Number Representations

123_{10}

$$= 100_{10} + 20_{10} + 3_{10}$$

$$= 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$$

Each place represents a power of 10. Hence, a Base 10 number

$$\begin{array}{ccc} 1 & 2 & 3_{10} \\ \uparrow & \uparrow & \uparrow \\ 10^2 & 10^1 & 10^0 \end{array}$$

A Base 10 number has 10 digits

$$\text{Base } 10 = \underbrace{\{0, 1, 2, 3, \ldots 9\}}_{10 \text{ digits}}$$

The digits in Base 10 are are 0 to 9!

Computers do not work in Base 10. They work in Base 2. We prefer using Base 8 and Base 16 since it saves space and can easily be converted.

Base 2 → 2 digits = {0, 1}

Base 8 → 8 digits = {0, 1, 2, ... 7}

Base 16 → {0, 1, 2, ... 9, A, B, C, D, E, F}

Equivalence
Number Representation

BASE 2	8	10	16	2	8	10	BASE 16
0	0	0	0	1000	10	8	8
1	1	1	1	1001	11	9	9
10	2	2	2	1010	12	10	A
11	3	3	3	1011	13	11	B
100	4	4	4	1100	14	12	C
101	5	5	5	1101	15	13	D
110	6	6	6	1110	16	14	E
111	7	7	7	1111	17	15	F

___	___	___	___	___	___	___
64	32	16	8	4	2	1

The ease of conversion

$$8 = 2^3 \qquad 16 = 2^4$$

Base 2 can easily be converted into Base 8 and Base 16.

Base 8 is 3 places of Base 2
Base 16 is 4 places of Base 2

$$10101011101_2$$

Look at previous table to help

$$\boxed{10}\,\boxed{101}\,\boxed{011}\,\boxed{101}_2 \qquad\qquad \boxed{101}\,\boxed{0101}\,\boxed{1101}_2$$

$$2 \quad 5 \quad 3 \quad 5_8 \qquad\qquad 5 \quad 5 \quad D_{16}$$

Every 3 Places   Every 4 Plases

Which is equivalent in Base 10

$$2\times 8^3 + 5\times 8^2 + 3\times 8^1 + 5\times 8^0 = 5\times 16^2 + 5\times 16^1 + D\times 16^0$$
$$1024 + 320 + 24 + 5 = 1280 + 80 + 13$$
$$\underline{\underline{1373_{10}}} = \underline{\underline{1373_{10}}}$$

The conversion should be equivalent!

It Checks

Additional Examples

$1 \quad 2 \quad 3_8 = ?$ in Base 2, 10, 16

$001 \quad 010 \quad 011_2$ Every 3 places from Base 2

$\boxed{0}\boxed{0101}\boxed{0011}_2$

$0 \quad 5 \quad 3_{16}$ Every 4 places from Base 2

$$123_8 = 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$$
$$64 + 16 + 3 = \underline{\underline{83_{10}}}$$

$$1010011_2 = 1 \times 2^6 + 1 \times 2^4 + 1 \times 2^1 + 1 \times 2^0$$
$$64 + 16 + 2 + 1 = \underline{\underline{83_{10}}}$$

$$53_{16} = 5 \times 16^1 + 3 \times 16^0$$
$$80 + 3 = \underline{\underline{83_{10}}}$$

The conversion back to Base 10 shows the equivalence of all 3 representations.

Another Example

$$123_{16} = ? \text{ in Base } 2,\ 8,\ 10$$

$1 \quad 2 \quad 3_{16}$

16→2 use 4 places

$0001 \quad 0010 \quad 0011_2$

Use 4 places for Base 2

2→8 every 3 places

$\underline{1\,00}\ \underline{10\,0}\ \underline{011}_2$

$4 \quad 4 \quad 3_8$

From Base 2 use every 3 places

$$443_8 = 100100011_2$$

$$4\times8^2+4\times8^1+3\times8^0 = 1\times2^8+1\times2^5+1\times2^1+1\times2^0$$

$$256+32+3 = 256+32+2+1$$

$$291_{10} = 291_{10}$$

The conversion to Base 10 and its equivalence shows how correct the results are by agreement!

$127_{10} =$ ___ ___ ___ ___ ___ ___ ___ ___

128 64 32 16 8 4 2 1

Base 10		Base 2		Base 8		Base 16
127	=	1111111_2	=	177_8	=	$7F_{16}$
21_{10}	=	10101	=	25_8	=	15_{16}
57_{10}	=	111001_2	=	71_8	=	39_{16}
171_{10}	=	10101011_2	=	253_8	=	AB