# HOMEWORK 6: DUE MONDAY NOVEMBER 11

MATH 196, SECTION 57 (VIPUL NAIK)

*Note*: I've kept the homework short because you have a number of lengthy quizzes due in the immediate neighborhood of the homework, and it's more worthwhile for you to concentrate on the quizzes. However, if you feel that the homework provides you with insufficient computational practice, please do additional practice problems from the relevant section of the book.

## 1. Routine problems

*Please write your solutions clearly, show relevant steps, but be concise. Underline, highlight, or box your final answers to make life easy for the grader.*

(1) Exercise 3.1.4 (Page 119) (was 3.1.2 in the 4th Edition): Find vectors that span the kernel of $A$, where $A$ is the matrix

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$$

(2) Exercise 3.1.7 (Page 119): Find vectors that span the kernel of $A$, where $A$ is the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

(3) Exercise 3.1.8 (Page 119): Find vectors that span the kernel of $A$, where $A$ is the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(4) Exercise 3.1.11 (Page 119): Find vectors that span the kernel of $A$, where $A$ is the matrix

$$\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 4 \end{bmatrix}$$

(5) Exercise 3.1.31 (Page 120): Give an example of a matrix $A$ such that $\operatorname{im}(A)$ is the plane with normal vector $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ in $\mathbb{R}^3$.

(6) Exercise 3.1.32 (Page 120): Give an example of a linear transformation whose image is the line spanned by

$$\begin{bmatrix} 7 \\ 6 \\ 5 \end{bmatrix}$$

(7) Exercise 3.1.33 (Page 120): Give an example of a linear transformation whose kernel is the plane $x + 2y + 3z = 0$ in $\mathbb{R}^3$.

(8) Exercise 3.1.34 (Page 120): Give an example of a linear transformation whose kernel is the line spanned by

$$\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

in $\mathbb{R}^3$.

## 2. Problems for your own review, not for submission

(1) Exercise 3.1.51 (Page 121): Consider a $n \times p$ matrix $A$ and a $p \times m$ matrix $B$ such that $\ker(A) = \{\vec{0}\}$ and $\ker(B) = \{\vec{0}\}$. Find $\ker(AB)$.

## 3. Advanced problems

(1) Exercise 3.1.35 (Page 120): Consider a nonzero vector $\vec{v}$ in $\mathbb{R}^3$. Arguing geometrically, describe the image and kernel of the linear transformations $T$ from $\mathbb{R}^3$ to $\mathbb{R}$ given by the dot product

$$T(\vec{x}) = \vec{v} \cdot \vec{x}$$

(2) Exercise 3.1.36 (Page 120): Consider a nonzero vector $\vec{v}$ in $\mathbb{R}^3$. Using a geometric argument, describe the kernel of the linear transformation $T$ from $\mathbb{R}^3$ to $\mathbb{R}^3$ given by

$$T(\vec{x}) = \vec{v} \times \vec{x}$$

(3) Exercise 3.1.37 (Page 120): For the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

describe the images and kernels of the matrices $A$, $A^2$, and $A^3$ geometrically.