Homework 4 - STAT 231

Due in class, Thursday, Sept 26

Some of the following problems are from the Devore textbook.

1. Chapter 4: Problem 3

2. Chapter 4: Problem 11

3. Chapter 4: Problem 36
   Note: Part(d) involves the Binomial distribution, where the Binomial probability p corresponds to the probability (using a normal distribution) that a (single) random droplet exceeds 1500 μm (i.e., get this "p" by modifying your answer to part (a), which is related).

4. Chapter 4: Problem 40

5. Chapter 4: Problem 42
   Note: If $X \sim \mathcal{N}(\mu, \sigma^2)$, the question is: what does σ^2 (or σ) have to be so that $P(\mu - 1 \leq X \leq \mu + 1) = 95\%$?

6. A continuous random variable X has probability density function

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{4} & \text{if } 0 \leq x < 2 \\ \frac{1}{2} & \text{if } 2 \leq x < 3 \\ 0 & \text{if } x \geq 3. \end{cases}$$

   (a) Find $P(X < 1)$.

   (b) Find $P(X < 2.5)$.

   (c) Find the cumulative distribution function $F(x) = P(X \leq x)$. Be sure to define the function for all real numbers x. (Hint: The cdf will involve four pieces, depending on an interval/range for x.)

   (d) Find the expected value of X.

7. An autopilot attempts to maintain an airplane's altitude at a constant 18000 feet. Because of random variations, the actual altitude X is a normal random variable with mean 18000 feet and variance=2500 feet^2.

   (a) What is the probability, $P(|X - 18000| > 100)$, that the altitude differs from the assigned 18000 value by more than 100 feet?

   (b) Suppose that by mechanically improving the autopilot, the variance of the airplane's altitude can actually be reduced. Find the value of the variance such that the probability that the altitude differs from the assigned 18000 value by more than 100 feet is at most 0.01.

8. Suppose that 30% of items produced by a certain machine require "reworking." A shipment includes 1000 items.

   (a) Use a binomial random variable Y to find the probability that 310 or more items require reworking.
   Note: You can do this in JMP. Create a new data table... right click on the "Column 1" heading and select "Formula"... then select "Binomial Distribution" (this

computes $P(X \leq k)$ for a given k when X is Binomial(n, p))... enter n, p here with $k = 309$ and find the probability $P(X \leq 309)$... then find "$1 - P(X \leq 309)$" (why 309 here?)

(b) Use a normal random variable X with the same mean and variance as the binomial random variable to find the probability that 310 or more items require reworking.

(c) Compare your answers for (a) and (b). Why is it appropriate to use a normal distribution to approximate the binomial random variable in this case?