CLASS QUIZ (TAKE-HOME): FEBRUARY 14: SEQUENCES AND MISCELLANEA

MATH 153, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

Please attempt these quiz questions prior to class and turn them in during class on Monday February 14.

(1) Consider the sequence $a_n = 2a_{n-1} - \alpha$, with $a_1 = \beta$, for α, β real numbers. What can we say about this sequence for sure?
  (A) (a_n) is eventually increasing for all values of α, β.
  (B) (a_n) is eventually decreasing for all values of α, β.
  (C) (a_n) is eventually constant for all values of α, β.
  (D) (a_n) is either increasing or decreasing, and which case occurs depends on the values of α and β.
  (E) (a_n) is increasing, decreasing, or constant, and which case occurs depends on the values of α and β.

  Your answer: ____________________________

(2) *This is a generalization of the preceding question.* Suppose f is a continuous increasing function on $\mathbb{R}$. Define a sequence recursively by $a_n = f(a_{n-1})$, with a_1 chosen separately. What can we say about this sequence for sure?
  (A) (a_n) is eventually increasing regardless of the choice of a_1.
  (B) (a_n) is eventually decreasing regardless of the choice of a_1.
  (C) (a_n) is eventually constant regardless of the choice of a_1.
  (D) (a_n) is either increasing or decreasing, and which case occurs depends on the value of a_1 and the nature of f.
  (E) (a_n) is increasing, decreasing, or constant, and which case occurs depends on the value of a_1 and the nature of f.

  Your answer: ____________________________

For a function $f : \mathbb{R} \to \mathbb{R}$ and a particular element $a \in \mathbb{R}$, define $g : \mathbb{N} \to \mathbb{R}$ by $g(n) = f(f(\dots(f(a))\dots))$ with the f occurring $n-1$ times. Thus, $g(1) = a$, $g(2) = f(a)$, and so on. Choose the right expression for g for each of these choices of f.

(3) $f(x) := x + \pi$.
  (A) $g(n) := a + n\pi$.
  (B) $g(n) := a + n\pi - 1$.
  (C) $g(n) := a + n(\pi - 1)$.
  (D) $g(n) := a + \pi(n - 1)$.
  (E) $g(n) := \pi + n(a - 1)$.

  Your answer: ____________________________

(4) $f(x) := mx$, $m \neq 0$.
  (a) $g(n) := mna$.
  (b) $g(n) := m^n a$.
  (c) $g(n) := n^m a$.
  (d) $g(n) := m^{n-1} a$.
  (e) $g(n) := n^{m-1} a$.

  Your answer: ____________________________

(5) $f(x) := x^2$.
(A) $g(n) := a^{2^n} - 1$.
(B) $g(n) := a^{2^n - 1}$.
(C) $g(n) := a^{2^{n-1}}$.
(D) $g(n) := a^{{2^n}^{-1}}$.
(E) $g(n) := (a^{2^n})^{-1}$.

Your answer: ____________________________

(6) One of these sequences can *not* be obtained using the procedure described in the previous questions (i.e., iterated application of a function). Identify this sequence. Only the first five terms of the sequence are presented:
(A) $1, 2, 3, 3, 3$
(B) $1, 2, 3, 2, 3$
(C) $1, 2, 3, 2, 1$
(D) $1, 2, 3, 4, 5$
(E) $1, 2, 3, 4, 3$

Your answer: ____________________________

(7) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function. Identify which of these definitions is *not* correct for $\lim_{x \to c} f(x) = L$, where c and L are both finite real numbers.
(A) For every $\epsilon > 0$, there exists $\delta > 0$ such that if $x \in (c - \delta, c + \delta) \setminus \{c\}$, then $f(x) \in (L - \epsilon, L + \epsilon)$.
(B) For every $\epsilon_1 > 0$ and $\epsilon_2 > 0$, there exist $\delta_1 > 0$ and $\delta_2 > 0$ such that if $x \in (c - \delta_1, c + \delta_2) \setminus \{c\}$, then $f(x) \in (L - \epsilon_1, L + \epsilon_2)$.
(C) For every $\epsilon_1 > 0$ and $\epsilon_2 > 0$, there exists $\delta > 0$ such that if $x \in (c - \delta, c + \delta) \setminus \{c\}$, then $f(x) \in (L - \epsilon_1, L + \epsilon_2)$.
(D) For every $\epsilon > 0$, there exist $\delta_1 > 0$ and $\delta_2 > 0$ such that if $x \in (c - \delta_1, c + \delta_2) \setminus \{c\}$, then $f(x) \in (L - \epsilon, L + \epsilon)$.
(E) None of these, i.e., all definitions are correct.

Your answer: ____________________________

(8) In the usual $\epsilon - \delta$ definition of limit for a given limit $\lim_{x \to c} f(x) = L$, if a given value $\delta > 0$ works for a given value $\epsilon > 0$, then which of the following is true?
(A) Every smaller positive value of δ works for the same ϵ. Also, the given value of δ works for every smaller positive value of ϵ.
(B) Every smaller positive value of δ works for the same ϵ. Also, the given value of δ works for every larger value of ϵ.
(C) Every larger value of δ works for the same ϵ. Also, the given value of δ works for every smaller positive value of ϵ.
(D) Every larger value of δ works for the same ϵ. Also, the given value of δ works for every larger value of ϵ.
(E) None of the above statements need always be true.

Your answer: ____________________________

(9) In the usual $\epsilon - \delta$ definition of limit, we find that the value $\delta = 0.2$ for $\epsilon = 0.7$ for a function f at 0, and the value $\delta = 0.5$ works for $\epsilon = 1.6$ for a function g at 0. What value of δ *definitely* works for $\epsilon = 2.3$ for the function $f + g$ at 0?
(A) 0.2
(B) 0.3
(C) 0.5
(D) 0.7

(E) 0.9

Your answer: ____________________________

(10) The sum of limits theorem states that $\lim_{x\to c}[f(x)+g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$ *if* the right side is defined. One of the choices below gives an example where the left side of the equality is defined and finite but the right side makes no sense. Identify the correct choice.
(A) $f(x) := 1/x$, $g(x) := -1/(x+1)$, $c = 0$.
(B) $f(x) := 1/x$, $g(x) := (x-1)/x$, $c = 0$.
(C) $f(x) := \arcsin x$, $g(x) := \arccos x$, $c = 1/2$.
(D) $f(x) := 1/x$, $g(x) = x$, $c = 0$.
(E) $f(x) := \tan x$, $g(x) := \cot x$, $c = 0$.

Your answer: ____________________________

(11) If $\lim_{x\to\infty} f(x) = L$ for some finite L, this tells us that the graph of f has a:
(A) vertical asymptote
(B) horizontal asymptote
(C) vertical tangent
(D) horizontal tangent
(E) vertical cusp

Your answer: ____________________________

(12) If $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to\infty} f'(x) = M$, where both L and M are finite, then:
(A) $L = 0$ but M need not be zero
(B) $M = 0$ but L need not be zero
(C) Both L and M must be zero.
(D) Neither L nor M need be zero.
(E) At least one of L and M must be zero, but it could be either one.

Your answer: ____________________________

Consider the following $\epsilon - \delta$ definition of limit at ∞: $\lim_{x\to\infty} f(x) = L$ if for all $\epsilon > 0$, there exists $a \in \mathbb{R}$ such that for all $x > a$, $|f(x) - L| < \epsilon$.

(13) What is the smallest a that can be picked for the function $f = \arctan$ with L being its limit at ∞ and $\epsilon = \pi$?
(A) $\sqrt{3}$
(B) 1
(C) 0
(D) -1
(E) There is no smallest a. Any $a \in \mathbb{R}$ will do.

Your answer: ____________________________

(14) What is the smallest a that can be picked for the function $f = \arctan$ with L being its limit at ∞ and $\epsilon = \pi/6$?
(A) $1/2$
(B) $1/\sqrt{3}$
(C) 1
(D) $\sqrt{3}$
(E) 2

Your answer: ____________________________

(15) Suppose $f(x) := p(x)/q(x)$ is a rational function in reduced form (i.e., the numerator and denominator are relatively prime) and $\lim_{x \to c} f(x) = \infty$. Which of the following can you conclude about f?

(A) $x - c$ divides $p(x)$, and the largest r such that $(x - c)^r$ divides $p(x)$ is even.
(B) $x - c$ divides $q(x)$, and the largest r such that $(x - c)^r$ divides $q(x)$ is even.
(C) $x - c$ divides $p(x)$, and the largest r such that $(x - c)^r$ divides $p(x)$ is odd.
(D) $x - c$ divides $q(x)$, and the largest r such that $(x - c)^r$ divides $q(x)$ is odd.
(E) $x - c$ does not divide either $p(x)$ or $q(x)$.

Your answer: ____________________________

(16) Suppose $f(x) := p(x)/q(x)$ is a rational function in reduced form (i.e., the numerator and denominator are relatively prime) and $\lim_{x \to c^-} f(x) = \infty$ and $\lim_{x \to c^+} f(x) = -\infty$. Which of the following can you conclude about f?

(A) $x - c$ divides $p(x)$, and the largest r such that $(x - c)^r$ divides $p(x)$ is even.
(B) $x - c$ divides $q(x)$, and the largest r such that $(x - c)^r$ divides $q(x)$ is even.
(C) $x - c$ divides $p(x)$, and the largest r such that $(x - c)^r$ divides $p(x)$ is odd.
(D) $x - c$ divides $q(x)$, and the largest r such that $(x - c)^r$ divides $q(x)$ is odd.
(E) $x - c$ does not divide either $p(x)$ or $q(x)$.

Your answer: ____________________________