (A): 1 person got this incorrect.

There were thus 9 full scores and 1 score of 4.

2. Solutions

(1) Consider the function $f(x) := |x+1| - |x|$. For which of the following values of x is $f(x)$ equal to 0?
(A) $-\frac{1}{2}$
(B) $-\frac{1}{3}$
(C) 0
(D) $\frac{1}{3}$
(E) $\frac{1}{2}$

Answer: Option (A)

Explanation: When we set $x = -1/2$, we get $f(x) = |(-1/2) + 1| - | - 1/2| = |1/2| - | - 1/2|$, which becomes $1/2 - 1/2$, which is equal to 0.

We can also solve the equation formally, but this is a little trickier, and we will get to it at a later stage.

The other choices: All the other choices are incorrect:
Option (B): $f(-1/3) = 2/3 - 1/3 = 1/3$.
Option (C): $f(0) = 1 - 0 = 1$.
Option (D): $f(1/3) = 4/3 - 1/3 = 1$.
Option (E): $f(1/2) = 3/2 - 1/2 = 1$.

Performance review: Everybody got it correct.

Historical note: When this same quiz question was asked last year, everybody got it correct.

(2) Consider the function $f(x) := x^2 + 1$. What is the polynomial describing $f(f(x))$?
(A) $x^2 + 2$
(B) $x^4 + x^2 + 1$
(C) $x^4 + x^2 + 2$
(D) $x^4 + 2x^2 + 1$
(E) $x^4 + 2x^2 + 2$

Answer: Option (E)

Explanation: We have $f(f(x)) = f(x^2 + 1) = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 1 + 1$, which simplifies to option (E).

The other choices:
Option (A) is $(x^2 + 1) + 1 = x^2 + 2$. The error here is is not squaring the $x^2 + 1$ expression.
Option (D) is $(x^2 + 1)^2 = x^4 + 2x^2 + 1$. The error here is in forgetting to add the 1 at the end.
Options (B) and (C) are like options (D) and (E), with an error in the coefficient of x^2.

Performance review: Everybody got it correct.

Historical note: When this same quiz question was asked last year, everybody got it correct.

(3) Consider the function $f(x) := \frac{x}{x^2+1}$. What is $f(f(1))$?
(A) 1/5
(B) 2/5
(C) 4/5
(D) 5/4
(E) 5/8

Answer: Option (B)

Explanation: We have:

$$f(1) = \frac{1}{1^2+1} = \frac{1}{2}$$

Thus, $f(f(1)) = f(1/2)$, and we get:

$$f(1/2) = \frac{1/2}{(1/2)^2+1} = \frac{1/2}{5/4} = \frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$$

Performance review: Everybody got it correct.

Historical note: When the same question appeared last year, 1 person chose (A), everybody else got this correct.

(4) Consider the function $f(x) := x + 1$. What is $f(f(x))$?
(A) x
(B) $x + 2$
(C) $2x + 1$
(D) $(x + 1)^2$
(E) $x^2 + 1$

Answer: Option (B)

Explanation: We have $f(f(x)) = f(x + 1) = (x + 1) + 1 = x + 2$.

Performance review: Everybody got it correct.

Historical note: When this same quiz question was asked last year, everybody got it correct.

(5) If a circle has radius r, the area of the circle is πr^2. What is the area of a circle with diameter d?
(A) $\pi d^2/4$
(B) $\pi d^2/2$
(C) πd^2
(D) $2\pi d^2$
(E) $4\pi d^2$

Answer: Option (A)

Explanation: The diameter is twice the radius, so the radius is half the diameter, i.e., $r = d/2$. Plugging this in, we get that the area is:

$$\pi r^2 = \pi(d/2)^2 = \pi d^2/4$$

The other choices: Option (B) is the best distractor. It could arise if we forget to square the 2 in the denominator in the above calculation.

The other options could arise through erroneous starting assumptions such as $r = d$ or $r = 2d$.

Performance review: 1 person chose (B), the best distractor. Everybody else got it correct.

Historical note: When this same quiz question was asked last year, everybody got it correct.