NASA and IEEE 754 Format

Differences

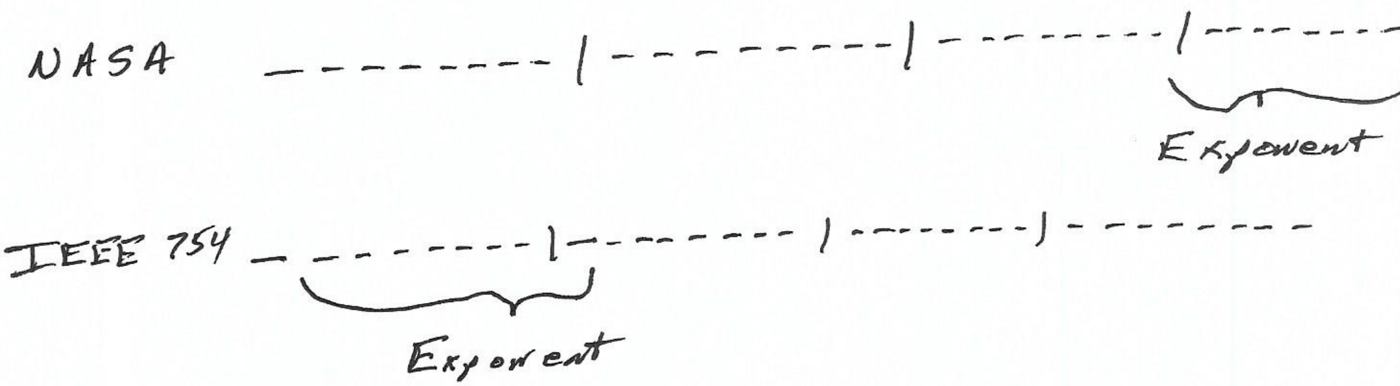
1) Scaling

$$NASA \rightarrow 0.1xxx \times 2^{E}$$

$$IEEE \rightarrow 1.xxx \times 2^{E-1}$$

The exponent has been shifted

2) The position of the Exponent is shifted. Moved to front Bits.



3) The exponent is biased by adding 127. No 2's complement

$$Nasa \rightarrow 0.1xxx \times 2^{E}$$

$$Vs. \quad 1.xxx \times 2^{E-1+127}$$

4) Lastly the 1 is dropped

NASA $\rightarrow$ $0.1xxx \times 2^{E}$

vs. $.xxx \times 2^{E-1+127}$

Gains an extra Bit of accuracy.

So, Let's start with an example

0.1_{10} $\rightarrow$ First convert to base 16

$$0.1 \times 16 = 1.\underline{6}$$
$$0.6 \times 16 = 9.\underline{6}$$
$$0.6 \times 16 = 9.\underline{6}$$
$$\vdots$$

$$0.1_{10} = 0.19999\ldots_{16} = 0.1\underline{9}_{16}$$

Let's have fun and add this infinite sequence to make sure it is 0.1_{10}

$$\sum_{i=0}^{\infty} c^{i} = \frac{1}{1-c}, \quad c < 1$$

Let's prove the previous formula

$$S_N = c^0 + c^1 + c^2 + c^3 + \ldots c^N$$

$$= \sum_{i=0}^{N} c^i$$

No multiply by c

$$c S_N = c \sum_{i=0}^{N} c^i = c^1 + c^2 + c^3 + \ldots c^{N+1}$$

Now subtract the first from the second

$$S_N = c^0 + \cancel{c^1} + \cancel{c^2} + \ldots \cancel{c^N}$$

$$-c S_N = \cancel{c^1} + \cancel{c^2} + \ldots \cancel{c^N} + c^{N+1}$$

$$(1-c) S_N = c^0 \qquad + c^{N+1}$$

$$S_N = \frac{c^0 + c^{N+1}}{1-c} \longrightarrow \begin{array}{c} \text{As } N \gg \\ c^{N+1} = 0 \end{array}$$

$$\underline{\underline{\lim_{N \to \infty} S_N = \frac{1}{1-c}}}$$

If we use ths formula then we can show equivalency

$$0.1\underline{9}_{16} = 1/16 + 9/16^2 + 9/16^3 + 9/16^4 + \cdots$$

$$= 1/16 + 9/16^2 \sum_{i=0}^{\infty} \left(1/16\right)^i$$

$$= 1/16 + 9/16^2 \left(1 + 1/16 + 1/16^2 + \cdots\right)$$

use formula

$$= 1/16 + 9/16^2 \left(\frac{1}{1 - 1/16}\right)$$

$$= 1/16 + 9/16^2 \left(16/15\right)$$

$$= 1/16 + 9/16 \cdot 15$$

$$= \frac{15 + 9}{16 \cdot 15} = \frac{24}{240} = .1_{10}$$

So $0.1_{10} = 0.1\underline{9}_{16}$

Now put this into WASA format

$$0.1\underline{9}_{16} = 0.0001\ 1001\ 1001 \cdots$$

$$= 0.11\,00\,\underline{1100} \times 2^{-4}$$

$$= 0.\underline{11\,00} \times 2^{-4} \rightarrow$$

2's complement

0000 0100
1111 1011
+ 1

$\underline{0}.\underline{1}\,\underline{1}\,\underline{0}\,\underline{0}\,\underline{1}\,\underline{1}\,\underline{0}\,|\,\underline{0}\,\underline{1}\,\underline{1}\,\underline{0}\,\underline{0}\,\underline{1}\,\underline{1}\,\underline{0}\,|\,\underline{0}\,\underline{1}\,\underline{1}\,\underline{0}\,\underline{0}\,\underline{1}\,\underline{1}\,\underline{0}\,|\,\underline{1}\,\underline{1}\,\underline{1}\,\underline{1}\,\underline{1}\,\underline{1}\,\underline{0}\,\underline{0}$

6 6 6 6 6 6 F C

We could actually round up if we wanted

$$\underline{\underline{6\ 6\ 6\ 6\ 6\ 7\ F\ C}}$$

Now convert to IEEE 754

$$0.\underline{1100} \times 2^{-4}$$

$$= 1.\underline{1001} \times 2^{-5}$$

The exponent -5 becomes

$$127 - 5 = \underline{\underline{122}}$$

We drop the 1. and plug in

122 $\downarrow$ 1001 $\downarrow$

0 | 0111101 | 0 | 1001100 | 11001100 | 11001100

Exponent

3 D 4 C C C C C

$\Uparrow$

Rounding Up

$$\underline{\underline{3D\ 4C\ CC\ CD}}$$