TAKE-HOME CLASS QUIZ: DUE NOVEMBER 2: SERIES CONVERGENCE

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________________

YOU ARE FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE MAKE SURE TO ONLY ENTER ANSWER CHOICES THAT YOU PERSONALLY ENDORSE

(1) Suppose p is a polynomial that take positive values on all nonnegative integers. Consider the summation $\sum_{k=1}^{\infty} \frac{(k^2+1)^{2/3}}{p(k)}$. Under what conditions does the summation converge? Note that the degree of p must be a nonnegative integer.
(A) The summation converges if and only if the degree of p is *at least* one
(B) The summation converges if and only if the degree of p is *at least* two
(C) The summation converges if and only if the degree of p is *at least* three
(D) The summation converges if and only if the degree of p is *at most* two
(E) The summation converges if and only if the degree of p is *at most* one

Your answer: ____________________________

(2) Suppose p is a polynomial that take positive values on all nonnegative integers. Consider the summation $\sum_{k=1}^{\infty} \frac{(-1)^k (k^2+1)^{2/3}}{p(k)}$. Under what conditions does the summation converge? Note that the degree of p must be a nonnegative integer.
(A) The summation converges if and only if the degree of p is *at least* one
(B) The summation converges if and only if the degree of p is *at least* two
(C) The summation converges if and only if the degree of p is *at least* three
(D) The summation converges if and only if the degree of p is *at most* two
(E) The summation converges if and only if the degree of p is *at most* one

Your answer: ____________________________

(3) Which of the following series converges? Assume for all series that the starting point of summation is large enough that the terms are well defined. *Two years ago:* 11/25 *correct*
(A) $\sum 1/(k \ln(\ln k))$
(B) $\sum 1/(k \ln k)$
(C) $\sum 1/(k(\ln(\ln k))^2)$
(D) $\sum 1/(k(\ln k)(\ln(\ln k)))$
(E) $\sum 1/(k(\ln k)(\ln(\ln k))^2)$

Your answer: ____________________________

(4) Which of the following series converges? *Two years ago:* 23/25 *correct*
(A) $\sum \frac{k+\sin k}{k^2+1}$
(B) $\sum \frac{k+\cos k}{k^3+1}$
(C) $\sum \frac{k^2-\sin k}{k+1}$
(D) $\sum \frac{k^3+\cos k}{k^2+1}$
(E) $\sum \frac{k}{\sin(k^3+1)}$

Your answer: ____________________________

(5) Consider the series $\sum_{k=0}^{\infty} \frac{1}{2^{2^k}}$. What can we say about the sum of this series? *Two years ago:* 14/26 *correct*

(A) It is finite and strictly between 0 and 1.
(B) It is finite and equal to 1.
(C) It is finite and strictly between 1 and 2.
(D) It is finite and equal to 2.
(E) It is infinite.

Your answer: ____________________________

(6) For one of the following functions f on $(0, \infty)$, the integral $\int_0^\infty f(x)\,dx$ converges but $\int_0^\infty |f(x)|\,dx$ does not converge. What is that function f? (Note that this is similar to, but not quite the same as, the absolute versus conditional convergence notion for series).

(A) $f(x) = \sin x$
(B) $f(x) = \sin(\sin x)$
(C) $f(x) = (\sin\sqrt{x})/\sqrt{x}$
(D) $f(x) = (\sin x)/x$
(E) $f(x) = (\sin^3 x)/x^3$

Your answer: ____________________________

(7) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Since it is a series of positive terms, this means that the partial sums get arbitrarily large. What is the approximate smallest value of N such that $\sum_{n=1}^{N} \frac{1}{n} > 100$? *Two years ago:* 14/26 *correct*

(A) Between 90 and 110
(B) Between 2000 and 3000
(C) Between 10^{40} and 10^{50}
(D) Between 10^{90} and 10^{110}
(E) Between 10^{220} and 10^{250}

Your answer: ____________________________

(8) Consider the series:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

Which of the following is **true** about the series?

(A) Every rearrangement of the series converges to $\pi^2/6$.
(B) Every rearrangement of the series converges to $\ln 2$.
(C) The series converges to $\pi^2/6$ and there is a rearrangement of the series that converges to $\ln 2$.
(D) The series converges to $\ln 2$ and there is a rearrangement of the series that converges to $\pi^2/6$.
(E) The series does not converge.

Your answer: ____________________________

(9) Consider a series $\sum a_n$ whose terms have alternating signs, with the first term (i.e., a_1) positive in sign, such that $|a_n|$ form a decreasing sequence and $\lim_{n\to\infty} |a_n| = 1$. Let $b_k = \sum_{n=1}^{2k-1} a_n$ (so these are the sums of odd numbers of initial terms), with $b = \lim_{k\to\infty} b_k$, and $c_k = \sum_{n=1}^{2k} a_n$ (so these are sums of even numbers of initial terms), with $c = \lim_{k\to\infty} c_k$. Which of the following is **true** about the sequences b_k and c_k and the limits b and c? *Hint: This is similar to the alternating series theorem. Make a picture of the number line and hop on it.*

(A) b_k form an increasing sequence, c_k form a decreasing sequence, and $b = c$.
(B) b_ks form an increasing sequence, c_k form a decreasing sequence, and $b - c = 1$
(C) b_k form a decreasing sequence, c_k form an increasing sequence, and $b - c = 1$
(D) b_k form an increasing sequence, c_k form a decreasing sequence, and $c - b = 1$

(E) b_k form a decreasing sequence, c_k form an increasing sequence, and $c - b = 1$

Your answer: ____________________________

For the next few questions, let $T : \mathbb{R} \to \mathbb{R}$ be a function defined as follows. For $a \in \mathbb{R}$, define:

$$T(a) := \sum_{n=1}^{\infty} \frac{1}{n^2 + a^2}$$

Note that T is well defined because the summation on the right converges for every a.

(10) Which of the following function types does T have?
(A) Constant
(B) Linear
(C) Periodic
(D) Odd
(E) Even

Your answer: ____________________________

(11) Which of the following is true about T?
(A) T attains its absolute maximum at 0 and has no absolute minimum
(B) T attains its absolute maximum at 0 and its absolute minimum at 1 and -1
(C) T attains its absolute minimum at 0 and has no absolute maximum
(D) T attains its absolute minimum at 0 and its absolute maximum at 1 and -1
(E) T has no absolute maximum or absolute minimum

Your answer: ____________________________

(12) How can we express the summation:

$$\sum_{n=1}^{\infty} \frac{1}{n^4 + 3n^2 + 2}$$

in terms of T?
(A) $T(1) - T(2)$
(B) $T(2) - T(1)$
(C) $T(1) + T(2)$
(D) $T(1) - T(\sqrt{2})$
(E) $T(\sqrt{2}) - T(1)$

Your answer: ____________________________