Homework 2 - STAT 231

Suggested Solution [*,†]

Due in class: 12 Sep 2019

The following problems except for the last one are from the Devore textbook.

1. **(Chapter 2: Problem 32)**

   (a) $N_a = 5 \times 4 \times 3 \times 4 = 240$

   (b) $N_b = 1 \times 1 \times 3 \times 4 = 12$

   (c) $N_c = 4 \times 3 \times 3 \times 3 = 108$

   (d) $N_d = N_a - N_c = 132$

   (e) $P(\text{at least 1 Sony}) = N_d/N_a = 11/20 = .55$,
   $P(\text{exactly 1 Sony}) = \dfrac{1 \times 3 \times 3 \times 3 + 4 \times 1 \times 3 \times 3 + 4 \times 3 \times 3 \times 1}{N_a} = .4125$

2. **(Chapter 2: Problem 34)**

   (a) $N_a = \binom{25}{5} = 53130$

   (b) $N_b = \binom{6}{2}\binom{19}{5-2} = 14535$

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[†] Please inform me if there is any error or possible improvement

(c) $P(\text{At least 4 out of 5 has mechanical defect})$

$$
\begin{aligned}
&= P(\{\text{4Mech 1Elec, 5Mech 0Elec}\}) \\
&= P(\{\text{4Mech 1Elec}\}) + P(\{\text{5Mech 0Elec}\}) \\
&= \frac{\binom{6}{1}\binom{19}{4}}{\binom{25}{5}} + \frac{\binom{6}{0}\binom{19}{5}}{\binom{25}{5}} \\
&= 34884/53130 \approx .6566
\end{aligned}
$$

3. **(Chapter 2: Problem 37)**

(a) $N_a = 3 \times 4 \times 5 = 60$

(b) $N_b = 3 \times 2 \times 5 = 10$

(c) $*P(\text{5 drawn (without replacement) experiments have different catelysts})$

$$
\begin{aligned}
&= \frac{\#\text{ 5 selected experiments has different catelysts}}{\#\text{ selecting 5 out of 60 experiments}} \\
&= \frac{(3 \times 4)^5 \times 5!}{\binom{60}{5} \times 5!} \\
&\approx .04556
\end{aligned}
$$

4. **(Chapter 2: Problem 39)**

(a) $P(\text{2 out of 3 is 23W}) = \dfrac{\binom{4}{2}\binom{11}{1}}{\binom{15}{3}} = \dfrac{66}{455} = .145$

(b) $P(\text{3 selected are the same}) = \dfrac{\binom{5}{3} + \binom{6}{3} + \binom{4}{3}}{\binom{15}{3}} = \dfrac{34}{455} \approx .0747$

(c) $P(\text{3 selected are all different}) = \dfrac{\binom{5}{1} \times \binom{6}{1} \times \binom{4}{1}}{\binom{15}{3}} = \dfrac{120}{455} \approx .264$

(d) $P(\text{at least 6 draws (without replacement) are needed to select a 23W})$

$= P(\text{First 5 selections are not 23W}) = \dfrac{\binom{11}{5}}{\binom{15}{5}} = \dfrac{462}{455} \approx .154$

5. **(Chapter 2: Problem 45)**

(a)

$$\begin{aligned} P(A) &= P(A \cap \text{Ethic Group 1}) + P(A \cap \text{Ethic Group 2}) + P(A \cap \text{Ethic Group 3}) \\ &= .106 + .141 + .200 = .447 \\ P(C) &= P(C \cap O) + P(C \cap A) + P(C \cap B) + P(C \cap AB) \\ &= .215 + .200 + .065 + .020 = .500 \end{aligned}$$

$P(A \cap C) = .200$

(b)

$$\begin{aligned} P(A|C) &= \frac{P(A \cap C)}{P(C)} = \frac{.2}{.5} = .4 \\ P(C|A) &= \frac{P(A \cap C)}{P(A)} = \frac{.2}{.447} \approx .447 \end{aligned}$$

(c) $P(\text{Ethic Group 1}|B') = P(\text{Ethic Group 1} \cap B')/P(B')$
$= (.082 + .106 + .004)/(1 - .008 - .018 - .065) \approx 0.211$

6. **(Chapter 2: Problem 59)** From the question, we have
$P(A_1) = .40$, $P(A_2) = .35$, $P(A_3) = .25$,
$P(B|A_1) = .30$, $P(B|A_2) = .60$, $P(B|A_3) = .50$.

(a) $P(A_2 \cap B) = P(B|A_2)P(A_2) = .60 \times .35 = .21$

(b) $P(B) = P(B \cap A_1) + P(B \cap A_2) + P(B \cap A_3) = .40 \times .30 + .35 \times .60 + .25 \times .50 = .455$

(c) $P(A_1|B) = \dfrac{P(A_1 \cap B)}{P(B)} = \dfrac{P(B|A_1)P(A_1)}{P(B)} = \dfrac{.30 \times .40}{.455} \approx .2637$

$P(A_2|B) = \dfrac{P(A_2 \cap B)}{P(B)} = \dfrac{P(B|A_2)P(A_2)}{P(B)} = \dfrac{.35 \times .60}{.455} \approx .4615$

$P(A_3|B) = \dfrac{P(A_3 \cap B)}{P(B)} = \dfrac{P(B|A_3)P(A_3)}{P(B)} = \dfrac{.25 \times .50}{.455} \approx .2747$

7. **(Chapter 2: Problem 71)**

   (a) $P(B'|A') = P(A' \cap B')/P(A') \overset{A \perp B}{=} P(A')P(B')/P(A') = P(B') = (1 - .7) = .3$

   (b) $P(A \cup B) \overset{DeMorgan}{=} P((A' \cap B')') = 1 - P(A' \cap B') = .82$

   (c) $P(A \cap B'|A \cup B) = \dfrac{P((A \cap B') \cap (A \cup B)}{P(A \cup B))} = \dfrac{P(A \cap B')}{P(A \cup B)} = \dfrac{.4 \times .3}{.82} \approx .146$

8. **(Chapter 2: Problem 78)**

   (a) $P(\text{At least 1 opens}) = 1 - P(\text{All fail}) = 1 - (1 - .96)^5 \approx .9999999$

   (b) $P(\text{At least 1 fails}) = 1 - P(\text{All open}) = 1 - .96^5 \approx .1846$

9. **(Chapter 2: Problem 79)** From the question, we have $P(\text{only the older one fail}) = .1$, $P(\text{only the newer one fail}) = .05$.
   Denote $p_1 := P(\text{The older one fails}) =: P(A_1)$, $p_2 := P(\text{The newer one fails}) =: P(A_2)$. By the independence between two pumps,

$$\begin{cases} p_1\,(1 - p_2) = .1 \\ (1 - p_1)\,p_2 = .05 \end{cases}$$

   Solve the system, we get two pair of solutions: $p_1 = .944, p_2 = .894$ or $p_1 = .1059, p_2 = .0559$. Therefore $P(\text{Both pumps fails}) = p_1 \cdot p_2 = .84$ or $.0059$. Note the first pair of solution does not satisfy $P(A_1) + P(A_2) - P(A_1 \cap A_2) \le 1$ therefore eliminated.

10. Denote A to be the event the laptop has a faulty keyboard. Denote B to be the event the laptop has a faulty screen.

    (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .09$

    (b) $P(B|A) = P(B \cap A)/P(A) = .01/.04 = .25$

    (c) $P(B'|A') = P(A' \cap B')/P(A') = (1 - P(A \cup B))/(1 - P(A)) = 91/96 \approx .948$