CLASS QUIZ: MARCH 4: SERIES CONVERGENCE

MATH 153, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Consider the series $\sum_{k=0}^{\infty} \frac{1}{2^{2^k}}$. What can we say about the sum of this series?
  (A) It is finite and strictly between 0 and 1.
  (B) It is finite and equal to 1.
  (C) It is finite and strictly between 1 and 2.
  (D) It is finite and equal to 2.
  (E) It is infinite.

  Your answer: ____________________________

(2) Consider the function $F(x,p) := \sum_{n=1}^{\infty} \frac{x^n}{n^p}$ with x and p both real numbers. For what values of x and what values of p does this summation converge?
  (A) For $|x| < 1$, it converges for all $p \in \mathbb{R}$. For $|x| \geq 1$, it does not converge for any p.
  (B) For $|x| \leq 1$, it converges for all $p \in \mathbb{R}$. For $|x| > 1$, it does not converge for any p.
  (C) For $|x| < 1$, it converges for all $p \in \mathbb{R}$. For $|x| > 1$, it does not converge for any p. For $|x| = 1$, it converges if and only if $p > 1$.
  (D) For $|x| < 1$, it converges for all $p \in \mathbb{R}$. For $|x| > 1$, it does not converge for any p. For $x = 1$, it converges if and only if $p > 0$. For $x = -1$, it converges if and only if $p > 1$.
  (E) For $|x| < 1$, it converges for all $p \in \mathbb{R}$. For $|x| > 1$, it does not converge for any $p \in \mathbb{R}$. For $x = 1$, it converges if and only if $p > 1$. For $x = -1$, it converges if and only if $p > 0$.

  Your answer: ____________________________

There is a result of calculus which states that, under suitable conditions, if $f_1, f_2, \dots, f_n, \dots$ are all functions, and we define $f(x) := \sum_{n=1}^{\infty} f_n(x)$, then $f'(x) = \sum_{n=1}^{\infty} f_n'(x)$. In other words, under suitable assumptions, we can differentiate a sum of countably many functions by differentiating each of them and adding up the derivatives.

We will not be going into what those assumptions are, but will consider some applications where you are explicitly told that these assumptions are satisfied.

(3) Consider the summation $\zeta(p) := \sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 1$. Assume that the required assumptions are valid for this summation, so that $\zeta'(p)$ is the sum of the derivatives of each of the terms (summands) with respect to p. What is the correct expression for $\zeta'(p)$?
  (A) $\sum_{n=1}^{\infty} \frac{-p}{n^{p+1}}$
  (B) $\sum_{n=1}^{\infty} \frac{-1}{(p+1)n^{p+1}}$
  (C) $\sum_{n=1}^{\infty} \frac{p}{n^{p-1}}$
  (D) $\sum_{n=1}^{\infty} \frac{-\ln n}{n^p}$
  (E) $\sum_{n=1}^{\infty} \frac{-\ln n}{n^{p+1}}$

  Your answer: ____________________________

(4) Going back to question 2, recall that we defined $F(x,p) := \sum_{n=1}^{\infty} \frac{x^n}{n^p}$ with x and p both real numbers. Assume that, for a particular fixed value of p, the summation satisfies the conditions as a function of x for $|x| < 1$. What is its derivative with respect to x, keeping p constant?
  (A) $\sum_{n=1}^{\infty} \frac{x^{n+1}}{n^{p+1}}$
  (B) $\sum_{n=1}^{\infty} \frac{x^{n+1}}{n^{p-1}}$

(C) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n^{p+1}}$
(D) $\sum_{n=1}^{\infty} \frac{x^{n-1} \ln n}{n^{p+1}}$
(E) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n^{p-1}}$

Your answer: ____________________________

(5) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Since it is a series of positive terms, this means that the partial sums get arbitrarily large. What is the approximate smallest value of N such that $\sum_{n=1}^{N} \frac{1}{n} > 100$?
(A) Between 90 and 110
(B) Between 2000 and 3000
(C) Between 10^{40} and 10^{50}
(D) Between 10^{90} and 10^{110}
(E) Between 10^{220} and 10^{250}

Your answer: ____________________________

(6) Consider the function $f(x) := \sum_{n=1}^{\infty} \frac{x^n}{n(n+2)}$ defined on the closed interval $[-1, 1]$. What are the values of $f(1)$ and $f(-1)$?
(A) $f(1) = 3/4$ and $f(-1) = 1/4$
(B) $f(1) = 3/4$ and $f(-1) = -1/4$
(C) $f(1) = 3/4$ and $f(-1) = -3/4$
(D) $f(1) = 1/4$ and $f(-1) = 3/4$
(E) $f(1) = 1/4$ and $f(-1) = -1/4$

Your answer: ____________________________

(7) Given that we have the following: $\sum_{n=1}^{\infty} x^n/n = -\ln(1-x)$ for all $-1 < x < 1$ and the series converges absolutely in the interval, what is an explicit expression for the summation $\sum_{n=1}^{\infty} x^n/(n(n+1))$ for $x \in (-1, 1) \setminus \{0\}$?
(A) $1 + \ln(1-x)$
(B) $1 - \ln(1-x)$
(C) $1 + \frac{(1+x)\ln(1-x)}{x}$
(D) $1 + \frac{(1-x)\ln(1-x)}{x}$
(E) $1 + \frac{(x-1)\ln(1-x)}{x}$

Your answer: ____________________________

(8) Given that we have the following: $\sum_{n=1}^{\infty} x^n/n = -\ln(1-x)$ for all $-1 < x < 1$ and the series converges absolutely in the interval, what is an explicit expression for the summation $\sum_{n=1}^{\infty} x^n/(n(n+2))$ for $x \in (-1, 1) \setminus \{0\}$?
(A) $\frac{1}{4} + \frac{1}{2x} + \frac{(1-x^2)\ln(1-x)}{2x^2}$
(B) $\frac{1}{4} + \frac{1}{2x} + \frac{(x^2-1)\ln(1-x)}{2x^2}$
(C) $\frac{1}{4} - \frac{1}{2x} + \frac{(x^2-1)\ln(1-x)}{2x^2}$
(D) $\frac{1}{4} - \frac{1}{2x} + \frac{(1-x^2)\ln(1-x)}{2x^2}$
(E) $\frac{1}{4} + \frac{1}{2x}$

Your answer: ____________________________