Answers for homework 4

3. $Pr(B) = Pr(B|A) \times Pr(A) + Pr(B/A^c) \times Pr(A^c)$

$= Pr(B|A_1) \times Pr(A_1) + Pr(B/A_2) \times Pr(A_2) + Pr(B/A_3) + Pr(A_3)$

$= 0.001 \times 0.99 + 0.90 \times 0.001 + 0.90 \times 0.009 = 0.00999$

4. a $Pr = 1 - \left[\binom{15}{0}(0.2)^0(0.8)^{15} + \binom{15}{1}(0.2)^1(0.8)^{14} + \binom{15}{2}(0.2)^2(0.8)^{13} + \binom{15}{3}(0.2)^3(0.8)^{12} \right.$

$\left. + \binom{15}{4}(0.2)^4(0.8)^{11} + \binom{15}{5}(0.2)^5(0.8)^{10} \right] = 0.061$

b $E(X) = np = 15 \times 0.2 = 3$

5. a $Pr(X < X_0) = Pr\left(\frac{X-\mu}{\sigma} < \frac{X_0-\mu}{\sigma}\right) = \Phi\left(\frac{200-219}{50}\right) = \Phi(-0.38)$

$= 0.352$

b $Pr(X > X_0) = 1 - Pr(X < X_0) = 1 - Pr\left(\frac{X-\mu}{\sigma} < \frac{X_0-\mu}{\sigma}\right) = 1 - \Phi\left(\frac{250-219}{50}\right)$

$= 1 - \Phi(0.62) = 0.2676$

c $Pr(X_1 < X < X_2) = Pr(X < X_2) - Pr(X < X_1) = Pr\left(\frac{X-\mu}{\sigma} < \frac{X_2-\mu}{\sigma}\right) - Pr\left(\frac{X-\mu}{\sigma} < \right.$

$\left.\frac{X_1-\mu}{\sigma}\right) = \Phi\left(\frac{250-219}{50}\right) - \Phi\left(\frac{200-219}{50}\right) = \Phi(0.62) - \Phi(-0.38) = 0.3804$

Answers for homework 5

1. a standard error $= \sigma_1/\sqrt{n_1} \approx 0.5/\sqrt{40} = 0.0791$ (Sd($\bar{X}_1$))

   $Sd(\bar{X}_2) = \sigma_2/\sqrt{n_2} \approx 0.4/\sqrt{32} = 0.0707$

   b If central limit theorem applies, $\bar{X} \sim N(\mu, \sigma^2/n)$

   which is $\bar{X}_1 \sim N(1.35, 0.00625)$, $\bar{X}_2 \sim N(0.92, 0.005)$

2. variable — Baseline — Follow up

Serum creatinine $CI = (\bar{X} - t_{n-1,1-\alpha/2} S/\sqrt{n}, \bar{X} + t_{n-1,1-\alpha/2} S/\sqrt{n})$

$= (0.97 - t_{101,0.975} \times \frac{0.22}{\sqrt{102}}, 0.97 + t_{101,0.975} \times \frac{0.22}{\sqrt{102}})$

$= (0.97 - 1.984 \times \frac{0.22}{\sqrt{102}}, 0.97 + 1.984 \times \frac{0.22}{\sqrt{102}})$

$= (0.9268, 1.0132)$

Follow up: $CI = (0.95436, 1.0456)$

variable	Baseline	Follow up
Serum potassium	$CI = (4.3042, 4.5557)$	$CI = (4.3195, 4.6605)$
Serum phosphate	$CI = (1.5876, 1.7723)$	$CI = (1.4739, 1.6661)$
PAIS	$CI = (33.3411, 39.6588)$	$CI = (19.9581, 26.5819)$

3. a. point estimator $\hat{p} = \frac{7}{525} = 13.33\%$

   b. $Sd(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.1333 \times (1 - 0.1333)}{525}} = 0.01483$

   $CI = (\hat{p} - Z_{1-\frac{\alpha}{2}} Sd(\hat{p}), \hat{p} + Z_{1-\frac{\alpha}{2}} Sd(\hat{p})) = (0.1043, 0.1623)$

Answers for homework 6

1. a/ $H_0: \mu = 1.2 \quad H_1: \mu \neq 1.2$

b/ Type I error = rejection of Ho that is actually true

c/ Type II error = fail to reject Ho that is actually wrong

d/ $t = \frac{\bar{X}-\mu}{S/\sqrt{n}} = \frac{1.05-1.2}{0.5/\sqrt{15}} = -1.1619$

$P_{value} = P(T \leq -1.1619 \text{ or } T \geq 1.1619) = 0.1324 \times 2 = 0.2648 > \alpha$

Fail to reject Ho

2. a/ $H_0: \mu = 10 \quad H_1: \mu \neq 10$

b/ Type I error = rejection of Ho $\mu = 9.1$ that is actually true

c/ Type II error = fail to reject Ho $\mu = 9.1$ that is actually wrong

d/ $t = \frac{\bar{X}-\mu}{S/\sqrt{n}} = \frac{9.1-10}{2.3/\sqrt{100}} = -3.9130$

$P_{value} = P(T \leq -3.9130 \text{ or } T \geq 3.9130) = 2 \times 0.0001 = 0.0002 < \alpha$

Reject Ho. which indicates μ is significantly different from 10

e/ $CI = (\bar{X} - t_{n-1,1-\frac{\alpha}{2}} S/\sqrt{n},\ \bar{X} + t_{n-1,1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}})$

$= (8.6437, 9.5563)$

3. a/ paired data

b/ $H_0: \mu_D = 0 \quad H_1: \mu_D \neq 0$

c/ For patient i, $D_i = Y_i - X_i$. $\bar{D} = \frac{\sum_{i=1}^{14} D_i}{14} = -0.3629 \quad t = \frac{-0.3629-0}{0.4059/\sqrt{14}} = -3.3453$

$P_{value} = P(T \leq -3.3453 \text{ or } T \geq 3.3453) = 2 \times 0.0026 = 0.0052 < \alpha$

Reject Ho.

d/ μ_D is significantly different from 0, which means two treatments are significantly different.

4. $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{(54.8 - 69.5) - 0}{\sqrt{S_p^2\left(\frac{1}{156} + \frac{1}{148}\right)}}$$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1) + (n_2-1)}$$

$$= \frac{(156-1) \times 28.1^2 + (148-1) \times 34.7^2}{(156-1) + (148-1)}$$

$$= -4.0687$$

$$= 991.3602$$

$P_{value} = P(T < -4.0687 \text{ or } T > 4.0687) \leq 0.0001 < \alpha$

The dietary fat intake between two groups of men are significantly different.

Answers for homework 8

1.a. $H_0: \mu_1 = \mu_2 = \mu_3$

$H_1: \mu_i \neq \mu_j$ for some i, j

b.

Source of variation	SS	df	MS	F value	p
between group	SST=180.067	2	MST=90.033	4.662	0.018
within group	SSE=521.459	27	MSE=19.313		
Total	701.526	29			

c. Since p value < 0.05, reject H_0. There is evidence that at least μ_j differs from the rest.

2.

a. $\hat{p}_1 = \frac{10}{10+90} = 0.1$ $\quad \hat{p}_2 = \frac{14}{14+86} = 0.14$ $\quad \hat{p}_3 = \frac{19}{19+81} = 0.19$

b. $H_0: P_1 = P_2 = P_3$ $\quad H_1: P_i \neq P_j$ for some i, j

$\hat{P} = \frac{10+14+19}{100+100+100} = 0.143$

	# of Tumors	
rat group	one or more	none
control	Observed = 10 expected = 0.143 x 100 = 14.3	observed = 90 expected = 85.7
Low dose	observed = 14 expected = 14.3	observed = 86 expected = 85.7
High dose	observed = 19 expected = 14.3	observed = 81 expected = 85.7

$$\chi^2 = \Sigma \frac{(O-E)^2}{E} = \frac{(10-14.3)^2}{14.3} + \frac{(14-14.3)^2}{14.3} + \frac{(19-14.3)^2}{14.3} + \frac{(90-85.7)^2}{85.7} + \frac{(86-85.7)^2}{85.7} + \frac{(81-85.7)^2}{85.7}$$

$= 3.3186$

Pvalue = 0.81 $> \alpha$, Fail to reject H_0

Answers for homework 9.

1. a. $\hat{Y_i} = \hat{\beta_0} + \hat{\beta_1} X_i$

$$\hat{\beta_1} = \frac{L_{xy}}{L_{xx}} = \frac{\sum(X_i-\bar{X})(y_i-\bar{y})}{\sum_i(X_i-\bar{X})^2} = \frac{\sum_i X_i y_i - \frac{\sum X_i \sum y_i}{n}}{\sum_i X_i^2 - \frac{(\sum X_i)^2}{n}} = \frac{423643.3 - \frac{23670 \times 214.9}{12}}{46689410 - \frac{(23670)^2}{12}} = -0.7372$$

$$\hat{\beta_0} = \bar{y} - \hat{\beta_1}\bar{x} = \frac{214.9}{12} + 0.7372 \times \frac{23670}{12} = 1471.965$$

b. $H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

$$t = \frac{\hat{\beta_1} - 0}{S_{\hat{\beta_1}}} \qquad S_{y.x} = \sum_i \frac{(y_i - \hat{y_i})^2}{n-2} = 0.5733 \qquad S_{\hat{\beta_1}} = \frac{S_{y\cdot x}}{\sqrt{\sum(X_i-\bar{X})^2}} = 0.03132$$

$$= \frac{-0.7372}{0.03122}$$

$= -23.54$ $\qquad Pr(>|t|) < 0.0001 < \alpha$. Reject H_0

c. $y = -0.7372 \times 1989 + 1471.965 = 5.6742$

d. It will be inaccurate to apply this model to make predictions or forecasts outside of relevant range

2. a. $H_0: \mu_y = 0$

$H_1: \mu_y \neq 0$

$$t = \frac{\bar{Y} - \mu_y}{S_y/\sqrt{n}} = \frac{\frac{290}{16} - 0}{15.0106/\sqrt{16}} = 4.82992$$

Pvalue $= 0.00022 < \alpha$

reject H_0

b, d, see attached plots.

c. $\hat{Y_i} = \hat{\beta_0} + \hat{\beta_1} X_i$

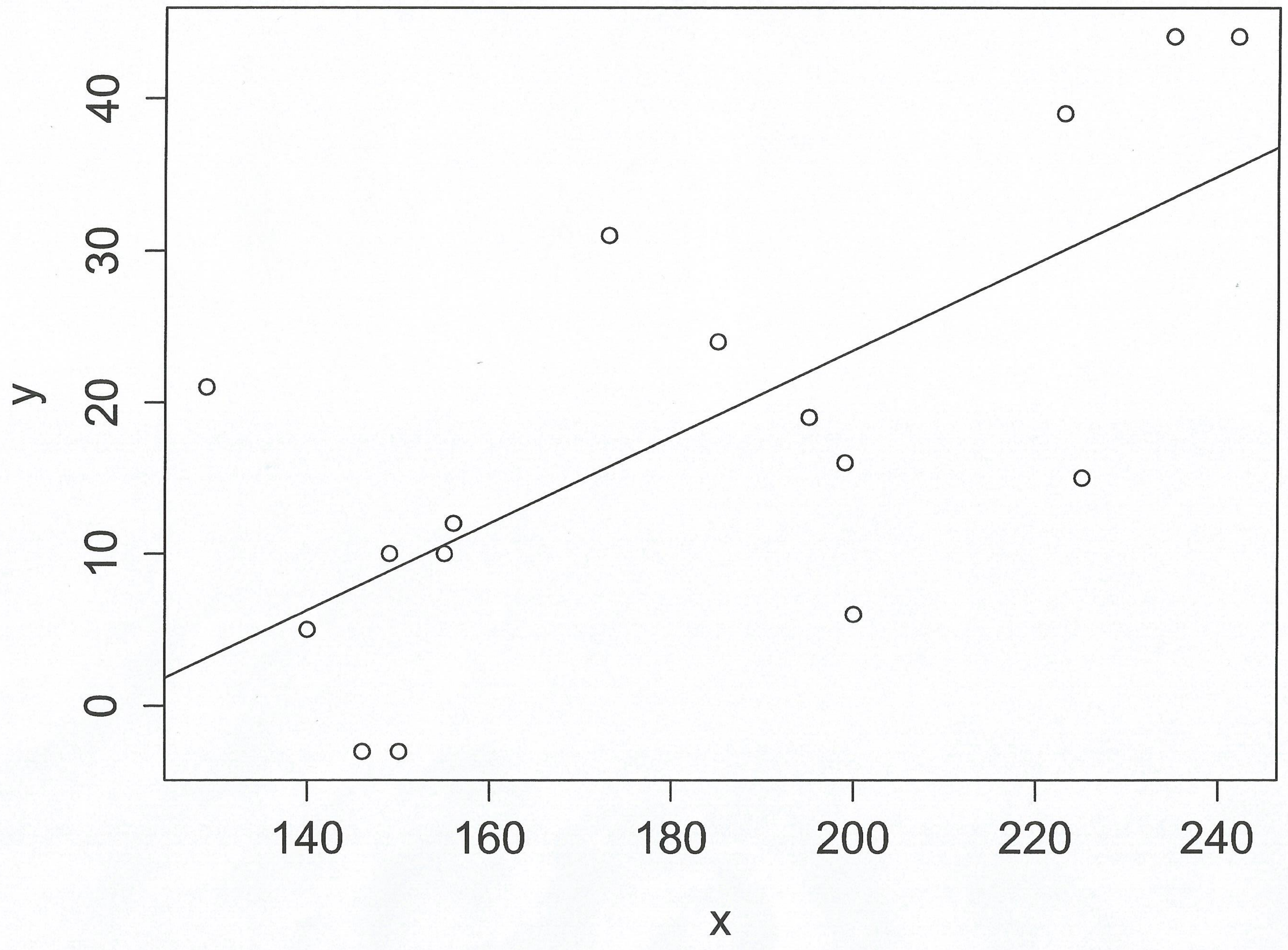
$$\hat{\beta_1} = \frac{\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n}}{\sum X_i^2 - \frac{(\sum X_i)^2}{n}} = 0.2866 \qquad \hat{\beta_0} = \bar{y} - \hat{\beta_1}\bar{x} = -33.8507$$

e. $H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

$$t = \frac{\hat{\beta}_1 - 0}{S_{\hat{\beta}_1}} = \frac{0.2866}{0.0780} = 3.672$$

$Pr(>|t|) = 0.00251 < \alpha$. Reject H_0



40
30
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