TAKE-HOME CLASS QUIZ: DUE OCTOBER 3: REFRESHER

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

YOU ARE FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE MAKE SURE TO ONLY ENTER ANSWER CHOICES THAT YOU PERSONALLY ENDORSE

Note that for most quizzes, you will be allowed to discuss only certain star-marked questions. However, this is a particularly hard set of questions and it is your first quiz, so you can discuss all the questions.

Note also that most quizzes will not be take-home – you will be given time to attempt questions in class, though the quiz will still be given to you a class or two in advance. Only a few select quizzes that are of a "refresher" or "review" nature will be take-home.

These questions are related to material ostensibly covered in the first two quarters of calculus. However, these are extremely hard questions, so doing poorly on this quiz is *not* an indication that you need to switch down.

(1) Which of the following statements is **always true**?
- (A) The range of a continuous nonconstant function on an open bounded interval (i.e., an interval of the form (a, b)) is an open bounded interval (i.e., an interval of the form (m, M)).
- (B) The range of a continuous nonconstant function on a closed bounded interval (i.e., an interval of the form $[a, b]$) is a closed bounded interval (i.e., an interval of the form $[m, M]$).
- (C) The range of a continuous nonconstant function on an open interval that may be bounded or unbounded (i.e., an interval of the form (a, b),(a, ∞), $(-\infty, a)$, or $(-\infty, \infty)$), is also an open interval that may be bounded or unbounded.
- (D) The range of a continuous nonconstant function on a closed interval that may be bounded or unbounded (i.e., an interval of the form $[a, b]$, $[a, \infty)$, $(-\infty, a]$, or $(-\infty, \infty)$) is also a closed interval that may be bounded or unbounded.
- (E) None of the above.

Your answer: ______________________

(2) For which of the following specifications is there **no continuous function** satisfying the specifications?
- (A) Domain $(0, 1)$ and range $(0, 1)$
- (B) Domain $[0, 1]$ and range $(0, 1)$
- (C) Domain $(0, 1)$ and range $[0, 1]$
- (D) Domain $[0, 1]$ and range $[0, 1]$
- (E) None of the above, i.e., we can get a continuous function for each of the specifications.

Your answer: ______________________

(3) Suppose f is a continuously differentiable function from the open interval $(0, 1)$ to $\mathbb{R}$. Suppose, further, that there are exactly 14 values of c in $(0, 1)$ for which $f(c) = 0$. What can we say is **definitely true** about the number of values of c in the open interval $(0, 1)$ for which $f'(c) = 0$?
- (A) It is at least 13 and at most 15.
- (B) It is at least 13, but we cannot put any upper bound on it based on the given information.
- (C) It is at most 15, but we cannot put any lower bound (other than the meaningless bound of 0) based on the given information.
- (D) It is at most 13.
- (E) It is at least 15.

Your answer: ______________________

(4) Consider the function

$$f(x) := \begin{cases} x, & 0 \le x \le 1/2 \\ x - (1/7), & 1/2 < x \le 1 \end{cases}$$

Define by $f^{[n]}$ the function obtained by iterating f n times, i.e., the function $f \circ f \circ f \circ \cdots \circ f$ where f occurs n times. What is the smallest n for which $f^{[n]} = f^{[n+1]}$?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ____________________________

(5) Suppose f and g are functions $(0,1)$ to $(0,1)$ that are both right continuous on $(0,1)$. Which of the following is *not* guaranteed to be right continuous on $(0,1)$?
(A) $f+g$, i.e., the function $x \mapsto f(x)+g(x)$
(B) $f-g$, i.e., the function $x \mapsto f(x)-g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be right continuous functions

Your answer: ____________________________

(6) Suppose f and g are increasing functions from $\mathbb{R}$ to $\mathbb{R}$. Which of the following functions is *not* guaranteed to be an increasing function from $\mathbb{R}$ to $\mathbb{R}$?
(A) $f+g$
(B) $f \cdot g$
(C) $f \circ g$
(D) All of the above, i.e., none of them is guaranteed to be increasing.
(E) None of the above, i.e., they are all guaranteed to be increasing.

Your answer: ____________________________

(7) Suppose F and G are two functions defined on $\mathbb{R}$ and k is a natural number such that the k^{th} derivatives of F and G exist and are equal on all of $\mathbb{R}$. Then, $F-G$ must be a polynomial function. What is the **maximum possible degree** of $F-G$? (Note: Assume constant polynomials to have degree zero)
(A) $k-2$
(B) $k-1$
(C) k
(D) $k+1$
(E) There is no bound in terms of k.

Your answer: ____________________________

(8) Suppose f is a continuous function on $\mathbb{R}$. Clearly, f has antiderivatives on $\mathbb{R}$. For all but one of the following conditions, it is possible to guarantee, without any further information about f, that there exists an antiderivative F satisfying that condition. **Identify the exceptional condition** (i.e., the condition that it may not always be possible to satisfy).
(A) $F(1) = F(0)$.
(B) $F(1) + F(0) = 0$.
(C) $F(1) + F(0) = 1$.
(D) $F(1) = 2F(0)$.
(E) $F(1)F(0) = 0$.

Your answer: ____________________________

(9) Suppose F is a function defined on $\mathbb{R} \setminus \{0\}$ such that $F'(x) = -1/x^2$ for all $x \in \mathbb{R} \setminus \{0\}$. Which of the following pieces of information is/are **sufficient** to determine F completely? Please see Options (D) and (E) before answering.

(A) The value of F at any two positive numbers.
(B) The value of F at any two negative numbers.
(C) The value of F at a positive number and a negative number.
(D) Any of the above pieces of information is sufficient, i.e., we need to know the value of F at any two numbers.
(E) None of the above pieces of information is sufficient.

Your answer: ____________________________

(10) Suppose F and G are continuously differentiable functions on all of $\mathbb{R}$ (i.e., both F' and G' are continuous). Which of the following is **not necessarily true**?
(A) If $F'(x) = G'(x)$ for all integers x, then $F - G$ is a constant function when restricted to integers, i.e., it takes the same value at all integers.
(B) If $F'(x) = G'(x)$ for all numbers x that are not integers, then $F - G$ is a constant function when restricted to the set of numbers x that are not integers.
(C) If $F'(x) = G'(x)$ for all rational numbers x, then $F - G$ is a constant function when restricted to the set of rational numbers.
(D) If $F'(x) = G'(x)$ for all irrational numbers x, then $F - G$ is a constant function when restricted to the set of irrational numbers.
(E) None of the above, i.e., they are all necessarily true.

Your answer: ____________________________

(11) Consider the four functions $\sin(\sin x)$, $\sin(\cos x)$, $\cos(\sin x)$, and $\cos(\cos x)$. Which of the following statements are true about their periodicity?
(A) All four functions are periodic with a period of π.
(B) All four functions are periodic with a period of 2π.
(C) $\cos(\sin x)$ and $\cos(\cos x)$ have a period of π, whereas $\sin(\sin x)$ and $\sin(\cos x)$ have a period of 2π.
(D) $\sin(\sin x)$ and $\sin(\cos x)$ have a period of π, whereas $\cos(\sin x)$ and $\cos(\cos x)$ have a period of 2π.
(E) $\sin(\sin x)$ has a period of 2π, the other three functions have a period of π.

Your answer: ____________________________

(12) Suppose f is a continuous one-to-one function with domain a closed interval $[a, b]$ and range a closed interval $[c, d]$. Suppose t is a point in (a, b) such that f has left hand derivative l and right-hand derivative r at t, with both l and r nonzero. What is the left hand derivative and right hand derivative to f^{-1} at $f(t)$? *Earlier score:* 6/15
(A) The left hand derivative is $1/l$ and the right hand derivative is $1/r$.
(B) The left hand derivative is $-1/l$ and the right hand derivative is $-1/r$.
(C) The left hand derivative is $1/r$ and the right hand derivative is $1/l$.
(D) The left hand derivative is $-1/r$ and the right hand derivative is $-1/l$.
(E) The left hand derivative is $1/l$ and the right hand derivative is $1/r$ if $l > 0$, otherwise the left hand derivative is $1/r$ and the right hand derivative is $1/l$.

Your answer: ____________________________

(13) Which of these functions is one-to-one?
(A) $f_1(x) := \begin{cases} x, & x \text{ rational} \\ x^2, & x \text{ irrational} \end{cases}$
(B) $f_2(x) := \begin{cases} x, & x \text{ rational} \\ x^3, & x \text{ irrational} \end{cases}$
(C) $f_3(x) := \begin{cases} x, & x \text{ rational} \\ 1/(x-1), & x \text{ irrational} \end{cases}$
(D) All of the above
(E) None of the above

Your answer: ____________________________

(14) Consider the following function $f : [0,1] \to [0,1]$ given by $f(x) := \begin{cases} \sin(\pi x/2), & 0 \leq x \leq 1/2 \\ \sqrt{x}, & 1/2 < x \leq 1 \end{cases}$.

What is the correct expression for $(f^{-1})'(1/2)$?

(A) It does not exist, since the two one-sided derivatives of f at $1/2$ do not match.

(B) $\sqrt{2}$

(C) $2\sqrt{2}/\pi$

(D) $4/\pi$

(E) $4/(\sqrt{3}\pi)$

Your answer: ______________________________