Robotics HW 1 Due: 9/9/2022

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1. a) $\underline{AB} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -\cos\alpha\sin\beta - \sin\alpha\cos\beta & 0 \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

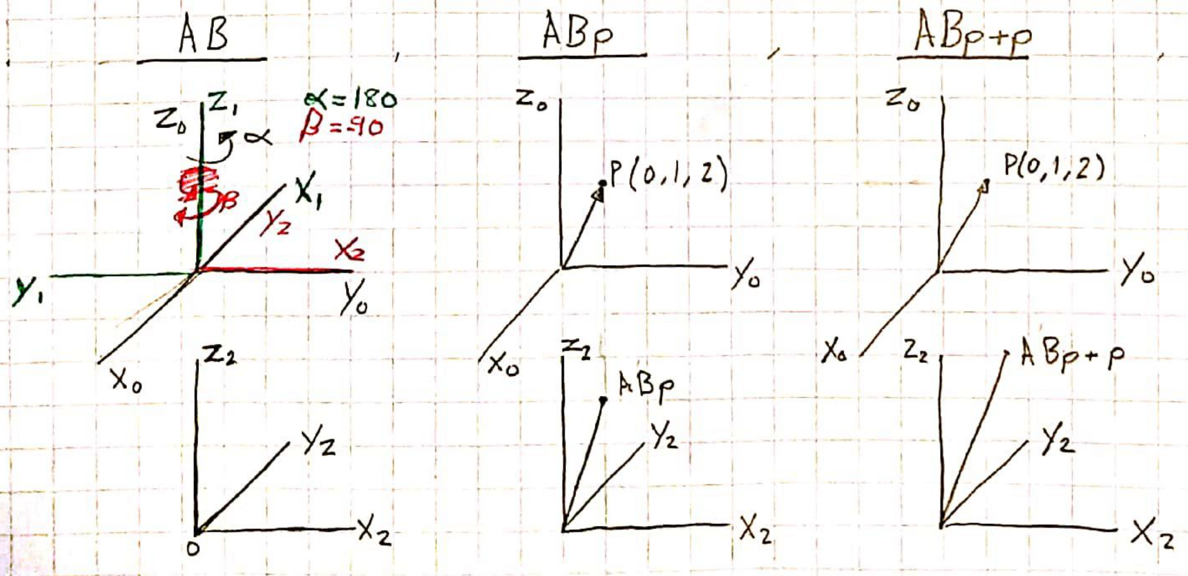
$$\underline{A\rho} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\sin\alpha \\ \cos\alpha \\ 2 \end{bmatrix}$$

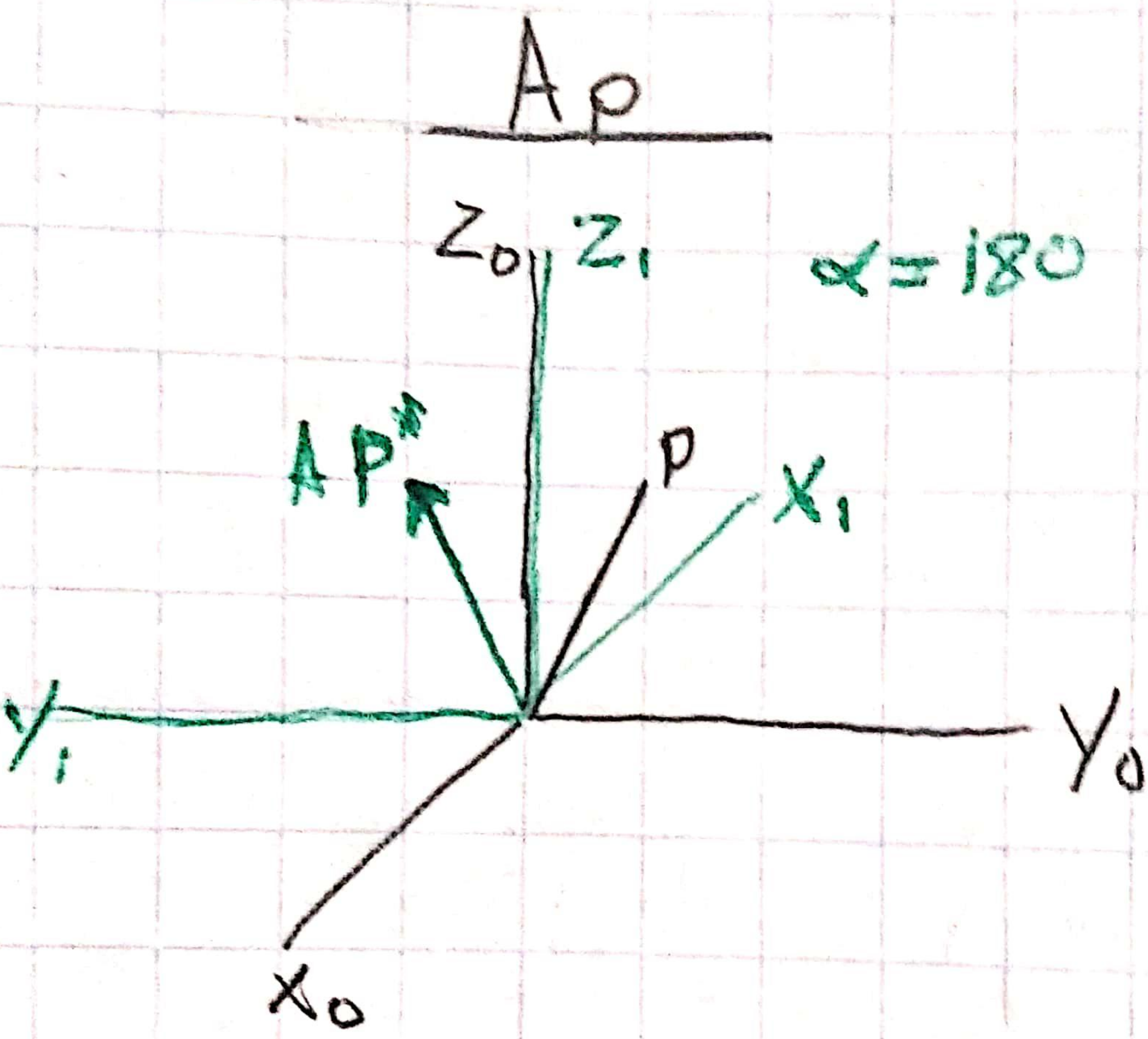
$$\underline{AB\rho} = \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -(\cos\alpha\sin\beta + \sin\alpha\cos\beta) & 0 \\ \cos\alpha\sin\beta + \sin\alpha\cos\beta & \cos\alpha\cos\beta - \sin\alpha\sin\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -(\cos\alpha\sin\beta + \sin\alpha\cos\beta) \\ \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ 2 \end{bmatrix}$$

$$\underline{AB\rho + \rho} = \nearrow + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -(\cos\alpha\sin\beta + \sin\alpha\cos\beta) \\ \cos\alpha\cos\beta - \sin\alpha\sin\beta + 1 \\ 4 \end{bmatrix}$$

A = Rotation in Z = B ρ = vector

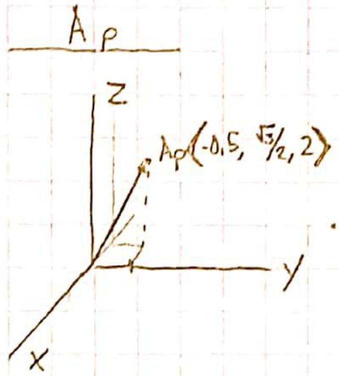


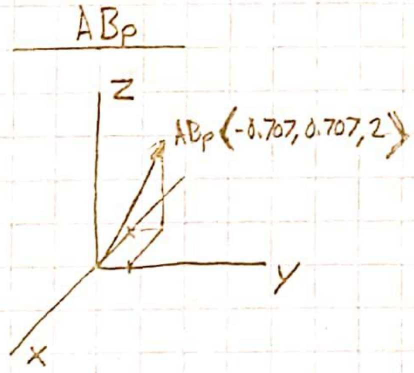

Ap
Z_0
Z_1
$\alpha = 180$
Ap''
p
X_1
Y_1
Y_0
X_0

1. b) $A = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0.966 & -0.259 & 0 \\ 0.259 & 0.966 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $p = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

$AB = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A_p = \begin{bmatrix} -0.5 \\ \sqrt{3}/2 \\ 2 \end{bmatrix}$

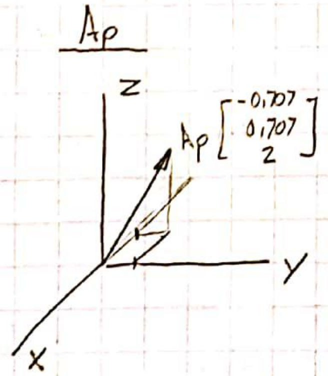
$AB_p = AB\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.707 \\ 0.707 \\ 2 \end{bmatrix}$ $\alpha = 30$ $\beta = 15$

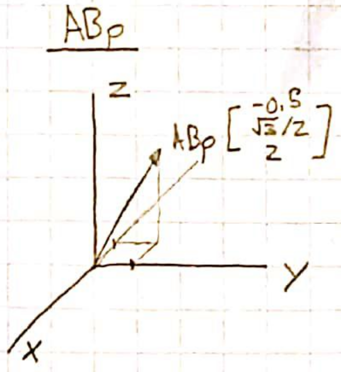




1.c) $\alpha = 45$ $\beta = -15$

$AB = \begin{bmatrix} \sqrt{3}/2 & -0.5 & 0 \\ 0.5 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A_p = \begin{bmatrix} -0.707 \\ 0.707 \\ 2 \end{bmatrix}$ $AB_p = \begin{bmatrix} -0.5 \\ \sqrt{3}/2 \\ 2 \end{bmatrix}$





1. d) $\alpha = 45 \quad \beta = 15$

$$A = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0.966 & -0.259 & 0 \\ 0.259 & 0.966 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B^T = \begin{bmatrix} 0.966 & 0.259 & 0 \\ -0.259 & 0.966 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \left[\begin{array}{ccc|ccc} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 1 & & \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & & 1 & \\ 0 & 0 & 1 & & & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 1 & 0 & -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$B^{-1} = \left[\begin{array}{ccc|ccc} 0.966 & -0.259 & 0 & 1 & & \\ 0.259 & 0.966 & 0 & & 1 & \\ 0 & 0 & 1 & & & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -0.268 & 0 & 0.966 & 0.254 & 0 \\ 0 & 1 & 0 & -0.259 & 0.966 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

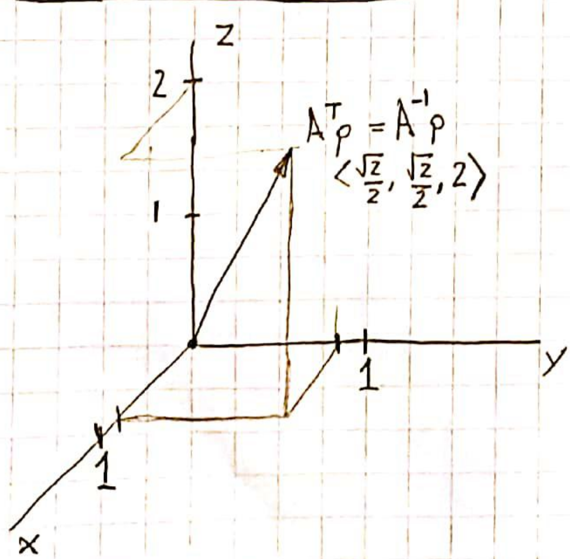
$$AB^{-1} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.966 & +0.259 & 0 \\ -0.259 & 0.966 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB^{-1}p = AB \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.866 \\ 2 \end{bmatrix}$$

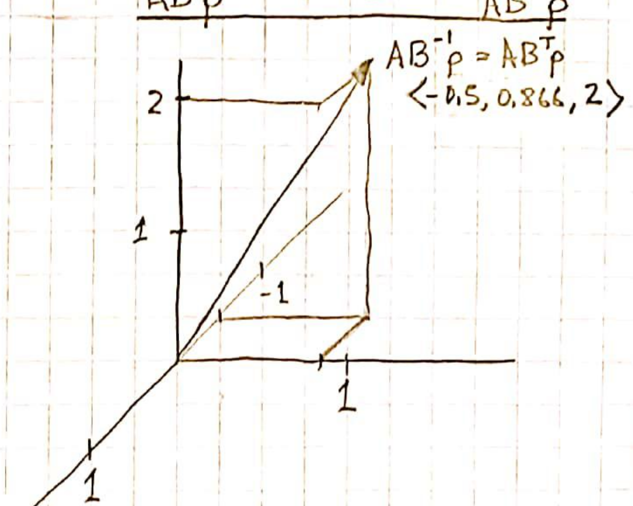
$$AB^T = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} = AB^T p = \begin{bmatrix} -0.5 \\ 0.866 \\ 2 \end{bmatrix} \quad \text{since } B^T = B^{-1}$$

$$A^{-1}p = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 2 \end{bmatrix} = A^T p = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 2 \end{bmatrix} \quad \text{since } A^{-1} = A^T$$

$A^{-1}p = A^T p$



$AB^{-1}p = AB^T p$



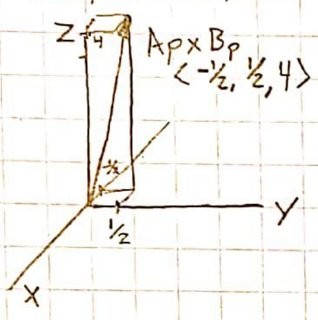
1.e) $\alpha = 45 \quad \beta = -45$

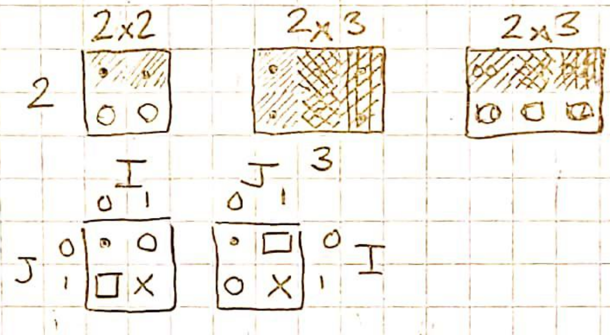
$$A_p = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \\ 2 \end{bmatrix} \qquad B_p = \begin{bmatrix} -\sin\beta \\ \cos\beta \\ 2 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 2 \end{bmatrix}$$

$$A_p \times B_p = \left\langle -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2},\ 2\cdot 2 \right\rangle = \boxed{\left\langle -\frac{1}{2}, \frac{1}{2}, 4 \right\rangle}$$

$$\| A_p \times B_p \| = \sqrt{\left((-\tfrac{1}{2})^2 + (\tfrac{1}{2})^2 + (4)^2 \right)} = \sqrt{(16.5)} = \boxed{4{,}062}$$

$\underline{A_p \times B_p}$





3.) H_1^0: the table w/ respect to the robot

$p^0 = R_1^0 p^1 + d^0 \rightarrow H_1^0 = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix}$

$d_1^0 = \langle d_x, d_y, d_z \rangle = \langle 0m, 1m, 1m \rangle$

$R_1^0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ since no rotation took place

$\underline{H_1^0} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\underline{H_2^0} = \begin{bmatrix} 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

H_2^0: the cube w/ respect to the robot

$d_2^0 = \langle -0.5m, 1.5m, 1m + 10cm \rangle = \langle -0.5, 1.5, 1.1 \rangle$

$R_2^0 =$ 3x3 matrix filled w/ 0's cause not rotation of the coordinates took place

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H_3^0: the camera w/ respect to the robot

$d_3^0 = \langle -0.5, 1.5, 1+2 \rangle = \langle -0.5, 1.5, 3 \rangle$

$R_3^0 = R_{z,\frac{\pi}{2}} R_{x,\pi} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & +1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$H_3^0 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

H_3^1 : the camera w/ respect to the table

$d_3^1 = \langle -0.5, 0.5, 2 \rangle$

$R_3^1 = R_{z,\frac{\pi}{2}} R_{x,\pi} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = R_3^0 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

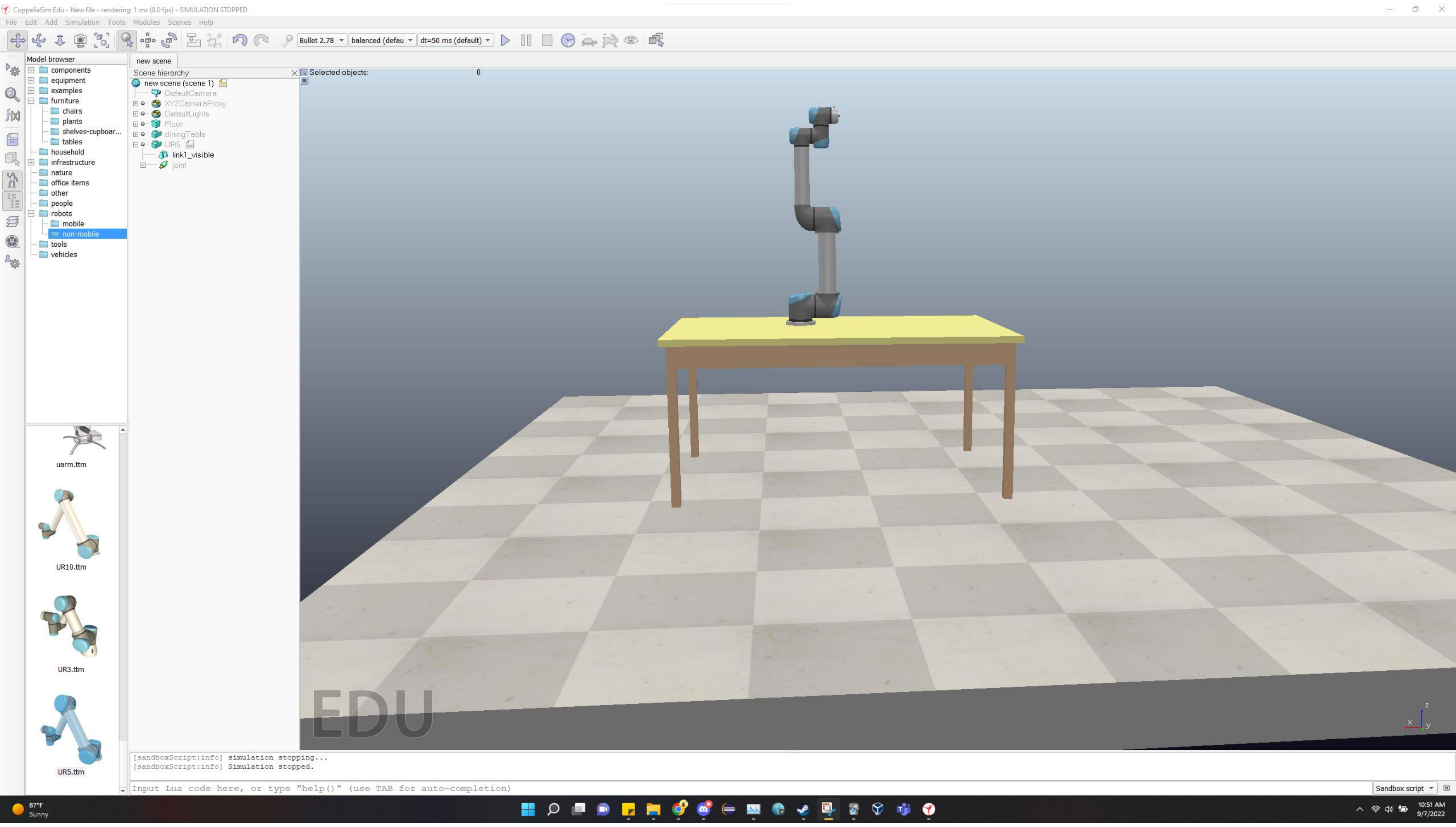
$H_3^1 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 0.5 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

H_3^2 : the camera w/ respect to the cube

$d_3^2 = \langle 0, 0, 2m - 10cm \rangle = \langle 0, 0, 1.9 \rangle$

$R_3^2 = R_3^0 = R_{z,\frac{\pi}{2}} R_{x,\pi} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$H_3^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$


CoppeliaSim Edu - New file - rendering: 1 ms (8.0 fps) - SIMULATION STOPPED
File Edit Add Simulation Tools Modules Scenes Help
Bullet 2.78
dt=50 ms (default)
Model browser
components
equipment
examples
furniture
chairs
plants
tables
household
infrastructure
nature
office items
other
people
robots
mobile
non-mobile
tools
vehicles
new scene
Scene hierarchy
new scene (scene 1)
DefaultCamera
XYZCameraProxy
DefaultLights
Floor
diningTable
UR5
link1_visible
joint
Selected objects:
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uarm.ttm
UR10.ttm
UR3.ttm
UR5.ttm
EDU
[sandboxScript:info] simulation stopping...
[sandboxScript:info] Simulation stopped.
Input Lua code here, or type "help()" (use TAB for auto-completion)
Sandbox script
87°F
Sunny
10:51 AM
9/7/2022