User's Guide for complexity: a LATEX package, Version 0.75

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March 22nd, 2005

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1 Introduction

1.1 What is complexity?

complexity is a IATEX package that typesets computational complexity classes such as P (deterministic polynomial time) and NP (nondeterministic polynomial time) as well as sets (languages) such as SAT (satisfiability). In all, over 350 commands are defined for helping you to typeset Computational Complexity constructs.

1.2 Why a complexity package?

A better question is why not? Complexity theory is a more recent, though mature area of Theoretical Computer Science. Each researcher seems to have his or her own preferences as to how to typeset Complexity Classes and has built up their own personal LATEX commands file. This can be frustrating, to say the least, when it comes to collaborations or when one has to go through an entire series of files changing commands for compatibility or to get exactly the look they want (or what may be required). I find it amazing that a package hasn't been written already! (Maybe there has been, but I've not found it).

I thought the best solution would be to develop a single LATEX package to handle all of this for us.

2 Installation

You should place the complexity directory and its files (complexity.sty and mycomplexity.sty as well the documentation files if you like) in a place that your T_FX distribution will be able to find them.

If you use MiKT_EX then you should place it in your localtexmf directory, making sure to refresh your T_EX tree (MiKT_EX options wizard \rightarrow General \rightarrow File Names

Database, Refresh Now).

In a unix TEX distribution, you may find it useful to use a local LATEX path (i.e. creating a directory /usr/local/username/mylatexfolder/ and tell your TEX distribution where to find it. Depending on your distribution, you might declare an environmental variable (say in your .cshrc file) like

setenv \$TEXINPUTS = /usr/local/texdistro//:/usr/local/username/mylatexfolder//
(note that the double slash will recursively search all subdirectories).

3 Package Options

The complexity package provides two general options—a *font* option (of which there are three classes) and a *mode* option. The font option specifies what font the complexity classes (as well as functions and languages) are typeset in while the mode option specifies *how many* complexity classes are defined.

One specifies these options in the usual manner. When you use the package, you can pass it the options you wish; for example, calling the package with

\usepackage[bold,full]{complexity}

specifies that classes (and languages) should be typeset in bold and that the full list of classes should be defined. Invalid options are ignored and only the last option (of each type) is used if multiple, conflicting options are given. The complete options are described in the next two subsections.

3.1 Mode Options

The mode options specify to what extent the package declares commands for complexity classes. By default, every (supported) class command is defined. Alternatively, you can limit the number of commands the complexity package defines (and perhaps limit conflicts with other packages or your own commands) by using the basic option. This option defines only the most commonly used complexity classes.¹

full (*Default*) This option will load *every* complexity class that the package has defined. See Section 4.5 for a complete list.

basic This option will only load the "standard" complexity classes so as to minimize the number of commands the package defines (i.e. standard classes like P and NP but not less well known classes like AWPP (Almost wide PP).

¹Deciding which classes were most "common" was purely based on my judgement, for better or worse. If I've not considered your favorite complexity class as "common," I humbly apologize.

3.2 Font Options

Different researchers and different publications have their own preferences for how to typeset complexity classes. The beauty of the complexity package is that it not only defines a whole sleigh of complexity classes for you, but it also allows you to change the font they are typeset in with a simple option call.

The complexity package defines three different font entities: a font for complexity classes (classfont), a font for languages (langfont), and a font for functions (funcfont). By default, all of these fonts are typeset using the mathsf font. You can change the font for all of them together or specify a font for each individually. To apply a single font to all three entities, simply pass the font (by itself) as an option. The supported font options are as follows.

sanserif (Default) This typesets the classes in a \mathsf (sans serif) font.

roman This option typesets the classes in a \mathrm (roman) font.

bold This option typesets the classes in a \mathbf (roman, bold) font.

typewriter This option typesets the classes in a \mathtt (typewriter) font.

italic This option typesets the classes in a \mathit (math italic) font.

caps This option typesets the classes in a \textsc (small caps font) font.

slant This option typesets the classes in a \textsl (slanted font) font.

As an alternative, you can specify a different font for each of the three entities. To do this, you simply qualify the font with a key-value pair: either classfont, langfont, or function. For example, if we want our complexity classes to be typeset in bold, our languages to be typeset in roman and our functions to be typeset in italic, we would call the package using:

Examples of how each of the fonts appears when typeset can be found in Table 1.

3.2.1 The small Option

A special option is the small option and pertains only to how complexity classes (classfont) are typeset. Since classes are typeset in uppercase letters, they tend to be more dominant. This is not so important for classes such as P and NP, but if you are referencing classes such as PSPACE or DTIME it can interrupt the normal

Table 1: An Example of each font Font classfont langfont funcfont $CVP \leq_m SAT$, $P \subseteq NP$, $polylog \in O(poly),$ sanserif $\mathsf{PSPACE} \subseteq \mathsf{EXP}$ $SAT <_T MaxSAT$ $\mathsf{polylog} \in \Omega(\mathsf{log})$ $CVP \leq_m SAT$, $polylog \in O(poly),$ roman $P \subseteq NP$, $PSPACE \subseteq EXP$ $SAT \leq_T MaxSAT$ $\operatorname{polylog} \in \Omega(\log)$ $P \subseteq NP$, $\mathbf{CVP} \leq_m \mathbf{SAT},$ bold $\operatorname{\mathbf{polylog}} \in O(\operatorname{\mathbf{poly}}),$ $SAT \leq_T MaxSAT$ $\mathbf{PSPACE}\subseteq\mathbf{EXP}$ $polylog \in \Omega(log)$ $CVP \leq_m SAT$, typewriter $P \subseteq NP$, $polylog \in O(poly),$ $\mathtt{PSPACE}\subseteq\mathtt{EXP}$ $\mathtt{SAT} \leq_T \mathtt{MaxSAT}$ $polylog \in \Omega(log)$ $P \subseteq NP$, $CVP \leq_m SAT$, $polylog \in O(poly),$ italic $SAT \leq_T MaxSAT$ $PSPACE \subseteq EXP$ $polylog \in \Omega(log)$ $\text{CVP} \leq_m \text{SAT},$ caps $P \subseteq NP$, $POLYLOG \in O(POLY),$ $PSPACE \subseteq EXP$ $SAT \leq_T MAXSAT$ $POLYLOG \in \Omega(LOG)$ Better example: $PROMISERP \subseteq PROMISEBPP$ $P \subseteq NP$, $CVP \leq_m SAT$, $polylog \in O(poly),$ slant $SAT \leq_T MaxSAT$ $PSPACE \subseteq EXP$ $polylog \in \Omega(log)$

flow of text layouts. One solution to this is to typeset classes 1pt smaller than the surrounding text. This is the approach taken in some texts (most notably, Papadimitriou's book *Computational Complexity*, 1994) and it works quite well. To illustrate, consider the following:

There are deterministic classes such as PSPACE PSPACE, nondeterministic classes such as NP NP, and functional classes such as GapP GapP. But I like them all.

In the preceding pairs, the first was typeset in the document's default font size while the second was typeset 1pt smaller (internally, the \small command is used). The difference is subtle but when used in a long text, flows more naturally.

To get the same effect using complexity, simply use the small option (i.e. \usepackage[small]{complexity} with any combination of the other options (it works for all fonts, but some do not look as good as others; typewriter for example looks bad with this option). Remember, however that this option only affects how classes are typeset, not languages.

It should be noted that this option only affects how classes are typeset in the display and in-line mathmodes. It has no effect in, say, a footnote or some special environment. Subscripts, superscripts (as well as subsubscripts and supersuperscripts) are not effected either—TEX is allowed to automatically change font sizes for these cases.

4 Using the Package

Each of the commands is defined using \ensuremath so that you need not be in LATEX's mathmode to use them. A word of warning, however, if you use a command outside of mathmode, TEX may not properly insert surrounding whitespace. Thus, its best to always use complexity commands inside mathmode. A complete list of commands for classes can be found in Section 4.5.

4.1 Overridden Commands

Three commands in the complexity package override built-in T_EX commands. Specifically, L (which typesets the symbol L), P (typesetting \P), and S (which typesets the symbol R) are all redefined for use in the package. The complexity package preserves these commands so that you may still use them. To use any of these symbols, use the commands $\det L$, $\det L$, and $\det L$.

Additionally, it may be the case that other LATEX packages are loaded that already define (or redefine) some of the commands in the complexity package. If this is the case, please email me so that I can work something out for future updates. The quick solution is to simply comment out the definition of the conflicting command in complexity.sty and directly use \ComplexityFont{} to typeset your complexity class (see also Section 5 - Customization).

4.2 Special Commands

In addition to complexity classes, the complexity package also conveniently defines several commands for commonly used functions and languages. In particular, co (ex: co) and parity (an alias for plus, typesetting ellip) can be placed preceding a class to refer to the complement or counting versions respectively.

4.3 Function Commands

complexity defines several general classes of functions such as logarithms and polynomials. Table 2 gives a complete list of these functions.

Command	Result	Table 2: func Commands Comment
\llog	log	Denotes logarithmic functions. Note that the command is invoked with <i>two</i> l's so as to not interfere with the LATEX \log command.
\poly	poly	Denotes polynomial functions
\polylog	polylog	Denotes polylogarithmic functions
\qpoly	qpoly	Denotes polynomial functions for quantum ad-
		vice
\qlog	qlog	Denotes logarithmic functions for quantum
		advice
\MOD	MOD	Used for Modular classes/functions
\Mod	Mod	Used for Modular classes/functions

4.4 Language Commands

complexity also defines commands to typeset languages (subsets of $\{0,1\}^*$). A complete list of predefined language commands can be found in Table 3 below. The number of commands is sparse; this was intentional. How one refers to languages is far less standard than how one refers to classes. Some people like to explicitly write every word (WeightedHamiltonianCycle, or WEIGHTED HAMILTONIAN CYCLE), while others have their own abbreviations. Keeping the number of languages complexity defines to a minimum allows for the maximum flexibility.

Command	Table 3: Result	Special complexity Commands Comment
\CVP	CVP	Used for the Circuit Value Problem (a P-
\SAT \MaxSAT	SAT MaxSAT	complete set) Used for Satisfiability (an NP-complete set) Used for the Lexicographically maximum sat-
		is fiability optimization problem (complete for OptP)

4.5 Complete List of Class Commands

A complete list (in alpha-numeric order according to the command name) of complexity commands is given below. The first item in each row is the command itself. The second is an example of how it is typeset using the default sanserif font. Finally, the third item indicates which mode the command is defined in.

	4.6	
\AC	AC	basic
\A	A	full
\ACC	ACC	basic
\AH	AH	basic
\AL	AL	basic
\AlgP	AlgP	full
\AM	AM	basic
\AMEXP	AM-EXP	basic
\Amp	Amp	full
\AmpMP	AmpMP	full
\AmpPBQP	AmpPBQP	full
\AP	AP	basic
\APP	APP	full
\APX	APX	full
\AUCSPACE	AUC-SPACE	full
\AuxPDA	AuxPDA	full
\AVBPP	AVBPP	full
\AvE	AvE	full
\AvP	AvP	full
\AW	AW	full
\AWPP	AWPP	full
\betaP	βP	full
\BH	BH	basic
\BP	BP	full
\BPE	BPE	basic
\BPEE	BPEE	basic
\BPHSPACE	BP _H SPACE	full
\BPL	BPL	full
\BPP	BPP	basic
\BPPOBDD	BPP-OBDD	full
\BPPpath	BPP _{path}	full
\BPQP	BPQP	full
\BPSPACE	BPSPACE	basic
\BPTIME	BPTIME	basic
\BQNC	BQNC	full
\BQNP	BQNP	full
•	BQP	basic
\BQP		full
\BQPOBDD	BQP-OBDD	
\BQTIME	BQTIME	basic

\C	C	basic
\cc	СС	basic
\CeL	$C_{=}L$	basic
\CeP	$C_{=}P$	basic
\CFL	CFL	basic
\CH	CH	basic
\CkP	C_kP	basic
\CLOG	CLOG	full
\CNP	CNP	full
\coAM	coAM	basic
\coBPP	coBPP	basic
\coCeP	$coC_{=}P$	basic
\cofrIP	cofrIP	full
\Coh	Coh	full
\coMA	coMA	basic
\compIP	compIP	full
\compNP	compNP	full
\coNE	coNE	basic
\coNEXP	coNEXP	basic
\coNL	coNL	basic
\coNP	coNP	basic
\coNQP	coNQP	basic
\coRE	coRE	basic
\coRNC	coRNC	basic
\coRP	coRP	basic
\coSL	coSL	basic
\coUCC	coUCC	full
\coUP	coUP	basic
\CP	CP	full
\CSIZE	CSIZE	basic
\CSL	CSL	full
\CZK	CZK	full
\D	D	full
\DCFL	DCFL	full
\DET	DET	basic
\DiffAC	DiffAC	full
\DisNP	DisNP	full
\DistNP	DistNP	full
\DP	DP	full
\DQP	DQP	full
\DSPACE	DSPACE	basic
\DTIME	DTIME	basic
\DTISP	DTISP	basic
\Dyn	Dyn	full
\DynF0	Dyn-FO	full
\E	E	basic

\EE ΕE basic \EEE **EEE** basic **EESPACE** basic **\EESPACE EEXP** \EEXP basic \EH EΗ basic \EL EL full **ELEMENTARY \ELEMENTARY** full \ELkP $\mathsf{EL}_k\mathsf{P}$ full **EPTAS \EPTAS** basic **EQBP** full \EQBP \EQP **EQP** full \EQTIME **EQTIME** full **\ESPACE ESPACE** basic ${\sf ExistsBPP}$ full \ExistsBPP \ExistsNISZK ExistsNISZK full \EXP **EXP** basic **\EXPSPACE EXPSPACE** basic **FBQP** \FBQP full \Few Few full full \FewP FewP FΗ full \FH FNL basic \FNL **FNP** \FNP basic FO full \F0 \FOLL **FOLL** full FΡ \FP basic \FPR FPR full **FPRAS** \FPRAS basic \FPT **FPT** full \FPTAS **FPTAS** full FPT_{nu} full \FPTnu $\mathsf{FPT}_{\mathsf{su}}$ \FPTsu full \FQMA **FQMA** basic \frIP frIP full F-TAPE \FTAPE full \FTIME F-TIME full full \G G \GA GA basic **GAN-SPACE** \GANSPACE full \Gap Gap basic \GapAC GapAC basic \GapL GapL basic\GapP GapP basic \GC GC full **GCSL** full \GCSL \GI GI basic \GPCD **GPCD** full \Heur basic Heur ${\sf HeurBPP}$ basic \HeurBPP HeurBPTIME basic \HeurBPTIME \HkP H_kP full \HSPACE **HSPACE** basic **HVSZK** full \HVSZK IC full \IC IΡ basic \IP \IPP **IPP** full \K Κ basic \kBQBP $k ext{-}\mathsf{BQBP}$ full \kBWBP $k ext{-BWBP}$ full full k-EQBP \kEQBP \kPBP k-PBP full \KT ΚT basic \L L basic \LIN LIN basic \LkP $\mathsf{L}_k\mathsf{P}$ full LOGCFL full \LOGCFL basic \LogFew LogFew basic \LogFewNL LogFewNL \LOGNP LOGNP full **LOGSNP** full \LOGSNP **LWPP** \LWPP full Μ full \M \MA MA basic MAC \MAC basic MA-E basic \MAE \MAEXP MA-EXP basic \mbox{mAL} mAL basic\MaxNP MaxNP basic MaxPB basic \MaxPB \MaxSNP MaxSNP basic $\mbox{\mbox{$\mbox{mcoNL}$}}$ comNLbasic MinPB basic \MinPB MIP \MIP basic \MkP $(M_k)P$ full mL basic \mbox{mL} \mNC mNCbasic mNL \mbox{mNL} basic mNP \mbox{mNP} basic\ModkL $\mathsf{Mod}_k\mathsf{L}$ basic \ModkP $\mathsf{Mod}_k\mathsf{P}$ basic ModP \ModP basic $\mathsf{ModZ}_k\mathsf{L}$ full \ModZkL

 \mbox{mP} $\mathsf{m}\mathsf{P}$ basic \MP MP basic MPC basic \MPC mTC basic \mTC \NAuxPDA NAuxPDA full \NC NC basic ΝE basic \NE NEE basic \NEE \NEEE NEEE basic **NEEXP** basic \NEEXP \NEXP NEXP basic NIPZK full \NIPZK \NIQPZK NIQPZK full full **NIQSZK** \NIQSZK \NISZK NISZK full \NL NLbasic \NLIN NLIN basic full \NLOG **NLOG** \NP NPbasic NPC \NPC basic NPI basic \NPI **NPMV** full \NPMV \NPMVsel NPMV-sel full NPO full \NPO \NPOPB **NPOPB** full **NPSPACE** \NPSPACE basic **NPSV** \NPSV full NPSV-sel full \NPSVsel basic \NQP NQP \NSPACE **NSPACE** basic NT full \NT \NTIME NTIME basic OBDD full \OBDD \OCQ OCQ full \Opt Opt basic \OptP OptP basic basic ****p p \P Ρ basic PAC \PAC basic PBP full \PBP PCD \PCD basic full \Pclose P-close \PCP PCP basic \PermUP PermUP full \PEXP **PEXP** basic PF \PF full

\PFCHK **PFCHK** full \PH РΗ basic PhP full \PhP **PINC** \PINC full \PIO PIO full \PKC **PKC** full PL\PL basic PLfull \PLF PLL full \PLL PLS full \PLS \POBDD P-OBDD full \PODN **PODN** full polyL full \polyL full \PostBQP PostBQP \PP PP basic \PPA PPA full \PPAD **PPAD** full \PPADS **PPADS** full P/poly basic \Ppoly PPP \PPP full **PPSPACE** \PPSPACE basic **PQUERY** full \PQUERY \PR PR full $Pr_{H}SPACE$ \PrHSPACE full Promise basic \Promise PromiseBPP \PromiseBPP basic \PromiseBQP PromiseBQP basic PromiseP \PromiseP basic \PromiseRP PromiseRP basic \PrSPACE **PrSPACE** basic P-Sel full \PSel \PSK **PSK** full **PSPACE** basic \PSPACE \PT РΤ basic **PTAPE** \PTAPE full \PTAS **PTAS** basic PT/WK basic \PTWK \PZK PZK full QAC basic \QAC \QACC QACC basic \QAM QAM basic QCFL \QCFL basic\QCMA **QCMA** basic \QH QH basic QIP \QIP basic \QMA QMA basic \QMAM QMAM basic \QMIP QMIP basic full \QMIPle $QMIP_{le}$ full \QMIPne $QMIP_{ne}$ \QNC QNC basic \QP QΡ basic **QPLIN** full \QPLIN full \Qpoly Qpoly **QPSPACE** \QPSPACE basic full \QSZK **QSZK** \R R basic \RE RE basic **REG** \REG basic RevSPACE full \RevSPACE \RHL R_HL full full \RHSPACE **R_HSPACE** \RL RLbasic \RNC RNC basic \RNP **RNP** full RP basic \RP \RPP **RPP** full **RSPACE** \RSPACE basic \S S basic SAC \SAC basic \SAPTIME **SAPTIME** full \SBP SBP full SC \SC basic SE \SE basic \SEH SEH basic \Sel Sel full SelfNP full \SelfNP SF \SF full SIZE basic \SIZE \SKC SKC basic \SL SL basic \SLICEWISEPSPACE SLICEWISEPSPACE full SNP full \SNP \SOE SO-E full SP \SP full **SPACE** \SPACE basic span-P full \spanP **SPARSE** \SPARSE basicSPL \SPL basic \SPP SPP basic \SUBEXP **SUBEXP** basic symPfull \symP

\SZK SZK basic **TALLY** \TALLY full TC \TC basic **TFNP** \TFNP full \ThC ThC full \TreeBQP TreeBQP full TREE-REGULAR **\TREEREGULAR** full **UAP** \UAP full \UCC UCC full \UE UE full \UL UL full \UP UP basic \US US full **VNC** \VNC full **VNP** \VNP full VΡ full \VP **VQP** \VQP full W \W basic **WAPP** \WAPP full WPP \WPP full $XOR-MIP^{*}[2,1]$ \XORMIP full XΡ \XP full \XPuniform $XP_{uniform} \\$ full \YACC YACC full ZPE \ZPE basic **ZPP** \ZPP basic \ZPTIME **ZPTIME** basic

5 Customization

The complexity package provides some 350 commands to typeset complexity classes. However, that should not mean that the commands here are the *only* ones you'll ever need. Expanding the list of commands to suit your needs is very easy. Please note, however, it is preferred that you not alter the base style file (complexity.sty). Instead, a file is provided for you to define your commands in (mycomplexity.sty).

5.1 Class Commands

To define a new complexity class, simply use the LATEX command, \newcommand as follows. Say that we want to define the new complexity class, "VCCC" ("very complex complexity class"). We would use something like

```
\newcommand{\VCCC}{\ComplexityFont{VCCC}}
```

Then, anytime we wanted to typeset our new class, we simply use \$\VCCC\$. Internally, complexity typesets everything using the command \ComplexityFont which is setup at the invocation of the package.

You also may have different preferences for typesetting the classes that complexity already defines. For instance, the class promiseBPP (typeset using the command \promiseBPP) is typeset with "promise" explicitly written. Preferring brevity over clarity, you wish to typeset the same class as pBPP. To do this, we use the LATEX command, \renewcommand as follows.

```
\renewcommand{\promiseBPP}{\ComplexityFont{pBPP}}}
```

However, this only changes what the command does, not how we invoke it—we would still use \$\promiseBPP\$.

As another example, say we want to change how the class $\mathsf{Mod}_k\mathsf{L}$ (typeset using the command ModkL) is typeset. By default, the subscript k is typeset in regular mathmode. Say we want to change it so that it is typeset in the same font as the rest of the classes. We may use something like the following.

```
\renewcommand{\ModkL}{ %
    {
        \ComplexityFont{Mod}_{\ComplexityFont{k}}\ComplexityFont{L}
    }
}
```

Note the use of "extra" brackets. In your commands, more is always better (or at least safer); since we are using subscripts and superscripts, we want to ensure that if we use the \ModkL command itself in a subscript or superscript (say as an oracle) are typeset correctly.

5.2 Language Commands

You can define languages (to be typeset in the langfont) in a similar manner. Instead of using \ComplexityFont, however, you would use the command \lang. You can use \lang as a stand alone command in your document (i.e. \lang{Matching} \in \P\) or you can define a command (using \lang) that can be reused throughout your document. Again, we give an example. Say we wanted to typeset the language "Graph Non-Isomorphism" using the abbreviation, "GNI". We could define something like the following.

```
\newcommand{\GNI}{\lang{GNI}}
```

In our document, we would would use something like \$\GNI \in \AM\$. We can also redefine any predefined language commands using the \renewcommand command as before.

5.3 Function Commands

Again, the procedure for typesetting your own functions is the same as for classes. Here, however, you use the \func command. You can use it as a stand alone command (\func{\lin}(n) \in \Theta(n)\func or you can define a command that can be reused. Say we wanted to typeset a class of subexponential functions, say "subexp". We could define something like the following.

```
\newcommand{\subexp}{\func{subexp}}
```

In our document, we could then use $\sum (n) = 2^{o(n)}$.

6 Extended Example

Here, we present an extended example using the package. Consider the following T_EX code.

```
\documentclass{article}
\usepackage{complexity}
\begin{document}
It follows immediately from the definitions of \Phi \ and \Phi \ that
$$\P \subseteq \NP$$
but the million dollar question is whether or not $\P
\stackrel{?}{=} \NP$. In an effort to resolve this question,
Stockmeyer (1976) defined a \emph{polynomial} hierarchy using
oracles.
\textbf{Definition}[Stockmeyer 1976] \\
Let \Delta_0P = \sigma_0P = Pi_0P = P. Then for i > 0, let
\begin{itemize}
   \t \ \Delta_i\P = \P\$ with a \Sigma_{i-1}\P\$ oracle.
   \star \ $\Sigma_i\P = \NP$ with $\Sigma_{i-1}\P$ oracle.
   \star \ item \pi_i P = \ with \pi_{i-1}\ oracle.
\end{itemize}
Then $\PH$ is the union of these classes for all nonnegative
constant $i$.
It has been shown that $\PH \subseteq \PSPACE$. Moreover, Toda
(1989) showed the following
\textbf{Theorem}
$$\PH \subseteq \P^\PP$$
and since since P^P = P^{\star} it follows that
\ \PH \subseteq \P^{\#\P}$$
\end{document}
```

Would produce something like the following:

It follows immediately from the definitions of P and NP that

$$\mathsf{P}\subseteq\mathsf{NP}$$

but the million dollar question is whether or not $P \stackrel{?}{=} NP$. In an effort to resolve this question, Stockmeyer (1976) defined a *polynomial* hierarchy using oracles.

Definition[Stockmeyer 1976]

Let $\Delta_0 P = \Sigma_0 P = \Pi_0 P = P$. Then for i > 0, let

- $\Delta_i P = P$ with a $\Sigma_{i-1} P$ oracle.
- $\Sigma_i P = NP$ with $\Sigma_{i-1} P$ oracle.
- $\Pi_i P = \text{coNP with } \Sigma_{i-1} P \text{ oracle.}$

Then PH is the union of these classes for all nonnegative constant i.

It has been shown that $PH \subseteq PSPACE$. Moreover, Toda (1989) showed the following.

Theorem

$$\mathsf{PH} \subset \mathsf{P^{PP}}$$

and since since $P^{PP} = P^{\#P}$ it follows that

$$\mathsf{PH} \subset \mathsf{P}^{\#\mathsf{P}}$$

For an even more complicated example, check out the LATEXed (PDF) version of the Complexity Zoo (http://www.ComplexityZoo.com) available on my webpage (http://www.cse.unl.edu/~cbourke)

7 Feedback

I'd very much appreciate feedback that would improve this package. Specifically, I'm looking for the following.

- Inconsistencies in (or suggestions for better) notation
- Errors, Typos, etc
- Incompatibilities with other packages
- Feature requests

You can email me at cbourke@cse.unl.edu

7.1 Acknowledgements

I'd like to thank Till Tantau for several useful suggestions and feature requests as well as some clever code segments for the ${\tt small}$ option.