CLASS QUIZ: NOVEMBER 5: SERIES SUMMATION

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ____________________________________

THIS IS AN IN-CLASS QUIZ. YOU MAY ATTEMPT THE QUESTIONS IN ADVANCE, BUT PLEASE DO NOT DISCUSS WITH OTHERS.

(1) Consider the function $f(x) := \sum_{n=1}^{\infty} \frac{x^n}{n(n+2)}$ defined on the closed interval $[-1, 1]$. What are the values of $f(1)$ and $f(-1)$?
(A) $f(1) = 3/4$ and $f(-1) = 1/4$
(B) $f(1) = 3/4$ and $f(-1) = -1/4$
(C) $f(1) = 3/4$ and $f(-1) = -3/4$
(D) $f(1) = 1/4$ and $f(-1) = 3/4$
(E) $f(1) = 1/4$ and $f(-1) = -1/4$

Your answer: ___________________________

(2) Given that we have the following: $\sum_{n=1}^{\infty} x^n/n = -\ln(1-x)$ for all $-1 < x < 1$ and the series converges absolutely in the interval, what is an explicit expression for the summation $\sum_{n=1}^{\infty} x^n/(n(n+1))$ for $x \in (-1, 1) \setminus \{0\}$?
(A) $1 + \ln(1-x)$
(B) $1 - \ln(1-x)$
(C) $1 + \frac{(1+x)\ln(1-x)}{x}$
(D) $1 + \frac{(1-x)\ln(1-x)}{x}$
(E) $1 + \frac{(x-1)\ln(1-x)}{x}$

Your answer: ___________________________

(3) Given that we have the following: $\sum_{n=1}^{\infty} x^n/n = -\ln(1-x)$ for all $-1 < x < 1$ and the series converges absolutely in the interval, what is an explicit expression for the summation $\sum_{n=1}^{\infty} x^n/(n(n+2))$ for $x \in (-1, 1) \setminus \{0\}$?
(A) $\frac{1}{4} + \frac{1}{2x} + \frac{(1-x^2)\ln(1-x)}{2x^2}$
(B) $\frac{1}{4} + \frac{1}{2x} + \frac{(x^2-1)\ln(1-x)}{2x^2}$
(C) $\frac{1}{4} - \frac{1}{2x} + \frac{(x^2-1)\ln(1-x)}{2x^2}$
(D) $\frac{1}{4} - \frac{1}{2x} + \frac{(1-x^2)\ln(1-x)}{2x^2}$
(E) $\frac{1}{4} + \frac{1}{2x}$

Your answer: ___________________________

(4) Suppose $p > 1$ and let $\zeta(p) = \sum_{n=1}^{\infty} \frac{1}{n^p}$. What is the value of $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^p}$ (this is the sum of the p^{th} powers of all odd positive integers) in terms of $\zeta(p)$? *Hint: Write the sum of the odd-numbered terms as the total sum minus the sum of the even-numbered terms.*
(A) $2\zeta(p) - 1$
(B) $\zeta(p)/3$
(C) $(2^p - 1)\zeta(p)$
(D) $(1 - 2^{-p})\zeta(p)$
(E) $\zeta(p)/(2^p + 1)$

Your answer: ____________________________

(5) Suppose $p > 1$ and $\zeta(p) = \sum_{n=1}^{\infty} \frac{1}{n^p}$. What is the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ in terms of $\zeta(p)$?
(A) $-\zeta(p)$
(B) $\zeta(p)/2$
(C) $\zeta(p)/3$
(D) $(2^{p-1} - 1)\zeta(p)$
(E) $(2^{1-p} - 1)\zeta(p)$

Your answer: ____________________________

There is a result of calculus which states that, under suitable conditions, if $f_1, f_2, \dots, f_n, \dots$ are all functions, and we define $f(x) := \sum_{n=1}^{\infty} f_n(x)$, then $f^{(r)}(x) = \sum_{n=1}^{\infty} f_n^{(r)}(x)$ for any positive integer r. In other words, under suitable assumptions, we can repeatedly differentiate a sum of countably many functions by repeatedly differentiating each of them and adding up the derivatives.

We will not be going into what those assumptions are, but will consider some applications where you are explicitly told that these assumptions are satisfied.

(6) Consider the summation $\zeta(p) := \sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 1$. Assume that the required assumptions are valid for this summation, so that $\zeta''(p)$ is the sum of the second derivatives of each of the terms (summands) with respect to p. What is the correct expression for $\zeta''(p)$?
(A) $\sum_{n=1}^{\infty} \frac{(\ln p)^2}{n^p}$
(B) $\sum_{n=1}^{\infty} \frac{(\ln n)(\ln p)}{n^p}$
(C) $\sum_{n=1}^{\infty} \frac{-(\ln n)(\ln p)}{n^p}$
(D) $\sum_{n=1}^{\infty} \frac{-(\ln n)^2}{n^p}$
(E) $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^p}$

Your answer: ____________________________

(7) What can you say about the nature of the function ζ on the interval $(1, \infty)$?
(A) Increasing and concave up
(B) Increasing and concave down
(C) Decreasing and concave up
(D) Decreasing and concave down
(E) Decreasing, initially concave down, then concave up

Your answer: ____________________________