Homework 10 - STAT 231

Suggested Solution *†

Due in class: 21 Nov 2019

Problem 1-5 are from the Devore textbook.

1. (Chapter 12: Problem 7)

(a)

$$\mu_{y|x=2500} = 1800 + 1.3 * 2500 = 5050$$

(b) We expect 28-strendth (y) to increase by 1.3 psi when accelerated strength (x) increases by 1 psi

$$dY/dX = 1.3$$

- (c) We expect 28-strendth (y) to increase by 1.3*100 = 130 psi when accelerated strength (x) increases by 100 psi
- (d) We expect 28-strendth (y) to decrease by 1.3*100 = 130 psi when accelerated strength (x) decreases by 100 psi
- 2. (Chapter 12: Problem 8 (a)(b))

(a)
$$\hat{y}|_{x=2000} = 1800 + 1.3 * 2000 = 4400$$

$$P(y > 5000|_{x} = 2000) = P((y - \mu_{y|x=2000})/350 > (5000 - \mu_{y|x=2000})/350)$$

$$= P(Z > (5000 - 4400)/350)) = 1 - N(600/350) = 1 - N(1.714286) = 0.043238$$
 (b)

$$\hat{y}|_{x=2500} = 1800 + 1.3 * 2500 = 5050$$

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[†]Please inform me if there is any error or possible improvement

$$P(Y > 5000 | x = 2500) = P((y - \mu_{y|x=2500})/350 > (5000 - \mu_{y|x=2500})/350)$$
$$= P(Z > (5000 - 5050)/350)) = 1 - N(-50/350) = N(0.1428571) = 0.5567985$$

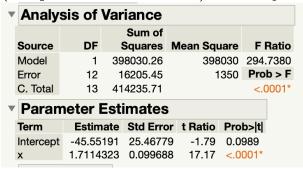
3. (Chapter 12: Problem 14)

(a) Let y denote Ratio, let x denote temperature, from the JMP output:

$$y = -15.24497 + .0942361x$$

(b)
$$\hat{y}|_{x=182} = -15.24497 + .0942361 * 182 \approx 1.906$$

- (c) Residuals = $y_i|x = 182 E(y|x = 182) = (.90, .181, 1.94, 2.68) 1.906 = (-1.006, -1.725, 0.034, 0.774)$ respectively. They are not on the same side of zero because there is random effect that can take both posotove and negative values.
- (d) $R^2 = SSR/SST = 4.4757442/9.9152958 \approx 0.451398$. Therefore around 45.14% of the variation in efficiency ratio can be attributed to the simple linear regression relationship between the two variables.
- 4. (Chapter 12: Problem 19) JMP output:



(a) From the JMP output:

$$y = -45.5519 + 1.7114x$$

- (b) $\hat{y}|_{x=225} = -45.5519 + 1.7114 * 225 = 339.5131$
- (c) dy/dx = 1.7114. We expect the NO_x to decrease by 50 * 1.7114 = 85.57 when x is decreased by 50.

(d) No. 500 is out of the range of x observations. Extrapolation is dangerous.

5. (Chapter 12: Problem 52 (b) - (f))

(a) From the JMP output, we see the F-statistic for

$$H_0: \beta_1 = 0, \quad H_1: \beta_1 \neq 0$$

is 112.7559, and the corresponding P-value < .0001. Therefore there is sufficient evidence to reject H_0 , i.e. the simple linear regression model specifies a useful relationship between chlotine flow and etch rate.

(b) From the JMP output, we have

$$y = 6.4487 + 10.6026x$$

therefore a unit change in flow rate (x), we expect β_1 unite change in etch rate. The 95% confidence interval for β_1 is $\hat{\beta}_1 \pm t_{n-2,\alpha/2} \hat{SE}(\beta_1) = 10.6025 \pm 2.4 * .998484 = (8.206, 12.999)$

(c) 95% CI for $\mu_{Y|x^*=3.0}$ is

$$\hat{y}|_{x=x^*} \pm t_{n-2,\alpha/2} \hat{\sigma} \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$$

where $\hat{y} = 6.4487 + 10.6026 * 3.0 = 38.2565, \hat{\sigma} = \sqrt{MSE} = \sqrt{6.48} = 2.545584, \sum (x_i - \bar{x})^2 = s_x^2 * (n-1) = .9013878^2 * (9-1) = 6.5.$

$$95\%CI = 38.2565 \pm 2.4 * 2.545584 * \sqrt{\frac{1}{9} + \frac{(3.0 - 2.6667)^2}{.9013878^2 * (9 - 1)}} = (36.06, 40.44)$$

(d) 95% Prediction Interval for $y|_{x^*=3.0}$ is $\hat{y} \pm t_{n-2,\alpha/2} \hat{\sigma} \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$, similarly we get

$$95\%PI = 38.2565 \pm 2.4 \times 2.545584 \times \sqrt{1 + \frac{1}{9} + \frac{(3.0 - 2.6667)^2}{.9013878^2 \times (9 - 1)}} = (31.76729, 44.74571)$$

(e) The 95% CI and PI when $x^* = 2.5$ will be narrower, since it is closer to

 $\bar{x} = 2.6667$

(f) No. 6 is not within the range of the observation of x. Extrapolation can be dangerous, and the prediction out of range can be meaningless.