Homework 9 - STAT 231

Suggested Solution [*,†]

Due in class: 11 Nov 2019

Problem 1-3 are from the Devore textbook.

1. **(Chapter 9: Problem 38(b))** Estimate the difference between true average times for the two types of retrieval in a way that conveys the information. Denote d as the difference between true averages times.

$$H_0 : \mu_d = 0, \quad H_1 : \mu_d \neq 0$$

From the jmp output, we have $\bar{d} = 20.54, s = 11.96, n = 13, df = n - 1 = 12$. Note

$$T = \frac{\bar{d} - 0}{s/\sqrt{n}} \sim t_{12}, \quad t = \frac{20.54 - 0}{11.96/\sqrt{13}} = 5.949218, \quad t^* = \pm t_{df=12,.025} = \pm 2.179$$

Since $t > 2.179$, therefore reject the null hypothesis and conclude that the true average difference is not zero. The 95% confidence interval is

$$\bar{d} \pm t_{df=12,.025} \frac{s}{\sqrt{n}} = 20.54 \pm 2.179 \frac{11.96}{\sqrt{13}} = (13.31202, 27.76798)$$

Since the confidence interval does not include zero, therefore there is sufficient evidence to reject the null hypothesis. $\square$

2. **(Chapter 9: Problem 52)** Denote p_1 to be the percentage of satisfied teachers in elementary schoool, p_2 be that of high schools teachers.

$$n_1 = 395, p_1 = 224/395 \quad n_2 = 266, p_2 = 126/266$$

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[†] Please inform me if there is any error or possible improvement

Since both n_1, n_2 are large, we can use z-statistics. the 95% confidence interval is

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\tilde{p}_1(1-\tilde{p}_1)}{n_1} + \frac{\tilde{p}_2(1-\tilde{p}_2)}{n_2}}$$
$$= 224/395 - 126/266 \pm 1.96\sqrt{\frac{0.566416*(1-0.566416)}{395} + \frac{0.4740741*(1-0.4740741)}{266}}$$
$$= (0.01601376, 0.170795) \quad \square$$

where

$$\tilde{p}_1 = \frac{n_1 p_1 + 2}{n_1 + 4} = \frac{224+2}{395+4} = 0.566416$$
$$\tilde{p}_2 = \frac{n_2 p_2 + 2}{n_2 + 4} = \frac{126+2}{266+4} = 0.4740741$$

3. **(Chapter 9: Problem 63)** Let σ_1^2 denote the variance of weight gains for low dose, let σ_2^2 denote that of control group.

$$H_0 : \sigma_1^2 = \sigma_2^2 \iff \sigma_1^2/\sigma_2^2 = 1, \quad H_1 : \sigma_1^2 > \sigma_2^2 \iff \sigma_1^2/\sigma_2^2 > 1$$

Use F-test, we know $F = \frac{s_1^2}{s_2^2} \sim F_{n_1-1,n_2-1}$. In this question, we have $f = \frac{54^2}{32^2} = 2.848, n_1 = 20, n_2 = 23$. Find the critical value $F_{.05,19,22} \approx 2.07$. Since $f > F_{.05,19,22}$, P-value: $P(F_{.05,19,22} > f) < .05 = \alpha$, there is sufficient evidence to reject H_0 and conclude with 95% confidence that the low dose leads to higher variability in weight gain. $\square$

4.

$$H_0 : \sigma_1^2 = \sigma_2^2 \iff \sigma_2^2/\sigma_1^2 = 1, \quad H_1 : \sigma_1^2 < \sigma_2^2 \iff \sigma_2^2/\sigma_1^2 > 1$$

Use F-test, we know $F = \frac{s_2^2}{s_1^2} \sim F_{n_2-1,n_1-1}$. In this question, we have $f = \frac{4.875^2}{4.533^2} = 1.156586, n_1 = 47, n_2 = 34$. Find the critical value $F_{.05,33,46} \approx 1.69(\nu_1 = 33, \nu_2 = 46)$. Since $f < F_{.05,33,46}$, P-value: $P(F_{.05,33,46} > f) > .05 = \alpha$, there is no sufficient evidence to reject H_0 and conclude with 95% confidence that the standard deviation of HwyMPG for small and midsized cars does not differ. $\square$

5. From the question, we know $n_1 = 47, p_1 = 9/n_1, \quad n_2 = 34, p_2 = 9/n_2$, we try to perform the hypothesis:

$$H_0 : p_1 = p_2, \quad H_1 : p_1 \neq p_2$$

Use Z-test, note

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \dot{\sim} N(0,1), \quad \hat{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

In this question, we have

$$z = \frac{9/47 - 9/34}{\sqrt{\frac{18}{81}(1 - \frac{18}{81})\left(\frac{1}{47} + \frac{1}{34}\right)}} = -0.7822282$$

P-Value $= P(Z < -0.7822282) + P(Z > 0.7822282) = 2P(Z < -0.7822282) \approx 2 \times .2177 = .4354 > .05 = \alpha$. Therefore there is no sufficient evidence to reject H_0. We conclude that there is no difference between the AWD proportion between small and midsized cars with 95% confidence. $\square$