The Calculus Concept Inventory

What's this for? This is not a "test" or a "quiz" and won't count for any course credit. It's simply a way for me to try to understand how thorough a grasp you have of some calculus concepts. Most of you would already be computationally proficient, but you may be a rusty on the concepts if you took your Math 130s/150s a while back. Your performance on this test will give me an idea of your background. So, if you cheat this test, you'll literally "only be cheating yourself" as it might mean I don't get to know of certain conceptual gaps and hence will not be able to fill them in during my teaching.

This is a confidential test (I got it from the test creator with the explicit promise of confidentiality). So please don't share these questions with anybody outside the classroom. Because of the confidential nature of the questions, I won't be able to share the final answers with you.

If you are interested in learning more about concept inventories, you can check out the Wikipedia article on the subject, which is at http://en.wikipedia.org/wiki/Concept_inventory

NAME: _____

Turn over to begin attempting the questions.

1. Given an arbitrary function *f*, if $\lim_{x\to 2} f(x) = 5$, what is $f(2)$?

   a. 2
   b. 5
   c. It must be close to 5.
   d. $f(2)$ is not defined.
   e. Not enough information is given. ANS:_____

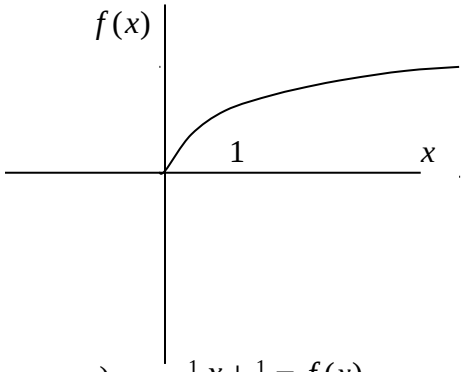
2. If a function is always positive, then what must be true about its derivative?

   a) The derivative is always positive.
   b) The derivative is never negative.
   c) The derivative is increasing.
   d) The derivative is decreasing.
   e) You can't conclude anything about the derivative. ANS:_____

3. (QUESTION CANCELED DUE TO UNCLEAR PICTURES)

4. We are growing a population of bacteria in a jar. At 11:00 a.m. there is one bacterium in the jar. The bacteria divide once every minute so that the population doubles every minute. At 12:00 noon the jar is full. At what time was the jar half full?

   a) 11:01
   b) 11:15
   c) 11:30
   d) 11:45
   e) 11:59 ANS: _____

5. 

   The tangent line to this graph at $x = 1$ is given by $y = \frac{1}{2}x + \frac{1}{2}$. Which of the following statements is true everywhere on the graph?

   (see choices, next page)

   a) $\frac{1}{2}x + \frac{1}{2} = f(x)$
   b) $\frac{1}{2}x + \frac{1}{2} \le f(x)$
   c) $\frac{1}{2}x + \frac{1}{2} \ge f(x)$

d) $\frac{1}{2}x = \frac{1}{2}f(x)$

e) None of these

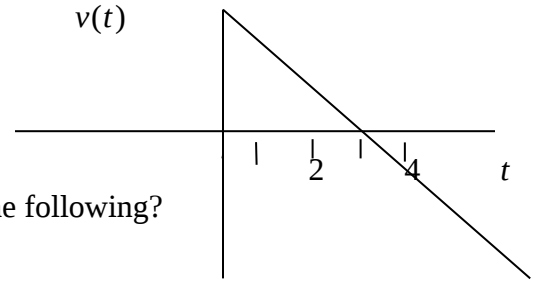
ANS:_____

6. If a number very close to zero is divided by another number very close (but not equal) to zero, the result ____(choose one)____

a) must be a number very close to zero
b) must be a number close to 1
c) could be any number
d) might not be a number at all.

ANS:_____

7. A graph of a moving object's velocity against time is shown.



The acceleration is the rate of change of the velocity with time.
The acceleration of the object is which of the following?

a) Positive everywhere
b) Negative everywhere
c) Zero everywhere
d) Zero at some times but not others
e) It cannot be determined from the given information.

ANS:_____

8. Yesterday's temperature at *t* hours past midnight was $T(t)$ degrees F. At noon the temperature was 50 deg F. The first derivative $T'(t)$ reached a low of +2 degrees/hour at 6:00 a.m., then increased for the rest of the day. Which of the following is correct?

a) Temperature fell in the morning and rose in the afternoon.
b) At 1 pm the temperature was 52 degrees.
c) At 1 pm the temperature was 48 degrees.
d) The temperature was lower at 6:00 a.m. than at any other time.
e) The temperature rose all day.

ANS: _____

9. The derivative, $f'(x)$, of a function $f(x)$, is negative everywhere. We know also that $f(0) = 0$.

What must be true about $f(-1)$?

a) $f(-1)$ is negative.
b) $f(-1)$ is positive.
c) $f(-1)$ is zero.
d) Not enough information to conclude anything about $f(-1)$

ANS: _____

10. The table shows the average cost per gallon of gasoline for different years in cents per gallon.

Year	1995	1996	1997	1998	1999	2000
Cost	112	120	122	101	121	144

For which span of years could the cost of gasoline per gallon be a function that is increasing and concave down?

a) 1995-1997
b) 1996-1998
c) 1997-1999
d) 1998-2000
e) None of these.

ANS:_____

11. As water is drained out of a bathtub, the volume of water, *V*, and its depth in the tub, *h*, change with time, *t*. Which of the following is true of $\frac{dV}{dh}$ and $\frac{dV}{dt}$?.

a) They must be equal.
b) They must have opposite signs.
c) Both must be negative.
d) Both must be positive.
e) None of these

ANS:_____

12. Let f be the function defined by $f(x) = x + 2$, and let g be the function defined by $g(u) = u + 2$ for all real numbers x and u. Then which of the following is true?

a) f and g are exactly the same function.
b) f and g are different functions because x and u are different variables.
c) f and g are different functions whenever x and u are different numbers.
d) Not enough information is given to determine if f and g are the same function.

ANS:_____

13. For which type of function does linear approximation always give the exact value of $f(x)$ for x near the starting point?

a) Linear
b) Quadratic
c) Exponential
d) Logarithmic
e) Linear approximation never gives the exact value. ANS:_____

14. Values at a few points are given below for two functions $f(x)$ and $g(x)$ that are defined for all real values of *x*.

x	f(x)	x	g(x)
1	101	2	101
2	102	4	102
3	103	6	103
4	104	8	104

Which of the following relations can be correct?

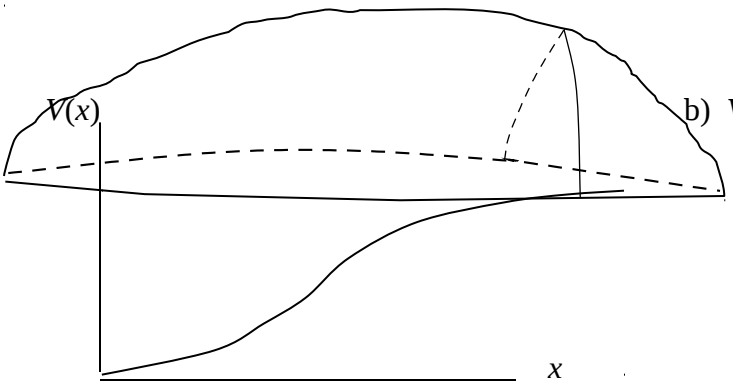
a) $g(x) = 2f(x)$
b) $g(x) = f(2x)$
c) $g(x) = \frac{f(x)}{2}$
d) $g(x) = f\left(\frac{x}{2}\right)$
e) $g(x) = \frac{f(2x)}{2}$ ANS:_____

15. The derivative of a function f is given by $f'(x) = ax^2 + b$. (that's supposed to read as f'(x) = ax^2 + b).
What is required of the values of a and b so that the slope of the tangent line to f will be positive at $x = 0$?

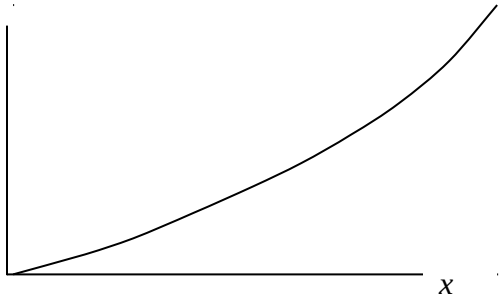
a) a and b must both be positive numbers.
b) a must be positive, while b can be any real number.
c) a can be any real number, while b must be positive.
d) a and b can be any real numbers. ANS:_____

16. The drawing represents a loaf of bread with a slice shown x inches from the left-hand end of the bread. Which of the following graphs could represent the volume V of the bread to the left of the slice as a function of the distance x from the left-hand end to the slice?

a) $V(x)$

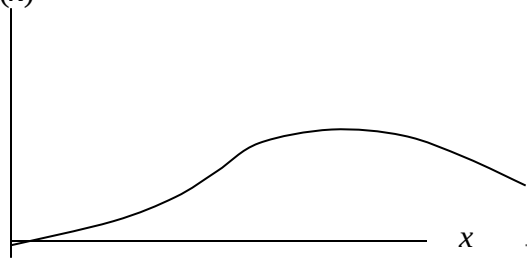


b) $V(x)$

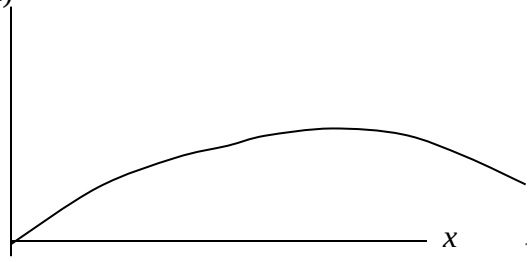


c)

$V(x)$



d) $V(x)$



ANS:_____

17. Suppose that from 1980 until 2000, the rate at which a certain redwood tree grew was constant. Let $h(t)$ represent the height of the redwood in feet t years after 1980. Which function could realistically represent $h(t)$ from 1980 to 2000?

a) $h(t) = 50$
b) $h(t) = 50 + \frac{1}{2}t$
c) $h(t) = 1980 + \frac{1}{2}t$
d) $h(t) = 50\left(\frac{3}{2}\right)^t$
e) None of these

ANS:_____

18. If the distance between 3 and a fixed point a on the number line is less than $\frac{1}{10}$, less than $\frac{1}{100}$, less than $\frac{1}{1000}$, in fact less than $\left(\frac{1}{10}\right)^N$ for all positive integers N, then we can conclude which of the following about the position of a?

a) $a = 3$
b) a is near, but not necessarily equal, to 3
c) a does not have to be close to 3
d) a is not equal to 3
e) There is not enough information to conclude anything about a

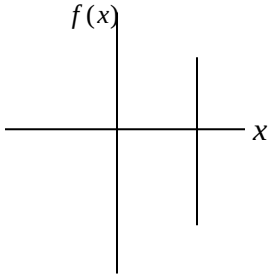
ANS:_____

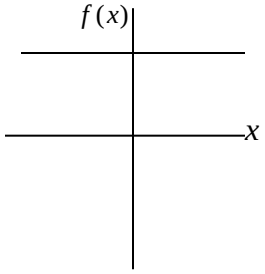
19. The duration of daylight L in minutes (sunrise to sunset), x kilometers north of the equator on May 17, is given by the function $L = f(x)$. What are the units of measurement of the second derivative $f''(4000)$?

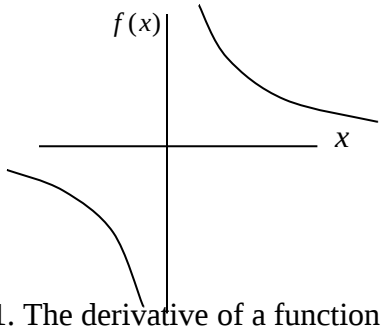
a) kilometers/minute
b) minutes/kilometer
c) kilometers/minute^2
d) minutes/kilometer^2
e) minutes^2/kilometer^2

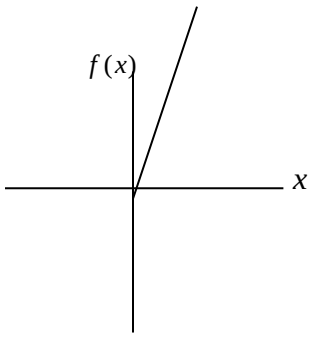
ANS:_____

20. A correct sketch of the graph of the function $f(x)$ defined by: $x \times f(x) = 12$, (that is supposed to read as xf(x) = 12, i.e., the product of x and f(x) is 12) could be which of the following?

a) 

b) 

c) 

d) 

e) None of the above

ANS:_____

21. The derivative of a function is negative everywhere on the interval $x = 2$ to $x = 3$. Where on this interval does the function have its maximum value?

a) At $x = 2$,

b) At $x = 3$,
c) Somewhere between $x = 2$ and $x = 3$,
d) It does not have a maximum, since the derivative is never zero,
e) We cannot tell if it has a maximum since we don't know where the second derivative is negative.

ANS:_____

22. From 1995 to 2000 a forester's grove of white pine trees grew from 80 feet to 90 feet tall. During the same period of time, his young grove of oak trees grew from 25 to 35 feet tall. Which of the following is true?

a) The average rate of change of height for the white pine and oak was the same.
b) The average rate of change for the white pine was greater than for the oak.
c) The average rate of change for the oak was greater than that for the white pine.
d) It is not possible to compare the rates of change for the two.

ANS:_____