Homework 1 - STAT 231

Suggested Solution *†

Due in class: 05 Sep 2019

Some of the following problems are from the Devore textbook. When using JMP in Problem 9 below, you can install or obtain a copy of JMP 13 or 14 at: https://www.stat.iastate.edu/statistical-software.

1. (Chapter 1: Problem 51) The article "Thin-Film Oxygen Uptake Test for the Evaluation of Automotive Crankcase Lubricants" (Lubric. Engr., 1984: 75 – 83) reported the following data on oxidation-induction time (min) for various commercial oils:

87	103	130	160	180	195	132	145	211	105	145
153	152	138	87	99	93	119	129			

(a) Calculate the sample variance and standard deviation.

Sample size: $n = 19$.
Sample mean: $\overline{x} = \sum_{i=1}^{n} x_i/n \approx 134.8947$.
Sample variance: $s^2 = \sum_{i=1}^{n}(x_i - \overline{x})^2/(n-1) = \left(\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2/n\right)/(n-1) \approx 1264.7661$.
Sample standard deviation: $s = \sqrt{s^2} \approx 35.5635$.

(b) If the observations were reexpressed in hours, what would be the resulting values of the sample variance and sample standard deviation? Answer without actually performing the reexpression.

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†Please inform me if there is any error or possible improvement

Consider a new dataset y_i with $y_i = cx_i$ for $i = 1, \ldots, n$ and $c = 1/60$. Then $s_y^2 = c^2 s_x^2 = 1/3600 * 1264.7661 \approx 0.3513$ hr^2 and $s_y = c * s_x \approx 0.5927$ hr.

2. **(Chapter 2: problem 2(a),(b),(c) only)** Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.

   (a) List all outcomes in the event A that all three vehicles go in the same direction.

   $A = \{RRR, LLL, SSS\}$

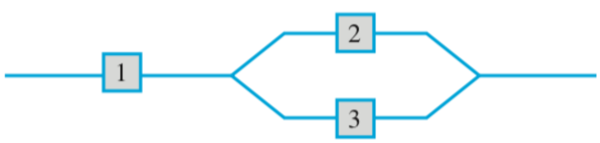
   (b) List all outcomes in the event B that all three vehicles take different directions.

   $B = \{RLS, RSL, LRS, LSR, SLR, SRL\}$

   (c) List all outcomes in the event C that exactly two of the three vehicles turn right.

   $C = \{RRS, RRL, RSR, RLR, SRR, LRR\}$

3. **(Chapter 2: problem 3)** Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2–3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2–3 subsystem.



The experiment consists of determining the condition of each component [S (success) for a functioning component and F (failure) for a nonfunctioning component].

   (a) Which outcomes are contained in the event A that exactly two out of the three components function?

   $A = \{SSF, SFS, FSS\}$

   (b) Which outcomes are contained in the event B that at least two of the components function?

   $B = A \cup \{SSS\} = \{SSF, SFS, FSS, SSS\}$

(c) Which outcomes are contained in the event C that the system functions?

$C = \{SSF, SFS, SSS\}$ (1 must function, one of 2 and 3 must function)

(d) List outcomes in C', $A \cup C$, $A \cap C$, $B \cup C$, and $B \cap C$.

$\Omega = \{SSS, SSF, SFS, SFF, FSS, FSF, FFS, FFF\}$
$C' = \{SFF, FSS, FSF, FFS, FFF\}$
$A \cup C = \{SSF, SFS, FSS, SSS\}$
$A \cap C = \{SSF, SFS\}$
$B \cup C = \{SSF, SFS, FSS, SSS\}$
$B \cap C = \{SSF, SFS, SSS\}$

4. (Chapter 2: problem 8) An engineering construction firm is currently working on power plants at three different sites. Let A_i denote the event that the plant at site i is completed by the contract date. Use the operations of union, intersection, and complementation to describe each of the following events in terms of A_1, A_2, and A_3, draw a Venn diagram, and shade the region corresponding to each one.

(a) At least one plant is completed by the contract date.

$A_1 \cup A_2 \cup A_3$

(b) All plants are completed by the contract date.

$A_1 \cap A_2 \cap A_3$

(c) Only the plant at site 1 is completed by the contract date.

$A_1 \cap A_2' \cap A_3'$

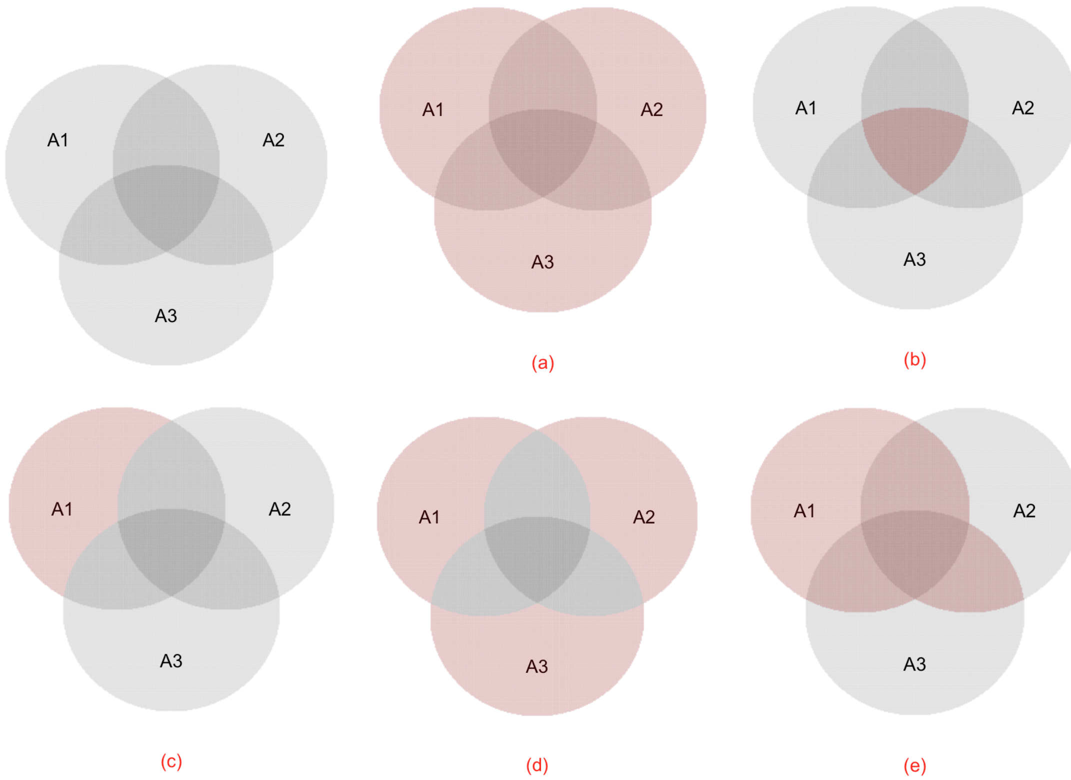
(d) Exactly one plant is completed by the contract date.

$(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$

(e) Either the plant at site 1 or both of the other two plants are completed by the contract date.

$A_1 \cup (A_2 \cap A_3)$

5. (Chapter 2: problem 12) Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = .6$ and $P(B) = .4$.



(a) Could it be the case that $P(A \cap B) = .5$? Why or why not?
No. Since $(A \cap B) \subset A$ and $(A \cap B) \subset B$, $P(A \cap B) \leq \min(P(A), P(B)) = .4$

(b) From now on, suppose that $P(A \cap B) = .3$. What is the probability that the selected student has at least one of these two types of cards?
$P(A \cup B) = P(A \cup (A' \cap B)) = P(A) + P(A' \cap B) = P(A) + P(B) - P(A \cap B) = .6 + .4 - .3 = .7$

(c) What is the probability that the selected student has neither type of card?
$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .7 = .3$

(d) Describe, in terms of A and B , the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.
$P(A \cap B') = P(A) - P(A \cap B) = .6 - .3 = .3$

(e) Calculate the probability that the selected student has exactly one of the two types of cards.

$P((A \cap B') \cup (A' \cap B)) = P(A \cap B') + P(A' \cap B) = .3 + (P(B) - P(A \cap B)) = .3 + (.4 - .3) = .4$

Hint: In parts (d) – (e), it's helpful to use that[1]

$$P(A \cap B') = P(A) - P(A \cap B)$$

and likewise

$$P(B \cap A') = P(B) - P(A \cap B)$$

6. (Chapter 2: problem 18) A wallet contains five \$10 bills, four \$5 bills, and six \$1 bills (nothing larger). If the bills are selected one by y one in random order, what is the probability that at least two bills must be selected to obtain a first \$10 bill?

Hint: This is just the probability that the first bill is not a \$10.
Denote $A = \{\text{The first bill is not a \$10 bill}\}$, $P(A) = 1 - 5/(5+4+6) = 2/3$

7. (Chapter 2: problem 19) Human visual inspection of solder joints on printed circuit boards can be very subjective. Part of the problem stems from the numerous types of solder defects (e.g., pad nonwetting, knee visibility, voids) and even the degree to which a joint possesses one or more of these defects. Consequently, even highly trained inspectors can disagree on the disposi- tion of a particular joint. In one batch of $10,000$ joints, inspector A found 724 that were judged defective, inspector B found 751 such joints, and 1159 of the joints were judged defective by at least one of the inspectors. Suppose that one of the $10,000$ joints is randomly selected.

(a) What is the probability that the selected joint was judged to be defective by neither of the two inspectors?
Denote $A = \{\text{Judged defective by } A\}$, $B = \{\text{Judged defective by } B\}$. From the information given, we have $P(A) = 724/10,000$, $P(B) = 751/10,000$, $P(A \cup B) = 1159/10,000$.
The event of judged to be defective by neither inspectors $= (A \cup B)'$, $P((A \cup B)') = 1 - P(A \cup B) = 1 - 1159/10,000 = .8841$

[1] To see this holds, note that $A \cap B'$ and $A \cap B$ are disjoint events, whose union is A(i.e., $A = [A \cap B'] \cup [A \cap B]$) so that $P(A) = P([A \cap B'] \cup [A \cap B]) = P(A \cap B') + P(A \cap B)$ by probability axiom 3; re-writing, we have $P(A \cap B') = P(A) - P(A \cap B)$.

(b) What is the probability that the selected joint was judged to be defective by inspector B but not by inspector A?
$P(B \cap A') = P(A \cup B) - P(A) = (1159 - 724)/10,000 = .0435$
Note $B = B \cap \Omega = (B \cap A) \cup (B \cap A')$

8. (Chapter 2: problem 22) The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The probability that he must stop at the first signal is .4, the analogous probability for the second signal is .5, and the probability that he must stop at at least one of the two signals is .7. What is the probability that he must stop

(a) At both signals?
Denote $A = \{$ Stop at the first signal$\}$, $B = \{$ Stop at the second signal$\}$, $P(A) = .4$, $P(B) = .5$, $P(A \cup B) = .7$
The event he must stop at both signals is $A \cap B$,
$P(A \cap B) = P(A) + P(B) - P(A \cup B) = .4 + .5 - .7 = .2$

(b) At the first signal but not at the second one?
$P(A \cap B') = P(A \cup B) - P(B) = .7 - .5 = .2$

(c) At exactly one signal?
$P((A \cap B') \cup (A' \cap B)) = P(A \cup B) - P(A \cap B) = .7 - .2 = .5$