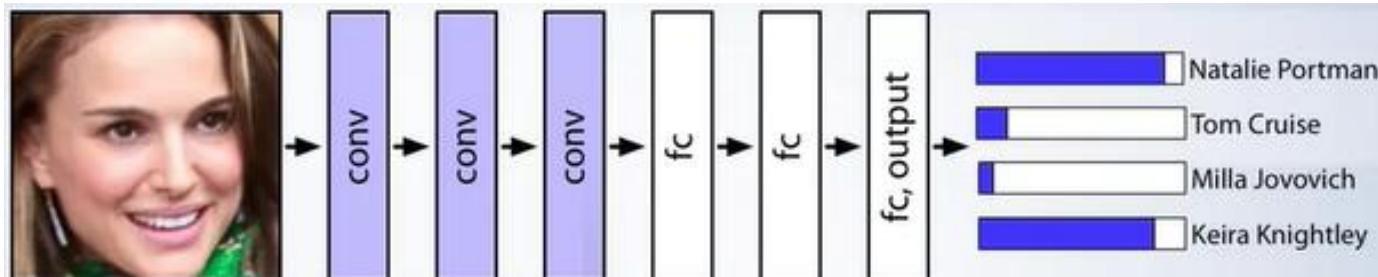


L4.1 CNNs and RNNs

Z. Gu 2021



Acknowledgement: some contents taken from UC Berkeley CS231n <https://cs231n.github.io>

Coursera MOOC on CNN: <https://www.coursera.org/learn/convolutional-neural-networks>

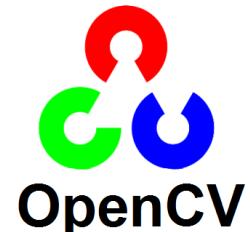
Hung-yi Lee: <https://speech.ee.ntu.edu.tw/~hylee/ml/2021-spring.html>

Outline

- CNN Convolution layers
- Pooling and Fully-Connected layers
- CNN case studies
- RNNs

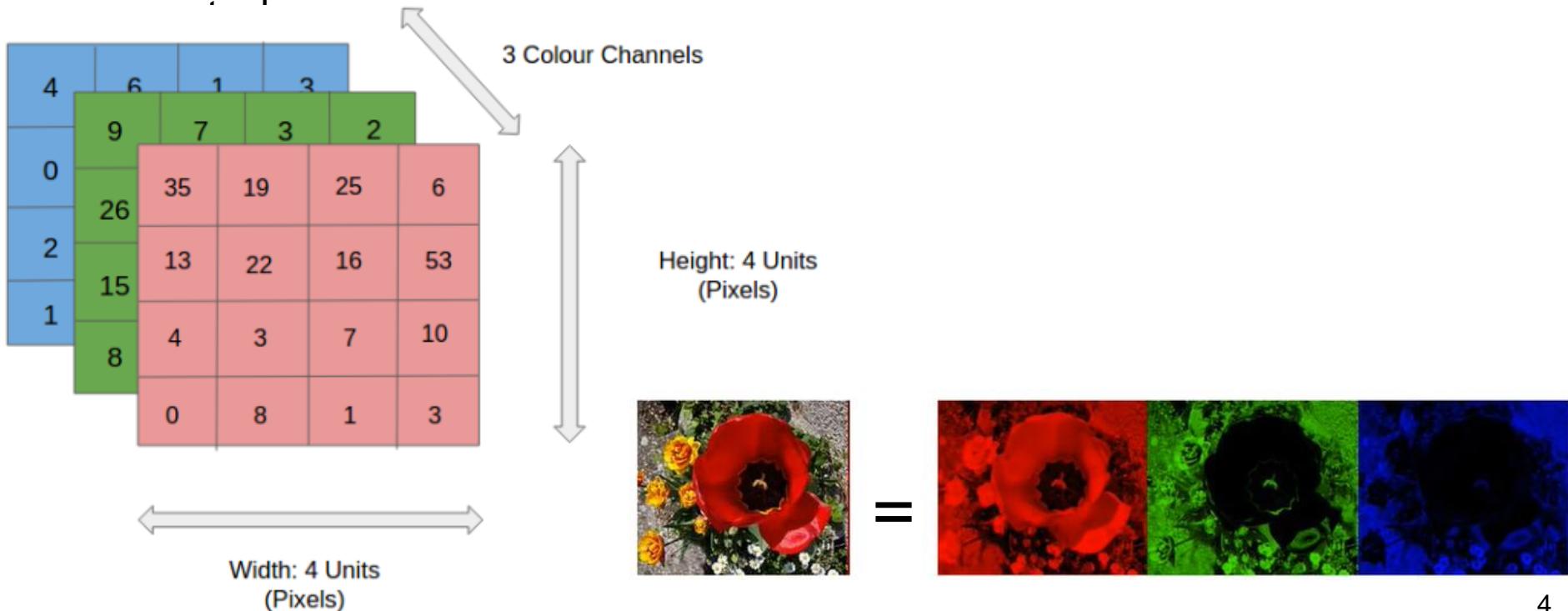
Classic Computer Vision

- Most “classic” (non-ML) CV algorithms are implemented in the OpenCV library, including
 - Core Operations:
 - basic operations on image like pixel editing, geometric transformations...
 - Image Processing
 - Thresholding, smoothing, edge detection, Hough Line Transform...
 - Feature Detection and Description
 - HOG, SIFT, SURF, BRIEF, ORB...
 - Video analysis
 - Object tracking w. optical flow
 - Camera Calibration and 3D Reconstruction
- They are simple, fast and reliable (e.g., for lane detection), and are often used in place of or in conjunction w. complex ML/DL algorithms, which may sometimes be unreliable and unpredictable.



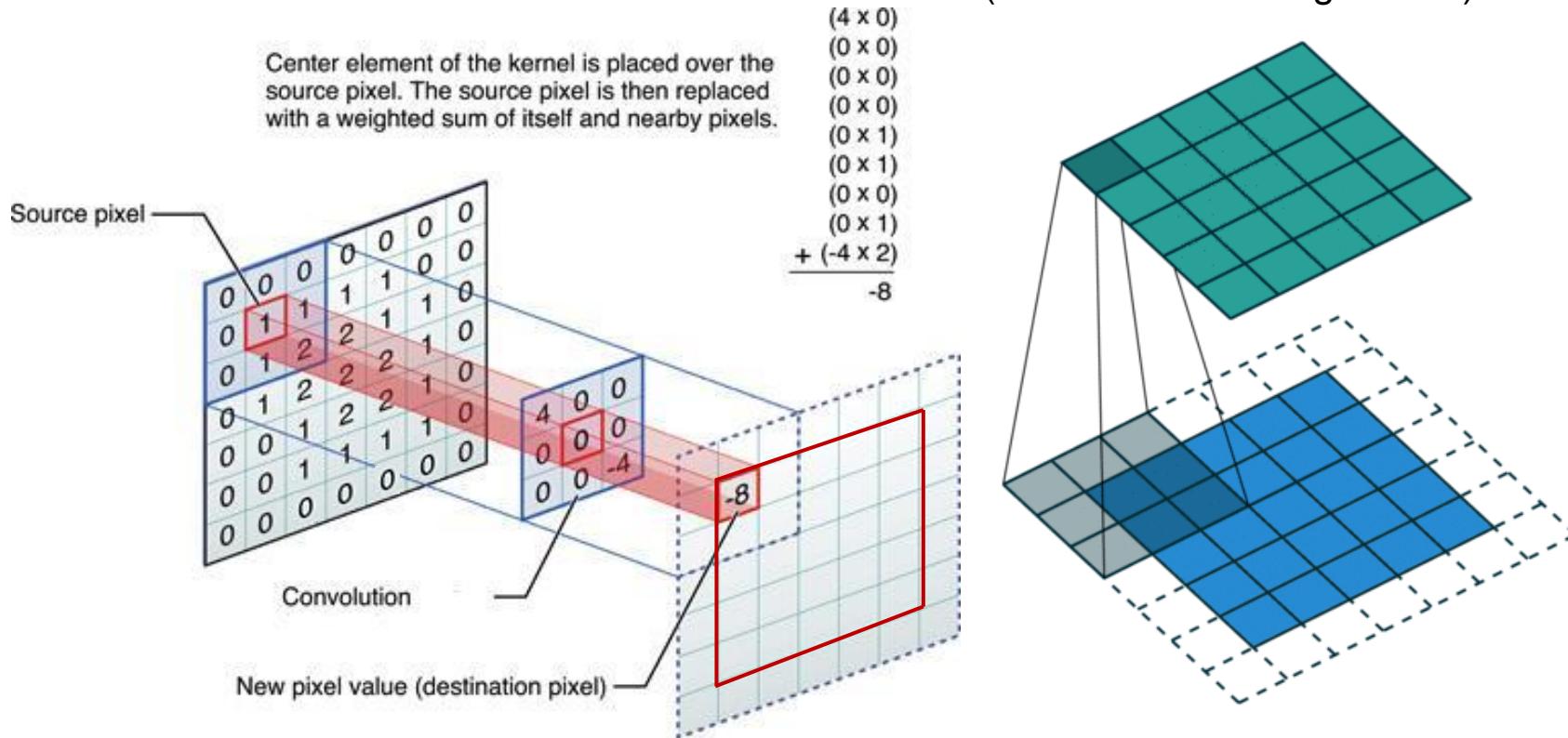
Input Image Encoding

- A size $N \times N$ color image has volume $N \times N \times 3$, w. $N \times N$ pixels and 3 color components (Red, Green, and Blue, RGB) for each pixel
- A size $N \times N$ greyscale image has volume $N \times N \times 1$
- Color depth, or bit depth, is number of bits used for each color component of a single pixel
 - Typical value is 8, so pixel value has range [0, 255]
 - Larger depth is possible, e.g., true color (24-bit) is used in computer and phone displays for human eyes, but 8-bit is typically enough for CV



Filters/Kernels in Computer Vision

- Convolution operation: we slide (convolve) each filter across the width and height of the input volume and compute dot products between the entries of the filter (kernel) and the input at any position. As we slide the filter over the width and height of the input volume we will produce a 2-dimensional activation map that gives the responses of that filter at every spatial position.
 - dot product operation: elementwise multiplication of a filter w. corresponding input values, then summing them to generate one output value
 - Used to extract features for downstream tasks (classification or regression)



A Filter for Vertical Edge Detection

| | | | | | |
|----|----|----|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |



$$\begin{matrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{matrix} * \begin{matrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \\ \hline \text{---} & \text{---} & \text{---} \end{matrix} = \begin{matrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ \hline \text{---} & \text{---} & \text{---} & \text{---} \end{matrix}$$



| | | | | | |
|---|---|---|----|----|----|
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |



$$\begin{matrix} 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \end{matrix} * \begin{matrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \\ \hline \text{---} & \text{---} & \text{---} \end{matrix} = \begin{matrix} 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ \hline \text{---} & \text{---} & \text{---} & \text{---} \end{matrix}$$



Sobel Filter for Vertical Edge Detection

| | | | | | |
|----|----|----|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |



$$\begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} =$$

| | | | |
|---|----|----|---|
| 0 | 40 | 40 | 0 |
| 0 | 40 | 40 | 0 |
| 0 | 40 | 40 | 0 |
| 0 | 40 | 40 | 0 |



| | | | | | |
|---|---|---|----|----|----|
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |

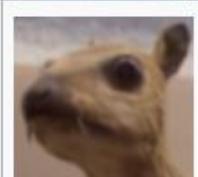
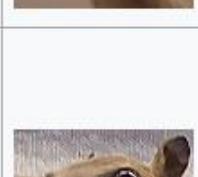


$$\begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} =$$

| | | | |
|---|-----|-----|---|
| 0 | -40 | -40 | 0 |
| 0 | -40 | -40 | 0 |
| 0 | -40 | -40 | 0 |
| 0 | -40 | -40 | 0 |



Common Filters in CV

| Operation | Kernel ω | Image result $g(x,y)$ | | |
|----------------|---|---|--|---|
| Identity | $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ |  | | |
| Edge detection | $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ |  | | |
| Sharpen | $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ |  | | |
| | | | Box blur (normalized) | $\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  |
| | | | Gaussian blur 3×3 (approximation) | $\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$  |
| | | | Gaussian blur 5×5 (approximation) | $\frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$  |
| | | | Unsharp masking 5×5 Based on Gaussian blur with amount as 1 and threshold as 0 (with no image mask) | $\frac{-1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & -476 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$  |

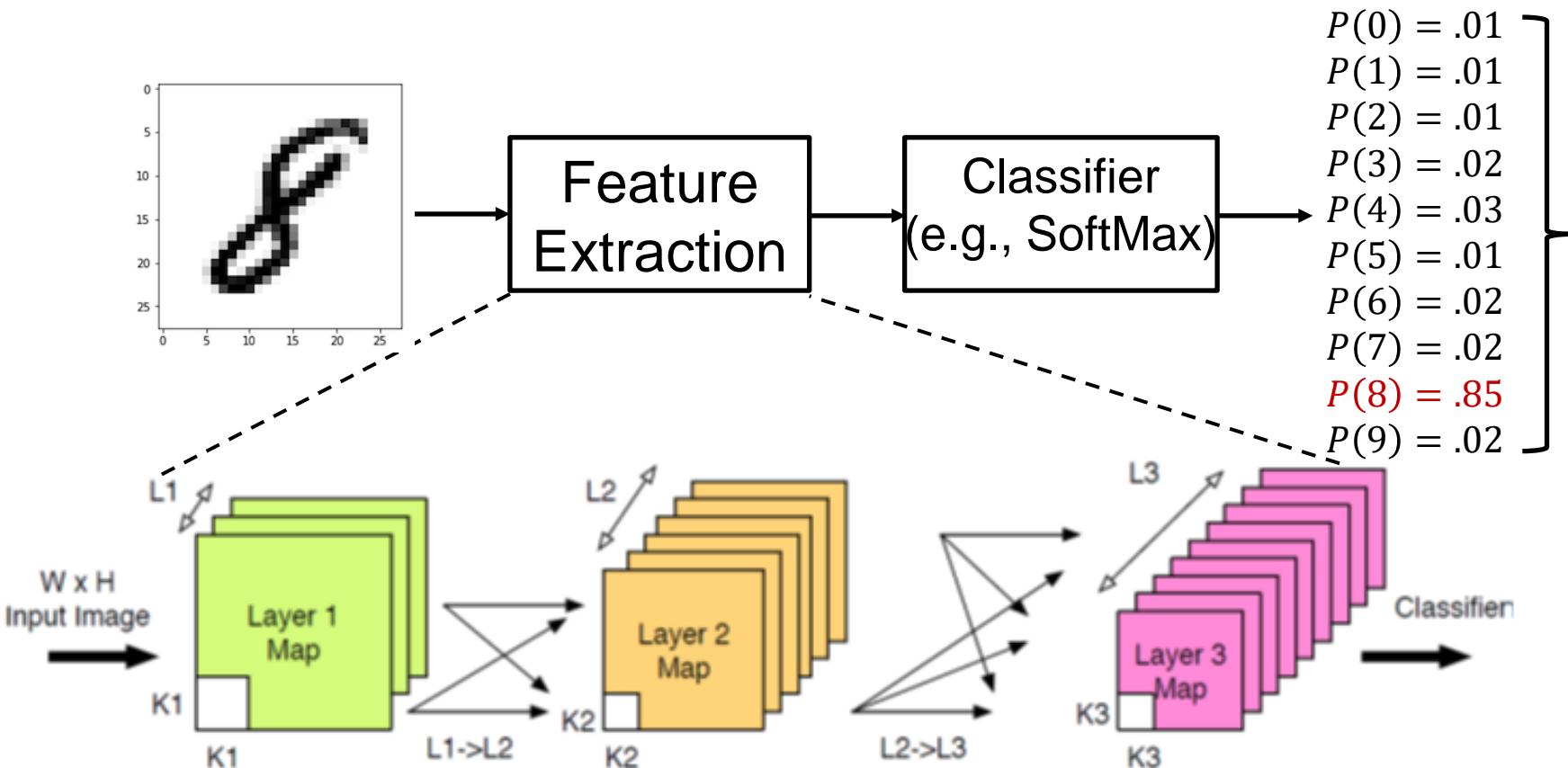
- These filters were designed, or “hand-crafted”, by CV researchers. They extract features used by downstream tasks such as classification, image segmentation, etc.

Machine Learning Meets CV

- Instead of hand-crafted filters in classic CV, why not learn custom convolutional filters from data by supervised learning?
 - For easy tasks like edge detection, learning may recover filters similar to hand-crafted ones.
 - For difficult CV tasks, learning is essential to achieving good results

$$\begin{array}{|c|c|c|c|c|} \hline & & & & \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline w_1 & w_2 & w_3 \\ \hline w_4 & w_5 & w_6 \\ \hline w_7 & w_8 & w_9 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|} \hline & & & & \\ \hline \end{array}$$

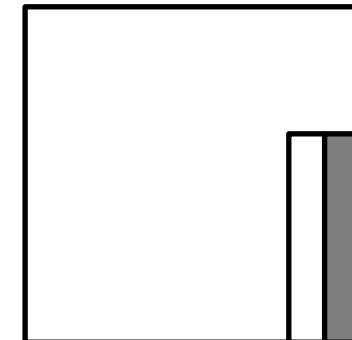
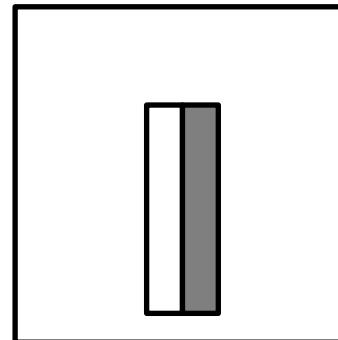
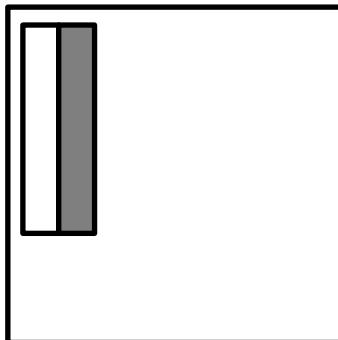
Convolutional Neural Networks (CNN)



- A CNN (also called ConvNet) is a sequence of Convolutional (CONV) Layers, Pooling (POOL) Layers and non-linear activation functions for feature extraction, followed by one or more Fully-Connected (FC) Layers for classification based on the extracted features

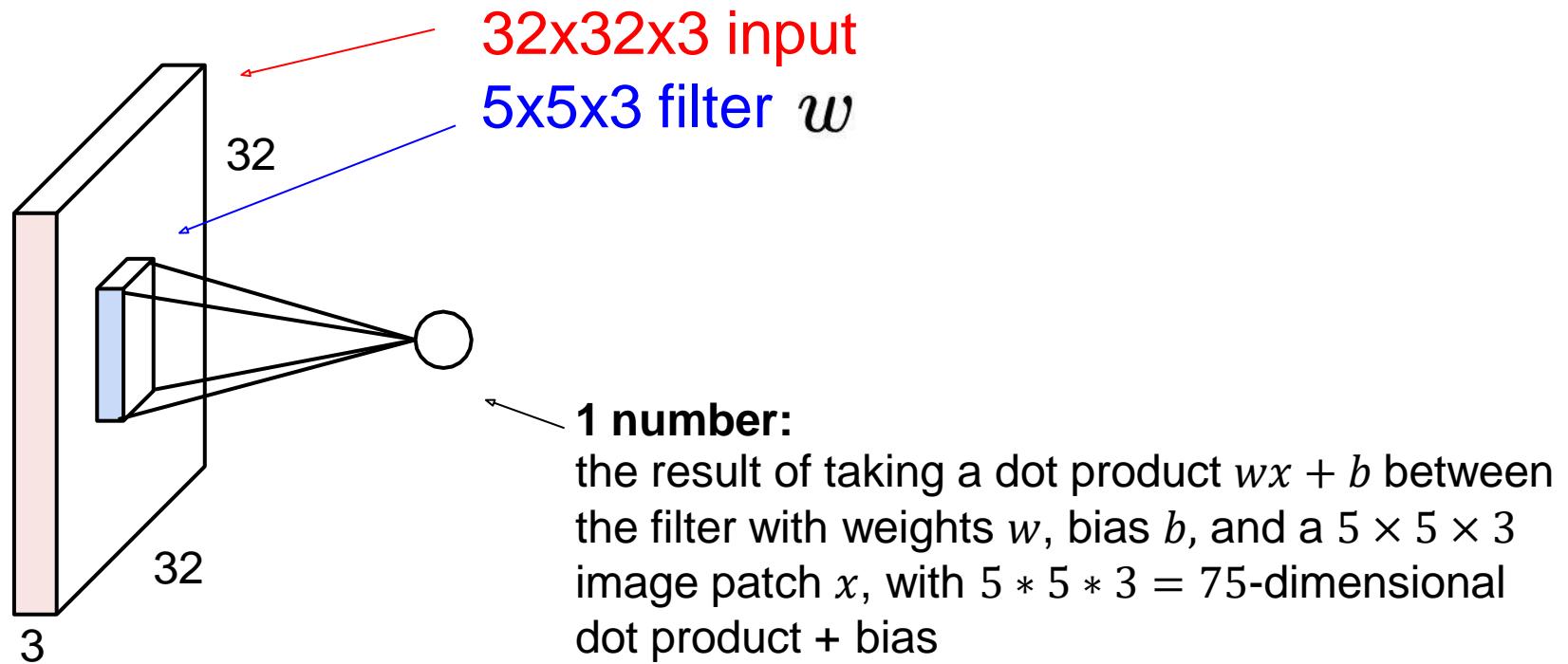
Receptive Field and Parameter Sharing

- Each neuron in a CONV layer has local, sparse connectivity to a small patch of the input volume w. size of the filter, called its Receptive Field
 - Each neuron covers a limited, narrow “field-of-view”
 - In contrast, each neuron in a FC layer has RF that covers the entire input volume
- Parameter sharing: all neurons in the same CONV layer share the same filter params w, b
 - It helps to reduce the number of params significantly compared to fully-connected networks
 - It gives translation invariance, e.g., an edge can be detected regardless of its location in the image

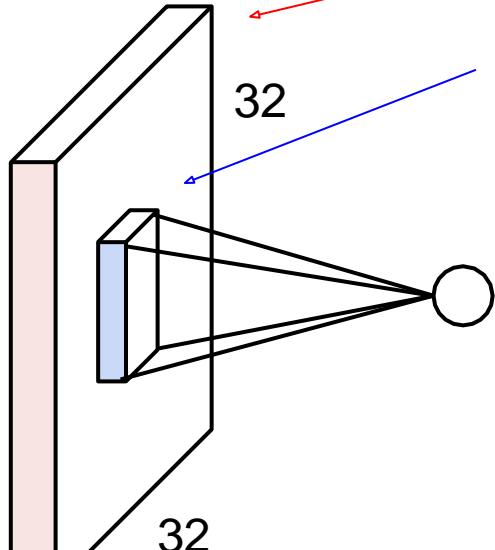


Convolution Operation

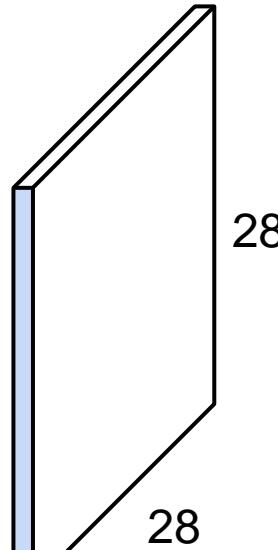
- Slide the filter over the image spatially, computing dot products $w^T x + b$ to generate an activation map as output
- The input may be an input RGB image w. 3 channels, hence depth=3, or intermediate activation maps generated by hidden layers of a CNN. We use the terms “input volume” and “output volume” to emphasize they may be 3D tensors



32x32x3 input
5x5x3 filter

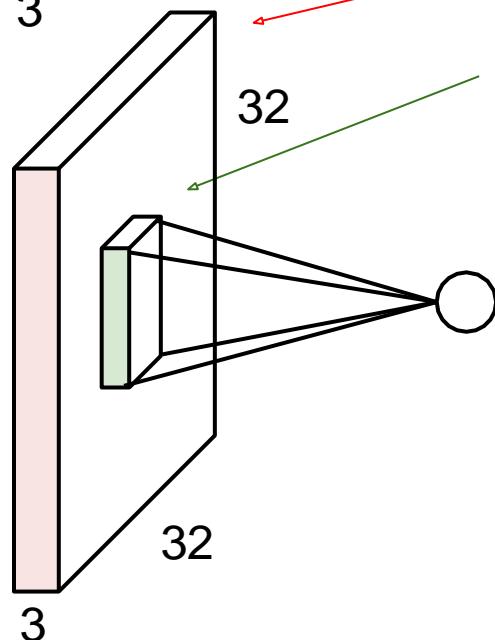


convolve (slide)
over all spatial
locations

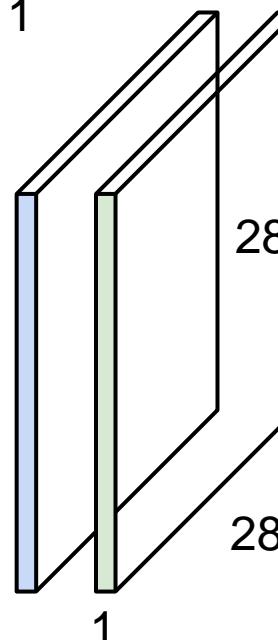


One (blue) filter
generates one
2D activation
map as output

32x32x3 input
5x5x3 filter



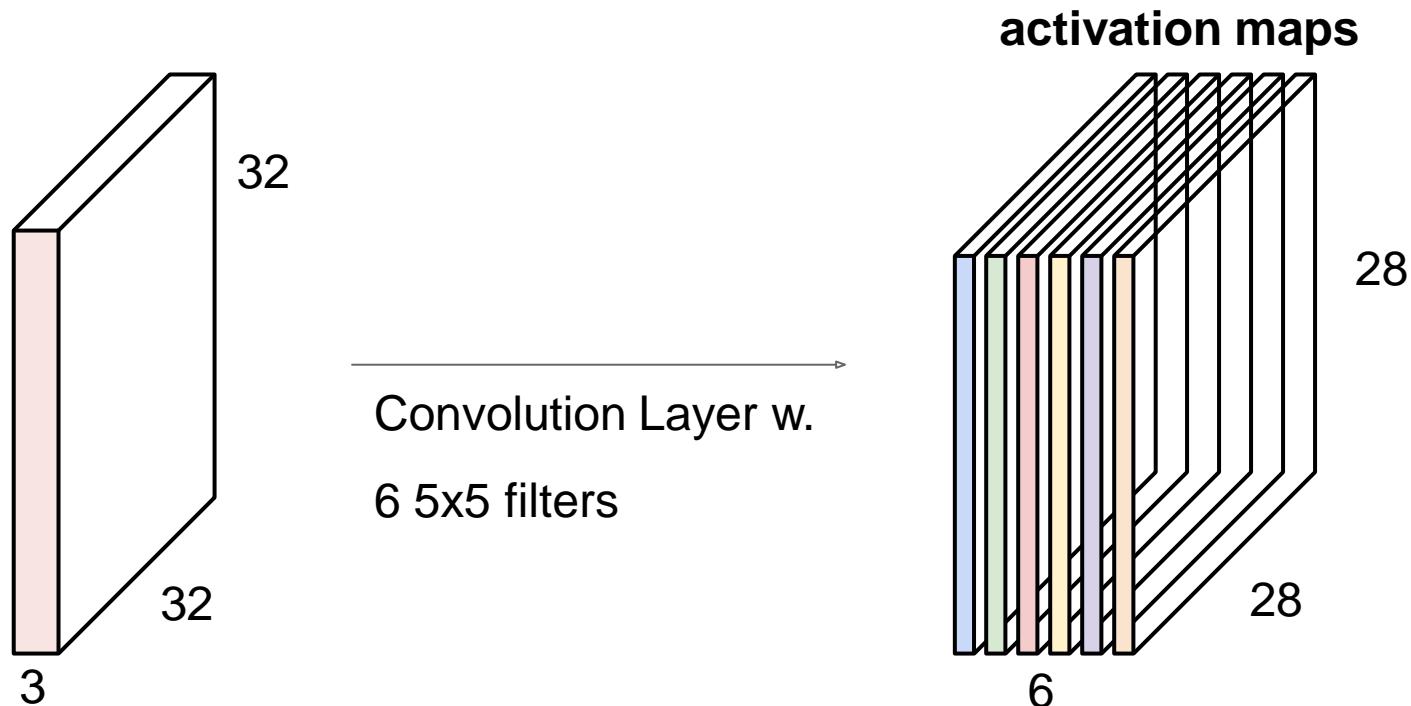
convolve
(slide) over all
spatial
locations



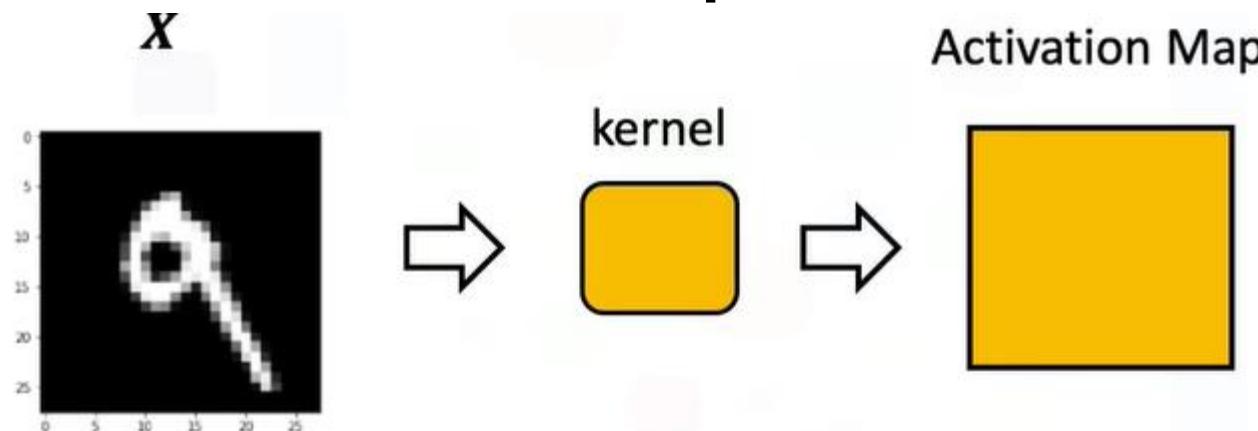
Multiple (blue
and green)
filters generate
multiple (blue
and green) 2D
activation maps,
stacked along
the depth
dimension to
produce the 3D
output volume

Stacked Activation Maps

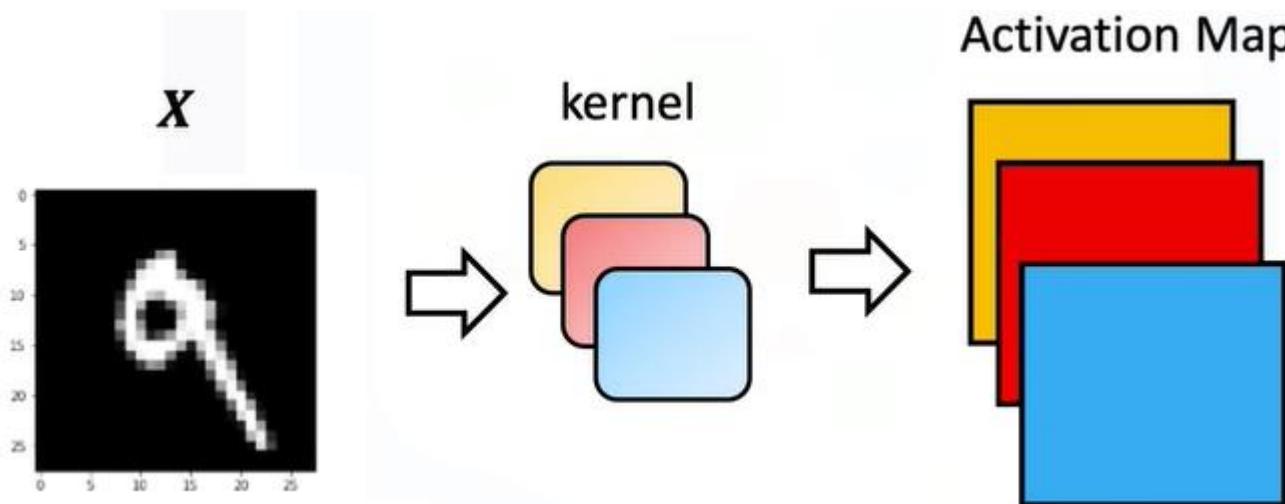
- If we have 6 5×5 filters, we'll get 6 separate activation maps (also called feature maps).
- We stack these up to get an output volume (a new “image”) of size $28 \times 28 \times 6$, an intermediate representation to be passed to subsequent layers



Activation Maps Illustration



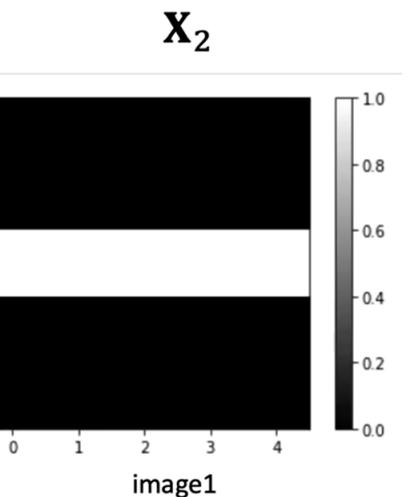
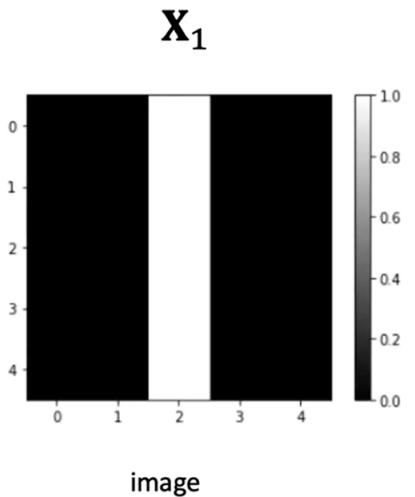
1 filter/kernel, 1 output activation map



3 filters/kernels, 3 output activation maps

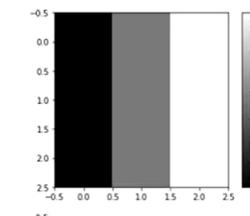
Concrete Example: 3 Filters

- 3 filters W_0, W_1, W_2 , each extracting different features. ($W_i * X_j$ denotes convolution of filter W_i w. input X_j)
- Upper left: filter W_0 extracts vertical line features Z_0 from input image X_1 . (the other 2 filters do not extract any meaningful features)
- Lower left: filter W_1 extracts horizontal line features Z_1 from input image X_2 (the other 2 filters do not extract any meaningful features)



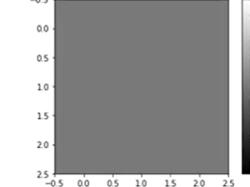
$$Z_0 = W_0 * X_1 + b_0$$

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -2 |
| 1 | 0 | -1 |



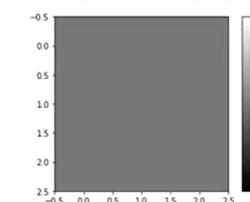
$$Z_1 = W_1 * X_1 + b_1$$

| | | |
|----|----|----|
| 1 | 2 | -1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |



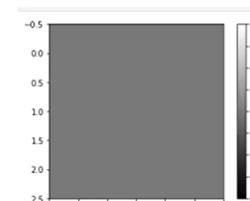
$$Z_2 = W_2 * X_1 + b_2$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |



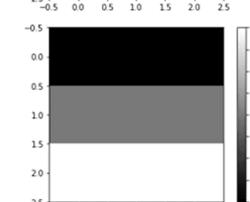
$$Z_0 = W_0 * X_2 + b_0$$

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -2 |
| 1 | 0 | -1 |



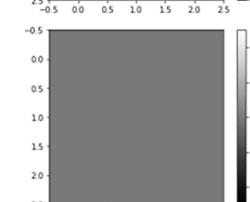
$$Z_1 = W_1 * X_2 + b_1$$

| | | |
|----|----|----|
| 1 | 2 | -1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

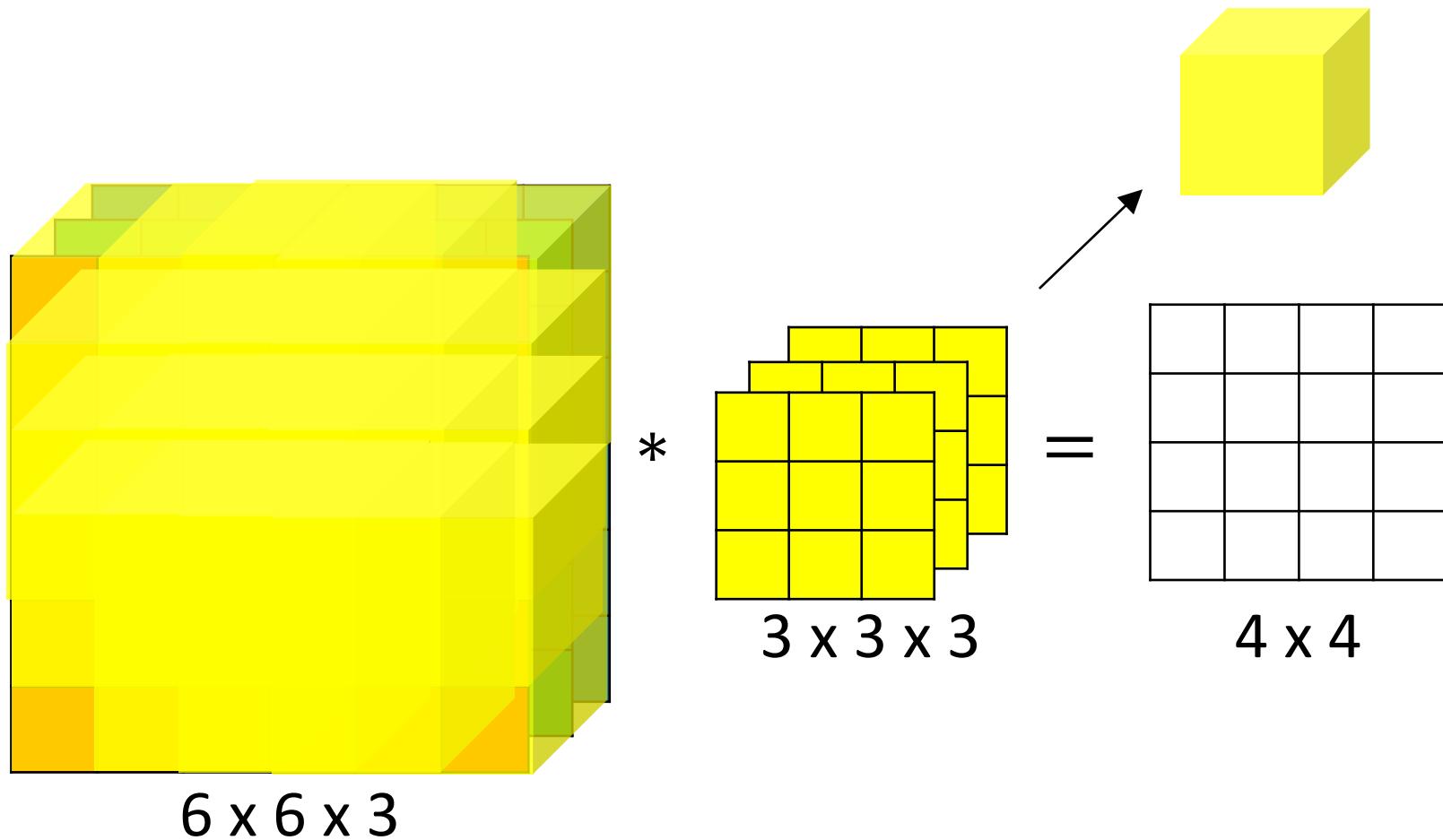


$$Z_2 = W_2 * X_2 + b_2$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

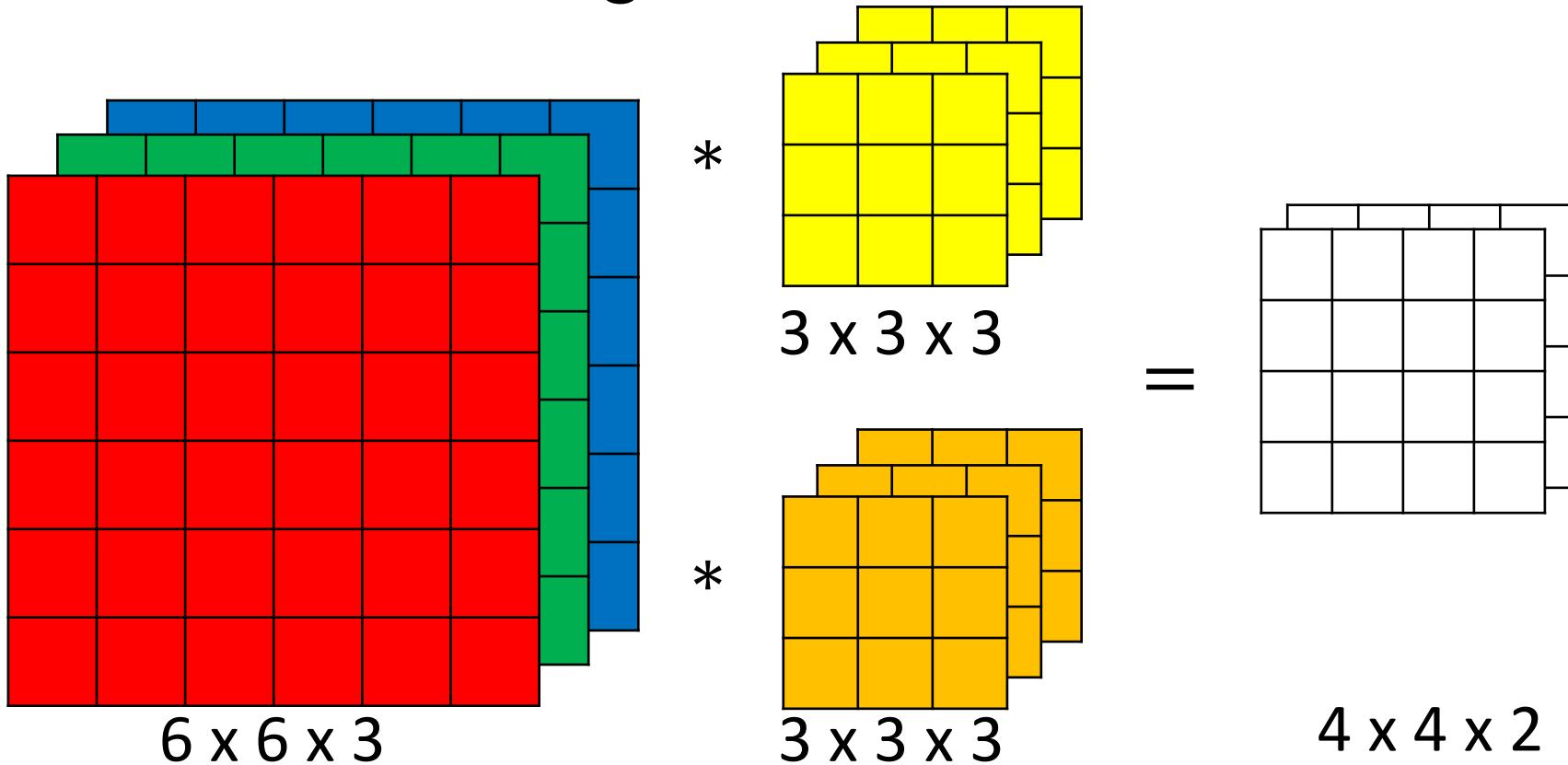


Convolution of a Filter on RGB Image w. 3 Channels





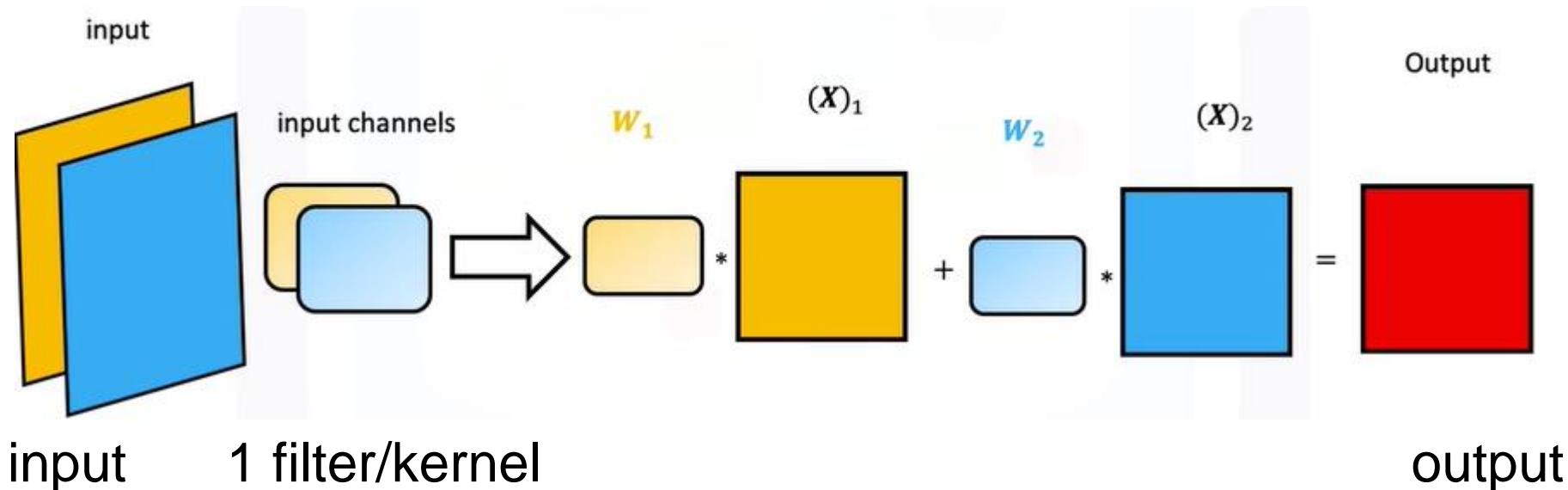
Convolution of 2 Filters on RGB Image w. 3 Channels



- Each filter always has the same depth (3) as its input volume, and the number of filters always equals the depth (2) of its output volume

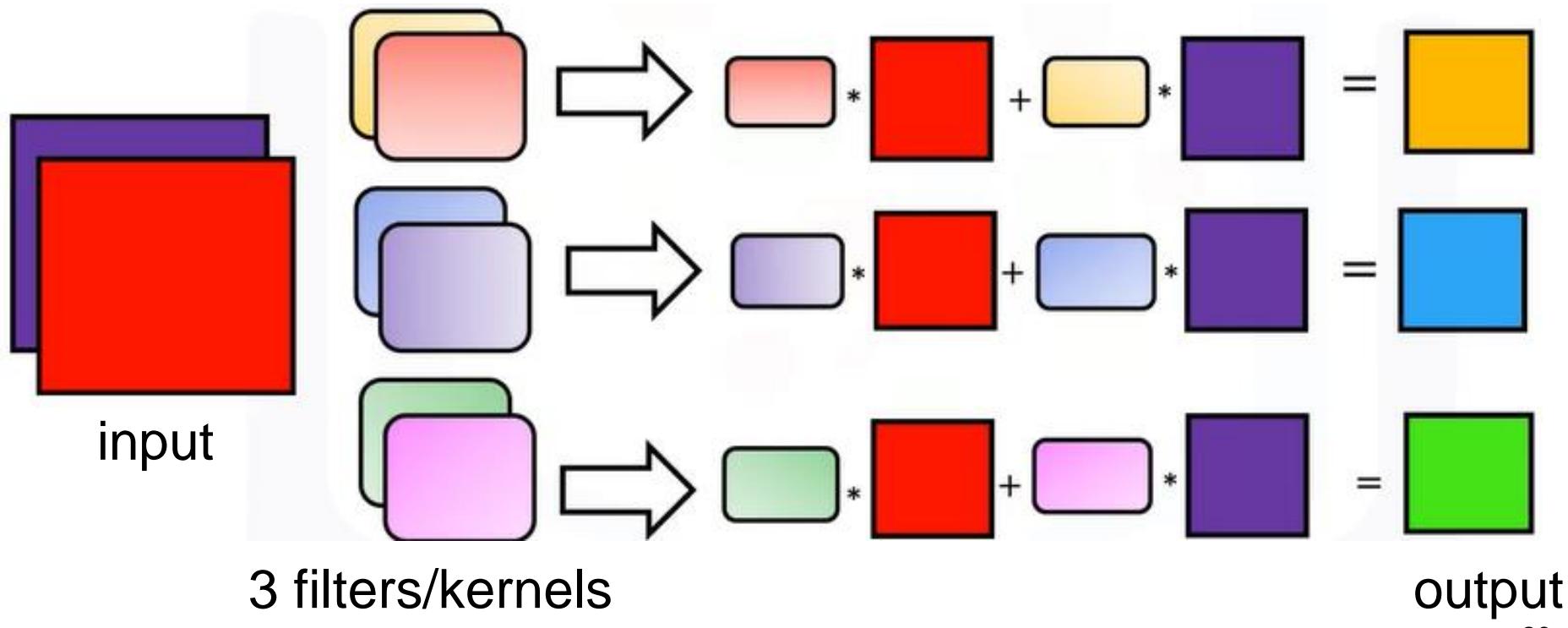
Convolution Example 1

- `conv=nn.Conv2d(in_channels=2, out_channels=1, kernel_size=3)`
 - Pytorch code for a CONV layer with an input image with 2 channels (`in_channels=2`), 1 3×3 filter, 1 output activation maps (`out_channels=1`).
 - The biases are assumed to be 0



Convolution Example 2

- `conv4=nn.Conv2d(in_channels=2, out_channels=3, kernel_size=3)`
 - Pytorch code for a CONV layer with an input image with 2 channels (`in_channels=2`), 3 3×3 filters, 3 output activation maps (`out_channels=3`)
 - The biases are assumed to be 0



Convolution Example 2: Filters and Input Image

`conv4.state_dict()['weight'][0][0]`

$$W_{0,0}$$

| | | |
|---|-----|---|
| 0 | 0 | 0 |
| 0 | 0.5 | 0 |
| 0 | 0 | 0 |

`conv4.state_dict()['weight'][0][1]`

$$W_{0,1}$$

| | | |
|---|-----|---|
| 0 | 0 | 0 |
| 0 | 0.5 | 0 |
| 0 | 0 | 0 |

`Image4[1,0,:,:]`

`Image4[1,1,:,:]`

`conv4.state_dict()['weight'][2][0]`

$$W_{2,0}$$

| | | |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -2 |
| 1 | 0 | -1 |

`conv4.state_dict()['weight'][2][1]`

$$W_{2,1}$$

| | | |
|----|----|----|
| 1 | 2 | -1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Channel 1

Channel 2

`conv4.state_dict()['weight'][1][0]`

$$W_{1,0}$$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

`conv4.state_dict()['weight'][1][1]`

$$W_{1,1}$$

| | | |
|---|----|---|
| 0 | 0 | 0 |
| 0 | -1 | 0 |
| 0 | 0 | 0 |

3 3 × 3 filters

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

input image with 2 channels

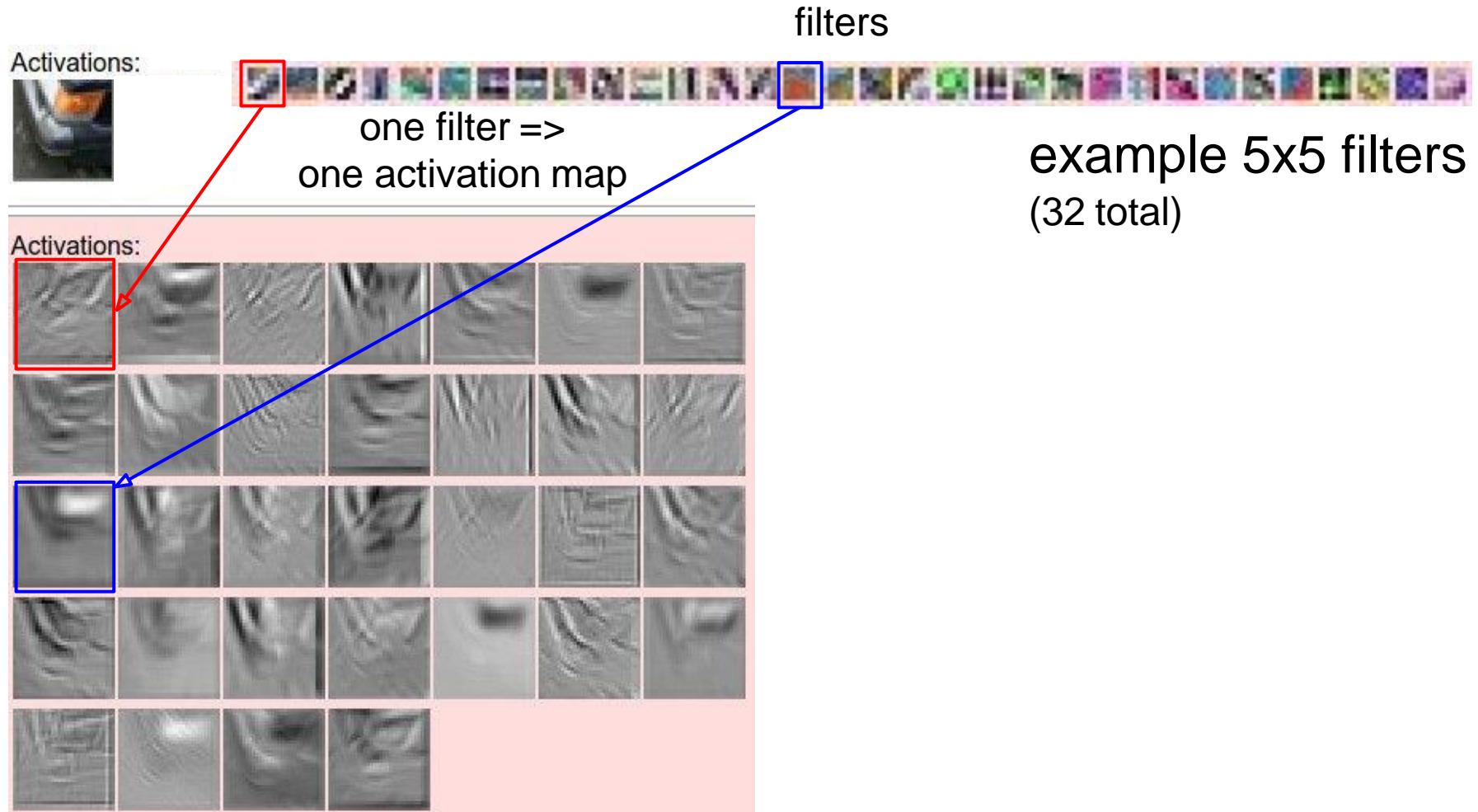
Convolution

Example 2: Output

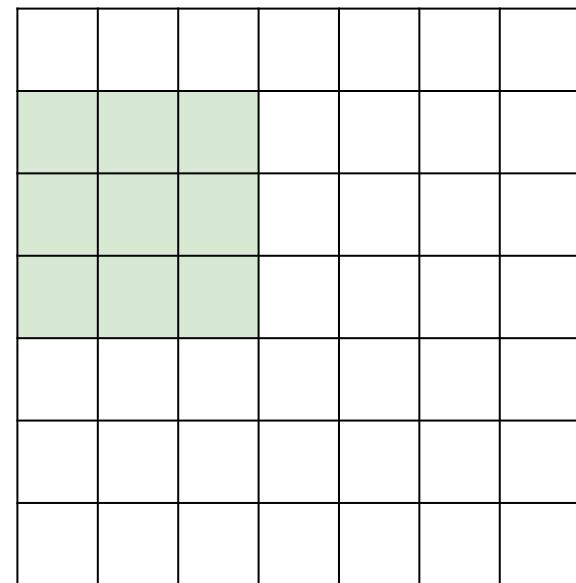
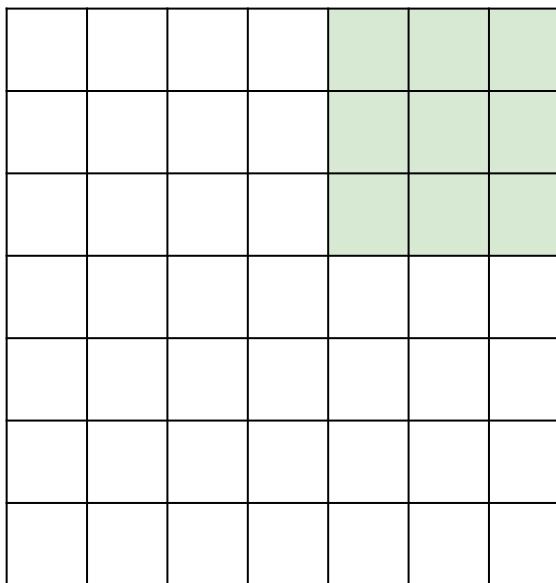
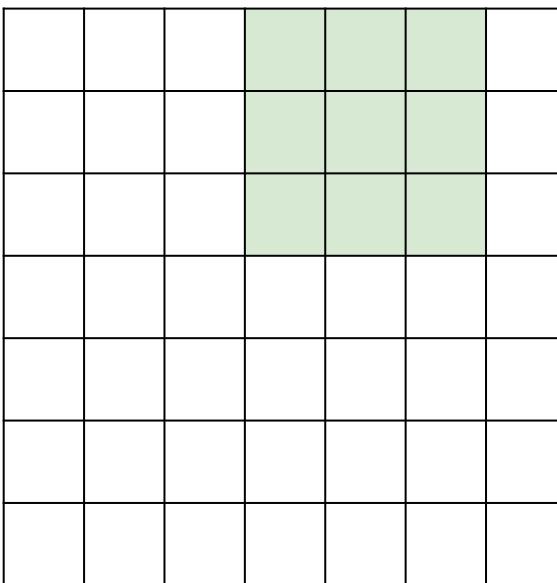
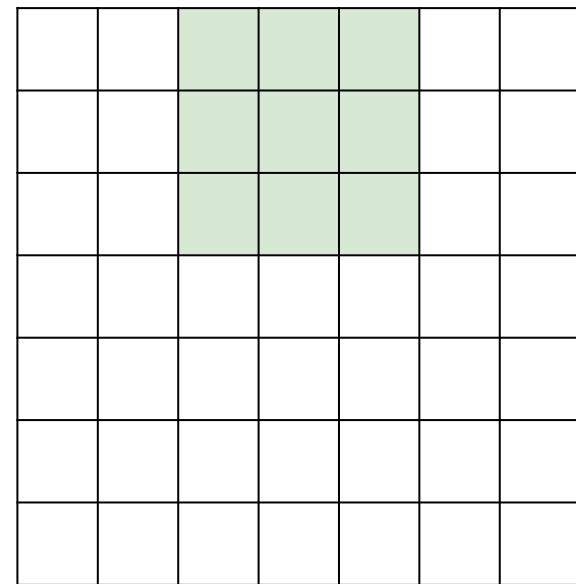
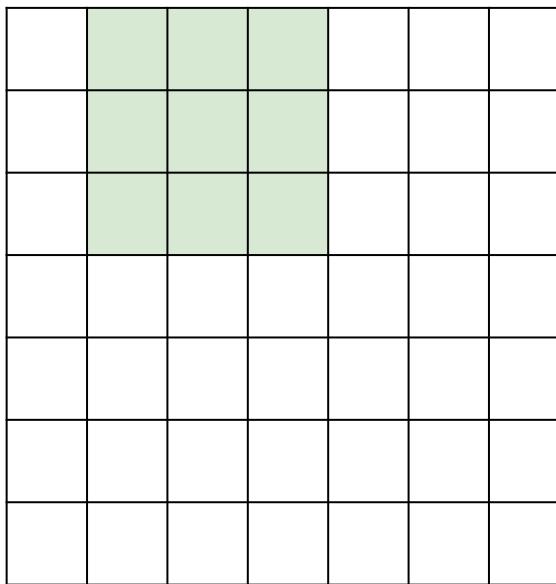
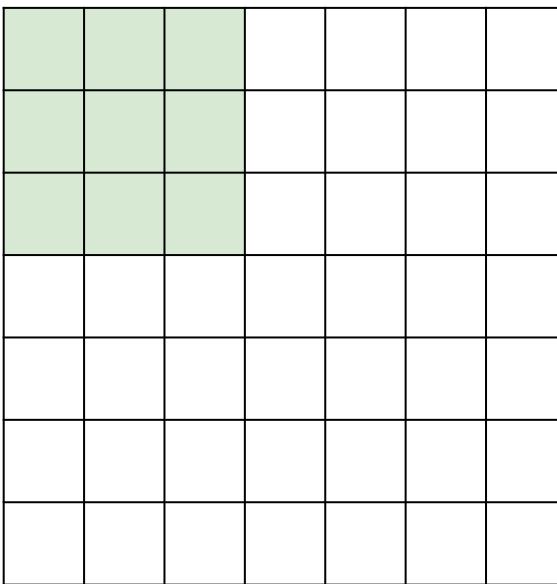
- Each of the 3 filters convolved with the input image generates an output activation map.
- The output volume consists of 3 3×3 activation maps, with volume $3 \times 3 \times 3$

$$\begin{aligned}
 (Z)_0 &= \begin{matrix} W_{0,0} \\ \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0.5 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} \end{matrix} * \begin{matrix} (X)_0 \\ \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline \end{array} \end{matrix} + \begin{matrix} W_{0,1} \\ \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0.5 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} \end{matrix} * \begin{matrix} (X)_1 \\ \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} \end{matrix} \\
 &= \begin{matrix} \begin{array}{|c|c|c|} \hline 0.5 & 0.5 & 0.5 \\ \hline 0.5 & 0.5 & 0.5 \\ \hline 0.5 & 0.5 & 0.5 \\ \hline \end{array} \end{matrix} + \begin{matrix} \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 0.5 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \end{matrix} = \begin{matrix} \begin{array}{|c|c|c|} \hline 0.5 & 0.5 & 0.5 \\ \hline 0.5 & 1 & 0.5 \\ \hline 0.5 & 0.5 & 0.5 \\ \hline \end{array} \end{matrix} \\
 (Z)_1 &= \begin{matrix} W_{1,0} \\ \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \end{matrix} * \begin{matrix} (X)_0 \\ \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline \end{array} \end{matrix} + \begin{matrix} W_{1,1} \\ \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & -1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \end{matrix} * \begin{matrix} (X)_1 \\ \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} \end{matrix} \\
 &= \begin{matrix} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \end{matrix} + \begin{matrix} \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & -1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \end{matrix} = \begin{matrix} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 0 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \end{matrix} \\
 (Z)_2 &= \begin{matrix} W_{2,0} \\ \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 1 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} \end{matrix} * \begin{matrix} (X)_0 \\ \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline \end{array} \end{matrix} + \begin{matrix} W_{2,1} \\ \begin{array}{|c|c|c|} \hline 1 & 2 & -1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array} \end{matrix} * \begin{matrix} (X)_1 \\ \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline \end{array} \end{matrix} \\
 &= \begin{matrix} \begin{array}{|c|c|c|} \hline -1 & -2 & -1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \end{matrix}
 \end{aligned}$$

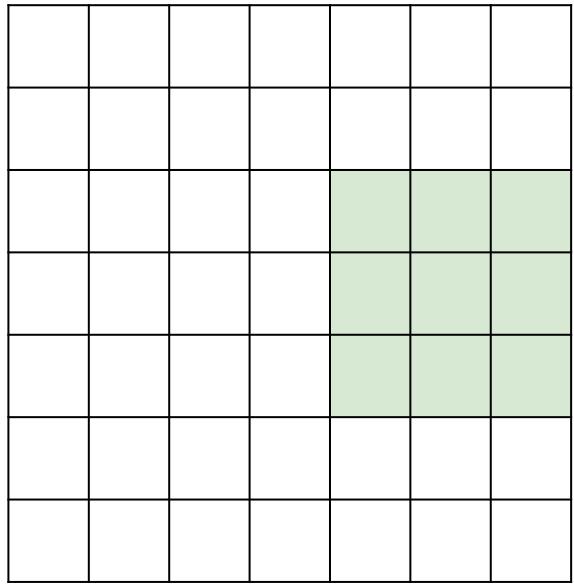
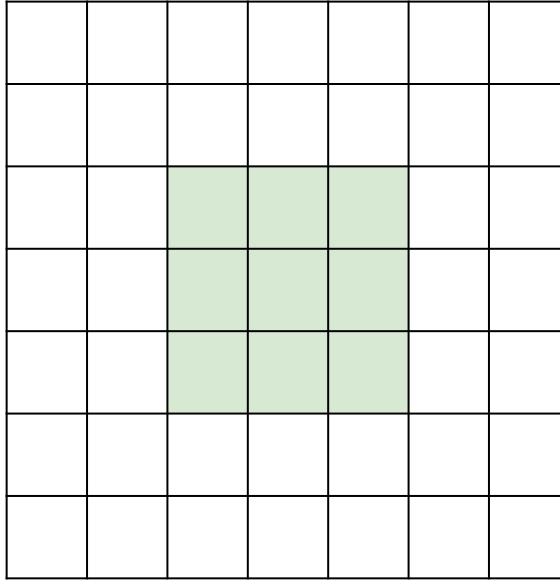
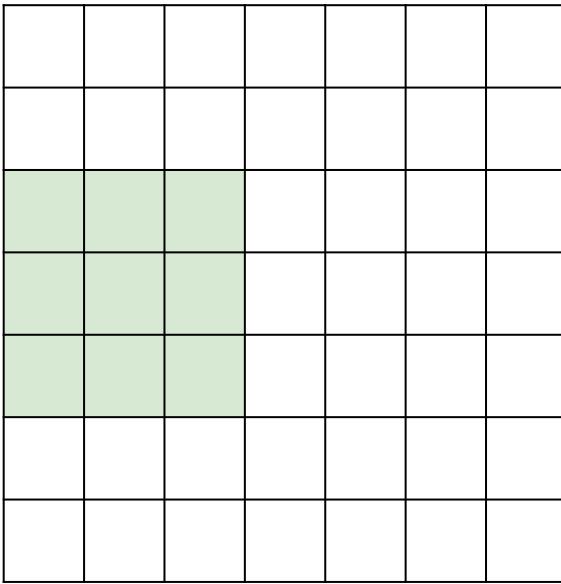
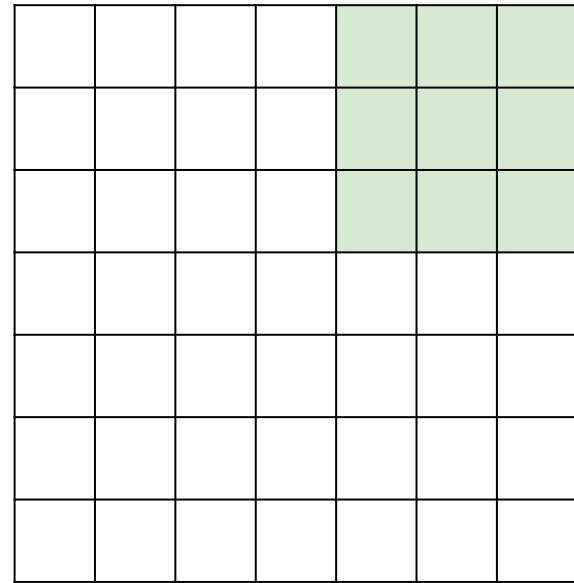
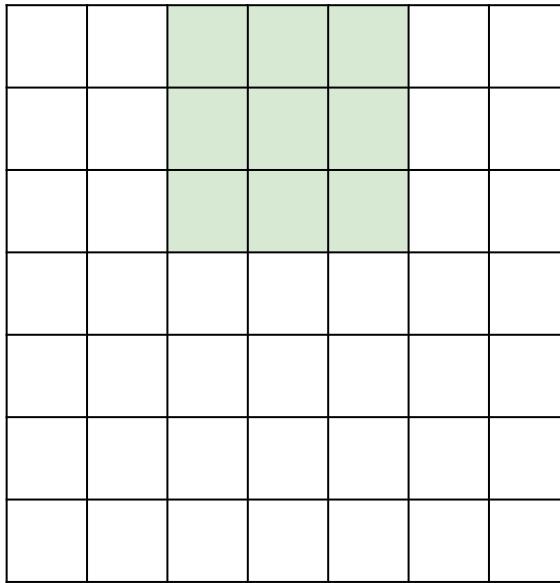
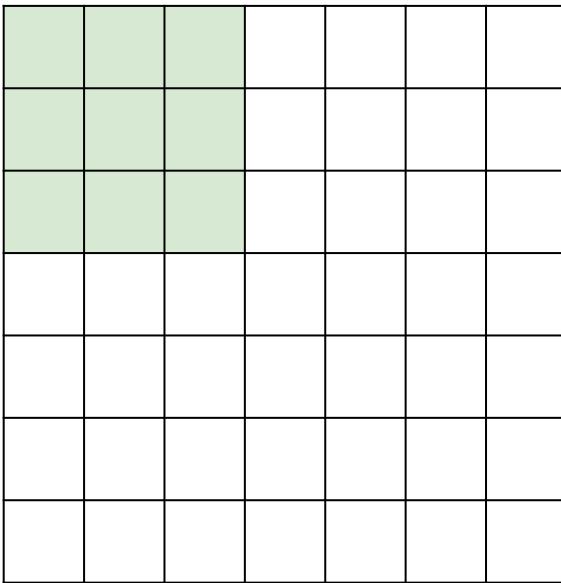
Filters and Activation Maps Example



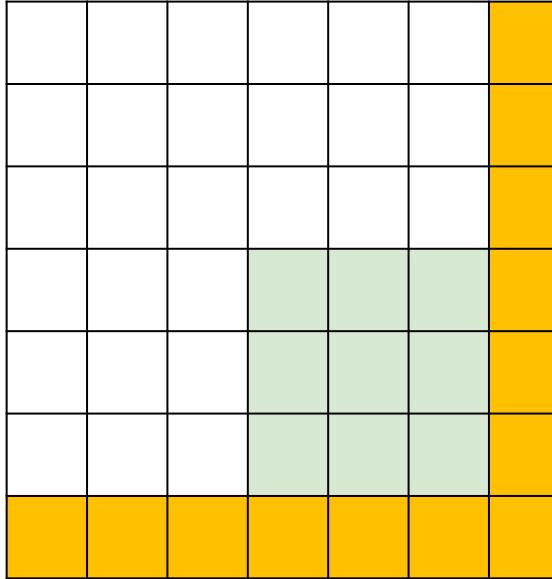
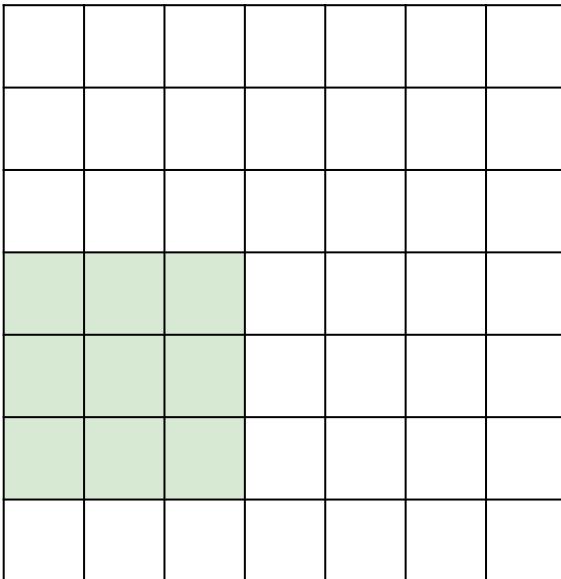
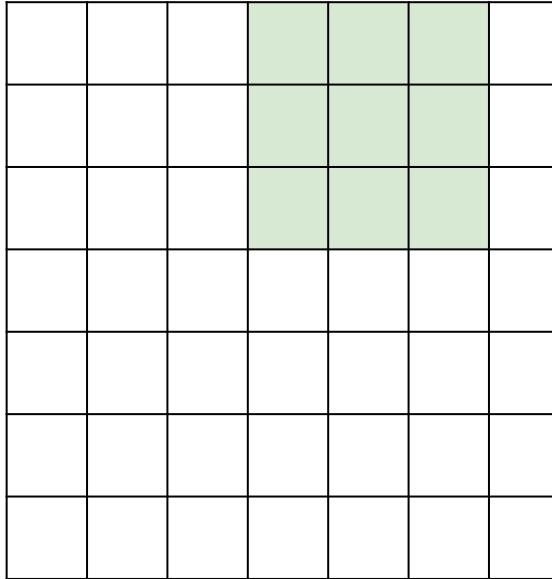
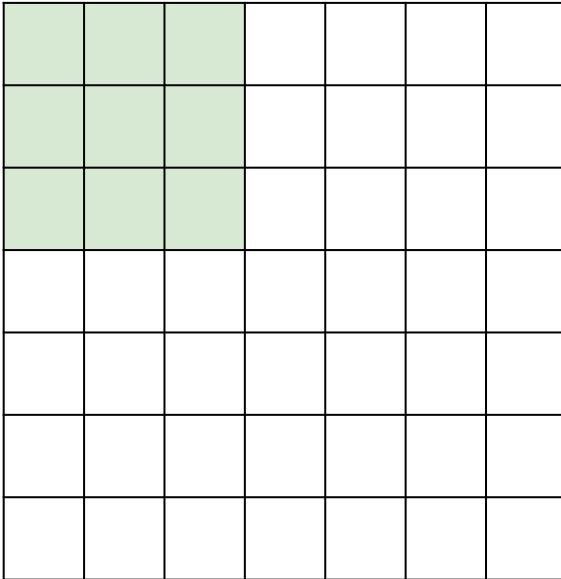
7×7 input, 3×3 filter, stride=1 \Rightarrow output: 5×5 filter



7×7 input, 3×3 filter, stride=2 \Rightarrow output: 3×3 filter



7×7 input, 3×3 filter, stride=3 \Rightarrow output: ???



The rightmost
and bottom
columns are not
processed!

Solution: Add Padding

- 7x7 input, 3x3 filter, stride=3, zero padding w. 1 \Rightarrow output: 3x3 filter

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |



Computation of CONV Layer Sizes

Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
 - Number of filters K ,
 - their spatial extent F ,
 - the stride S ,
 - the amount of zero padding P .
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
 - $W_2 = (W_1 - F + 2P)/S + 1$
 - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
 - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d -th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d -th filter over the input volume with a stride of S , and then offset by d -th bias.
- It is common to have square input shape, i.e., $W_1 = H_1 = N_1$, but general rectangle shape size is possible; a filter always has square shape
- Each filter always has the same depth D_1 as its input volume, and the number of filters K always equals the depth D_2 of its output volume
- In practice, it is common to have stride $S = 1$, filter size $F \times F$, and zero-pad $P = \frac{1}{2}(F - 1)$. Then output activation map has same spatial size as input. This is called “same padding”
 - $W_2 = \frac{1}{S}(W_1 + 2P - F) + 1 = \frac{1}{1}(W_1 + F - 1 - F) + 1 = W_1$; similarly, $H_2 = H_1$
 - e.g., $F < 3 \Rightarrow P = 0$; $F = 3 \Rightarrow P = 1$; $F = 5 \Rightarrow P = 2$

Common settings:

- $K = (\text{powers of 2, e.g. } 32, 64, 128, 512)$
- $F = 3, S = 1, P = 1$
 - $F = 5, S = 1, P = 2$
 - $F = 1, S = 1, P = 0$

CONV Example 1: No Pad

- Input volume: $5 \times 5 \times 1$ ($W_1 = H_1 = N_1 = 32, D_1 = 3$) (e.g., a greyscale image)
- A $3 \times 3 \times 1$ filter $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ ($K = 1, F = 3$) w. stride $S = 1$, no pad
- Output activation map:
 - Spatial size: $W_2 = H_2 = N_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(5 - 3) + 1 = 3$
 - Depth: $D_2 = K = 1$
- Output volume: $3 \times 3 \times 1$
- Even though the fig shows sequential computation, convolution operations are inherently parallel, hence suitable for efficient implementation on parallel hardware, e.g., GPU, FPGA...

| | | | | |
|---------|---------|---------|---|---|
| 1 x1 | 1 x0 | 1 x1 | 0 | 0 |
| 0 x0 | 1 x1 | 1 x0 | 1 | 0 |
| 0 x1 | 0 x0 | 1 x1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |

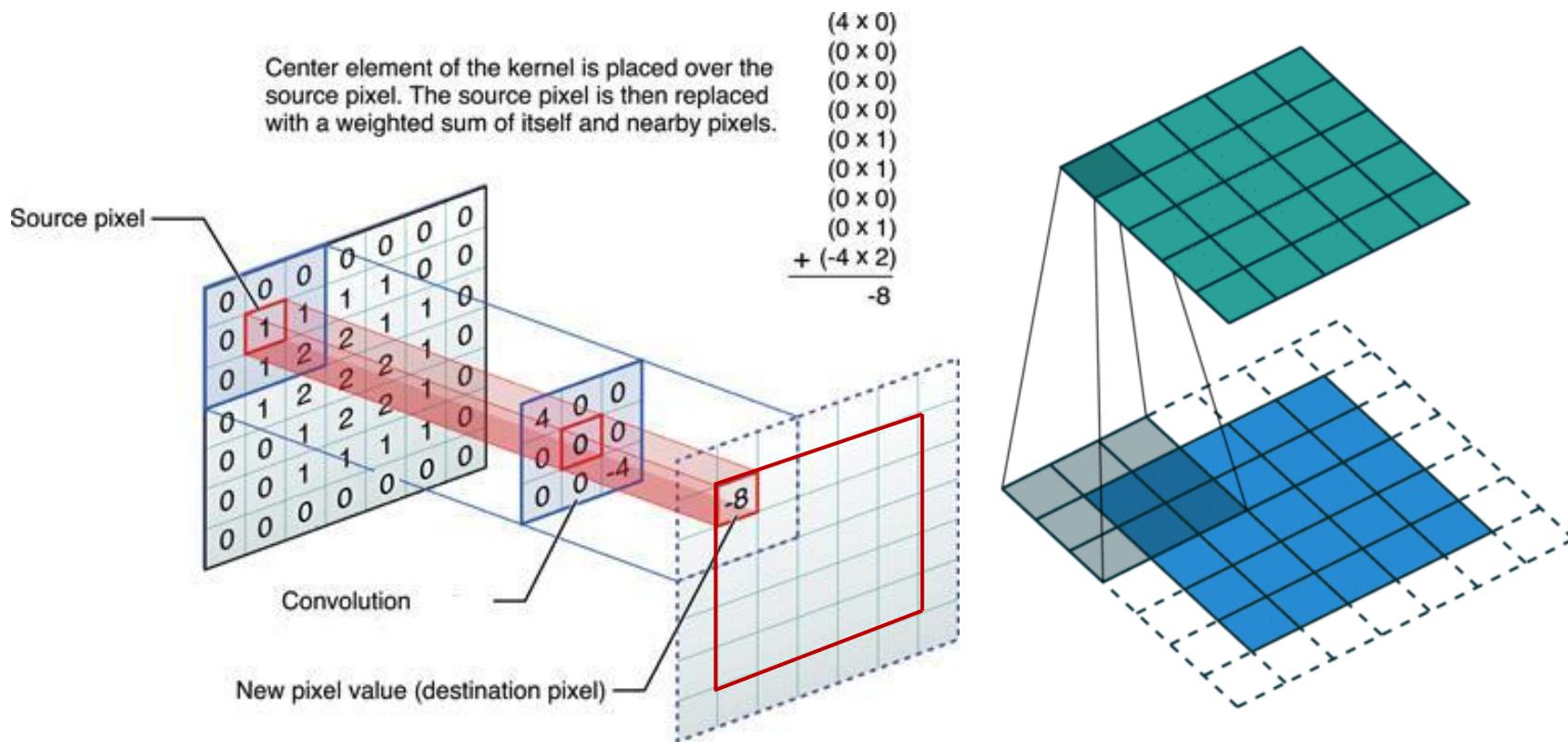
Image

| | | |
|---|--|--|
| 4 | | |
| | | |
| | | |
| | | |

Convolved
Feature

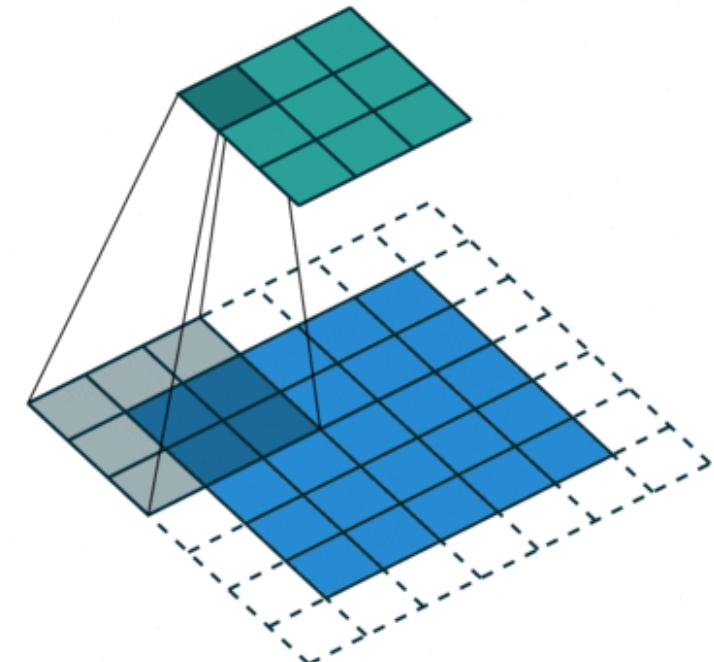
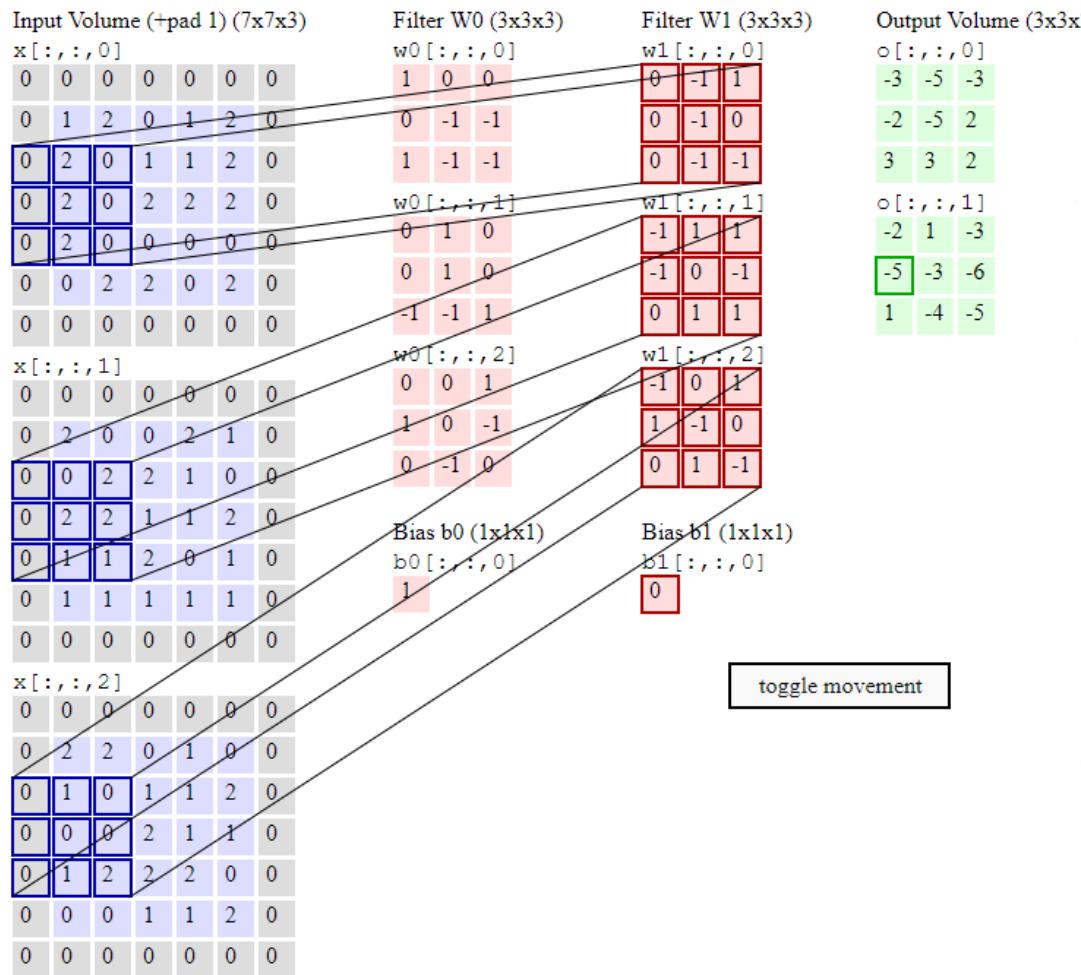
CONV Example 2: Same Padding

- Input volume: $5 \times 5 \times 1$
- A $3 \times 3 \times 1$ filter ($K = 1, F = 3$) w. stride $S = 1$, pad $P = 1$
- Output volume: $5 \times 5 \times 1$ (since $\frac{1}{1}(5 + 2 - 3) + 1 = 5$)
- Output activation map has the same spatial dimension as input (5×5)



CONV Example 3: Stride $S = 2$

- Input volume: $5 \times 5 \times 3$
- 2 $3 \times 3 \times 3$ filters ($K = 2, F = 3$) w. **stride $S = 2$** , pad $P = 1$
- Output volumes: 2 $3 \times 3 \times 1$ (since $\frac{1}{2}(5 + 2 * 1 - 3) + 1 = 3$)
 - Animation: <https://cs231n.github.io/convolutional-networks/>



CONV Example 4: Input Depth $D_1 = 3$

| | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|
| 0 | 0 | 0 | 0 | 0 | 0 | ... |
| 0 | 156 | 155 | 156 | 158 | 158 | ... |
| 0 | 153 | 154 | 157 | 159 | 159 | ... |
| 0 | 149 | 151 | 155 | 158 | 159 | ... |
| 0 | 146 | 146 | 149 | 153 | 158 | ... |
| 0 | 145 | 143 | 143 | 148 | 158 | ... |
| ... | ... | ... | ... | ... | ... | ... |

Input Channel #1 (Red)

| | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|
| 0 | 0 | 0 | 0 | 0 | 0 | ... |
| 0 | 167 | 166 | 167 | 169 | 169 | ... |
| 0 | 164 | 165 | 168 | 170 | 170 | ... |
| 0 | 160 | 162 | 166 | 169 | 170 | ... |
| 0 | 156 | 156 | 159 | 163 | 168 | ... |
| 0 | 155 | 153 | 153 | 158 | 168 | ... |
| ... | ... | ... | ... | ... | ... | ... |

Input Channel #2 (Green)

| | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|
| 0 | 0 | 0 | 0 | 0 | 0 | ... |
| 0 | 163 | 162 | 163 | 165 | 165 | ... |
| 0 | 160 | 161 | 164 | 166 | 166 | ... |
| 0 | 156 | 158 | 162 | 165 | 166 | ... |
| 0 | 155 | 155 | 158 | 162 | 167 | ... |
| 0 | 154 | 152 | 152 | 157 | 167 | ... |
| ... | ... | ... | ... | ... | ... | ... |

Input Channel #3 (Blue)

| | | |
|----|----|----|
| -1 | -1 | 1 |
| 0 | 1 | -1 |
| 0 | 1 | 1 |

Kernel Channel #1

| | | |
|---|----|----|
| 1 | 0 | 0 |
| 1 | -1 | -1 |
| 1 | 0 | -1 |

Kernel Channel #2

| | | |
|---|----|---|
| 0 | 1 | 1 |
| 0 | 1 | 0 |
| 1 | -1 | 1 |

Kernel Channel #3



308

+



-498

+

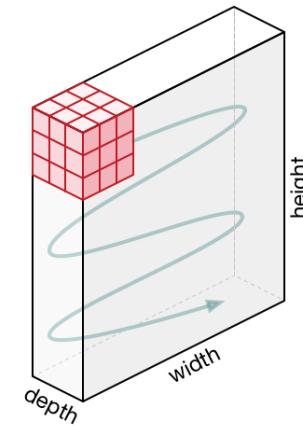
164

+

+ 1 = -25



Bias = 1



Movement of the filter

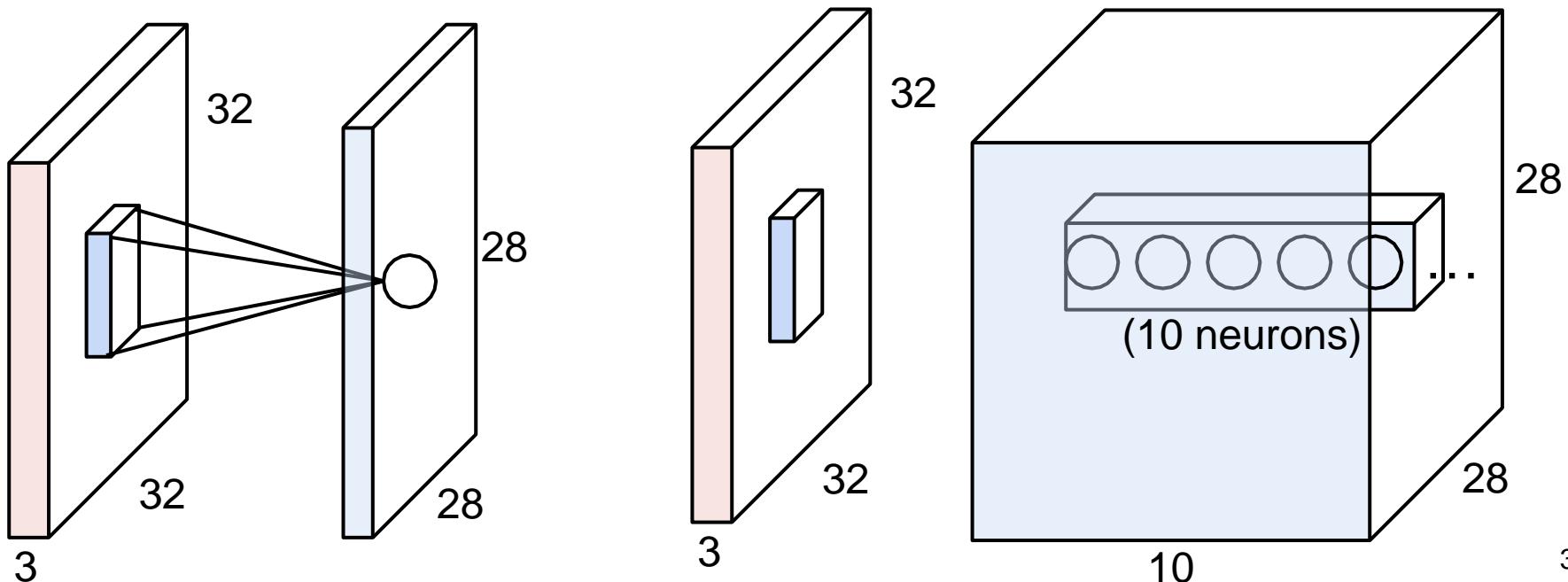
Output

| | | | | |
|-----|-----|-----|-----|-----|
| -25 | | | | ... |
| | | | | ... |
| | | | | ... |
| | | | | ... |
| ... | ... | ... | ... | ... |

- Input volume: $M \times N \times 3$
- A $3 \times 3 \times 3$ filter ($K = 1, F = 3$) w. stride $S = 1$, pad $P = 1$
- Output volume: $M \times N \times 3$ (since $\frac{1}{1}(M + 2 * 1 - 3) + 1 = M$, $\frac{1}{1}(N + 2 * 1 - 3) + 1 = N$)

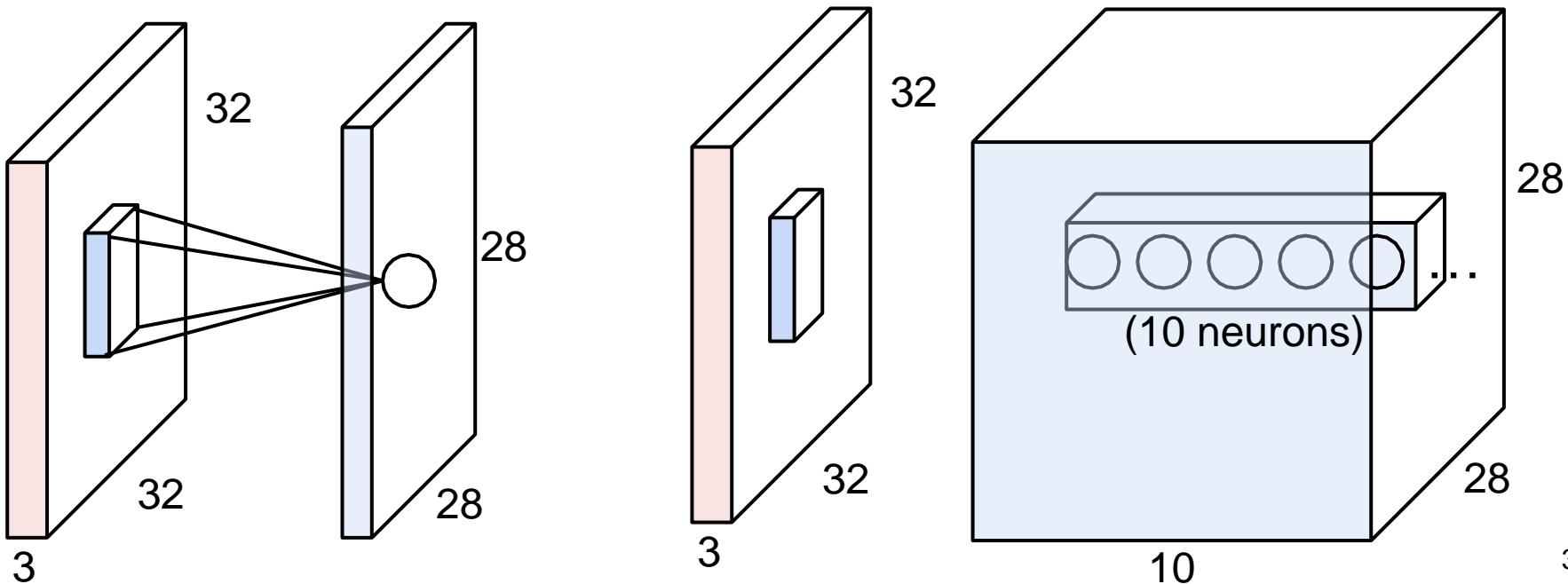
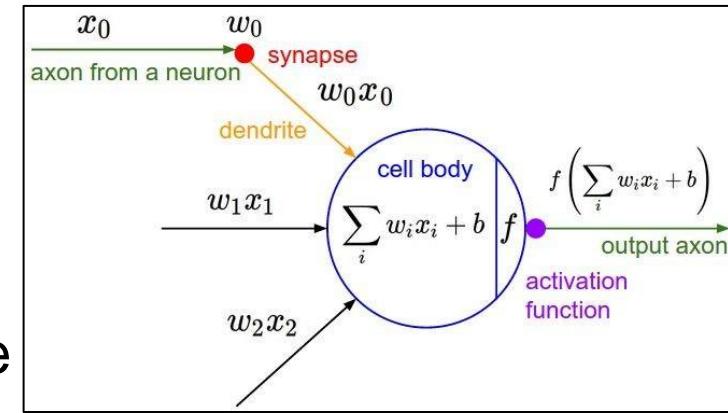
CONV Example 5: Multiple Filters $K = 10$

- Input volume: $32 \times 32 \times 3$ ($W_1 = H_1 = N_1 = 32, D_1 = 3$)
- 10 $5 \times 5 \times 3$ filters ($K = 10, F = 5$) w. stride $S = 1$, no pad ($P = 0$)
- Each output activation map:
 - Spatial size: $W_2 = H_2 = N_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(32 - 5) + 1 = 28$
 - Depth: $D_2 = K = 10$
- Output volume: $28 \times 28 \times 10$
- No. params (weights and biases) in this layer: each filter has $5 * 5 * 3 + 1 = 76$ params, so 10 filters add up to $76 * 10 = 760$ params



CONV Example 5: Neuron View

- One activation map is a 28×28 sheet of neuron outputs.
- With 10 filters, the CONV layer consists of neurons arranged in a 3D grid ($28 \times 28 \times 10$).
 - For each 5×5 patch of the input, there are 10 different neurons all looking at it, each extracting different features

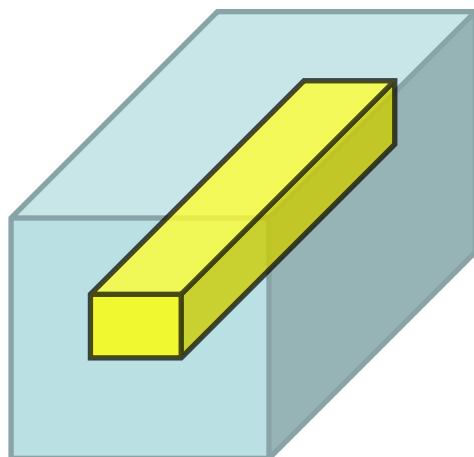
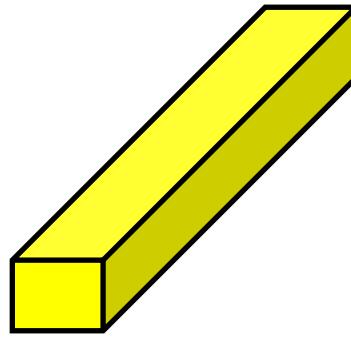
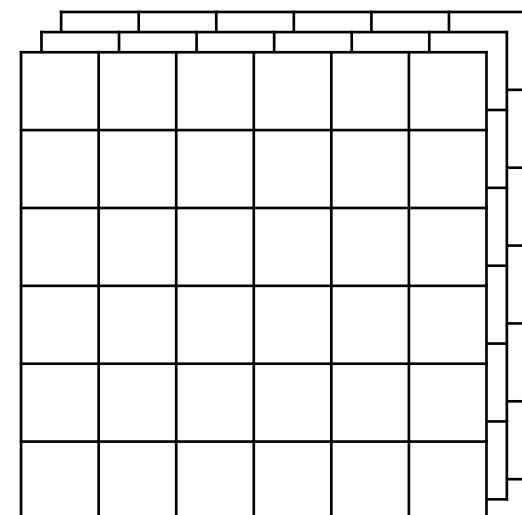


CONV Example 6: Pad $P = 2$

- Input volume: $32 \times 32 \times 3$ ($W_1 = H_1 = N_1 = 32, D_1 = 3$)
- 10 $5 \times 5 \times 3$ filters ($K = 10, F = 5$) w. stride $S = 1$,
pad $P = 2$
- Each activation map:
 - Spatial size: $W_2 = H_2 = N_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(32 + 2 * 2 - 5) + 1 = 32$
 - Depth: $D_2 = K = 10$
- Output volume: $32 \times 32 \times 10$
- No. params: each filter has $5 * 5 * 3 + 1 = 76$ params, so 10 filters add up to $76 * 10 = 760$ params

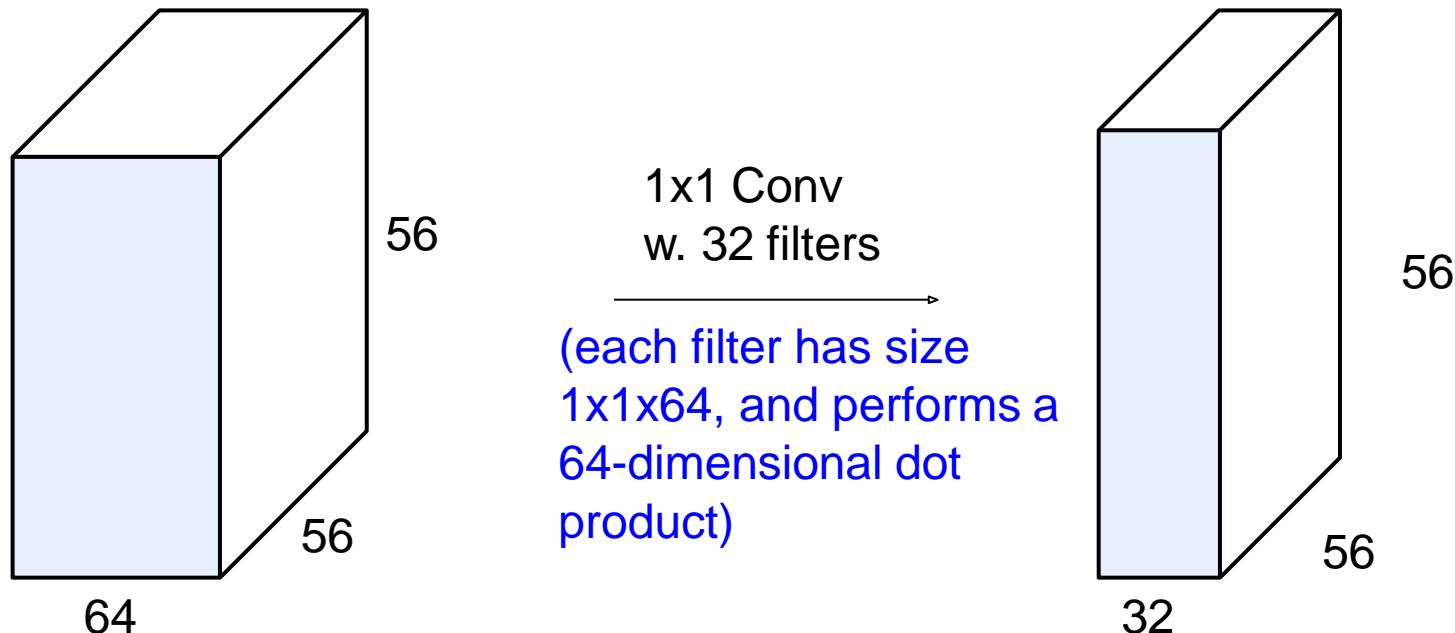
Pointwise Convolution with 1×1 Filter

- A 1×1 filter performs “mixing” of the input channels, then applies a non-linear activation function
- Can be used to reduce the number of channels (volume depth); the non-linear activation function also helps increase model capacity

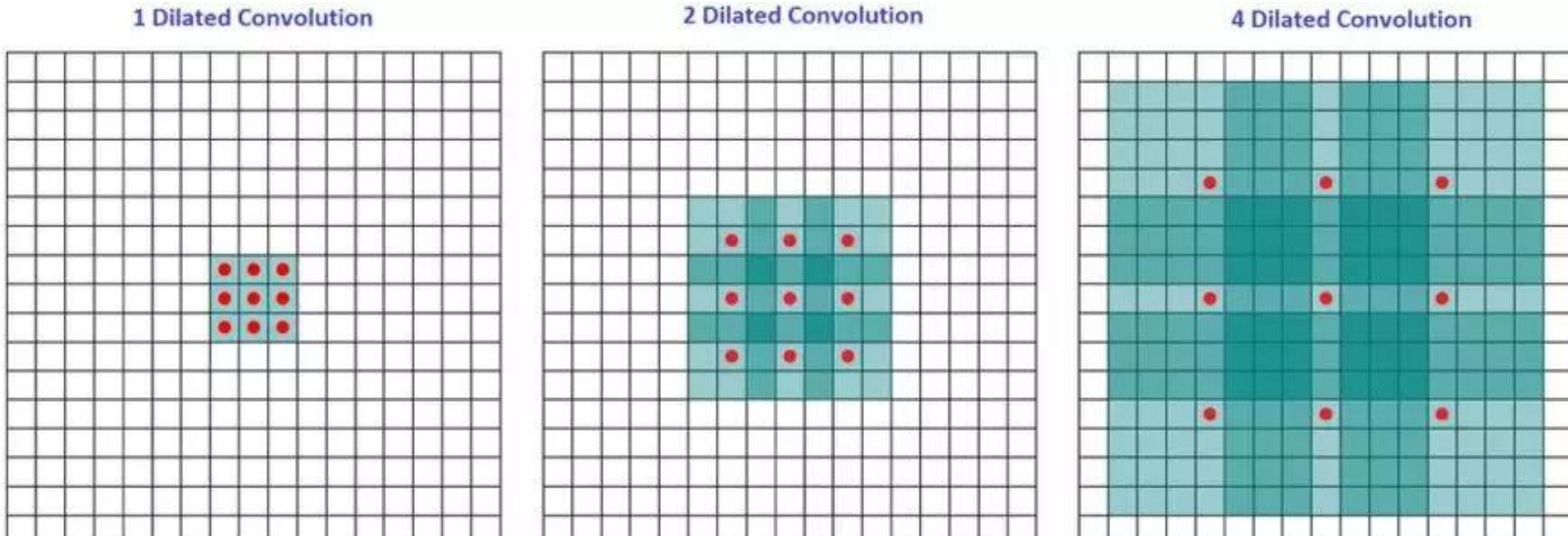
 $*$  $=$  $6 \times 6 \times 32$ $1 \times 1 \times 32$ $6 \times 6 \times \# \text{ filters}$

1×1 Filter Example

- Input volume: $56 \times 56 \times 64$ ($W_1 = H_1 = N_1 = 56, D_1 = 64$)
- 32 $1 \times 1 \times 64$ filters ($K = 32, F = 1$) w. stride $S = 1$, no pad
- Each activation map:
 - Spatial size: $W_2 = H_2 = N_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(56 - 1) + 1 = 56$
 - Depth: $D_2 = K = 32$
- Output volume: $56 \times 56 \times 32$
- No. params: each filter has $1 * 1 * 64 + 1 = 65$ params, so 32 filters add up to $65 * 32 = 2080$ params

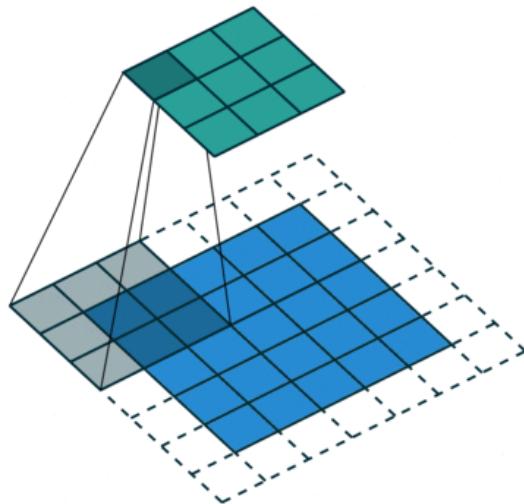


Dilated Convolution

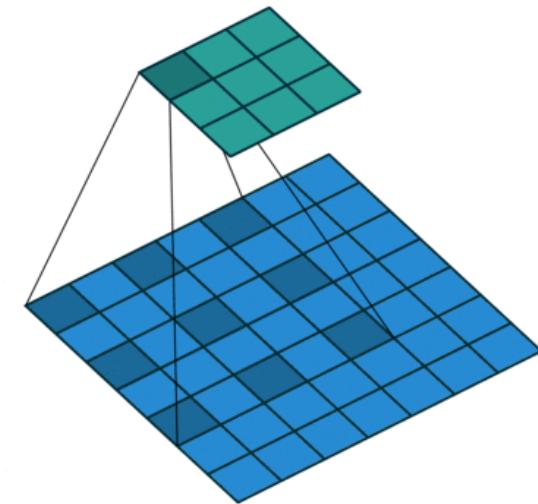


(a)

- Insert 0s between input elements to increase receptive field size without increasing # params



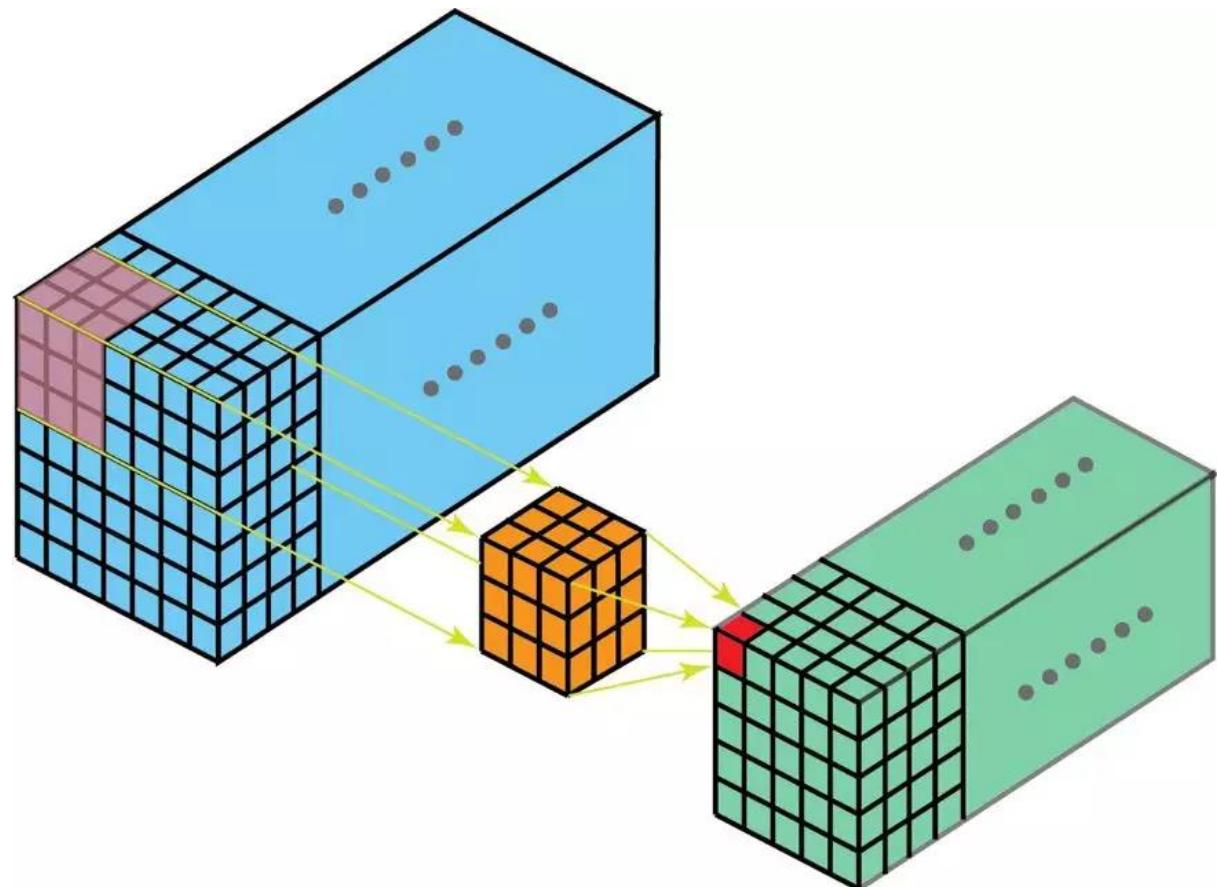
Regular convolution
(1-dilated)



2-dilated
convolution

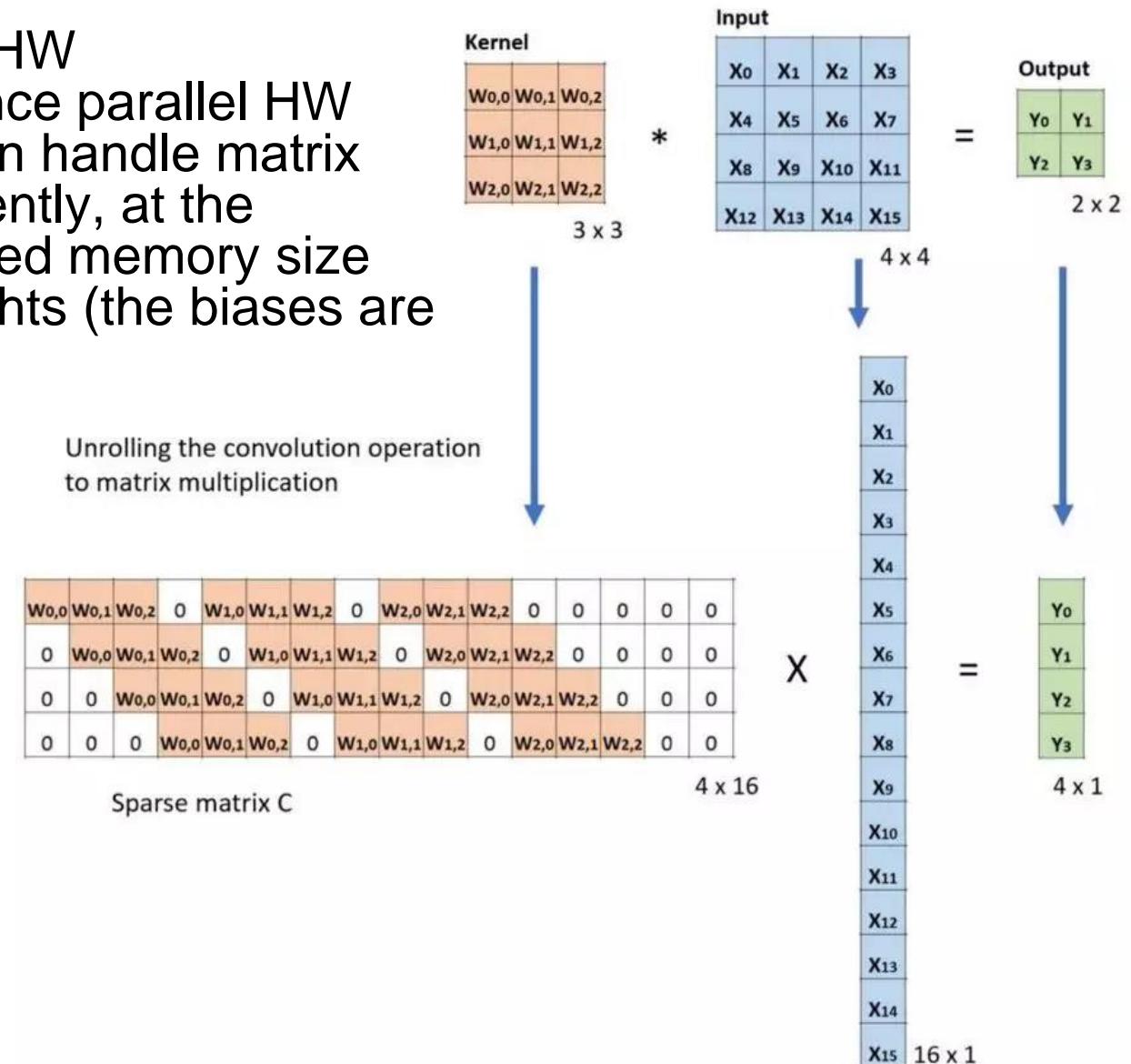
3D Convolution

- 3D filter slides along all 3 axes (width, height, depth). Very computation intensive
- Useful for 3D images such as medical CT/MRI images, or Point Clouds from Lidar



Converting Convolution to Matrix Multiplication

- Facilitates parallel HW implementation, since parallel HW (GPU, FPGA...) can handle matrix multiplication efficiently, at the expense of increased memory size for storing the weights (the biases are not shown in fig)

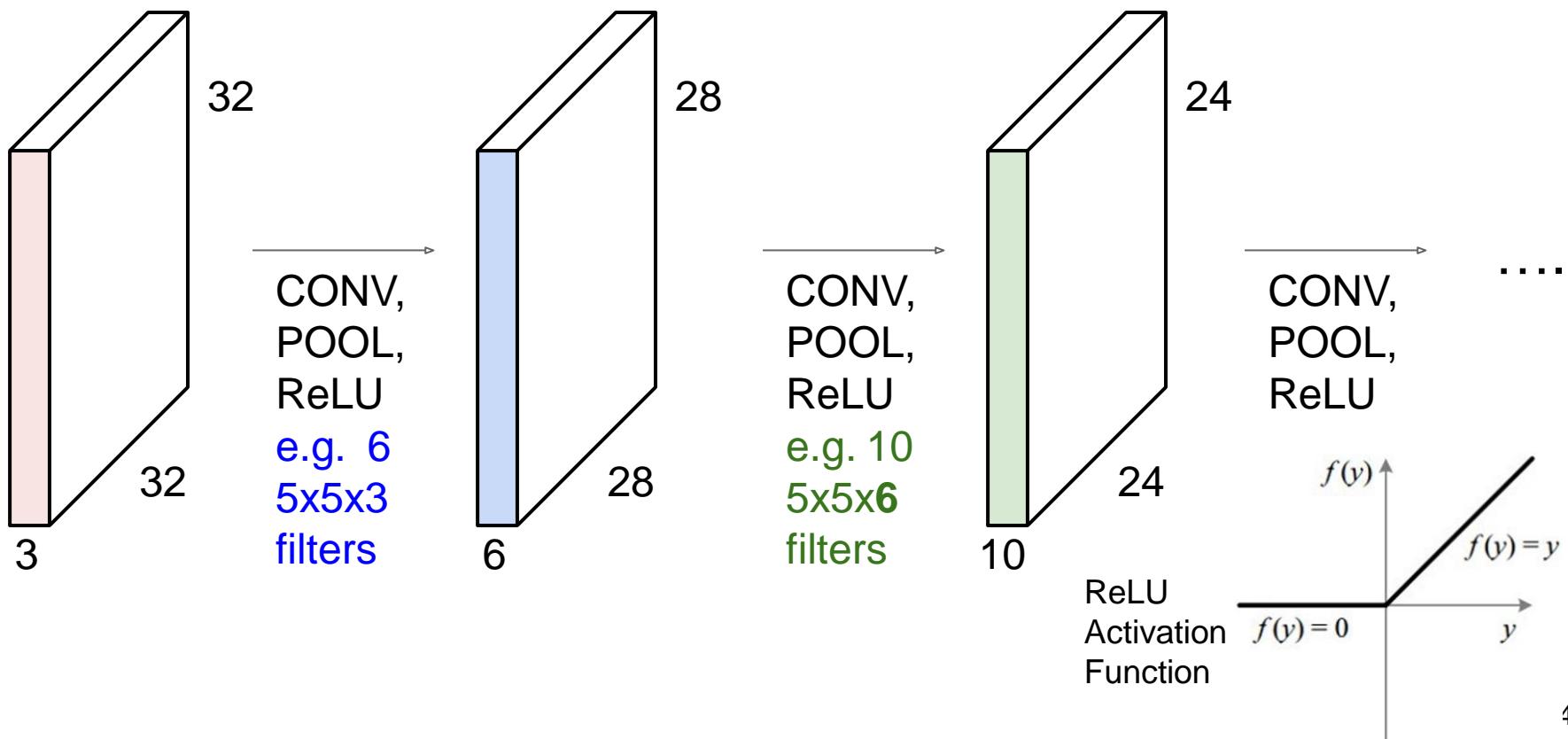


Outline

- CNN Convolution layers
- Pooling and Fully-Connected layers
- CNN case studies
- RNNs

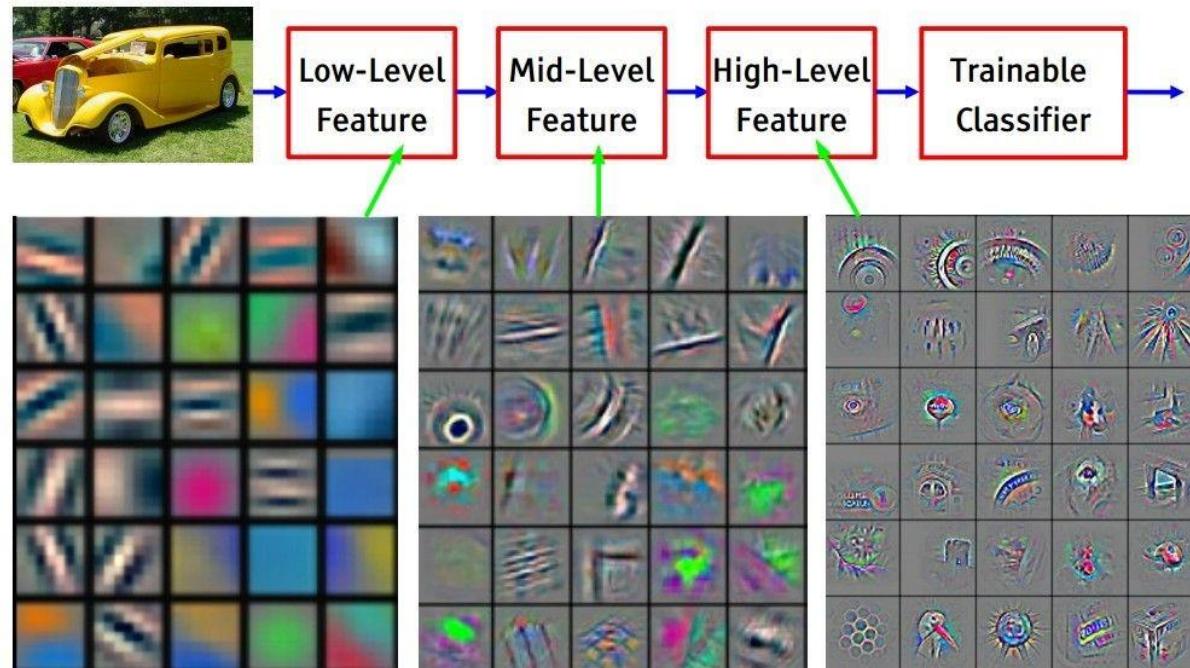
Typical CNN Architecture

- Multiple layers, each consisting of CONV, POOL and non-linear activation functions (e.g., ReLU), are stacked into a deep network
 - Many variants possible, e.g., multiple CONV layers can be stacked without POOL and activation functions in-between



Feature Hierarchy

- Multiple hidden layers can extract a hierarchy of increasingly-abstract features layer-by-layer, until the last layer produces a classification result



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

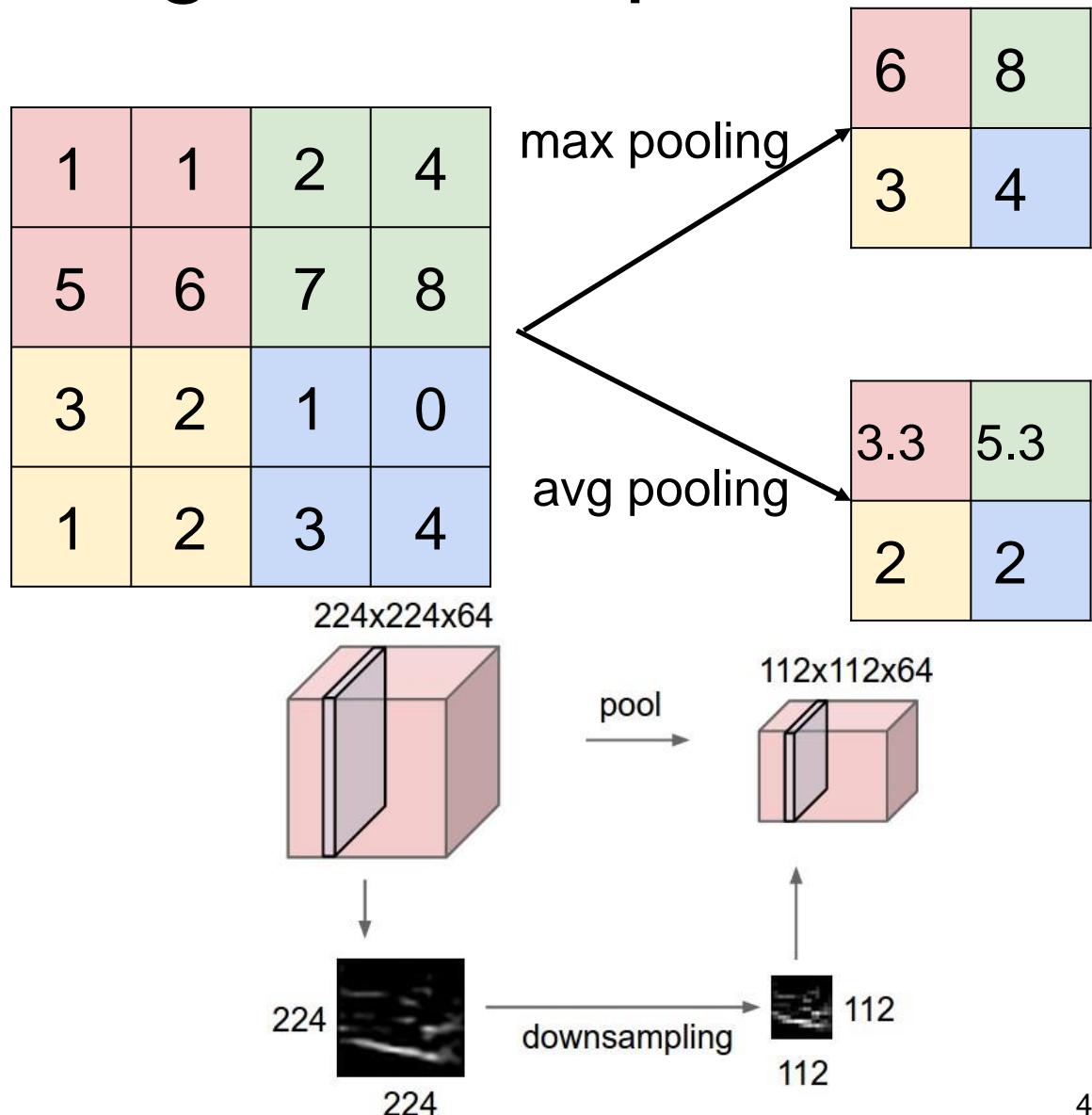


Pooling (Sub-Sampling) Layer

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires two hyperparameters:
 - their spatial extent F ,
 - the stride S ,
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
 - $W_2 = (W_1 - F)/S + 1$
 - $H_2 = (H_1 - F)/S + 1$
 - $D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- For Pooling layers, it is not common to pad the input using zero-padding.
- A pooling filter has depth 1, and operates over each activation map independently, hence the input volume and output volume always have the same depth $D_1 = D_2$
 - In contrast, a CONV filter always has the same depth D_1 as its input volume, and the number of filters K always equals the depth D_2 of its output volume
 - Common settings: $F = 3, S = 2$, or $F = 3, S = 2$
- Example: pooling w. a 2×2 filter w. stride $S = 2$, no pad
- Output volume: $\frac{W_1}{2} \times \frac{H_1}{2} \times D_1$ (since $\frac{1}{2}(W_1 - 2) + 1 = \frac{W_1}{2}, \frac{1}{2}(H_1 - 2) + 1 = \frac{H_1}{2}$)

Max Pooling w. Examples

- Max pooling: take the max element among the $F * F$ elements in each $F \times F$ patch of each input activation map to reduce its dimension
 - Alternative: average pooling is less commonly used
- Pooling is also called subsampling



Overlapping Pooling

- Input volume: $N \times N \times D_1$
- A 3×3 pooling filter w. stride $S = 1$, no pad
- Output volume: $(N - 2) \times (N - 2) \times D_1$ (since $\frac{1}{1}(N - 3) + 1 = N - 2$)
 - (In practice, it is more common to have $F = 3, S = 2$ for overlapping pooling)

| | | | | |
|---|---|---|---|---|
| 1 | 3 | 2 | 1 | 3 |
| 2 | 9 | | 1 | 5 |
| 1 | | | | 2 |
| 8 | 3 | | 1 | 0 |
| 5 | 6 | 1 | 2 | 9 |

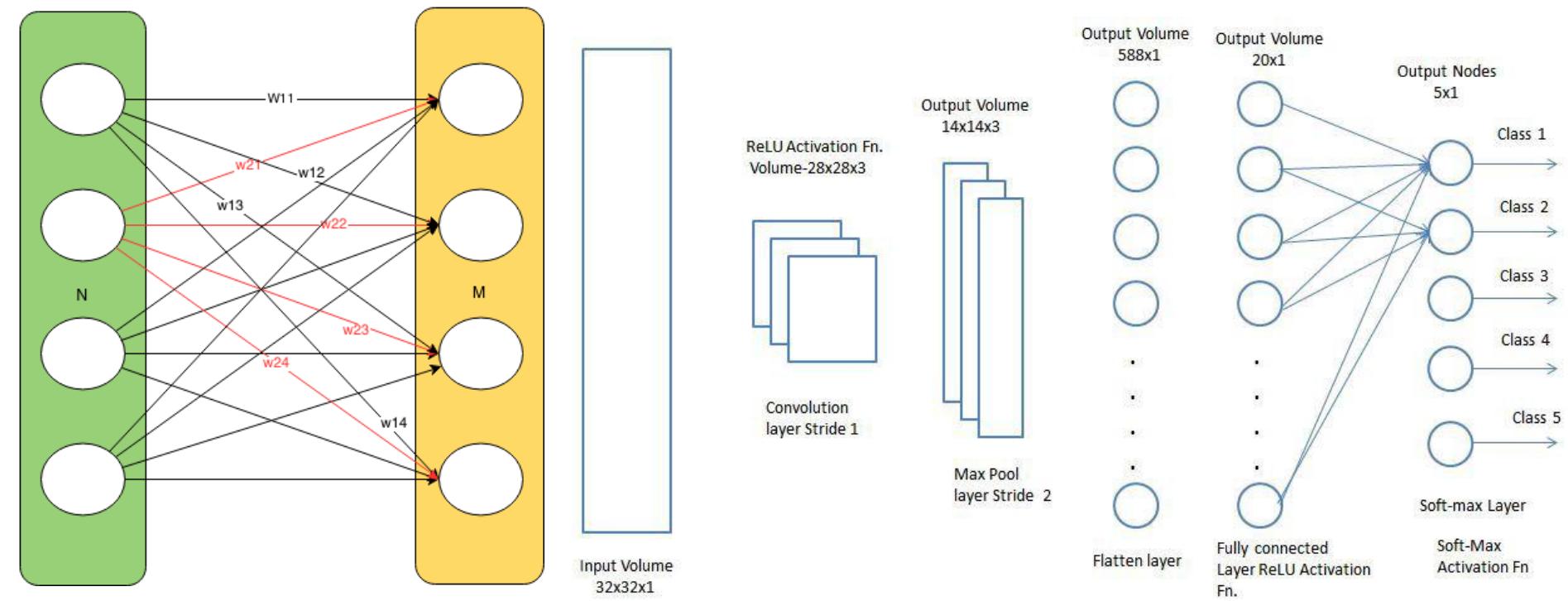
max pool w. 3×3
filter and stride 1



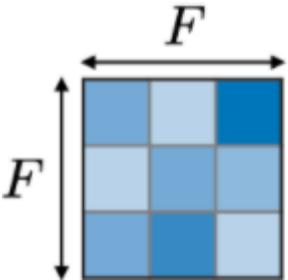
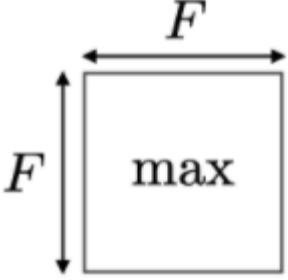
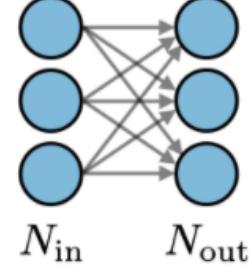
| | | |
|---|---|---|
| 9 | 9 | 5 |
| 9 | 9 | 5 |
| 8 | 6 | 9 |

FC Layer

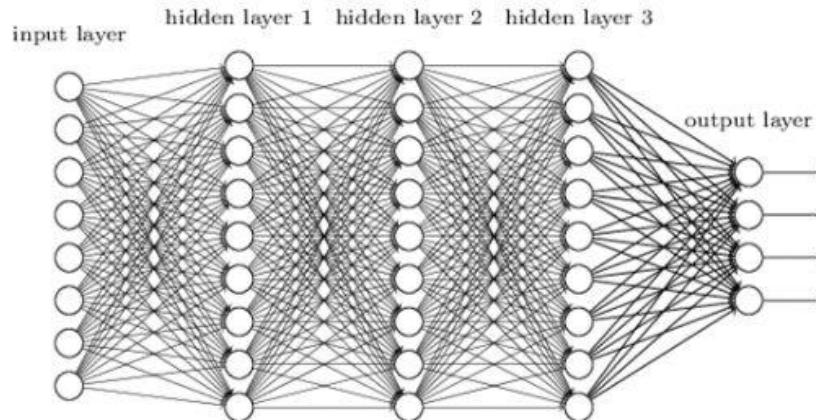
- Contains neurons that connect to the entire input volume w. no weight sharing
 - No. params for FC layer of size N_{out} connected to input layer of size N_{in} is $(N_{in} + 1) * N_{out}$



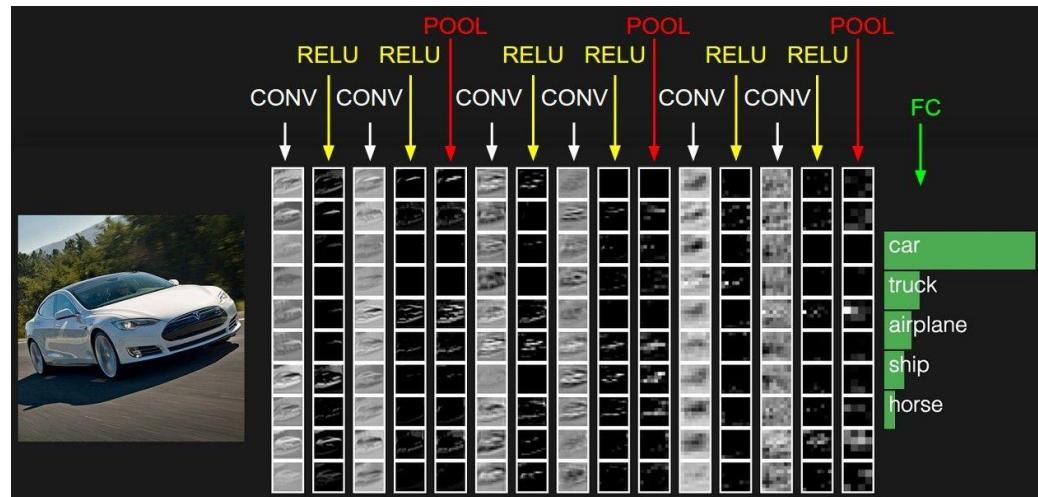
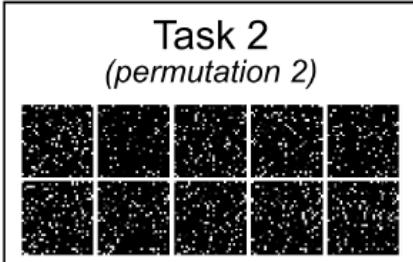
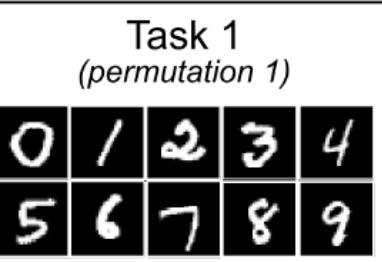
No. Params in Each Layer

| | CONV | POOL | FC |
|-------------|--|--|---|
| |  $\times K$ |  \max |  |
| Input size | $N_1 \times N_1 \times D_1$ | $N_1 \times N_1 \times D_1$ | N_{in} |
| Output Size | $N_2 \times N_2 \times K$ | $N_2 \times N_2 \times D_1$ | N_{out} |
| No. params | $(F * F * D_1 + 1) * K$ | 0 | $(N_{in} + 1) * N_{out}$ |

Fully-Connected NN vs. CNN



- In a FCNN, all layers are Fully-Connected
- Cannot alter input image size
- Permutation invariance
 - Cannot recognize geometry/locality of input, hence not useful for image recognition
 - Below two images, with same set of pixels but different permutations, will generate the exact same output in an MLP
- Num. params can grow very large

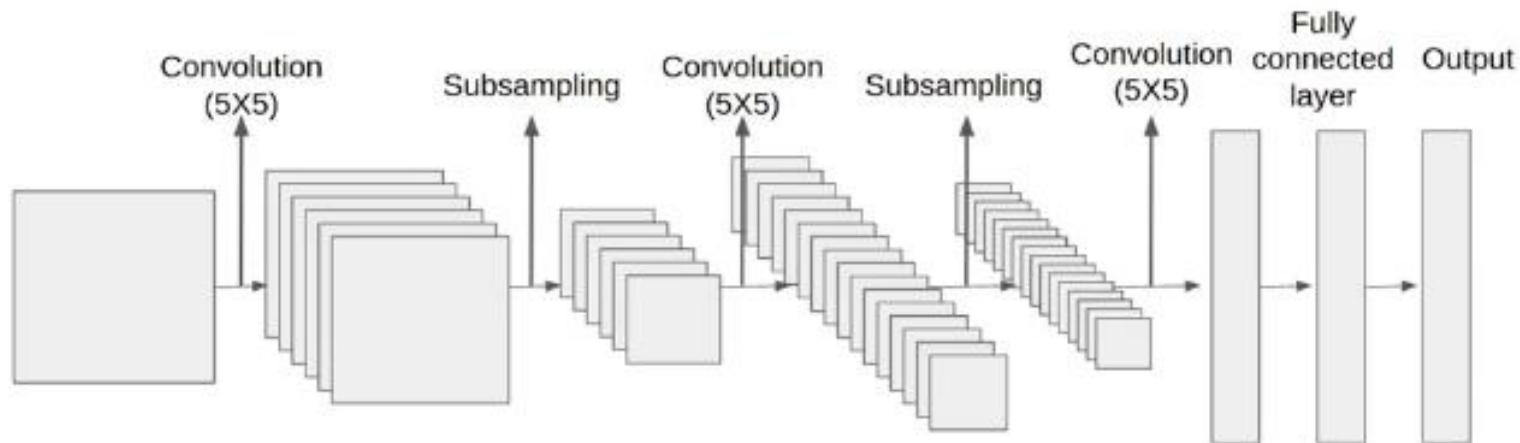


- In a CNN, only the last few (typically $<=3$) layer(s) are FC
- CONV layers can handle images of arbitrary size (but not the last FC layer(s))
- Not permutation invariance
 - Can recognize geometry/locality of input
- Translation invariance
- Fewer params than MLP (less prone to overfitting)

Outline

- CNN Convolution layers
- Pooling and Fully-Connected layers
- **CNN case studies**
- RNNs

LeNet-5



| Input | Feature Map | Feature Map | Feature Map | Feature Map | | | |
|-------------------------|-------------------------|-------------------------|--------------------------|------------------------|-----|----|----|
| $32 \times 32 \times 1$ | $28 \times 28 \times 6$ | $14 \times 14 \times 6$ | $10 \times 10 \times 16$ | $5 \times 5 \times 16$ | 120 | 84 | 10 |

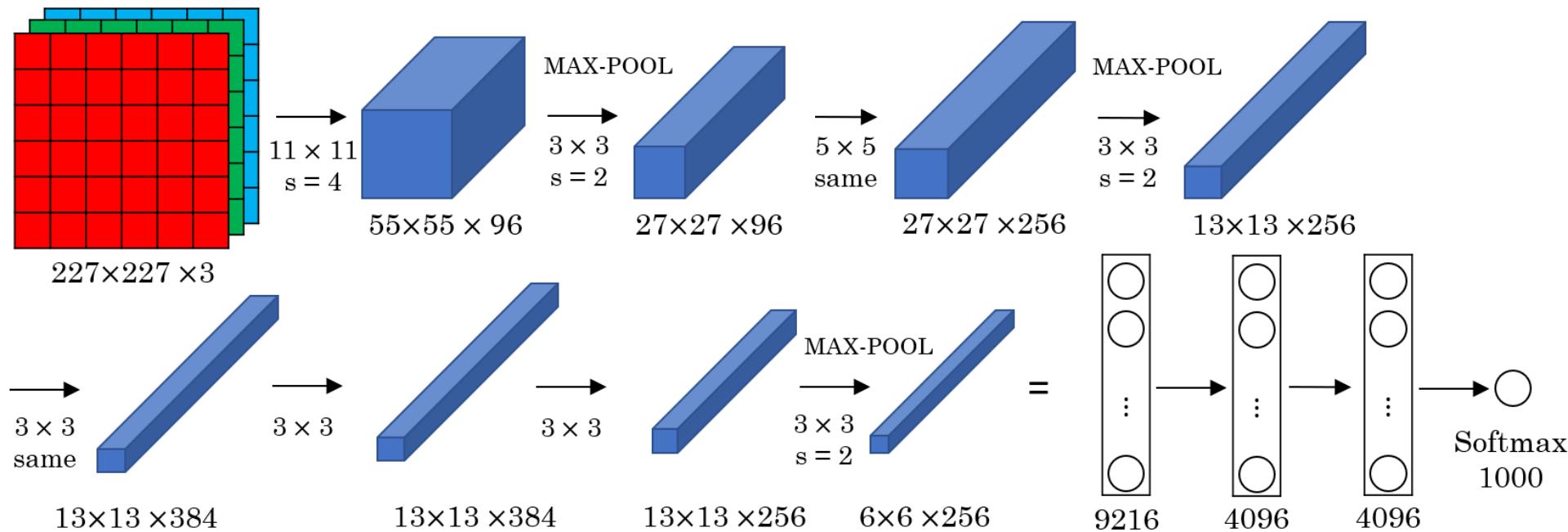
| Layer | Input $W_1 \times H_1 \times D_1$ | No. Filters | Filter $K \times K \times D/S$ | Output $W_2 \times H_2 \times D_2$ | No. params |
|---------|--------------------------------------|----------------|-----------------------------------|---------------------------------------|------------|
| C1:CONV | $32 \times 32 \times 1$ | 6 | $5 \times 5 \times 1$ | $28 \times 28 \times 6$ | 156 |
| S2:POOL | $28 \times 28 \times 6$ | 6 | $2 \times 2 \times 1/2$ | $14 \times 14 \times 6$ | 0 |
| C3:CONV | $14 \times 14 \times 6$ | 16 | $5 \times 5 \times 6$ | $10 \times 10 \times 16$ | 2416 |
| S4:POOL | $10 \times 10 \times 16$ | 16 | $2 \times 2 \times 1/2$ | $5 \times 5 \times 16$ | 0 |
| C5:CONV | $5 \times 5 \times 16$ | 120 | $5 \times 5 \times 16$ | $1 \times 1 \times 120$ | 48120 |
| F6 | FC | - | - | 84 | 10164 |
| Output | FC | | | 10 | 850 |

LeNet-5 Details

- Input image: $32 \times 32 \times 1$ (grey-scale images of hand-written digits w. size 32×32 pixels)
- Conv filters $5 \times 5 \times 1$ w. stride 1; Pooling filters 2×2 w. stride 2
- Conv layer C1 maps from input volume $32 \times 32 \times 1$ to 6 feature maps w. volume $28 \times 28 \times 6$ (since $\frac{1}{1}(32 - 5) + 1 = 28$). No params: $(5 * 5 * 1 + 1) * 6 = 156$
- Pooling layer S2 maps from input volume $28 \times 28 \times 6$ to 6 feature maps w. volume $14 \times 14 \times 6$ (since $\frac{1}{2}(28 - 2) + 1 = 14$).
- Conv layer C3 maps from input volume $14 \times 14 \times 6$ to 16 feature maps w. volume $10 \times 10 \times 16$ (since $\frac{1}{1}(14 - 5) + 1 = 10$). No params: $(5 * 5 * 6 + 1) * 16 = 2416$
- Pooling layer S4 maps from input volume $10 \times 10 \times 16$ to 16 feature maps w. volume $5 \times 5 \times 16$ (since $\frac{1}{2}(10 - 2) + 1 = 5$)
- Conv layer C5 maps from input volume $5 \times 5 \times 16$ to 120 feature maps w. volume $1 \times 1 \times 120$ (since $\frac{1}{1}(5 - 5) + 1 = 1$). No params: $(5 * 5 * 16 + 1) * 120 = 48120$
 - You can also view it as an equivalent Fully-Connected layer that maps from the flattened input of size 400×1 ($5 * 5 * 16 = 400$) to output of size 120×1 . For details, refer to L4.2 “Turning FC layer into CONV Layers”
- FC layer F6 maps from input of size 120×1 to output of size 84×1 . No params: $(120 + 1) * 84 = 10164$
- Output layer (SoftMax) maps from input of size 84×1 to output of size 10. No params: $(84 + 1) * 10 = 850$

AlexNet [Krizhevsky et al. 2012]

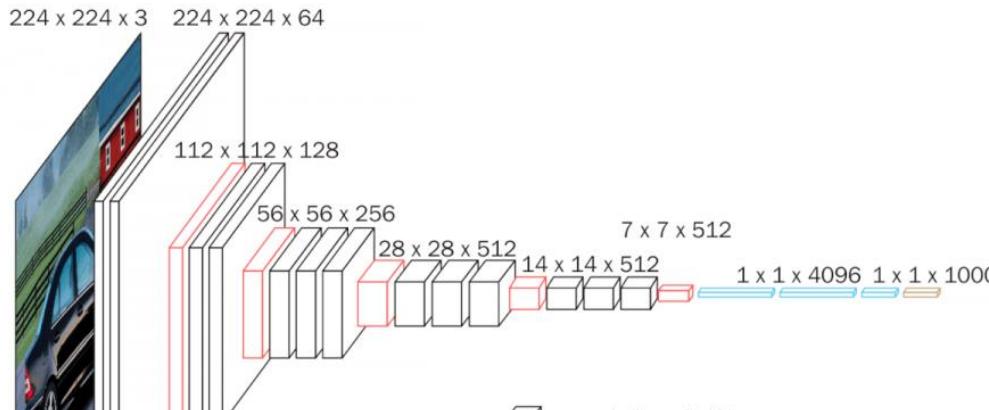
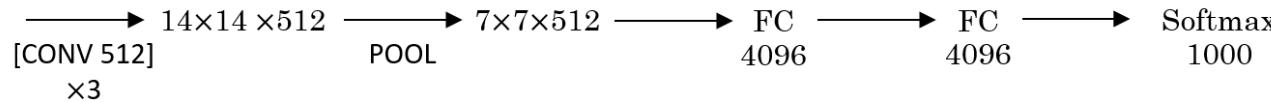
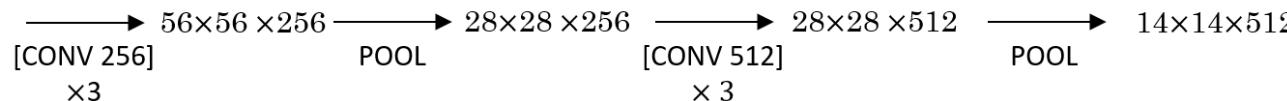
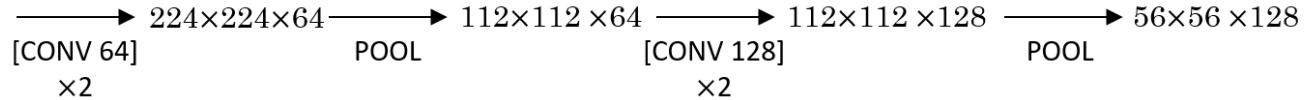
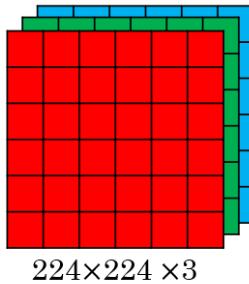
- Input image: $227 \times 227 \times 3$
- 1st layer (CONV1): 96 11×11 filters w. stride $S = 4$, w. ReLU activation function
- Output volume: $55 \times 55 \times 96$ (since $\frac{1}{4}(227 - 11) + 1 = 55$).
- 2nd layer (POOL1): 3×3 filters w. stride $S = 2$ (overlapping)
- Output volume: $27 \times 27 \times 96$ (since $\frac{1}{2}(55 - 3) + 1 = 27$)
- ...
- Total No. params: 60M
- Introduced ReLU activation function



VGGNet [Simonyan 2014] (the best performing variant VGG-16)

CONV = 3×3 filter, s = 1, same

MAX-POOL = 2×2 , s = 2



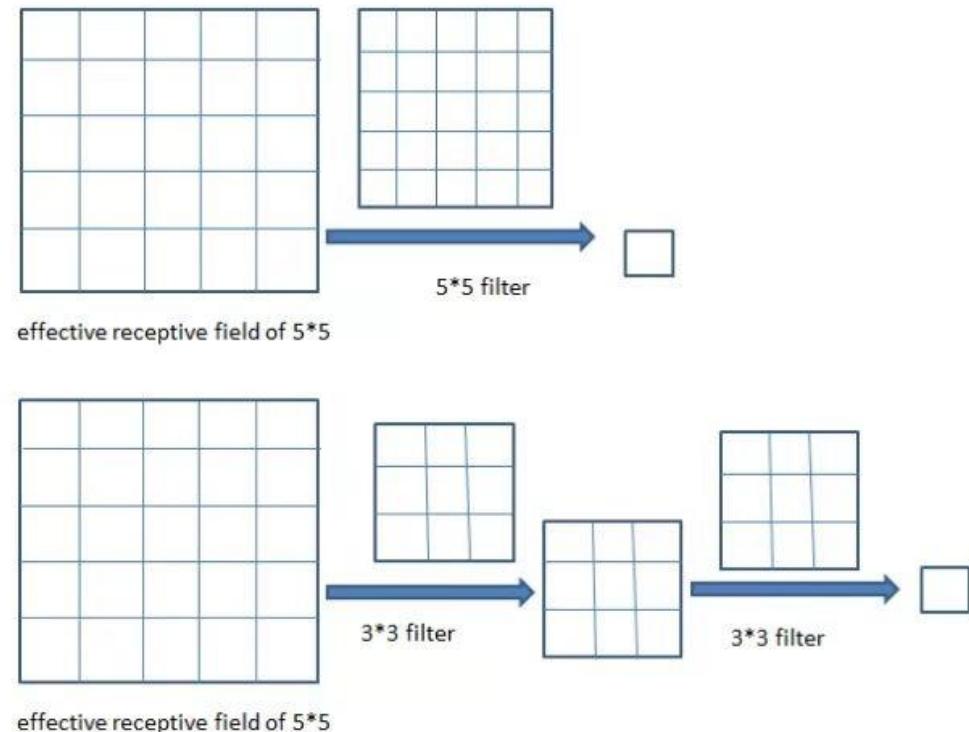
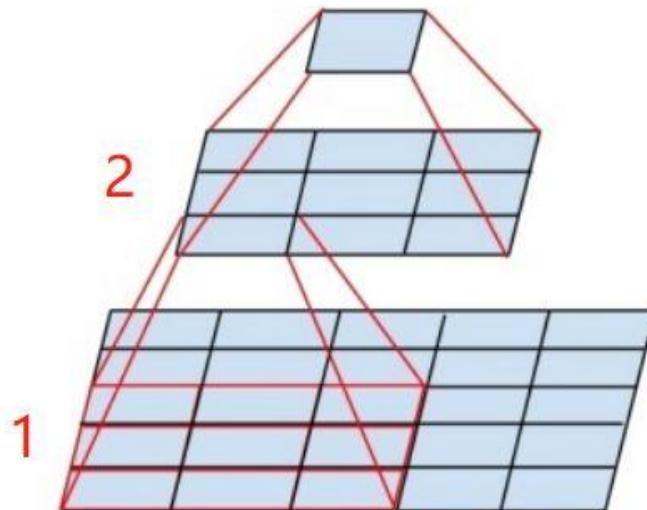
- ◻ convolution+ReLU
- ◼ max pooling
- ◼ fully connected+ReLU
- ◼ softmax

VGG-16 Details

- VGG-16 has 16 weight layers, not including POOL layers w. 0 weight
- Input image: $224 \times 224 \times 3$
- 1st and 2nd CONV layers: 64 3×3 filters w. stride $S = 1$, pad $P = 1$
 - Output volume: $224 \times 224 \times 64$ (since $\frac{1}{1}(224 + 2 * 1 - 3) + 1 = 224$)
- 3rd POOL layer: 2 \times 2 filters w. stride $S = 2$
 - Output volume: $112 \times 112 \times 64$ (since $\frac{1}{2}(224 - 2) + 1 = 112$)
- 4th and 5th CONV layers: 128 3×3 filters w. stride $S = 1$, pad $P = 1$
 - Output volume: $112 \times 112 \times 128$ (since $\frac{1}{1}(112 + 2 * 1 - 3) + 1 = 112$)
- 6th POOL layer: 2 \times 2 filters w. stride $S = 2$
 - Output volume: $56 \times 56 \times 128$ (since $\frac{1}{2}(112 - 2) + 1 = 56$)
- Total No. params: 60M
- ImageNet top 5 error: 7.3%

Stacked 3×3 CONV Layers

- 2 stacked 3×3 CONV layers w. pad $P = 1$ have the same effective receptive field as a 5×5 CONV layer; 3 stacked 3×3 CONV layers w. pad $P = 1$ have RF of 7×7 ; L stacked 3×3 CONV layers w. pad $P = 1$ have RF of $1 + 2L$
- Benefits:
 - Fewer params. Suppose all volumes have the same depth D , then a 7×7 CONV layer has $(7 * 7 * D + 1) * D \approx 49D^2$ params, while three stacked 3×3 CONV layers have only $(3 * 3 * D + 1) * D * 3 \approx 27D^2$ params
 - Two layers of non-linear activation functions increases CNN depth, hence larger model capacity



VGGNet No. Params

```
INPUT: [224x224x3]           memory: 224*224*3=150K  weights: 0
CONV3-64: [224x224x64]      memory: 224*224*64=3.2M   weights: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64]      memory: 224*224*64=3.2M   weights: (3*3*64)*64 = 36,864
POOL2: [112x112x64]         memory: 112*112*64=800K   weights: 0
CONV3-128: [112x112x128]    memory: 112*112*128=1.6M   weights: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128]    memory: 112*112*128=1.6M   weights: (3*3*128)*128 = 147,456
POOL2: [56x56x128]          memory: 56*56*128=400K   weights: 0
CONV3-256: [56x56x256]      memory: 56*56*256=800K   weights: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256]      memory: 56*56*256=800K   weights: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256]      memory: 56*56*256=800K   weights: (3*3*256)*256 = 589,824
POOL2: [28x28x256]          memory: 28*28*256=200K   weights: 0
CONV3-512: [28x28x512]      memory: 28*28*512=400K   weights: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512]      memory: 28*28*512=400K   weights: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512]      memory: 28*28*512=400K   weights: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512]          memory: 14*14*512=100K   weights: 0
CONV3-512: [14x14x512]      memory: 14*14*512=100K   weights: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512]      memory: 14*14*512=100K   weights: (3*3*512)*512 = 2,359,296
CONV3-512: [14x14x512]      memory: 14*14*512=100K   weights: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512]             memory: 7*7*512=25K    weights: 0
FC: [1x1x4096]               memory: 4096   weights: 7*7*512*4096 = 102,760,448
FC: [1x1x4096]               memory: 4096   weights: 4096*4096 = 16,777,216
FC: [1x1x1000]               memory: 1000   weights: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~= 93MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters
```

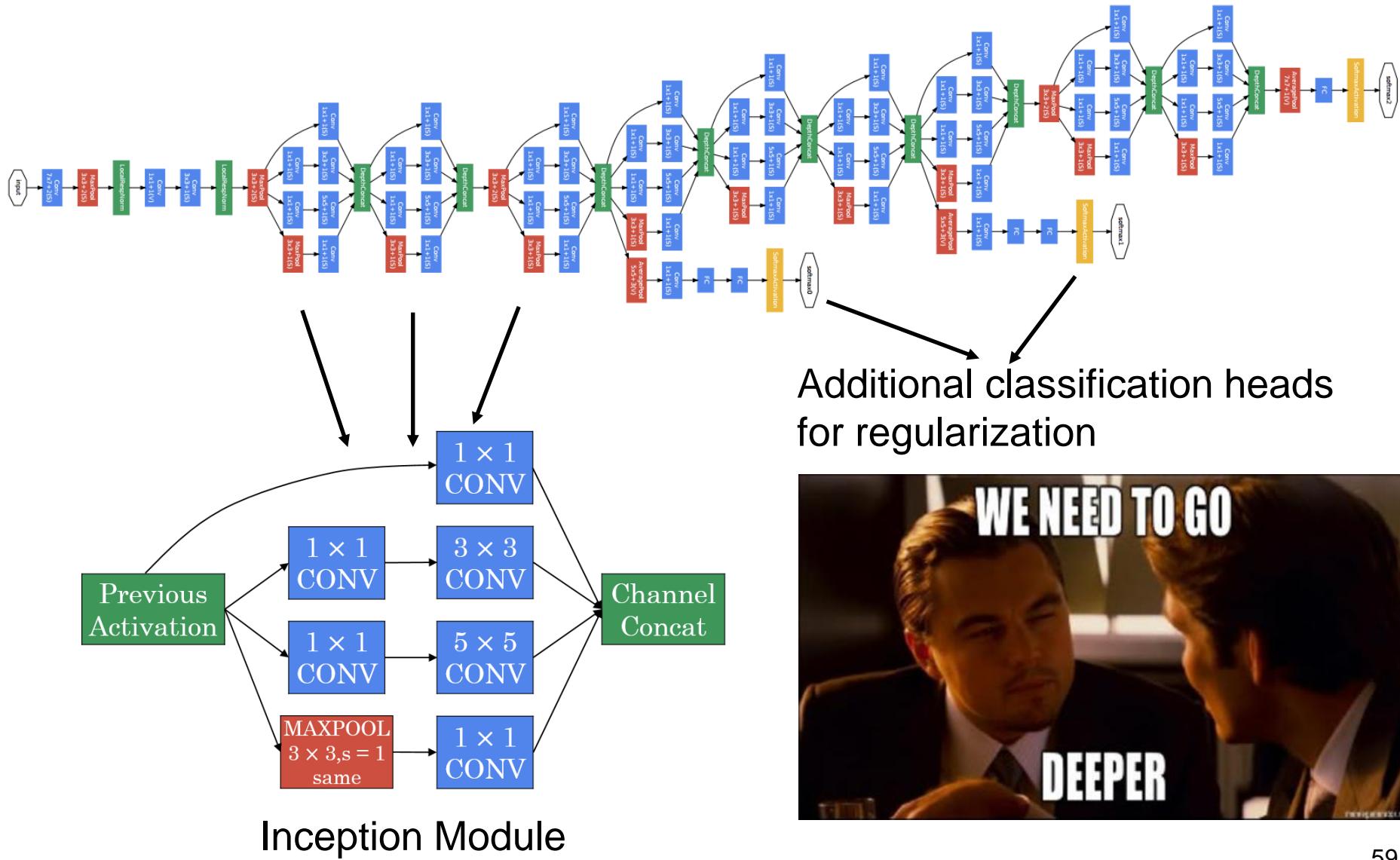
- Memory refers to memory size of activation maps
- For ease of calculation, only the No. weights are counted, not the biases

VGGNet Variants

Best performing variant
VGG-16

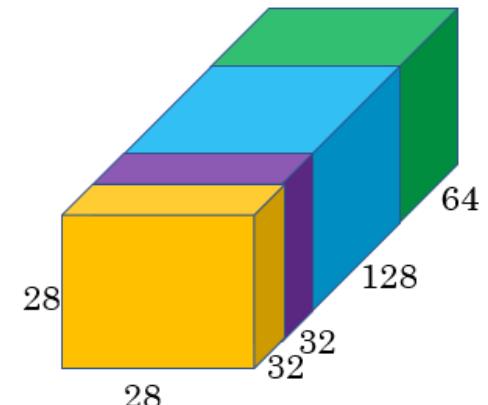
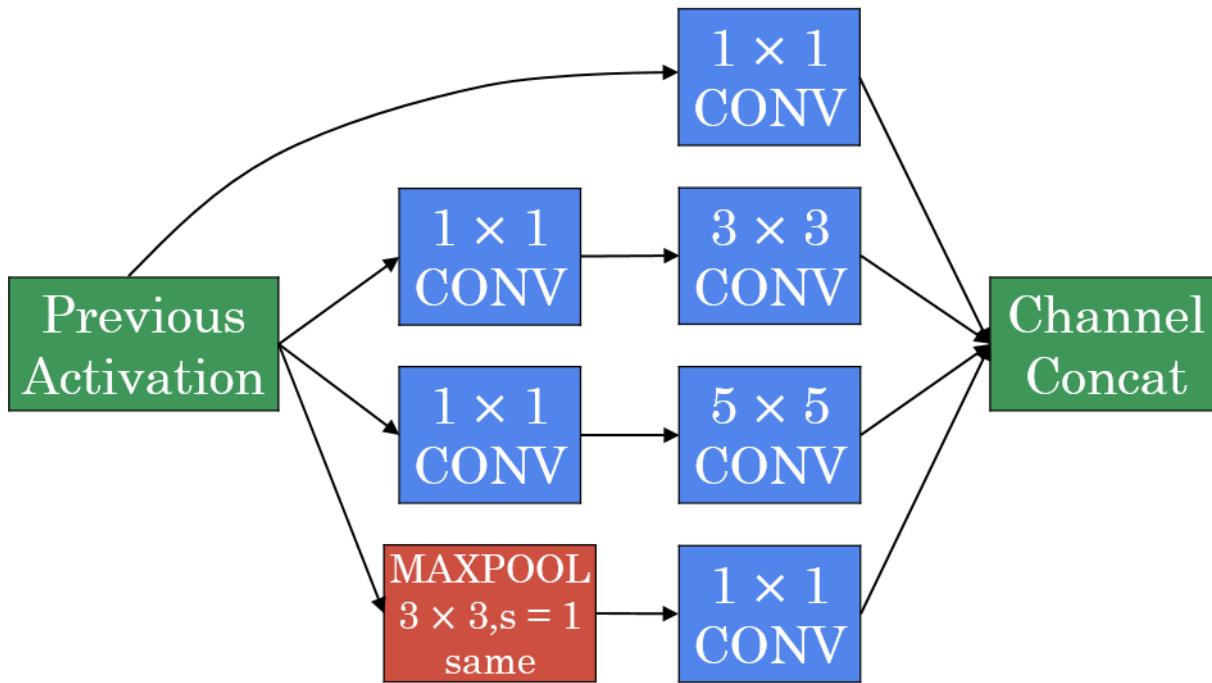
| ConvNet Configuration | | | | | |
|-----------------------------|------------------------|-------------------------------|--|--|---|
| A | A-LRN | B | C | D | E |
| 11 weight layers | 11 weight layers | 13 weight layers | 16 weight layers | 16 weight layers | 19 weight layers |
| input (224 × 224 RGB image) | | | | | |
| conv3-64 | conv3-64 LRN | conv3-64 conv3-64 | conv3-64 conv3-64 | conv3-64 conv3-64 | conv3-64 conv3-64 |
| maxpool | | | | | |
| conv3-128 | conv3-128 | conv3-128 conv3-128 | conv3-128 conv3-128 | conv3-128 conv3-128 | conv3-128 conv3-128 |
| maxpool | | | | | |
| conv3-256 conv3-256 | conv3-256 conv3-256 | conv3-256 conv3-256 | conv3-256 conv3-256 conv1-256 | conv3-256 conv3-256 conv3-256 | conv3-256 conv3-256 conv3-256 conv3-256 |
| maxpool | | | | | |
| conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 conv1-512 | conv3-512 conv3-512 conv3-512 | conv3-512 conv3-512 conv3-512 conv3-512 |
| maxpool | | | | | |
| conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 conv1-512 | conv3-512 conv3-512 conv3-512 | conv3-512 conv3-512 conv3-512 conv3-512 |
| maxpool | | | | | |
| FC-4096 | | | | | |
| FC-4096 | | | | | |
| FC-1000 | | | | | |
| soft-max | | | | | |

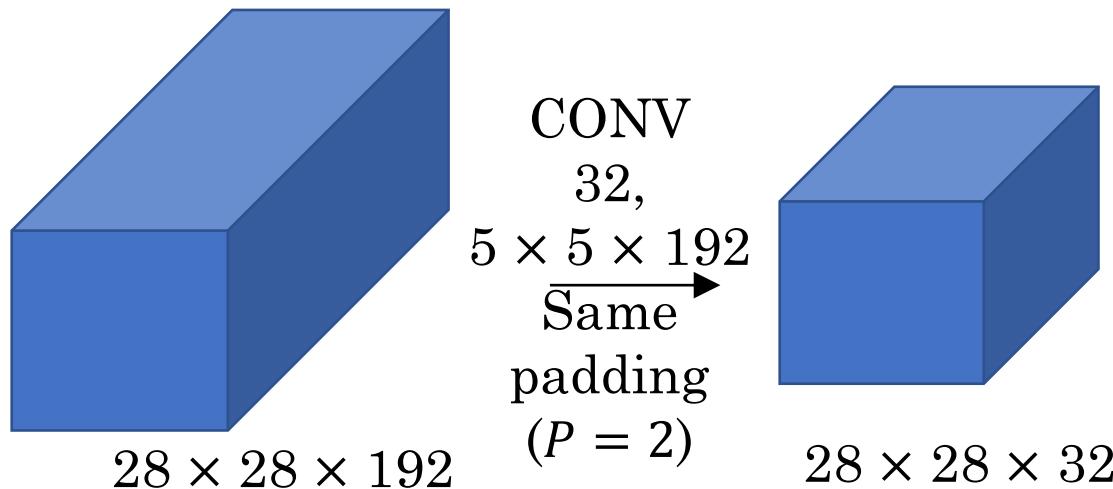
GoogLeNet [Szegedy et al., 2014]



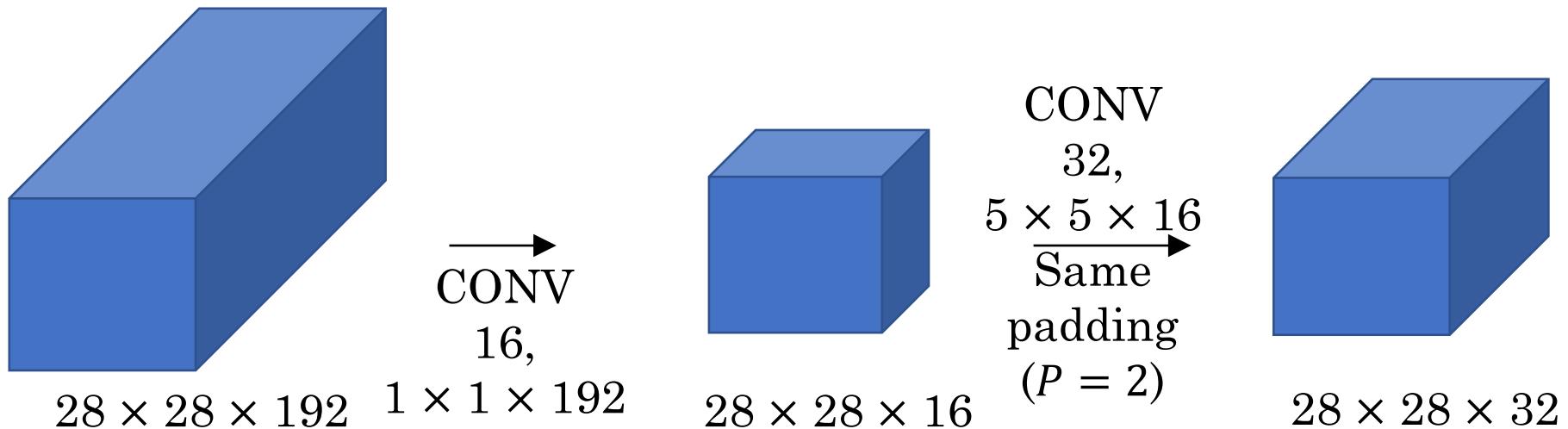
Inception Module

- Can't make up your mind about filter size? Have them all in the Inception Module!
 - But this increases computation load
- Additional 1×1 CONV layers serve as bottleneck to reduce number of parameters and computation load





- Without the bottleneck layer: No. params: $5 * 5 * 192 * 32 = 153600$; No. multiplications: $5 * 5 * 192 * 32 * 28 * 28 = 120M$



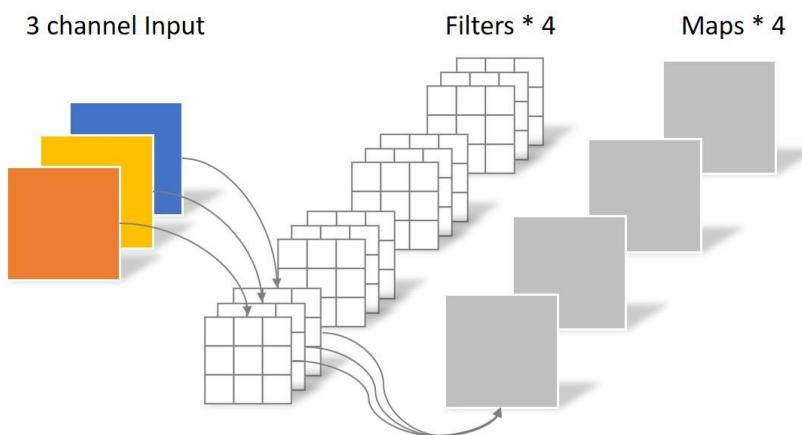
- With the bottleneck layer: No. params: $1 * 1 * 192 * 16 + 5 * 5 * 16 * 32 = 15872$; No. multiplications: $1 * 1 * 192 * 16 * 28 * 28 + 5 * 5 * 16 * 32 * 28 * 28 = 12.4M$

GoogLeNet Size

- Compared to AlexNet:
 - 12x less params (only 5M, due to no FC layers), 2x more compute (due to more CONV layers)

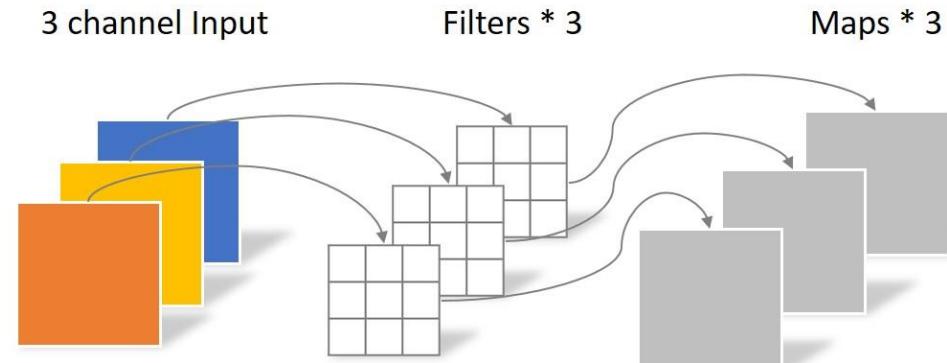
| type | patch size/ stride | output size | depth | #1×1 | #3×3 reduce | #3×3 | #5×5 reduce | #5×5 | pool proj | params | ops |
|----------------|-----------------------|----------------|-------|------|----------------|------|----------------|------|--------------|--------|------|
| convolution | 7×7/2 | 112×112×64 | 1 | | | | | | | 2.7K | 34M |
| max pool | 3×3/2 | 56×56×64 | 0 | | | | | | | | |
| convolution | 3×3/1 | 56×56×192 | 2 | | 64 | 192 | | | | 112K | 360M |
| max pool | 3×3/2 | 28×28×192 | 0 | | | | | | | | |
| inception (3a) | | 28×28×256 | 2 | 64 | 96 | 128 | 16 | 32 | 32 | 159K | 128M |
| inception (3b) | | 28×28×480 | 2 | 128 | 128 | 192 | 32 | 96 | 64 | 380K | 304M |
| max pool | 3×3/2 | 14×14×480 | 0 | | | | | | | | |
| inception (4a) | | 14×14×512 | 2 | 192 | 96 | 208 | 16 | 48 | 64 | 364K | 73M |
| inception (4b) | | 14×14×512 | 2 | 160 | 112 | 224 | 24 | 64 | 64 | 437K | 88M |
| inception (4c) | | 14×14×512 | 2 | 128 | 128 | 256 | 24 | 64 | 64 | 463K | 100M |
| inception (4d) | | 14×14×528 | 2 | 112 | 144 | 288 | 32 | 64 | 64 | 580K | 119M |
| inception (4e) | | 14×14×832 | 2 | 256 | 160 | 320 | 32 | 128 | 128 | 840K | 170M |
| max pool | 3×3/2 | 7×7×832 | 0 | | | | | | | | |
| inception (5a) | | 7×7×832 | 2 | 256 | 160 | 320 | 32 | 128 | 128 | 1072K | 54M |
| inception (5b) | | 7×7×1024 | 2 | 384 | 192 | 384 | 48 | 128 | 128 | 1388K | 71M |
| avg pool | 7×7/1 | 1×1×1024 | 0 | | | | | | | | |
| dropout (40%) | | 1×1×1024 | 0 | | | | | | | | |
| linear | | 1×1×1000 | 1 | | | | | | | 1000K | 1M |
| softmax | | 1×1×1000 | 0 | | | | | | | | |

Xception [Chollet 2017] MobileNets [Howard et al. 2017] : Depthwise Separable Convolution

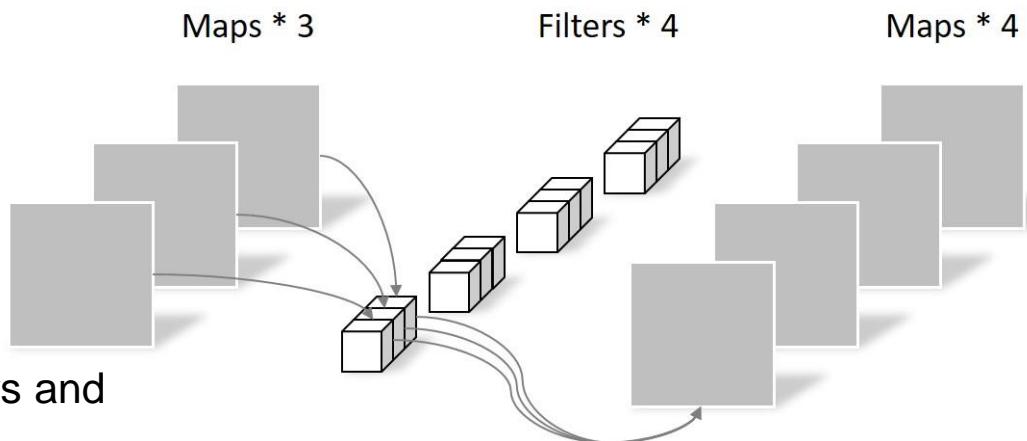


Each filter is convolved with all input channels

Regular Convolution



Each filter is convolved with one input channel



The intermediate feature maps serve as bottleneck to reduce number of parameters and computation load

(Optional) Depthwise Separable Convolution - A FASTER CONVOLUTION!

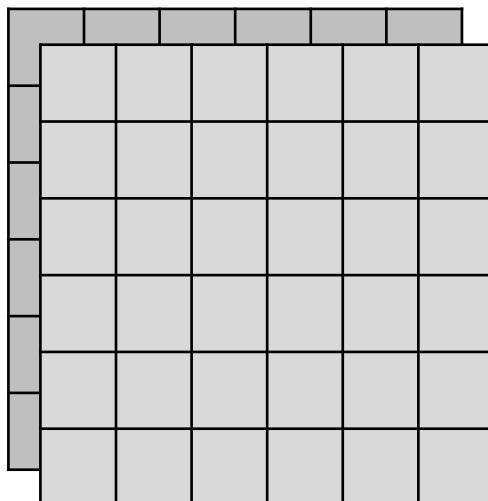
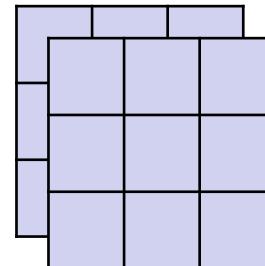
<https://www.youtube.com/watch?v=T7o3xvJLuHk>

Followed by pointwise convolution

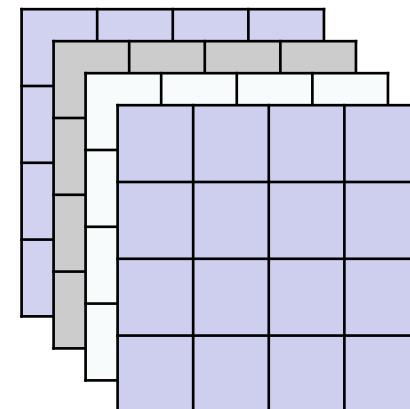
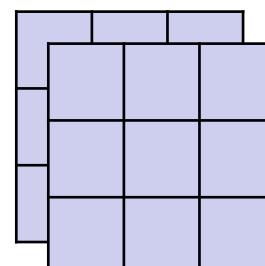
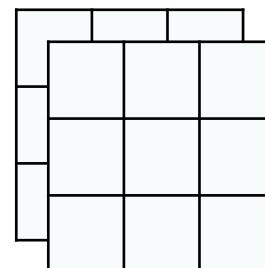
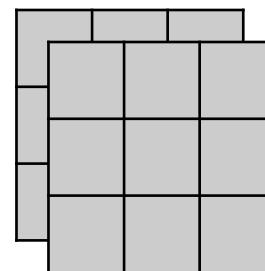
Depthwise Separable Convolution:

Example: Regular Convolution

Input feature map



2 channels

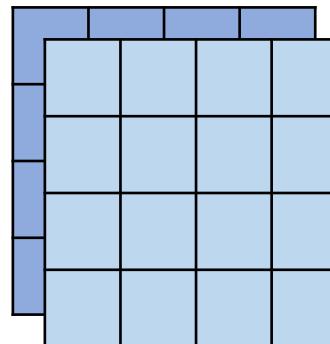
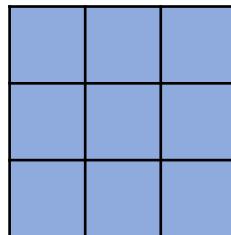
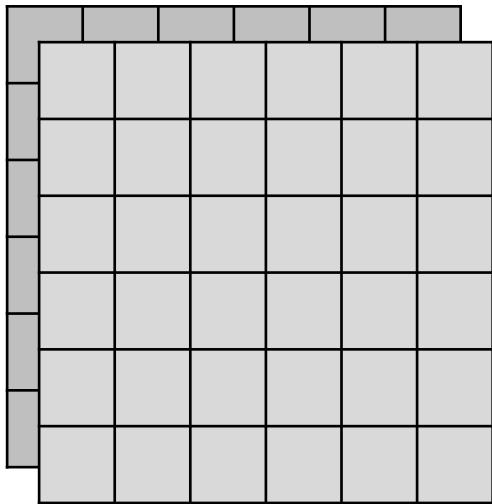


No. params: $3 * 3 * 2 * 4 = 72$;

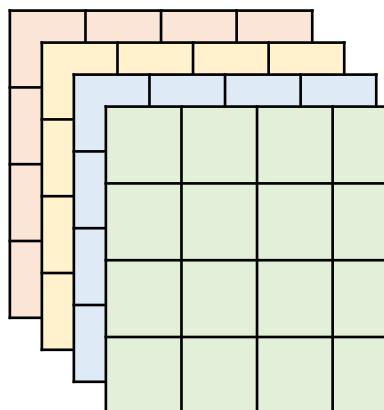
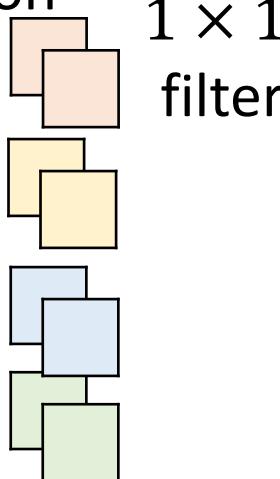
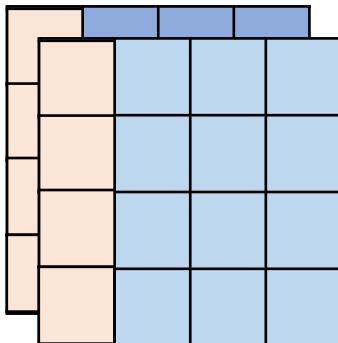
No. MULs: $3 * 3 * 2 * 4 * 4 * 4 = 1152$

Depthwise Separable Convolution Example

1. Depthwise Convolution



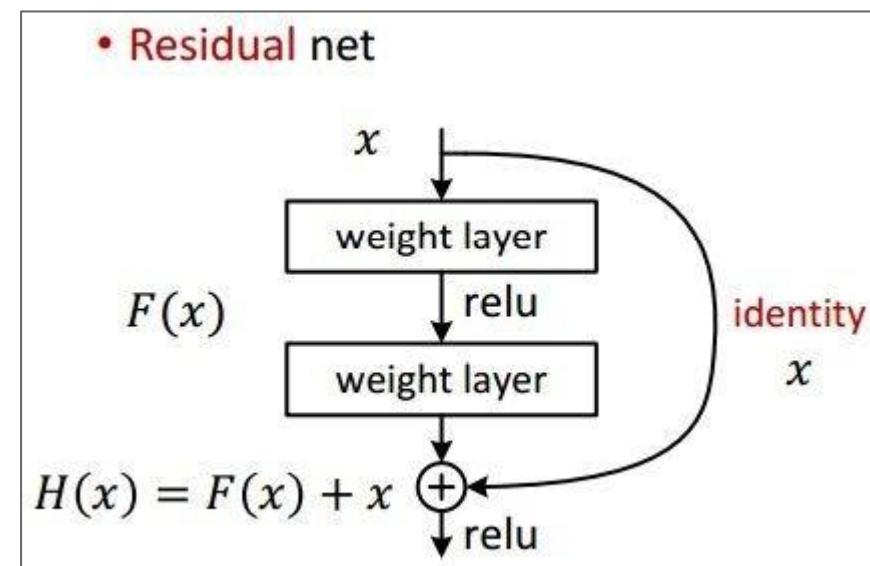
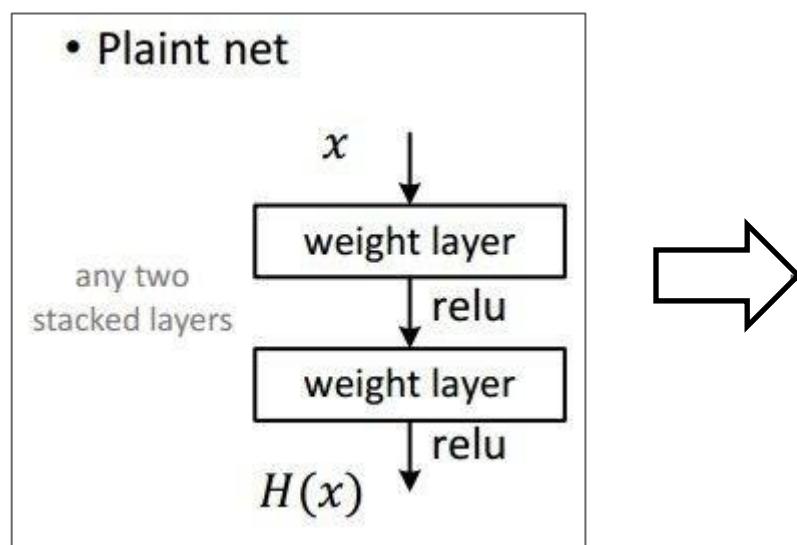
2. Pointwise Convolution



No. params: $3 * 3 * 2 + 2 * 4 = 26$;
No. MULs: $3 * 3 * 1 * 2 * 4 * 4 + 1 * 1 * 2 * 4 * 4 = 416$

Residual Networks (ResNet) [He et al. 2015]

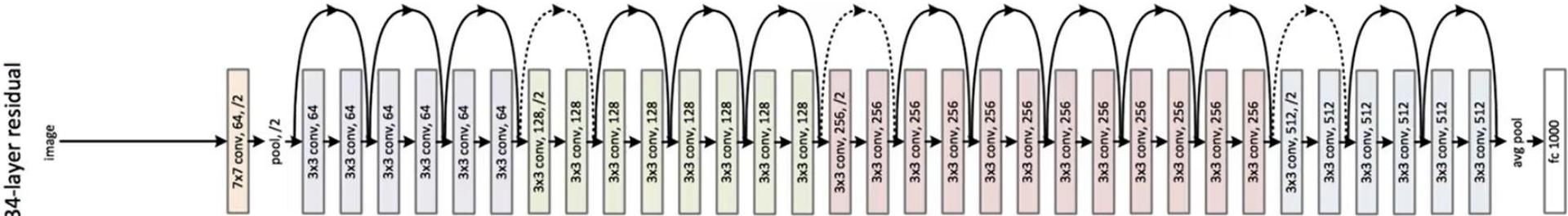
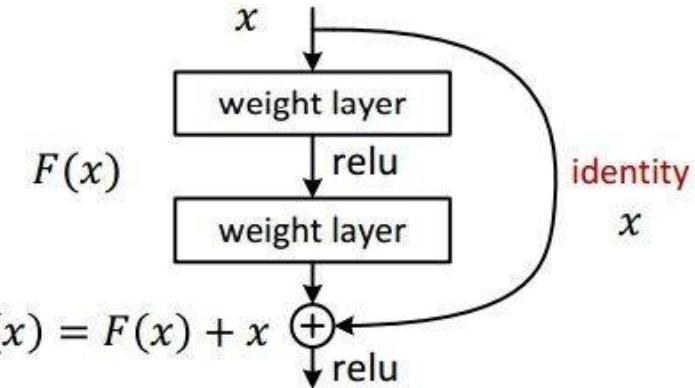
- Based on VGG-19, adding more layers and skip connections
- ImageNet top 5 error: 3.6%



ResNet Skip Connection

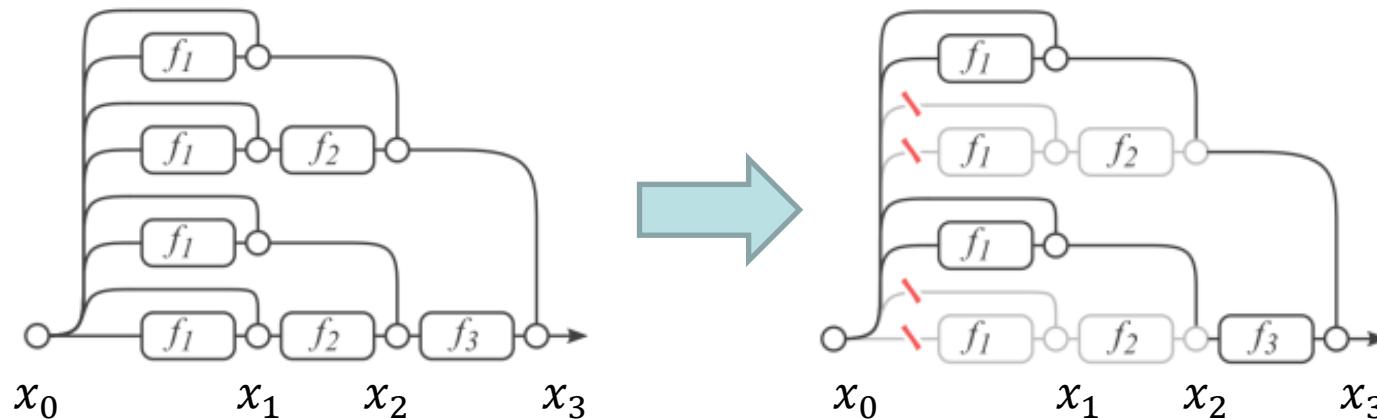
- In a standard network, output from a given layer is $F(x)$
- In ResNet w. the identity skip (or short-cut) connection, output from a given layer is $H(x) = F(x) + x$
- Benefits:
 - Residual connections help in handling the vanishing gradient problem in very deep NNs
 - If identity mapping is close to optimal, then weights can be small to capture minor differences only, in other words, “unnecessary layers” can learn to be identity mapping. This allows stacking many layers (e.g., 152) without overfitting

• Residual net



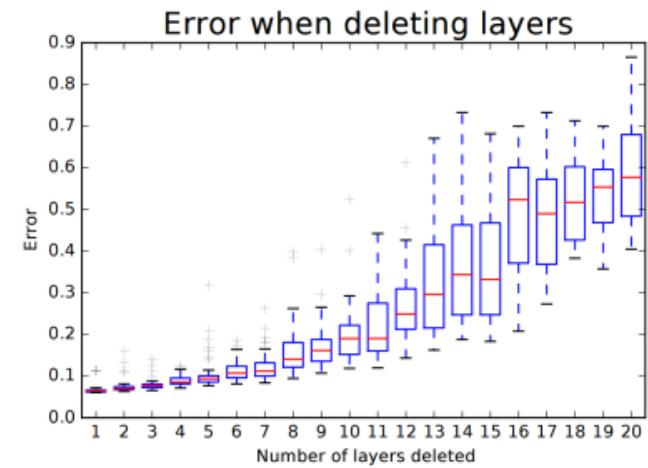
Consider a 3-layer Network

- Standard NN:
 - $x_3 = f_3(f_2(f_1(x_0)))$
- ResNet:
 - $x_1 = f_1(x_0) + x_0$
 - $x_2 = f_2(x_1) + x_1 = f_2(f_1(x_0) + x_0) + f_1(x_0) + x_0$
 - $x_3 = f_3(x_2) + x_2 = f_3(f_2(f_1(x_0) + x_0) + f_1(x_0) + x_0) + f_2(f_1(x_0) + x_0) + f_1(x_0) + x_0$
- Suppose $f_2(x_1)$ is a vector of very small values (layer 2 is “off”/skipped), then it looks like the input x_0 bypassed the second layer completely on its way to the output x_3



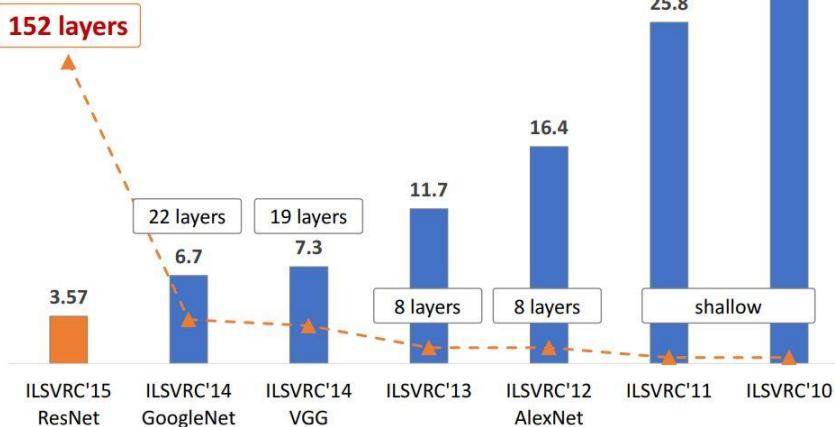
ResNet is an Ensemble of Models

- Every input x_0 to ResNet may activate a unique path to the output. Total number of possible paths is 2^N , where N is the total number of layers in the network, since each layer may be either “on” or “off” for a given input x_0
 - Compare w. a standard network, where there is only one single path for any input corresponding to all layers being “on”, and no layer is skipped
- Consequences:
 - Resilience to layer deletion: deleting 1-3 layers in a large ResNet introduces only around 6-7% error
 - Shortening of effective paths: w. 152-layer ResNet, most paths are only 20-30 levels deep!



Deeper Nets have Better Performance

Revolution of Depth



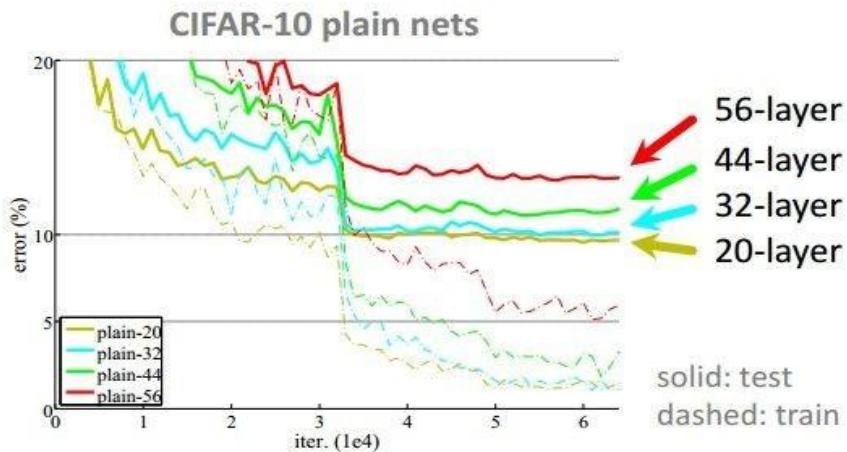
Revolution of Depth

AlexNet, 8 layers (ILSVRC 2012) VGG, 19 layers (ILSVRC 2014) ResNet, 152 layers (ILSVRC 2015)

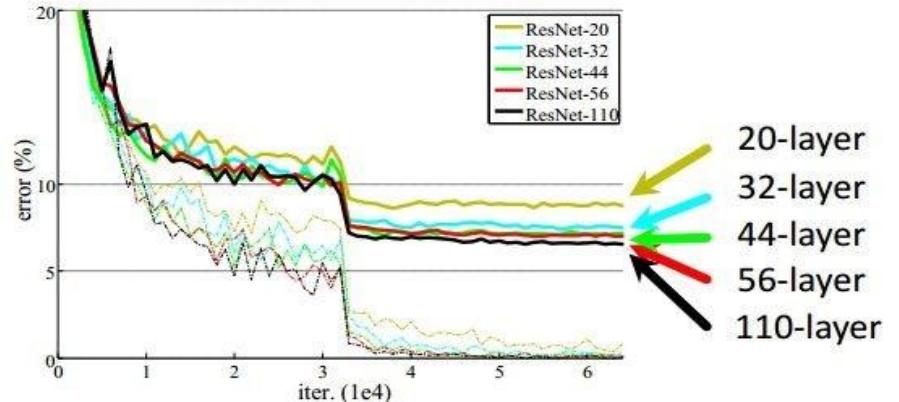


Kaiming He, Xiangyu Zhang, Shaoqing Ren, & Jian Sun. "Deep Residual Learning for Image Recognition". arXiv 2015.

CIFAR-10 experiments

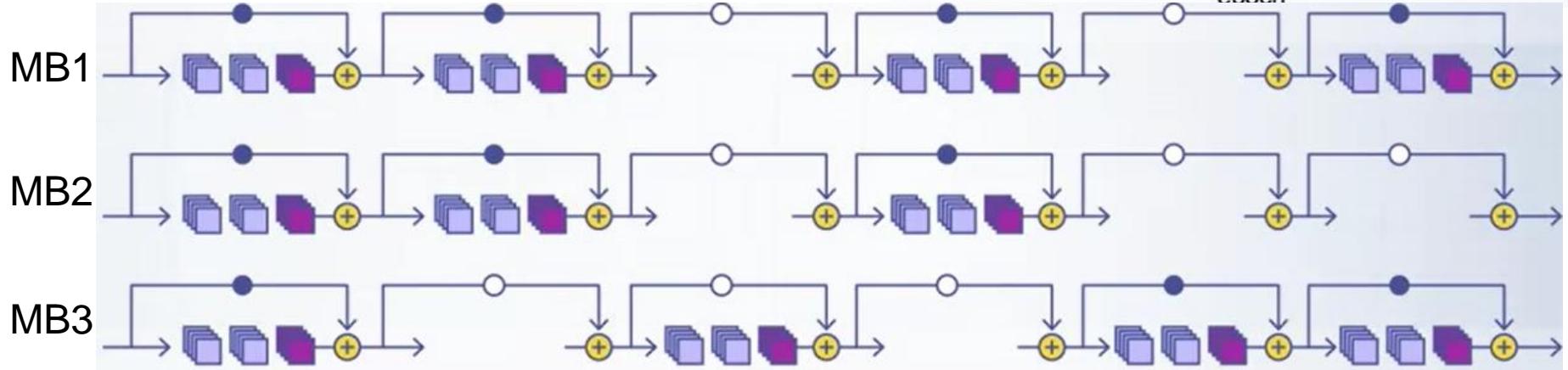
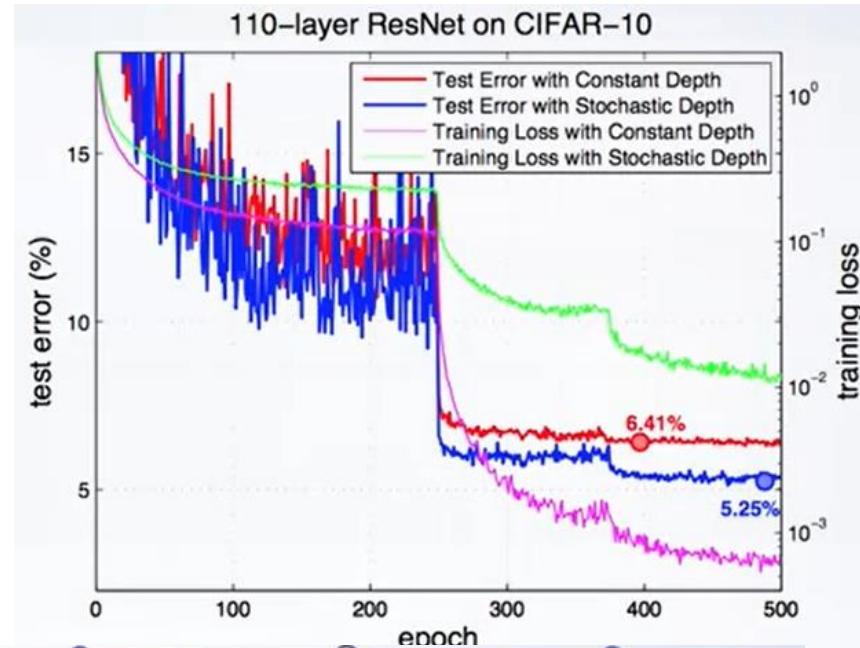


CIFAR-10 ResNets



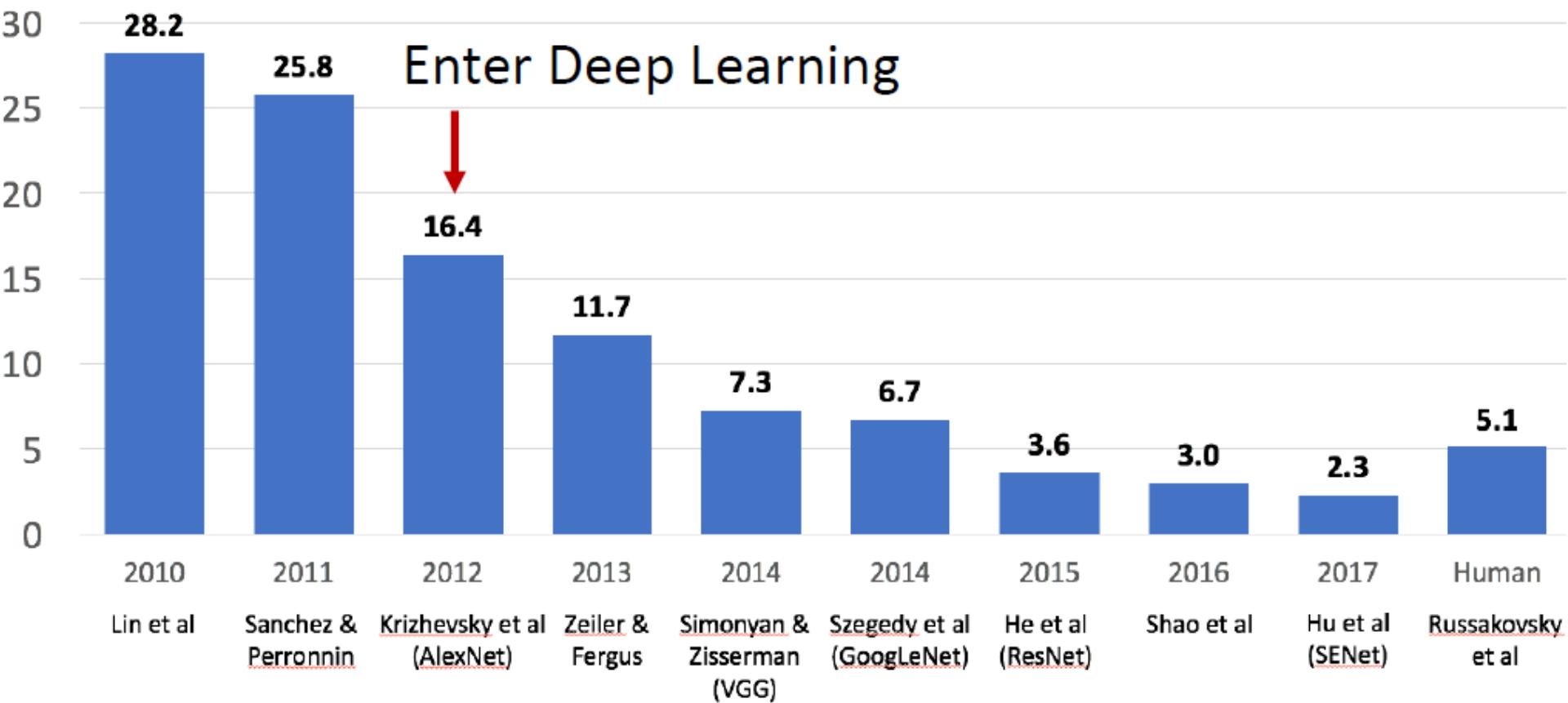
ResNet Training with Stochastic Depth

- For each minibatch of inputs, randomly skip some layers (replaced w. identity mapping)
- Reduced network depth during training; full depth during inference



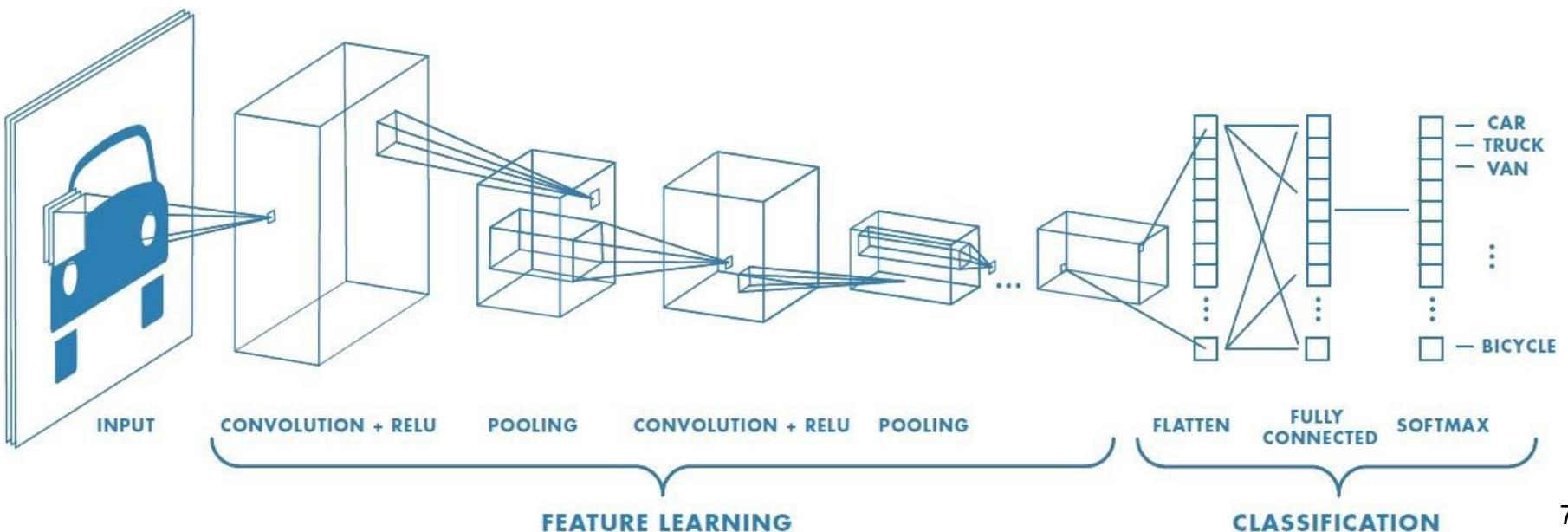
ImageNet Large Scale Visual Recognition Challenge

- 1,000 object classes, 1.4 M labeled images



CNN Layer Patterns

- A typical CNN architecture looks like: INPUT-> [[CONV->RELU] $*N$ ->POOL?] $*M$ -> [FC->RELU] $*K$ ->FC
 - where $*$ indicates repetition, and POOL? indicates an optional pooling layer. $N \geq 0$ (usually $N \leq 3$), $M \geq 0$, $K \geq 0$ (and usually $K < 3$)
- Some common architectures:
 - INPUT->FC, implements a linear classifier. Here $N = M = K = 0$.
 - INPUT->CONV->RELU->FC
 - INPUT-> [CONV->RELU->POOL] $*2$ ->FC->RELU->FC (fig below). There is a single CONV layer between every POOL layer.
 - INPUT-> [CONV->RELU->CONV->RELU->POOL] $*3$ -> [FC->RELU] $*2$ ->FC There are two CONV layers stacked before every POOL layer. This is generally a good idea for larger and deeper networks, because multiple stacked CONV layers can develop more complex features of the input volume before the destructive pooling operation.



Layer Sizing Rules-of-Thumb

- The input layer (that contains the image) should be divisible by 2 many times. Common numbers include 32 (e.g. CIFAR-10), 64, 96 (e.g. STL-10), or 224 (e.g. ImageNet), 384, and 512.
- The CONV layers should use small filters (e.g. 3x3 or at most 5x5), stride $S=1$. The input volume should have “same padding”, i.e., the conv layer does not alter the spatial size of the input. For any F , pad $P=(F-1)/2$ preserves the input size, e.g., when $F=3$, $P=1$; when $F=5$, $P=2$. This means the CONV layers only transform the input volume depth-wise, but do not perform downsampling. (c.f. CONV Example 3 and VGGNet).
- The POOL layers are in charge of downsampling the spatial dimensions of the input. The most common setting is to use max-pooling with 2x2 receptive fields ($F=2$), with stride of 2 ($S=2$). A less common setting is to use $F=3$, $S=2$. It is uncommon to see receptive field sizes for max pooling that are larger than 3, because the pooling is then too lossy and aggressive.
- In some cases (especially in early layers), the amount of memory can build up very quickly with the rules of thumb presented above. For example, filtering a 224x224x3 image with three 3x3 CONV layers with 64 filters each and padding 1 would create 3 activation volumes, each with size 224x224x64. This amounts to a total of about 10 million activations, or 72MB of memory (per image, for both activations and gradients). Since GPUs are often bottlenecked by memory, it may be necessary to compromise. In practice, make the compromise at only the first CONV layer that is looking at the input image. For example, AlexNet uses filter size of 11x11 and stride of 4 in the first CONV layer.

Memory Size Considerations

- From the intermediate volume sizes:
 - These are the raw number of activations at every layer of the CNN, and also their gradients (of equal size). Usually, most of the activations are on the earlier CONV layers of a CNN. These are kept around because they are needed for backpropagation during training, but for inference, we can store only the current activations at the current layer and discarding the activations from previous layers.
- From the parameter sizes:
 - These are the weights and biases, and their gradients during backpropagation, and also a step cache if the optimization is using momentum, Adagrad, or RMSProp. Therefore, the memory to store the parameter vector alone usually should be multiplied by a factor of at least 3 or so.
- Each number may need 4 B storage space for floating point, 8 B for double, or 1 B or smaller for optimized fixed-point implementations.

Transfer Learning

- Instead of training your CNN from scratch, start from a pre-trained CNN, e.g., ResNet, and fine-tune it for your task
- First, replace the SoftMax classification head with your own
- Next, train the CNN while keeping frozen
 - all CONV layers and only train the SoftMax layer
 - or part of the earlier CONV layers close to the input layer (since earlier layers extract lower-level features that are more likely to be common among different tasks)
 - or none of the layers
 - The decision depends on how much training data you have, and how similar your task is to that of the pre-trained CNN

Outline

- CNN Convolution layers
- Pooling and Fully-Connected layers
- CNN case studies
- RNNs

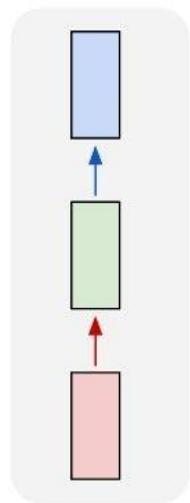
Recurrent Neural Network (RNN)

- An RNN has connections between nodes that form a directed graph along a temporal sequence. This allows it to process variable-length input sequences and take into account dynamic temporal behavior
 - e.g., To take into account temporal sequence of consecutive frames in a video clip, we can either stack together a fixed number of frames as input to a CNN, or we can use an RNN (combined w. CNN) to process any variable-length sequence of frames
 - (Optional) Michael Phi, Illustrated Guide to Recurrent Neural Networks: Understanding the Intuition

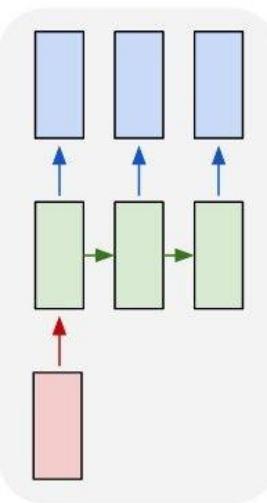
<https://www.youtube.com/watch?v=LHXXI4-lEms>

RNN Architecture Variants

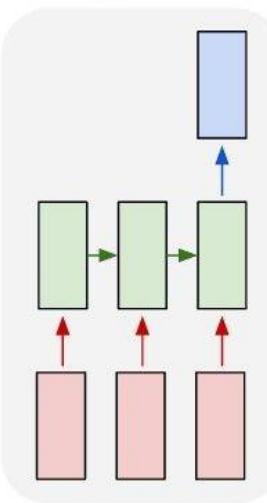
one to one



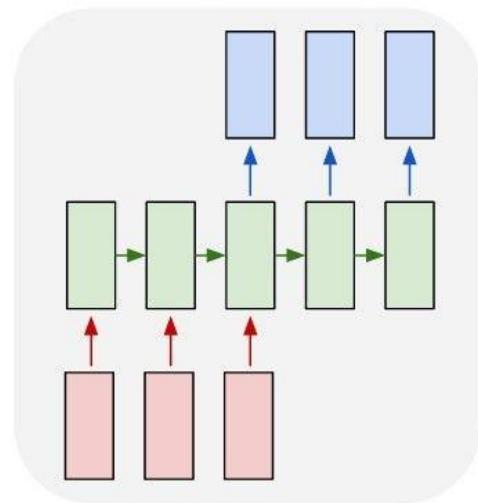
one to many



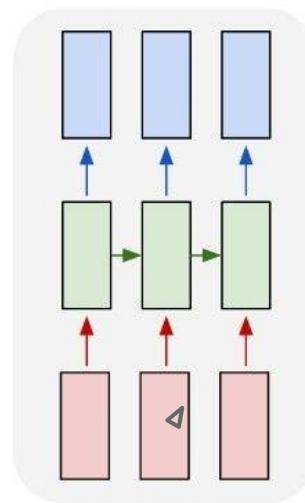
many to one



many to many



many to many



Regular
Feedforwar
d NN

e.g. Image
Captioning
image ->
sequence of
words

e.g. Sentiment
Classification
sequence of
words ->
sentiment

e.g. Machine Translation
seq of words -> seq of
words

e.g., video
classification
on frame level

RNN Many-to-Many Architecture

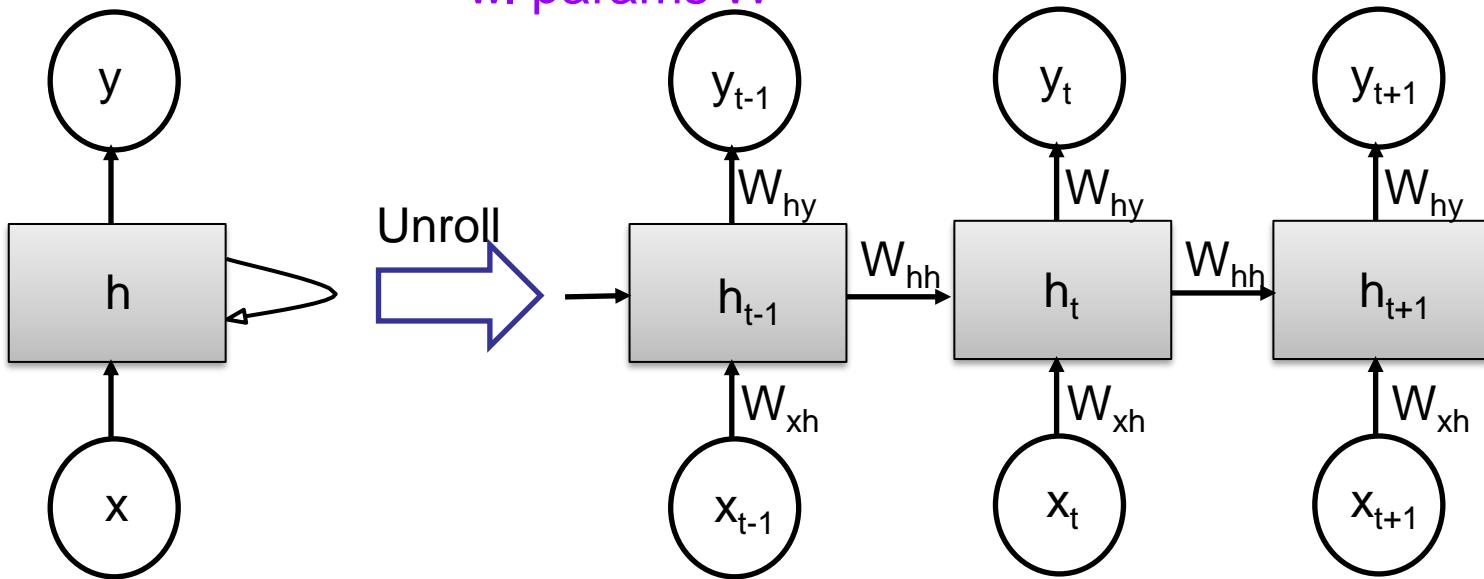
- RNN can process a sequence of inputs x recurrently at every time step, w. the same activation function and parameters f_W
 - e.g. $h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$, $y_t = W_{hy}h_t$
 - h_t can even be a large CNN (without the last classification layer)

$$h_t = f_W(h_{t-1}, x_t)$$

Activation function

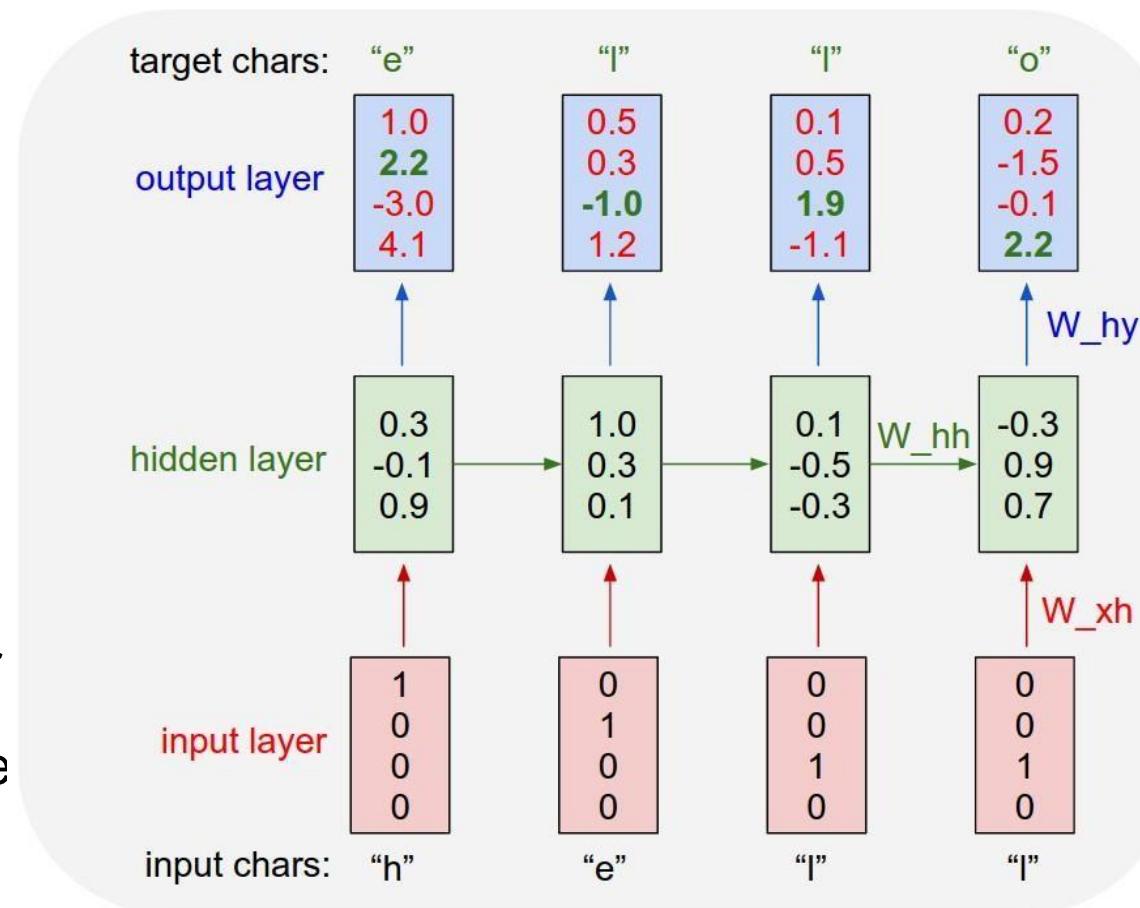
new state at timestep t old state at $t - 1$ input at timestep t

w. params W



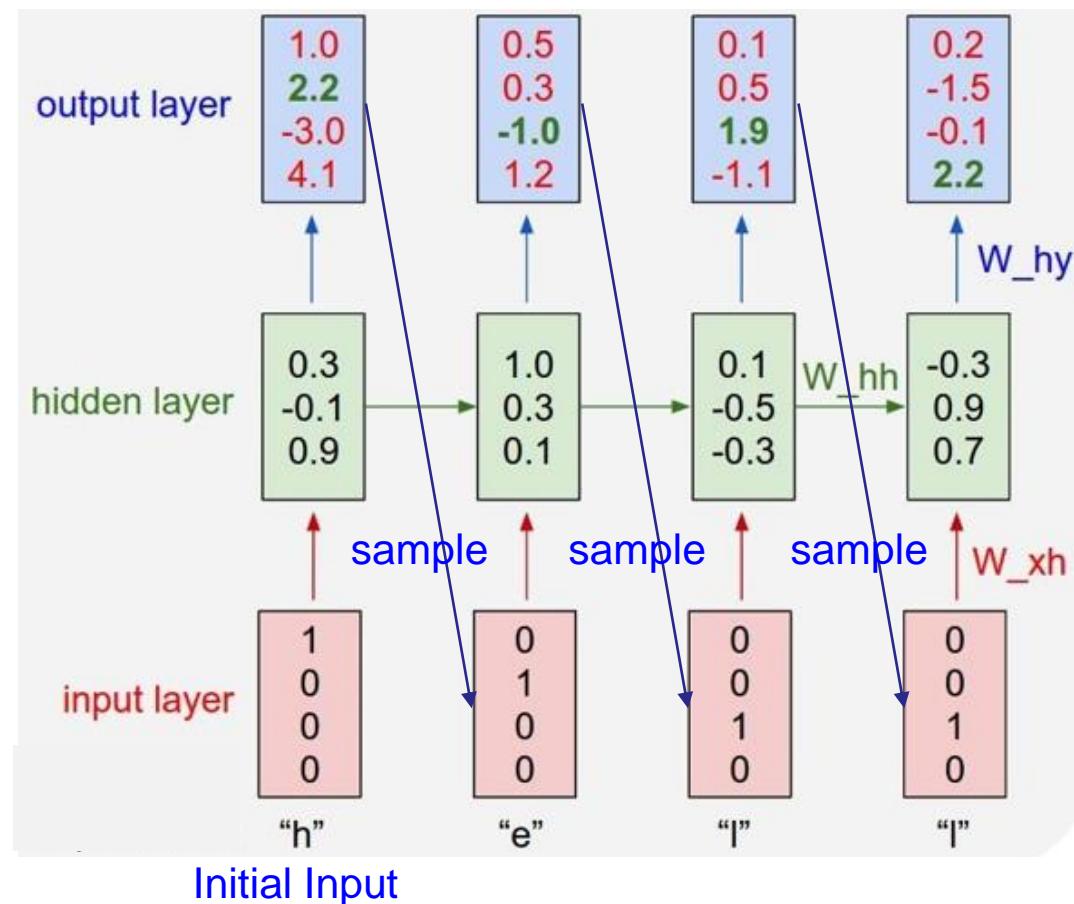
Example: Character-Level Language Model, Training Time

- Task: predict the next char from current char sequence
- Example training sequence: “hello” w. vocabulary: [h,e,l,o]
 - Input is one-hot encoding of each char
 - Hidden layer is learned embedding
 - Output layer is a probability vector w. size 4, denoting prob distribution of next char (Fig shows the activation values before applying the SoftMax function for computing probabilities).



Example: Character-Level Language Model, Inference Time

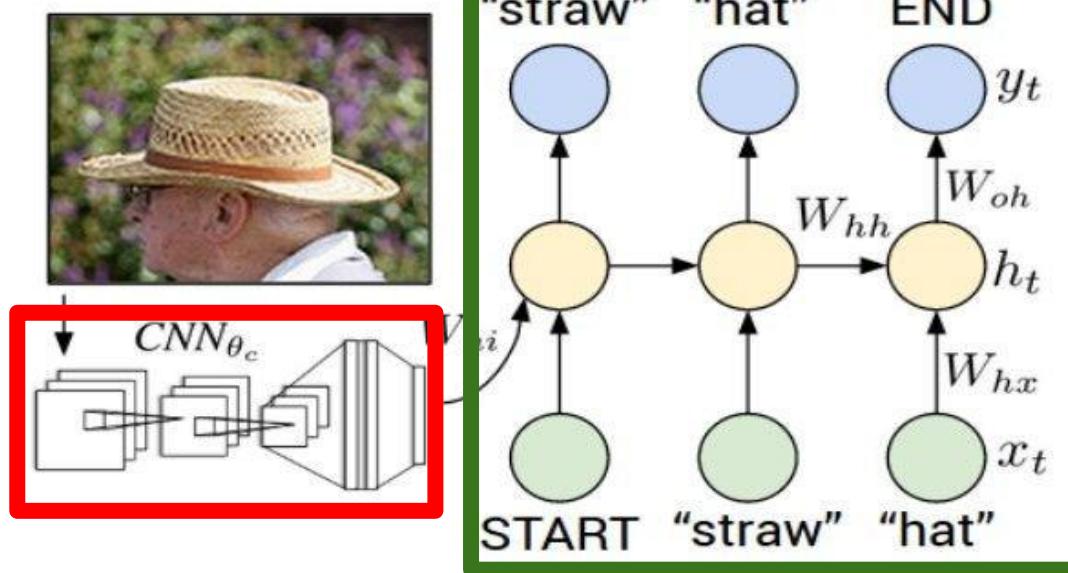
- Initial input is char “h”
- At each timestep, sample from the prob vector of the output y_t to generate the next input char
 - Here we assume the char w. highest prob is always selected as output at each timestep, i.e., “e”, “l”, “l” in sequence, but it is possible to select the other choices, e.g., “l”, “e”, “e”)



Example: Image Captioning

- CNN processes the input image and generates a feature vector as input to RNN

Recurrent Neural Network



Convolutional Neural Network

Image Captioning: Word-Level Language Model, Inference Time

- The last 2 layers of the CNN for classification (FC-1000 and SoftMax) are not used, since we only need the extracted features from the layer FC-4096
- At each timestep, sample from the prob vector of the output y_t to generate the next input word

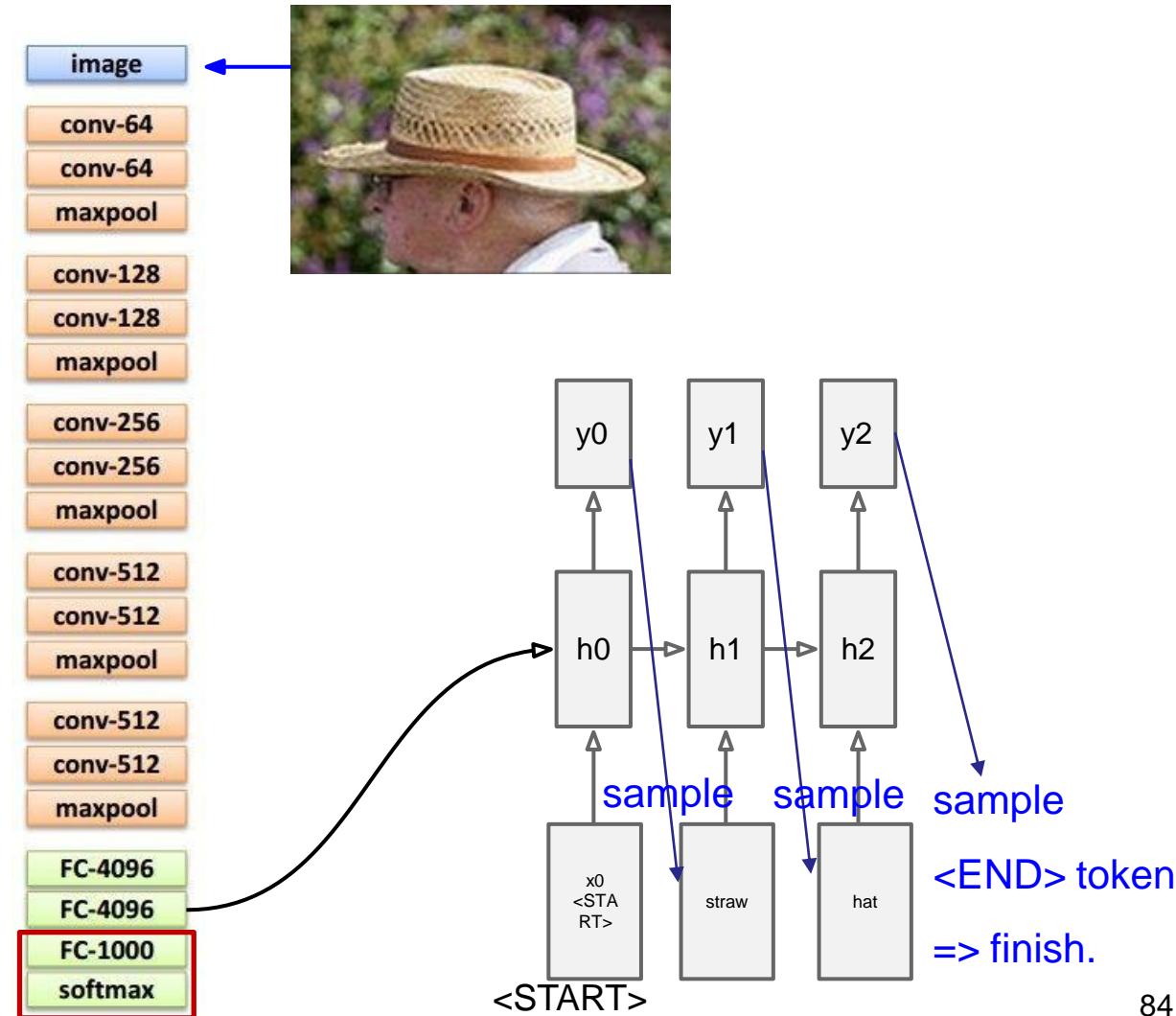


Image Captioning Examples



"man in black shirt is playing guitar."



"construction worker in orange safety vest is working on road."



"two young girls are playing with lego toy."



"boy is doing backflip on wakeboard."



"a young boy is holding a baseball bat."



"a cat is sitting on a couch with a remote control."



"a woman holding a teddy bear in front of a mirror."

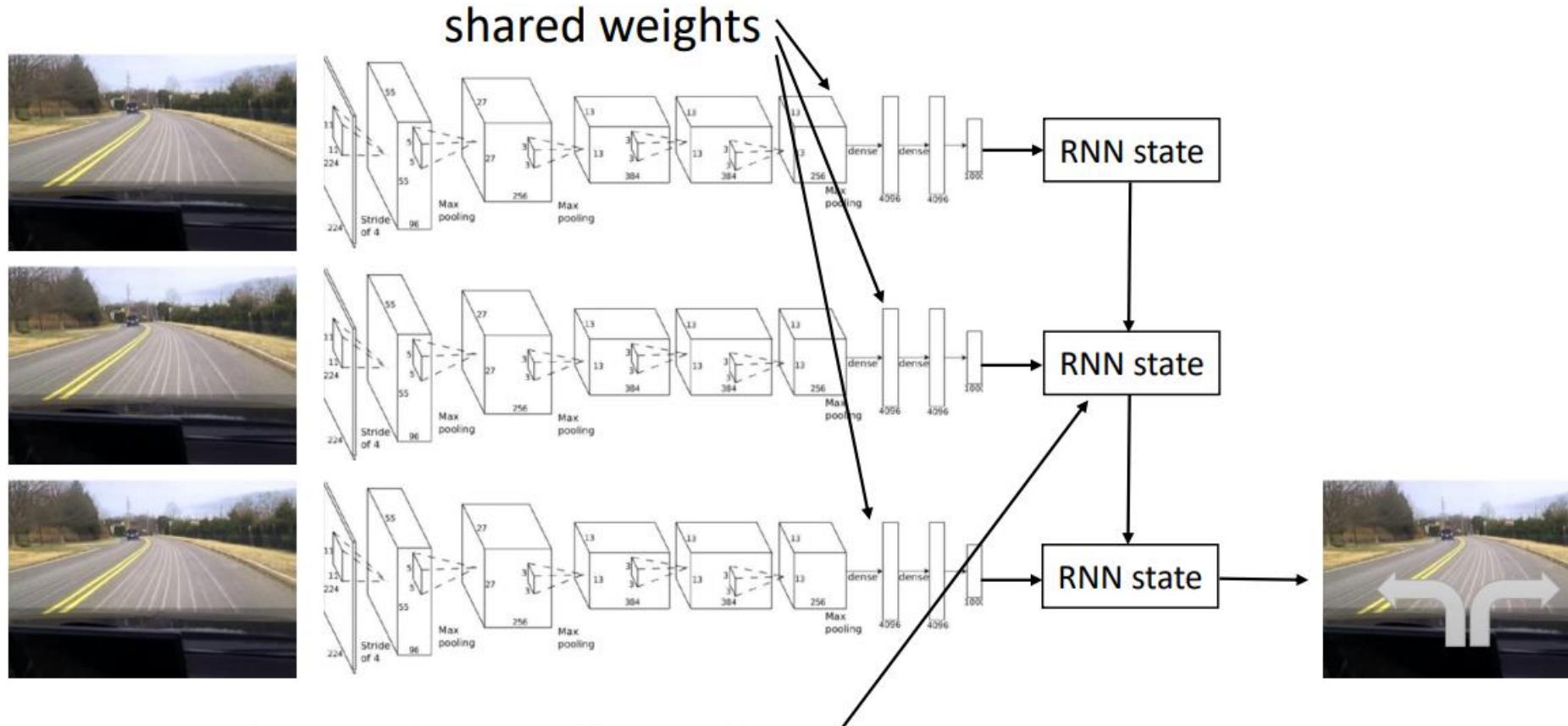


"a horse is standing in the middle of a road."

Bottom row shows failure cases

RNN in AD

- Combined with CNN, RNN can handle Non-Markovian behavior, i.e, the current action depends not just on the current observation (input image), but on a recent history of observations



Typically, LSTM cells work better here

RNN Summary

- Training of RNNs requires back propagation through time, which may cause exploding or vanishing gradient problems
- More sophisticated architectures are more practical
 - LSTM (Long Short-Term Memory Model) or GRU (Gated Recurrent Unit)
- RNNs are most widely used in Natural Language Processing, but it is also useful for processing videos, w. applications in autonomous driving