Lab 4. Conversions

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Convert the following Base 10 numbers to both NASA and IEEE-754 Format.

NASA Format Steps
1. decimal → hex
2. hex → binary
3. sci notation 0..
4. 2's complement for (-) exp
5. NASA format
6. hex rep

1a) 0.125_{10} NASA Format

$0.125_{10} = 0.2_{16}$ ①

$0.125 \times 16 = 2.0$

$0.2_{16} = 0.0010_2$ ②

③ $0.0010 \times 2^0 = 0.10 \times 2^{-2}$

④ $2 = 00000010$

11111101

$+1$

$-2 = 11111110$

⑤ 0.1 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 | 1 1 1 1 1 1 1 0

⑥ $\boxed{4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad F \quad E}$

1b) 0.125_{10} IEEE 754 Format

IEEE Format Steps
- decimal → hex
- hex → binary
1. sci notation 1....
2. bias exponent
3. IEEE format (drop 1)
4. hex rep

① $0.0010 \times 2^0 = 1.0 \times 2^{-3}$

② $-3 + 127 = 124_{10} = 01111100_2$

③ 0 0 1 1 1 1 1 0 | 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0 | 0 0 0 0 0 0 0 0

④ $\boxed{3 \quad E \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0}$

2a) 0.3_{10} Nasa Format

(i) Convert decimal to hex

$0.3 \times 16 = 4.8$

$0.8 \times 16 = C.8$

$0.8 \times 16 = C.8$

$\vdots$

$0.3_{10} = 0.4CCC\ldots_{16} = 0.4\overline{C}_{16}$

* Infinite sequence to check $0.3_{10} = 0.4\overline{C}_{16}$

$$\sum_{i=0}^{\infty} C^i = \frac{1}{1-C}, \quad C < 1$$

Infinite Seq Steps

1. Formula to prove
2. Multiply 1 by C
3. Subtract 1 − 2

1. $S_N = C^0 + C^1 + C^2 + \cdots + C^N = \sum_{i=0}^{N} C^i$

2. $CS_N = C\sum_{i=0}^{N} C^i = C(C^0 + C^1 + C^2 + \cdots + C^N)$
$= C^1 + C^2 + \cdots + C^{N+1}$

3.
$$\begin{aligned} S_N &= C^0 + \cancel{C^1} + \cancel{C^2} + \cancel{C^3} + \cdots + \cancel{C^N} \\ -CS_N &= \cancel{C^1} + \cancel{C^2} + \cancel{C^3} + \cdots + \cancel{C^N} + C^{N+1} \end{aligned}$$

$S_N - CS_N = C^0 \qquad C^{N+1}$

$(1-C)S_N = C^0 + C^{N+1}$

$S_N = \dfrac{C_0 + \cancel{C^{N+1}}}{(1-C)}$ ← as $N \to \infty$, $C^{N+1} = 0$

$$\lim_{N\to\infty} S_N = \frac{1}{1-C}$$

$0.4\overline{C}_{16} = 0.3_{10}$ ✓

$$0.4\overline{C}_{16} = \frac{4}{16} + \frac{12}{16^2} + \frac{12}{16^3} + \frac{12}{16^4} + \cdots$$

$$= \frac{4}{16} + \frac{12}{16^2} \cdot \sum_{i=0}^{\infty} \left(\frac{1}{16}\right)^i$$

$$= \frac{4}{16} + \frac{12}{16^2}\left(1 + \frac{1}{16} + \frac{1}{16^2} + \cdots\right) = \frac{4}{16} + \frac{12}{16^2}\left(\frac{1}{\frac{16}{16} - \frac{1}{16}}\right)$$

$$= \frac{4}{16} + \frac{12}{16^2}\left(\frac{16}{15}\right) = \frac{4}{16} + \frac{12}{16}\left(\frac{1}{15}\right) = \frac{60+12}{16 \cdot 15} = \frac{72}{240} = 0.3 \checkmark$$

(2) Convert hex to binary

$0.4\overline{C}_{16} = 0.0100\overline{1100}_2$

③ Put in sci notation

$0.0.1001\overline{1100}_2 \times 2^0$

$0.1001\overline{1100} \times 2^{-1}$

$1 = 00000001$

11111110

$+1$

$-1 = 11111111$

④ Exponent to binary

⑤ Nasa Format

0.1 0 0 1 1 0 0 | 1 1 0 0 1 1 0 0 | 1 1 0 0 1 1 0 0 | 1 1 1 1 1 1 1 1

$\boxed{4 \quad C \quad C \quad C \quad C \quad C \quad F \quad F}$

2b) 0.3_{10} IEEE Format

① Scientific notation $0.01001\overline{1100} \times 2^0 = 1.001\overline{100} \times 2^{-2}$

② Bias exponent $-2 + 127 = 125_{10} = 01111101_2$

$1001 + 1 = 1010$

③ IEEE format

0 0 1 1 1 1 1 0 | 1 . 0 0 1 1 0 0 1 | 1 0 0 1 1 0 0 1 | 1 0 0 1 1 0 1 0

④ hex rep $\boxed{3 \quad E \quad 9 \quad 9 \quad 9 \quad 9 \quad 9 \quad A}$