Homework 8 - STAT 231

Suggested Solution *†

Due in class: 4 Nov 2019

Problem 1-8 are from the Devore textbook.

1. **(Chapter 8: Problem 1)**

   (a) Yes

   (b) No

   (c) No

   (d) Yes

   (e) No

   (f) Yes

   We only need to determine whether the variables involved in the hypothesis are paramters. They need to be a fixed number instead of a random variable to form a reasonanle hypothesis.

2. **(Chapter 8: Problem 20)** From the question, we know the null and one-sided alternative hypotheses as follows:

   - H_0: The true average lifetime is the same as advertised
   - H_1: The true average lifetime is smaller than what is advertised

   (a) P-value $= .016 < \alpha_1 = .05$: Null hypothesis rejected, therefore the conclusion is the true average lifetime is smaller than what is advertised.

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†Please inform me if there is any error or possible improvement

(b) P-value $= .016 > \alpha_2 = .01$: Null hypotheses not rejected, therefore the conclusion is the true average lifetime is the same as advertised.

3. **(Chapter 8: Problem 24)** $\overline{X} = 191, s = 89, n = 58, \alpha = .001$. From the problem

$$H_0 : \mu = 153, \quad H_1 : \mu > 153$$

This is an one-sided hypotheses testing. Recall that the sample mean $t = \frac{\overline{X}-\mu}{s/\sqrt{n}}$ follows t-distribution, with the critical value $t(df = 57, \alpha = .001) = 3.232$. Note $t = \frac{\overline{X}-\mu}{s/\sqrt{n}} = \frac{191-153}{89/\sqrt{58}} = 3.251678 > 3.232$ ($\Longleftrightarrow$ P-value $P(t_{57} > 3.25167) < .001 = \alpha$). Therefore there is sufficient evidence to reject H_0 and conclude that the true calorie level in population exceeds the actual calorie level. $\square$

4. **(Chapter 8: Problem 35(a))** $\overline{X} = 249.7, s = 145.1, n = 12, \alpha = .001$. From the problem

$$H_0 : \mu = 200, \quad H_1 : \mu > 200$$

Critical value $t(df = 11, \alpha = .05) = 1.796$. Note $t = \frac{\overline{X}-\mu}{s/\sqrt{n}} = \frac{249.7-200}{145.1/\sqrt{12}} = 1.186532 < 1.796$ ($\Longleftrightarrow$ P-value $P(t_{11} > 1.186532) = .128 > .05 = \alpha$). There is no sufficient evidence to reject H_0 and conclude that the true repair time is not higher than 200 minutes. $\square$

5. **(Chapter 8: Problem 48(a)-(b))** $\hat{p} = 41/51, n = 51, \alpha = .01$. Recall

$$\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0, 1)$$

(a) From the problem

$$H_0 : p = .5, \quad H_1 : p > .5$$

Since $\alpha = .01$, we find the one-sided critical value $z^* = z_{(1-\alpha)} = 2.33$. $z = \frac{41/51-.5}{\sqrt{\frac{.5(1-.5)}{51}}} = 4.340868 > 2.33$ ($\Longleftrightarrow$ $P(z > 4.340868) < .0002 < .01 = \alpha$). Therefore there is sufficient evidence to reject the null hypotheses. Conclusion: more than 50% of all homes with Chinese drywall have electrical/environmental problem with 99% confidence. $\square$

(b) $\tilde{p} = \frac{n\hat{p}+2}{n+4} = \frac{41+2}{51+4} = .7818$. Lower bound $= \hat{p} - z^*\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n}} = \frac{41}{51} - 2.33 \times \sqrt{\frac{.7818\times(1-.7818)}{51}} \approx 0.6692$. $\square$

6. **(Chapter 9: Problem 8)** Let μ_1 denote 'true average of tensile strength for AISI1064'. Let μ_2 denote 'true average of tensile strength for AISI1078'.

   (a) From the problem

   $$H_0 : \mu_1 - \mu_2 = 10, \quad H_1 : \mu_1 - \mu_2 < -10$$

   Note $m = n = 129 \geq 30$, we can use z-test, i.e. $Z = \frac{\overline{x}-\overline{y}-(-10)}{\sqrt{\frac{s_1^2}{m}+\frac{s_2^2}{n}}} \dot\sim N(0,1)$. Note $\alpha = .01$, the critical value $z^* = z_\alpha = -2.33$. Plug in the statistics, we have $z = \frac{107.6-123.6-(-10)}{\sqrt{\frac{1.3^2}{129}+\frac{2.0^2}{129}}} = -28.56867 < z^*$ ($\iff P(z < -28.57) \approx 0 < .01 = \alpha$). Therefore there is sufficient evidence to reject H_0 and conclude with 99% confidence that the true average of tensile strength for AISI1078 exceeds that of AISI1064 by more than 10 kg/mm^2. $\square$

   (b) A two-sided confidence interval with confidence level $100(1-\alpha)\%$ for $\mu_1 - \mu_2$ can be constructed by $\overline{x}-\overline{y} \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{m}+\frac{s_2^2}{n}}$. Let $\alpha = .1$, $z_{\alpha/2} = 1.64$, then 90% CI $= 107.6 - 123.6 \pm 1.64 \times \sqrt{\frac{1.3^2}{129}+\frac{2.0^2}{129}} = (-16.34443, -15.65557)$ $\square$

7. **(Chapter 9: Problem 18 Use test statistic from our formula sheet)** Let μ_1 denote 'CO2 loss at traditional pour'. Let μ_2 denote 'CO2 loss at slanted pour.

   (a) At 18 Celcius, we conduct a two-sided hypotheses test:

   $$H_0 : \mu_1 = \mu_2, \quad H_1 : \mu_1 \neq \mu_2$$

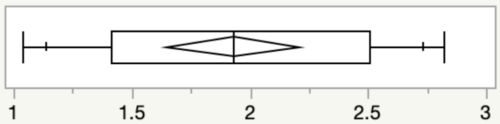
   Note $n = 4 < 30$, we should use T-test. $t = \frac{\overline{x_1}-\overline{x_2}-0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} = \frac{4.0-3.7}{\sqrt{\frac{.5^2}{4}+\frac{.3^2}{4}}} = 1.028992$. From the question, $df = \min(n_1-1, n_2-1) = 3, \alpha = .01$, critical value $t^* = t_{3,.01} = 5.841$. Since $t \in (-5.841, 5.841) \iff P(t_{df=3} < -1.028992 \text{ or } t_{df=3} > 1.028992) \approx 2 \times .196 > .01 = \alpha$, there is no sufficient evidence to reject H_0, therefore we conclude with 99% confidence that there is no difference in CO2 loss between two pour types. $\square$

   (b) At 12 Celcius, a similar test can be conducted. $t = \frac{\overline{x_1}-\overline{x_2}-0}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} = \frac{3.3-2.0}{\sqrt{\frac{.2^2}{4}+\frac{.3^2}{4}}} = 7.211103 > t^* \iff P(t_{df=3} < -7.211103 \text{ or } t_{df=3} > 7.211103) \approx 0 < .01 = \alpha$. There is sufficient evidence to reject H_0 and conclude with 99% confidence that there is significance difference in CO2 loss between two pour types. $\square$

8. **(Chapter 9: Problem 22)**

(a) The high group has distribution skewed to the right while the non-high group has symmetric distribution.

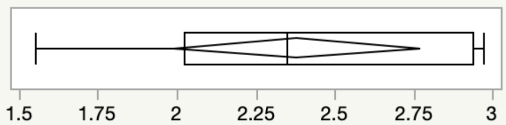
Non-high



Summary Statistics

Mean	1.9261111
Std Dev	0.5694284
Std Err Mean	0.1342156
Upper 95% Mean	2.2092812
Lower 95% Mean	1.642941
N	18

High



Summary Statistics

Mean	2.3777778
Std Dev	0.5072174
Std Err Mean	0.1690725
Upper 95% Mean	2.7676595
Lower 95% Mean	1.987896
N	9

(b) From the boxplot, we see a difference in the median between two plots. It is reasonable to conduct a test for if there is a difference.

(c) Let μ_1 denote 'the true average depth (mm) of the HAZ under non-high setting'. Let μ_2 denote 'the true average depth (mm) of the HAZ under high setting'.

$$H_0 : \mu_1 = \mu_2, \quad H_1 : \mu_1 < \mu_2$$

From question we know, $n_1 = 18, n_2 = 9, \overline{x_1} = 1.926, \overline{x_2} = 2.3777, s_1 = .5694, s_2 = .5072$, the test statitic

$$t = \frac{\overline{x_1} - \overline{x_2} - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1.926 - 2.3777}{\sqrt{\frac{.5695^2}{18} + \frac{.5072^2}{9}}} \approx -2.092418$$

Critical value $t^* = t_{df=\min(18-1,9-1)=8,\alpha=.01} = -2.896$. Note $t > t^* \iff P(t_{df=8} > -2.09) = .034 > .01 = \alpha$. Therefore there is no sufficient evidence to reject H_0 therefore conclude with 99% confidence that the true average depth (mm) of the HAZ under non-high setting does not differ from that of high setting. $\square$

9. (a) From the question

$$H_0 : \mu = 75, \quad H_1 : \mu < 75$$

Since we know the population variance, we here apply z-test, test statistic $z = \frac{\overline{X}-\mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{72.5-75}{\sqrt{\frac{9^2}{25}}} = -1.3889$. Note critical value $z^* = z_\alpha = z_{.01} = -2.33$. Since $z > z^* \iff P(z_{.01} < -1.3889) = .0823 > .01\alpha$, there is no sufficient evidence to reject H_0 and thus we conclude with 99% confidence that the true average drying time is not less than 75 min. $\square$

(b) $z = \frac{\overline{X}-\mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{69.816-75}{\sqrt{\frac{9^2}{25}}} = -2.88$. Since $P(z < -2.88) = .0020$, any confidence level $\alpha < .0020$ will lead to rejecting H_0. $\square$

(c)

$$\begin{aligned}
\beta(70) &= P(\text{Accept } H_0 | \mu = 70) \\
&= P(\overline{X} > 69.816 | \mu = 70) \\
&= 1 - \Phi\left(\frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}}\right) \\
&= 1 - \Phi\left(\frac{69.816 - 70}{\sqrt{\frac{9^2}{25}}}\right) \\
&= 1 - \Phi(-.1022) \\
&= 1 - .4620 = .5380 \quad \square
\end{aligned}$$

10. From the question we conduct an one-sided test, where $n = 16, \sigma = 1500$,

$$H_0 : \mu = 30,000, \quad H_1 : \mu > 30,000$$

(a) Apply Z-test, $z = \frac{\overline{X}-\mu}{\sqrt{\sigma^2/n}} = \frac{30{,}960-30{,}000}{\sqrt{1500^2/16}} = 2.56 > z^* = z_{1-\alpha=.99} = 2.33 \iff$ P-value $P(z > 2.56) = 1-.9948 = .0052 < .01 = \alpha$, therefore there is sufficient evidence to reject H_0 and conclude that the true average tread life is higher than 30,000. $\square$

(b) The critical value of $\overline{X}$ to reject H_0 is

$$\frac{\overline{X}^* - 30,000}{\sqrt{1500^2/16}} = z^* = 2.33 \implies \overline{X}^* = 30873.75$$

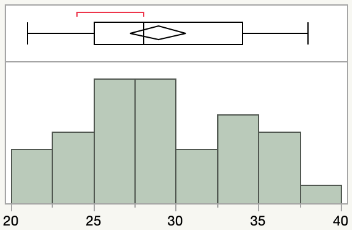
$$\begin{aligned}
\beta(30,500) &= P(\text{Accept } H_0 \mid \mu = 30,500) \\
&= P\left(\overline{X} < \overline{X}^* | \mu = 30,500\right) \\
&= P\left(\frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} < \frac{\overline{X}^* - \mu}{\sqrt{\frac{\sigma^2}{n}}} | \mu = 30,500\right) \\
&= \Phi\left(\frac{30873.75 - 30500}{\sqrt{\frac{1500^2}{16}}}\right) \\
&= \Phi(.9967) \\
&= .8413 \quad \square
\end{aligned}$$

(c) To reject H_0, $z = \frac{\overline{X} - \mu}{\sqrt{\sigma^2/n}} = \frac{30{,}960 - 30{,}000}{\sqrt{1500^2/16}} = 2.56 > z_{1-\alpha} \iff$ P-value $P(z > 2.56) = 1 - .9948 = .0052 < \alpha$. Therefore the smallest α is .0052. $\square$

11. See 'q11.jmp' for the scripts. Let $(\cdot)_1$ denote statistics for midsized, let $(\cdot)_2$ denote statistics for small cars. From the figure we know $n_1 = 34, n_2 = 47$, both larger than 30, Z-test can be applied. $s_1 = 4.7849, s_2 = 4.5331, \bar{X}_1 = 28.8824, \bar{X}_2 = 32.8723$

Distributions Size=Midsized

HwyMPG



Quantiles

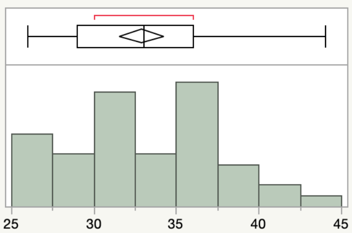
100.0%	maximum	38
99.5%		38
97.5%		38
90.0%		36
75.0%	quartile	34
50.0%	median	28
25.0%	quartile	25
10.0%		23
2.5%		21
0.5%		21
0.0%	minimum	21

Summary Statistics

Mean	28.882353
Std Dev	4.7848543
Std Err Mean	0.8205957
Upper 95% Mean	30.551868
Lower 95% Mean	27.212838
N	34

Distributions Size=Small

HwyMPG



Quantiles

100.0%	maximum	44
99.5%		44
97.5%		43.4
90.0%		39
75.0%	quartile	36
50.0%	median	33
25.0%	quartile	29
10.0%		26.8
2.5%		26
0.5%		26
0.0%	minimum	26

Summary Statistics

Mean	32.87234
Std Dev	4.5330526
Std Err Mean	0.6612137
Upper 95% Mean	34.203295
Lower 95% Mean	31.541386
N	47

(a) One-sided test with $\alpha = .05$

$$H_0 : \mu_1 = \mu_2, \quad H_1 : \mu_1 < \mu_2$$

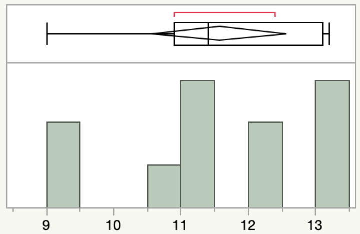
$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_1^2}{n_1}}} = \frac{28.8824 - 32.8723}{\sqrt{4.7849^2/34 + 4.5331^2/47}} = -3.786 < -1.64 = z^* = z_\alpha$. Therefore there is sufficient evidence to reject H_0 and conclude that the small cars has better highway mileage. $\square$

(b) 90% CI for (mean HwyMG of midsized car - mean HwyMG of smallsized car)= $\bar{X}_1 - \bar{X}_2 \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_1^2}{n_1}} = (28.8824 - 32.8723) \pm 1.64\sqrt{4.7849^2/34 + 4.5331^2/47} = (-5.718216, -2.261584)$ $\square$

12. See 'q11.jmp' for the scripts. Let $(\cdot)_1$ denote statistics for Hatchback, let $(\cdot)_2$ denote statistics for Wagon. From the figure we know $n_1 = 11, n_2 = 9$, both smaller than 30, t-test can be applied with $df = \min(n_1 - 1, n_2 - 1) = 8$. $s_1 = 1.4841, s_2 = 1.7843, \bar{X}_1 = 11.5636, \bar{X}_2 = 16.5889$

Distributions Type=Hatchback

FuelCap



Quantiles

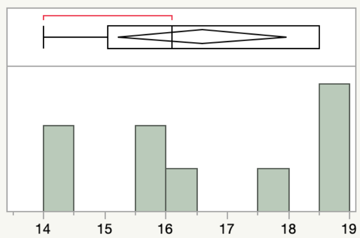
100.0%	maximum	13.2
99.5%		13.2
97.5%		13.2
90.0%		13.2
75.0%	quartile	13.1
50.0%	median	11.4
25.0%	quartile	10.9
10.0%		9.04
2.5%		9
0.5%		9
0.0%	minimum	9

Summary Statistics

Mean	11.563636
Std Dev	1.4840975
Std Err Mean	0.4474722
Upper 95% Mean	12.560667
Lower 95% Mean	10.566606
N	11

Distributions Type=Wagon

FuelCap



Quantiles

100.0%	maximum	18.5
99.5%		18.5
97.5%		18.5
90.0%		18.5
75.0%	quartile	18.5
50.0%	median	16.1
25.0%	quartile	15.05
10.0%		14
2.5%		14
0.5%		14
0.0%	minimum	14

Summary Statistics

Mean	16.588889
Std Dev	1.7842677
Std Err Mean	0.5947559
Upper 95% Mean	17.960398
Lower 95% Mean	15.217379
N	9

(a) Two-sided test with $\alpha = .05$

$$H_0 : \mu_1 = \mu_2, \quad H_1 : \mu_1 \neq \mu_2$$

$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_1^2}{n_1}}} = \frac{11.5636 - 16.5889}{\sqrt{1.4841^2/11 + 1.7843^2/9}} = -6.751731 < -2.306 = t^* = t_{df=8,\alpha/2=.025}$.
Therefore there is sufficient evidence to reject H_0 and conclude that Fuel cap of Hatchback differs from that of Wagon. $\square$

(b) 95% CI for (mean FuelCap of Hatchback - meanFuelCap of Wagon)$= \bar{X}_1 - \bar{X}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_1^2}{n_1}} = (11.5636 - 16.5889) \pm 2.306\sqrt{1.4841^2/11 + 1.7843^2/9} = (-6.741651, -3.308949)$. $\square$