Homework 1 - STAT 231

Due in class: 05 Sep 2019

Some of the following problems are from the Devore textbook. When using JMP in Problem 9 below, you can install or obtain a copy of JMP 13 or 14 at: https://www.stat.iastate.edu/statistical-software.

1. **(Chapter 1: Problem 51)** The article "Thin-Film Oxygen Uptake Test for the Evaluation of Automotive Crankcase Lubricants" (Lubric. Engr., 1984: 75 – 83) reported the following data on oxidation-induction time (min) for various commercial oils:

87	103	130	160	180	195	132	145	211	105	145
153	152	138	87	99	93	119	129			

   (a) Calculate the sample variance and standard deviation.

   (b) If the observations were reexpressed in hours, what would be the resulting values of the sample variance and sample standard deviation? Answer without actually performing the reexpression.

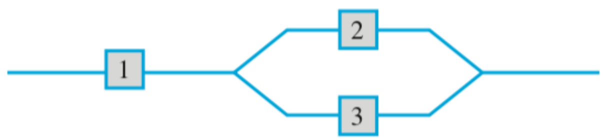
2. **(Chapter 2: problem 2(a),(b),(c) only)** Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.

   (a) List all outcomes in the event A that all three vehicles go in the same direction.

   (b) List all outcomes in the event B that all three vehicles take different directions.

   (c) List all outcomes in the event C that exactly two of the three vehicles turn right.

3. (Chapter 2: problem 3) Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2–3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2–3 subsystem.



The experiment consists of determining the condition of each component [S (success) for a functioning component and F (failure) for a nonfunctioning component].

(a) Which outcomes are contained in the event A that exactly two out of the three components function?

(b) Which outcomes are contained in the event B that at least two of the components function?

(c) Which outcomes are contained in the event C that the system functions?

(d) List outcomes in C', $A \cup C$, $A \cap C$, $B \cup C$, and $B \cap C$.

4. (Chapter 2: problem 8) An engineering construction firm is currently working on power plants at three different sites. Let A_i denote the event that the plant at site i is completed by the contract date. Use the operations of union, intersection, and complementation to describe each of the following events in terms of A_1, A_2, and A_3, draw a Venn diagram, and shade the region corresponding to each one.

(a) At least one plant is completed by the contract date.

(b) All plants are completed by the contract date.

(c) Only the plant at site 1 is completed by the contract date.

(d) Exactly one plant is completed by the contract date.

(e) Either the plant at site 1 or both of the other two plants are completed by the contract date.

5. **(Chapter 2: problem 12)** Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = .6$ and $P(B) = .4$.

   (a) Could it be the case that $P(A \cap B) = .5$? Why or why not?

   (b) From now on, suppose that $P(A \cap B) = .3$. What is the probability that the selected student has at least one of these two types of cards?

   (c) What is the probability that the selected student has neither type of card?

   (d) Describe, in terms of A and B , the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.

   (e) Calculate the probability that the selected student has exactly one of the two types of cards.

   Hint: In parts (d) – (e), it's helpful to use that[1]

$$P(A \cap B') = P(A) - P(A \cap B)$$

   and likewise

$$P(B \cap A') = P(B) - P(A \cap B)$$

6. **(Chapter 2: problem 18)** A wallet contains five \$10 bills, four \$5 bills, and six \$1 bills (nothing larger). If the bills are selected one by y one in random order, what is the probability that at least two bills must be selected to obtain a first \$10 bill?

   Hint: This is just the probability that the first bill is not a \$10.

7. **(Chapter 2: problem 19)** Human visual inspection of solder joints on printed circuit boards can be very subjective. Part of the problem stems from the numerous types of solder defects (e.g., pad nonwetting, knee visibility, voids) and even the degree to which a joint possesses one or more of these defects. Consequently, even highly trained inspectors can disagree on the disposi- tion of a particular joint. In one batch of $10,000$ joints, inspector A found 724 that were judged defective, inspector B found 751 such

[1] To see this holds, note that $A \cap B'$ and $A \cap B$ are disjoint events, whose union is A(i.e., $A = [A \cap B'] \cup [A \cap B]$) so that $P(A) = P([A \cap B'] \cup [A \cap B]) = P(A \cap B') + P(A \cap B)$ by probability axiom 3; re-writing, we have $P(A \cap B') = P(A) - P(A \cap B)$.

joints, and 1159 of the joints were judged defective by at least one of the inspectors. Suppose that one of the $10,000$ joints is randomly selected.

(a) What is the probability that the selected joint was judged to be defective by neither of the two inspectors?

(b) What is the probability that the selected joint was judged to be defective by inspector B but not by inspector A?

8. (Chapter 2: problem 22) The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The probability that he must stop at the first signal is .4, the analogous probability for the second signal is .5, and the probability that he must stop at at least one of the two signals is .7. What is the probability that he must stop

(a) At both signals?

(b) At the first signal but not at the second one?

(c) At exactly one signal?

9. (Not included) The `cars2015.jmp` dataset contains information on 110 new 2015 cars. Read the data into `JMP`. The columns `CityMPG` and `HwyMPG` give the typical gas mileage for each car when driving in a city or on the highway, respectively. *Create histograms to compare the distributions of these two variables & include a printout of the output.* [In JMP, click "Analyze" → "Distribution." If the histogram appear vertically & hard to read, click the red triangle & then under "Histogram options," deselect "vertical." You can copy these plots into Word® or your preferred software to compare and/or print these.]

Do cars typically get better gas mileage when driving in cities or on the highway? Briefly explain, referring to the graphs and giving statistics to support your answer.

10. A shipment of steel bars is inspected for cracks, scratches, and stains.

- 10% contain cracks
- 15% contain scratches
- 4% contain stains

- 19% contain either a crack or a scratch
- 12% contain either a crack or a stain
- 16% contain either a scratch or a stain
- 2% contain all three defects

Some bars contain more than one of these defects.

(a) Find the probability that a randomly selected bar does not contain a crack.

(b) Find the probability that a randomly selected bar contains both a crack and a scratch.

(c) Find the probability that a randomly selected bar contains both a crack and a scratch, but not a stain.

(d) Find the probability that a randomly selected bar has at most two of these defects.
Hint: "at most two defects" is the same as "not all three defects".

(e) Find the probability that a randomly selected bar has no defects.

Hint: If A_1 is the event of a bar having a crack, A_2 is the event of a bar having a scratch, and A_3 is the event of a bar having a stain, then

$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$.

This is the generalization of $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$ to three events.