Self-Driving Cars

Lecture 6 – Vehicle Control

Prof. Dr.-Ing. Andreas Geiger

Autonomous Vision Group

University of Tübingen / MPI-IS

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

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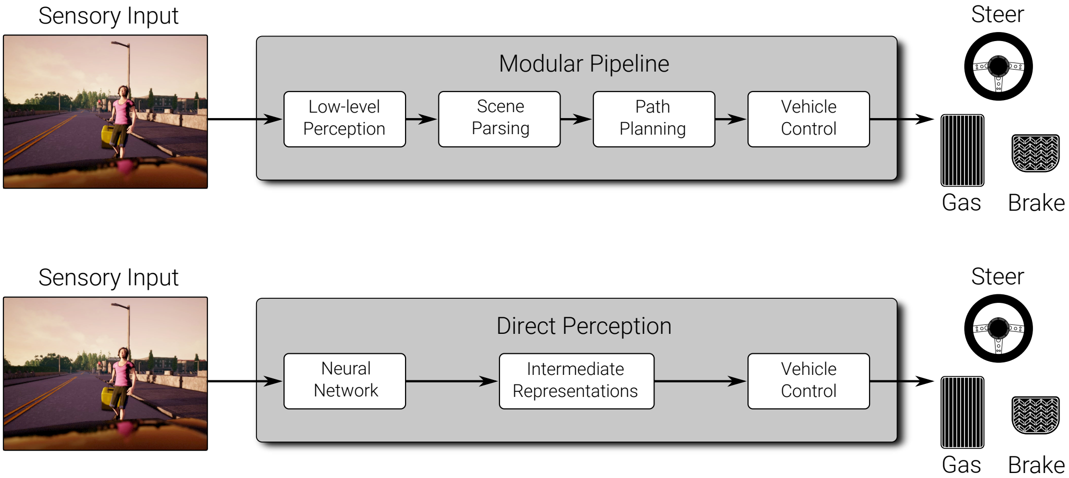
ellis

European Laboratory for Learning and Intelligent Systems

Agenda

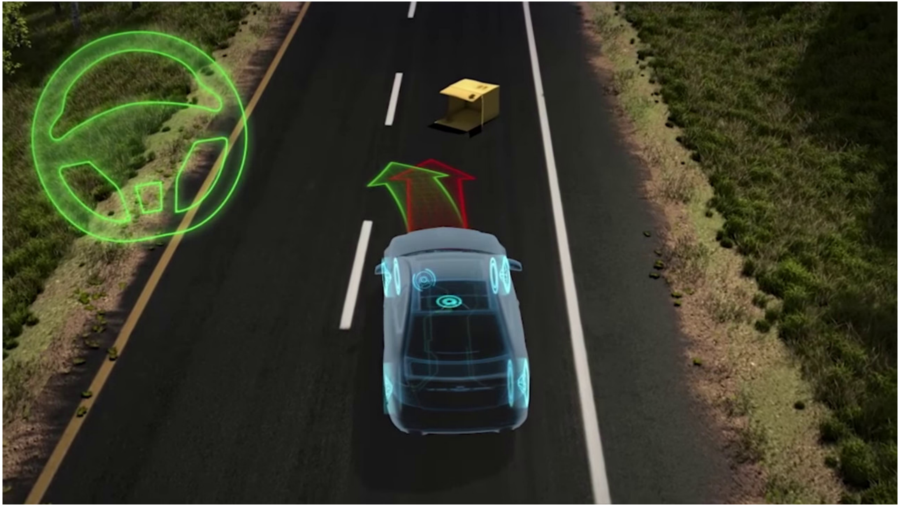
6.1
Introduction

Approaches to Self-Driving



- Self-driving cars and driver assistance systems require **vehicle control**

Electronic Stability Program

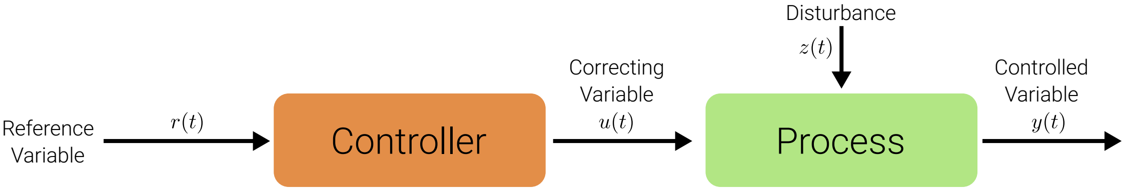


Sophisticated controllers are already in your car today!

Brief History of Driver Assistance Systems

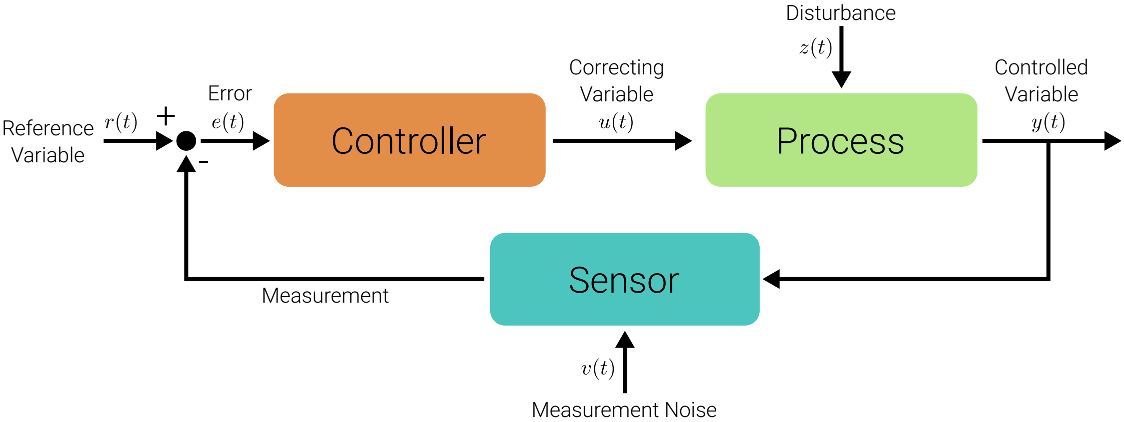
- 1926: Servo braking (Pierce-Arrow)
- 1951: Servo steering (Chrysler)
- 1958: Cruise control (Chrysler)
- 1978: Anti-lock braking system ABS (Bosch)
- 1986: Traction control system ASR (Bosch)
- 1995: Electronic stability program ESP (Bosch/BMW)
- 2000: Adaptive cruise control ACC (Mitsubishi/Toyota/Bosch)
- 2002: Emergency brake assistant (Mercedes Benz)
- 2003: Lane-keeping assistant (Honda)
- 2007: Automatic park assistant (Valeo)

Open-Loop Control



- Requires **precise knowledge** of the plant and the influence factors
- **No feedback** about the controlled variable
- Cannot handle unknown disturbances, resulting in **drift**

Closed-Loop Control



- Exploit feedback loop to minimize error between reference and measurement

Centrifugal Governor

https://www.youtube.com/watch?v=B01LgS8S5C8

Closed-Loop Control

- We will be considering **closed-loop control** in this lecture
- A vehicle needs to be controlled both **longitudinally** and **laterally**
- We consider 3 different types of controllers:
  - **Black box controllers** don't require knowledge about the process
  - **Geometric controllers** exploit geometric relationships between the vehicle and the path, resulting in compact control laws for path tracking
  - **Optimal controllers** use knowledge of the system and minimize an objective function over future time steps

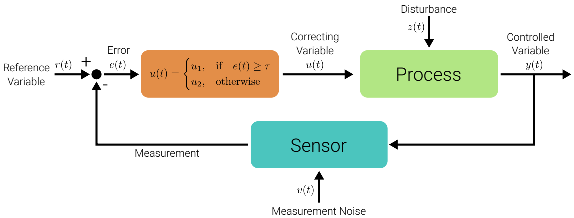
6.2
Black Box Control

Bang-Bang Control

Bang-Bang Control

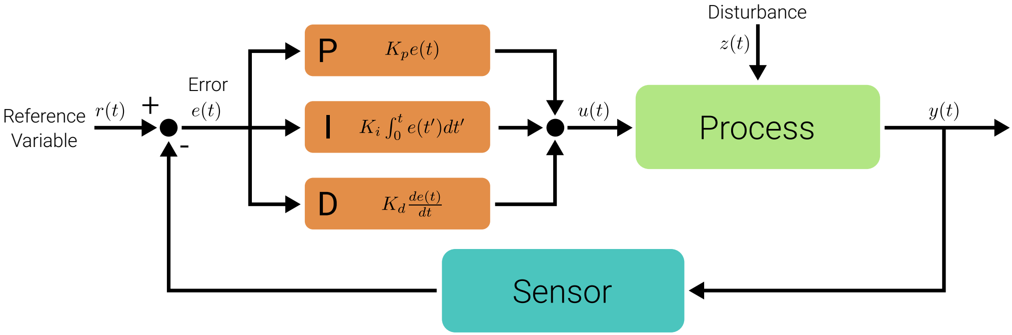
- Also called: hysteresis controller
- Often applied, e.g. in household thermostats
- Switches abruptly between two states
- Mathematical formulation:

$$u(t) = \begin{cases} u_1, & \text{if} \quad e(t) \geq \tau \\ u_2, & \text{otherwise} \end{cases}$$



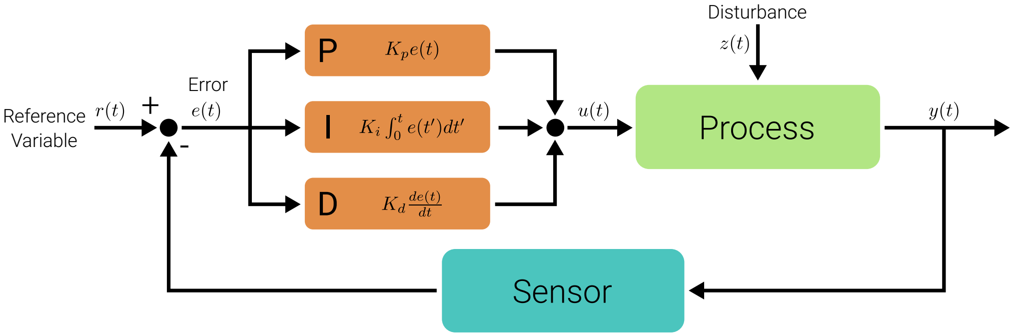
PID Control

PID Control



- Most common controller in current industrial applications
- Doesn’t require knowledge about plant/process to be controlled
- Comprises proportional (**P**), integration (**I**) and differential (**D**) element
- Reference variable $r(t)$, correcting variable $u(t)$, controlled variable $y(t)$, error $e(t)$

PID Control

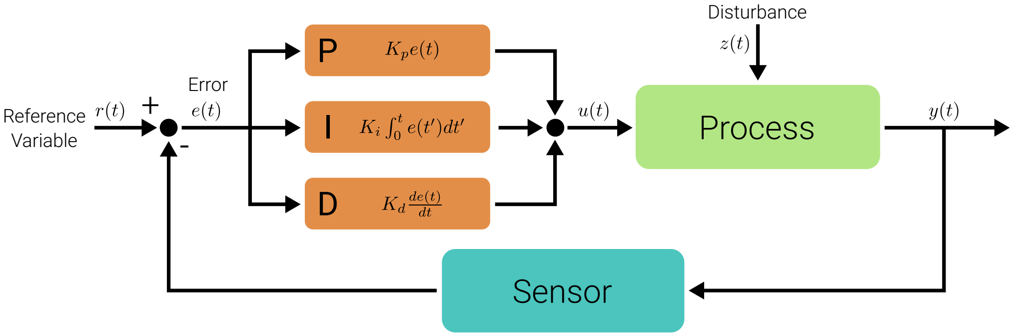


Mathematical formulation:

$$u(t) = K_p\, e(t) + K_i \int_0^t e(t')dt' + K_d\, \frac{de(t)}{dt}$$

with parameters K_p, K_i and K_d

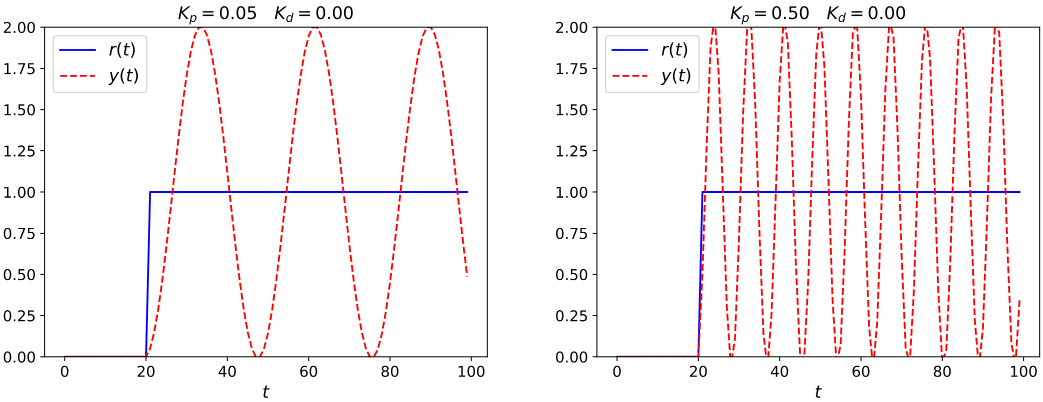
PID Control



- Using the **P** element alone leads to overshooting / oscillation
- Adding a **D** element alleviates this problem by introducing damping behavior
- The **I** element corrects residual errors by integrating past error measurements

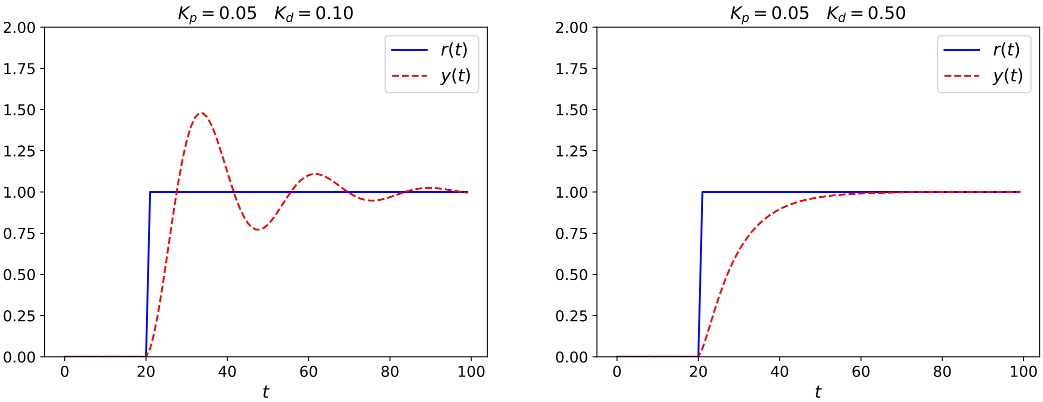
https://tttapa.github.io/Pages/Arduino/Control-Theory/Motor-Fader/PID-Controllers.html

P Control



- Controlled Variable: Position $y(t) = x(t)$
- Correcting Variable: Acceleration $u(t) = a(t) = \ddot{x}(t)$

PD Control



- Controlled Variable: Position $y(t) = x(t)$
- Correcting Variable: Acceleration $u(t) = a(t) = \ddot{x}(t)$

PID Control

As PID controllers are often used to control black box processes, various heuristics have been proposed for setting the parameters. **Ziegler-Nichols** is most common:

- Set $K_i = K_d = 0$
- Increase K_p until the ultimate gain $K_p = K_u$ where the system oscillates
- Measure the oscillation period T_u at K_u
- Set $K_p = 0.6K_u$, $K_i = 1.2K_u/T_u$ and $K_d = 3K_uT_u/40$

These heuristics have been derived empirically using computer simulations. While they provide a good starting point, manual fine-tuning is often required in practice. Note: Ziegler-Nichols does not apply to double integrators as in the previous example.

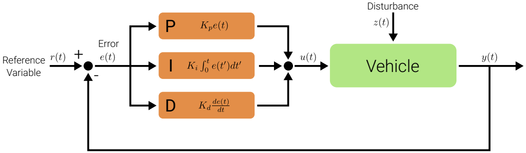
Ziegler and Nichols: Optimum settings for automatic controllers. Transactions of the ASME, 1942.

PID Control

Example: Longitudinal Vehicle Control

$$v(t) = v_{\text{max}}\left(1 - \exp\left(-\theta_1\, d(t) - \theta_2\right)\right)$$

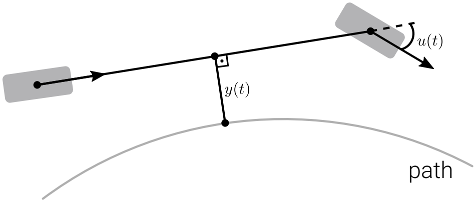
- $v(t)$: target velocity at time t
- $d(t)$: distance to preceding car

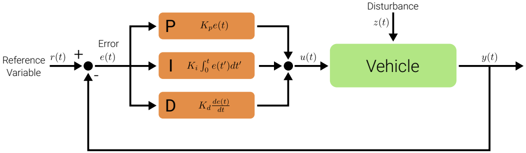


- Reference variable: $r(t) = v(t) =$ target velocity
- Correcting variable: $u(t) =$ gas/brake pedal
- Controlled variable: $y(t) =$ current velocity
- Error: $e(t) = v(t) - y(t)$

PID Control

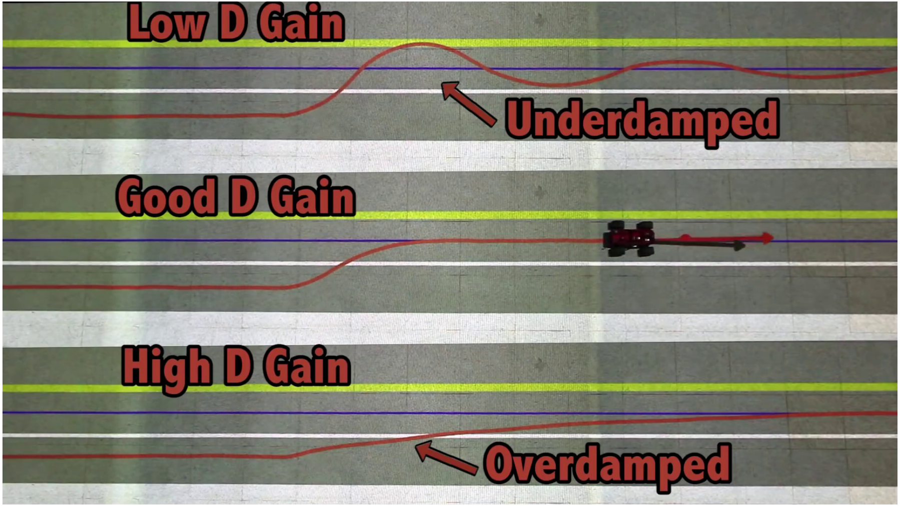
Example: Lateral Vehicle Control





- Reference variable: $r(t) = 0 =$ no cross track error
- Correcting variable: $u(t) = \delta =$ steering angle
- Controlled variable: $y(t) =$ cross track error
- Error: $e(t) = -y(t) =$ cross track error

PID Control



https://www.youtube.com/watch?v=4Y7zG48uHRo

PID Control

The **steady-state error** is defined as the difference between the input (command) and the output of a system in the limit as time goes to infinity (i.e., when the response has reached steady state). Steady-state error analysis is only useful for **stable** systems (a linear system is stable if its output will stay bounded for any bounded input).

PID Control

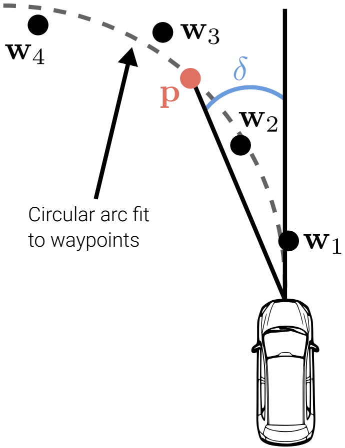
Example: Waypoint-based Vehicle Control

- **Input:** Waypoints $\mathbf{w} = \{\mathbf{w}_1, \dots, \mathbf{w}_K\}$
- **Velocity:** (Longitudinal PID control)

$$v = \frac{1}{K} \sum_{k=1}^{K} \frac{\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2}{\Delta t}$$

- **Steering angle:** (Lateral PID control)

$$\delta = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$



PID Control

Chitta, Prakash and Geiger: Neural Attention Fields for End-to-End Autonomous Driving. ICCV, 2021.

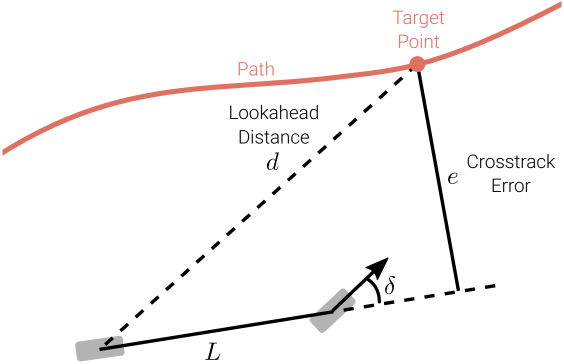
6.3
Geometric Control

Pure Pursuit Control

Pure Pursuit Control

Goal:

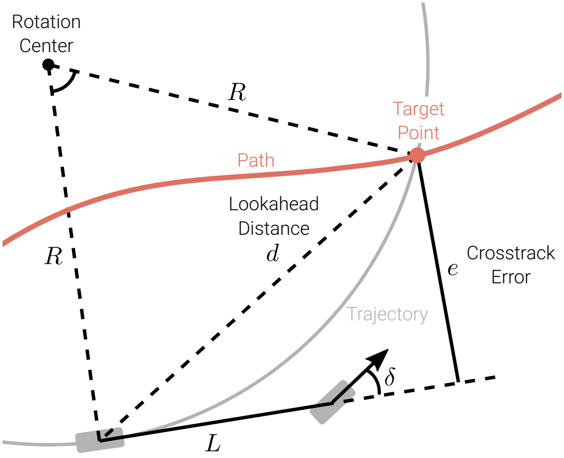
- Track **target point** at **lookahead distance** d to follow path
- Exploit geometric relationship between vehicle and path to follow



Pure Pursuit Control

Goal:

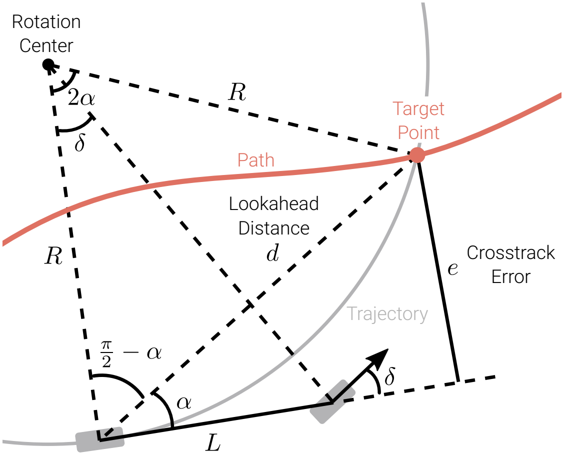
- Track **target point** at **lookahead distance** d to follow path
- Exploit geometric relationship between vehicle and path to follow
- Minimize **crosstrack error** e by following circular trajectory



Pure Pursuit Control

Goal:

- Track **target point** at **lookahead distance** d to follow path
- Exploit geometric relationship between vehicle and path to follow
- Minimize **crosstrack error** e by following circular trajectory
- Steering angle δ determined by angle α between vehicle heading direction and lookahead direction: $\delta(\alpha) = ?$



Pure Pursuit Control

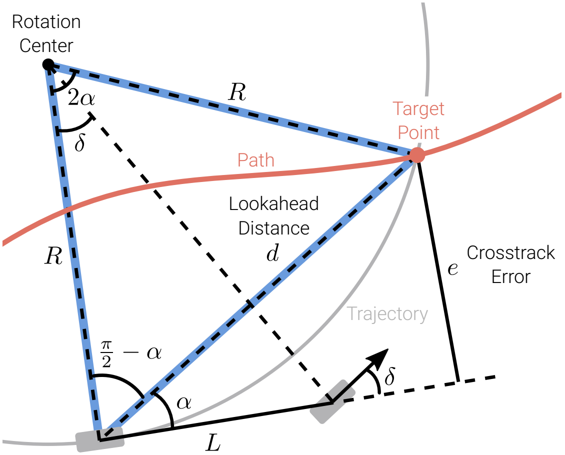
From the **law of sines:**

$$\frac{d}{\sin(2\alpha)} = \frac{R}{\sin\left(\frac{\pi}{2} - \alpha\right)}$$

$$\frac{d}{2\sin\alpha\cos\alpha} = \frac{R}{\cos\alpha}$$

$$\kappa = \frac{1}{R} = \frac{2\sin\alpha}{d}$$

with κ the curvature of trajectory.



Pure Pursuit Control

The **steering angle** is calculated as:

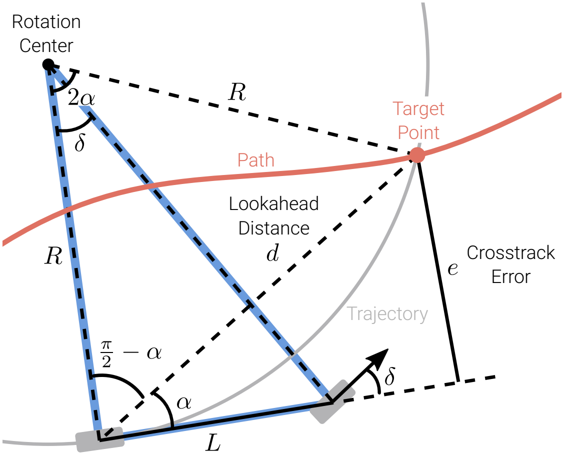
$$\tan(\delta) = \frac{L}{R} = \frac{2L\sin(\alpha)}{d}$$

$$\delta = \tan^{-1}\left(\frac{2L\sin(\alpha)}{d}\right)$$

$$\delta \approx \frac{2L\sin(\alpha)}{d}$$

- d is often based on vehicle speed v:

$$d = Kv \quad \text{with constant } K$$



Pure Pursuit Control

In terms of **cross track error** we obtain:

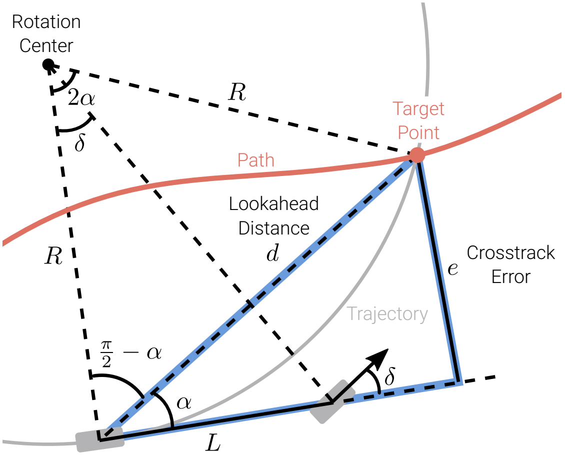
$$\sin\alpha = \frac{e}{d}$$

$$\delta = \tan^{-1}\left(\frac{2L\sin(\alpha)}{d}\right)$$

$$\delta = \tan^{-1}\left(\frac{2Le}{d^2}\right) \approx \frac{2L}{d^2}e$$

- Pure pursuit acts as a **proportional** controller wrt. the crosstrack error
- d is often based on vehicle speed v:

$$d = Kv \quad \text{with constant } K$$



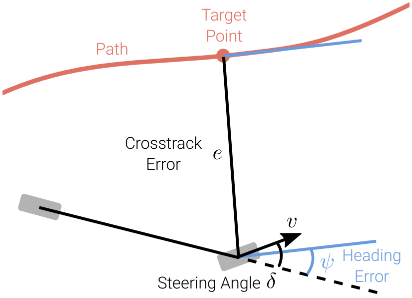
Stanley Control

Stanley Control

Control Law used by **Stanley** in **DARPA Challenge:**

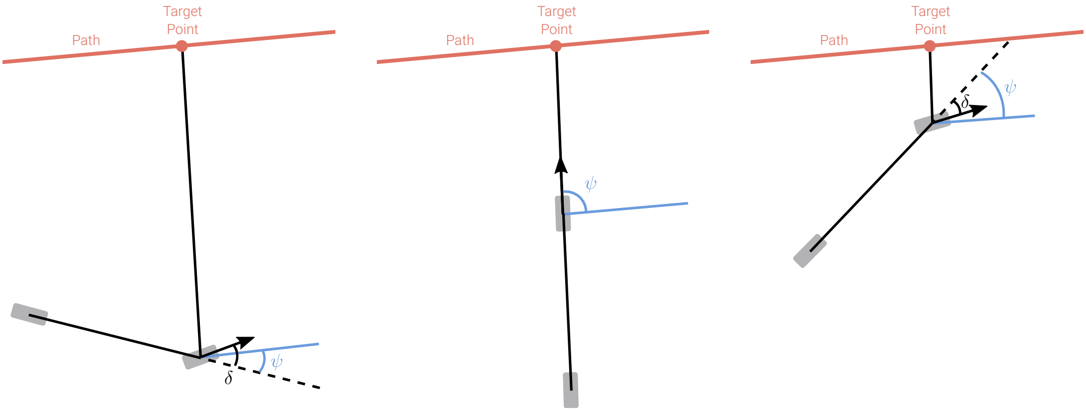
$$\delta = \psi + \tan^{-1}\left(\frac{k\,e}{v}\right)$$

- v = speed, ψ = heading err., e = crosstrack err.
- Reference at **front axle, no lookahead**
- Combines **heading** and **crosstrack error**
- It can be shown that the crosstrack error **converges** exponentially to 0 (indep. of v)
- Works for small velocities w/o disturbances



Thrun et al.: Stanley: The robot that won the DARPA Grand Challenge. Journal of Field Robotics, 2006.

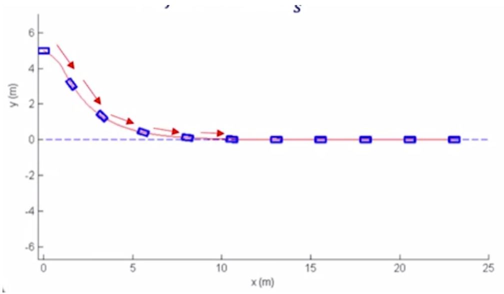
Stanley Control

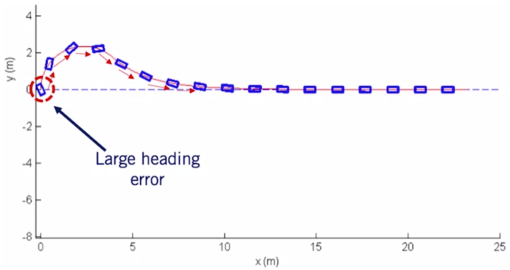


- As heading changes, heading correction **counteracts** crosstrack correction

Thrun et al.: Stanley: The robot that won the DARPA Grand Challenge. Journal of Field Robotics, 2006.

Stanley Control





- The Stanley controller can correct **large crosstrack** and **large heading errors**
- **Global stability:** Independent of initial conditions guides vehicle back (proven)
- But does not consider noisy observations, actuator dynamics, tire force effects
- Softening/dampening terms and curvature information can be added

Stanley Control

https://www.youtube.com/watch?v=M2AcMnfzpNg

6.4
Optimal Control

Optimal Control

Recap: Dynamic Bicycle Model

$$\begin{bmatrix} \dot{v}_y \\ \dot{\psi} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{c_r + c_f}{m v_x} & 0 & \frac{c_r l_r - c_f l_f}{m v_x} - v_x \\ 0 & 0 & 1 \\ \frac{l_r c_r - l_f c_f}{I_z v_x} & 0 & -\frac{l_f^2 c_f + l_r^2 c_r}{I_z v_x} \end{bmatrix} \underbrace{\begin{bmatrix} v_y \\ \psi \\ \omega \end{bmatrix}}_{\text{State } \mathbf{x}} + \begin{bmatrix} \frac{c_f}{m} \\ 0 \\ \frac{c_f}{I_z} l_f \end{bmatrix} \underbrace{\delta}_{\text{Input}}$$

With state $\mathbf{x}$ and front steering angle δ.

We can rewrite this equation as the following linear system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\delta \qquad \text{with} \qquad \mathbf{x} = (v_y, \psi, \omega)^T$$

Optimal Control

Linear Quadratic Regulator (LQR): For the continuous-time linear system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\delta \qquad \text{with} \qquad \mathbf{x}(0) = \mathbf{x}_{\text{init}}$$

and quadratic cost functional defined as ($\mathbf{Q}$ is a diagonal weight matrix)

$$J = \frac{1}{2}\int_0^\infty \Delta\mathbf{x}^T(t)\mathbf{Q}\Delta\mathbf{x}(t) + q\delta(t)^2\,dt$$

with $\Delta\mathbf{x}(t) = \mathbf{x}(t) - \mathbf{x}_{\text{target}}$. The feedback control $\delta(t)$ that minimizes J is given by

$$\delta(t) = -\mathbf{k}^T(t)\Delta\mathbf{x}(t)$$

with $\mathbf{k}(t) = \frac{1}{q}\mathbf{b}^T\mathbf{P}(t)$ and $\mathbf{P}(t)$ the solution to a Ricatti equation (no details here).

Model Predictive Control

Model Predictive Control

Generalizes LQR to:

- **Non-linear** cost function and dynamics (consider straight road leading into turn)
- **Flexible:** allows for receding window & incorporation of constraints
- **Expensive:** non-linear optimization required at every iteration

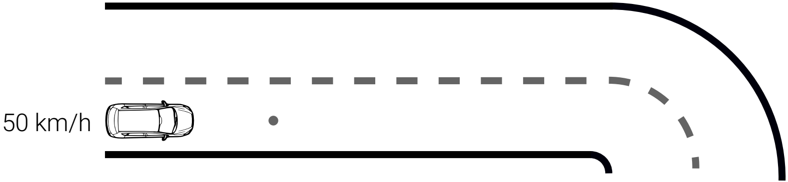
Formally:

$$\begin{aligned}
\underset{\delta_1,\ldots,\delta_T}{\operatorname{argmin}} \quad & \sum_{t=1}^{T} C_t(\mathbf{x}_t, \delta_t) && \text{(sum of costs)} \\
s.t. \quad & \mathbf{x}_1 = \mathbf{x}_{\text{init}} && \text{(initialization)} \\
& \mathbf{x}_{t+1} = f(\mathbf{x}_t, \delta_t) && \text{(dynamics model)} \\
& \underline{\delta} \leq \delta_t \leq \overline{\delta} && \text{(constraints)}
\end{aligned}$$

- Unroll dynamic model T times $\Rightarrow$ apply non-linear optimization to find $\delta_1, \ldots \delta_T$

Model Predictive Control

PID Control / Path Tracking



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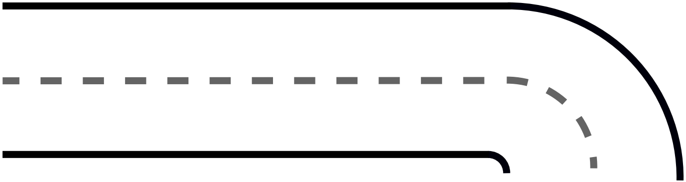
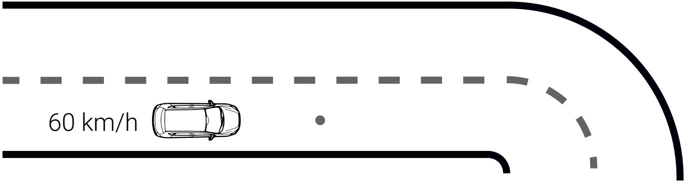
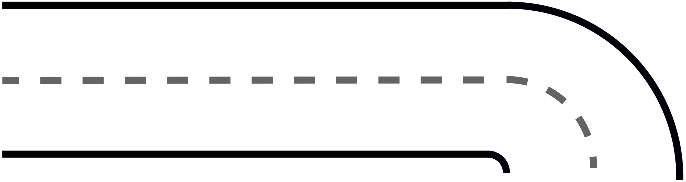


Model Predictive Control

PID Control / Path Tracking

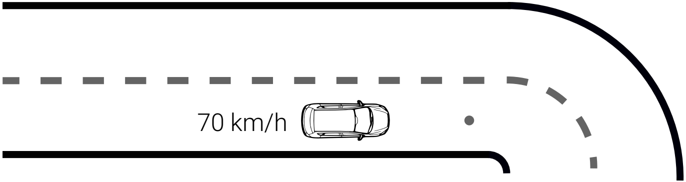


Model Predictive Control



Model Predictive Control

PID Control / Path Tracking



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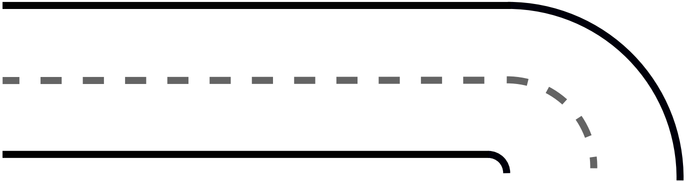
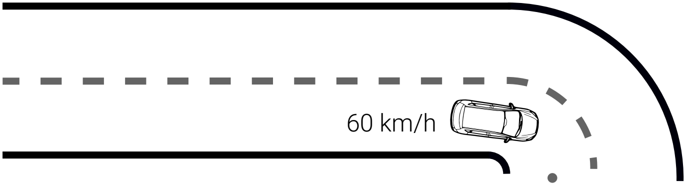


Model Predictive Control

PID Control / Path Tracking



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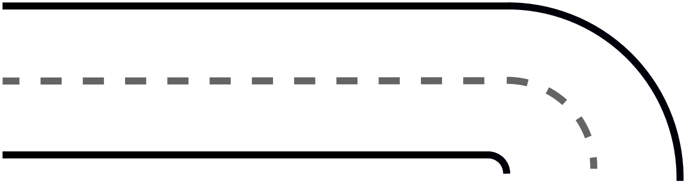

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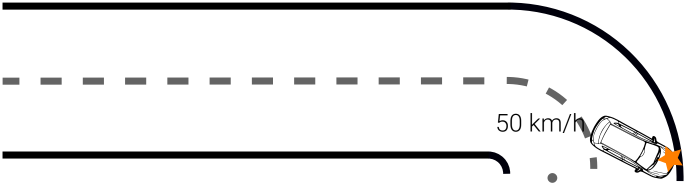
PID Control / Path Tracking



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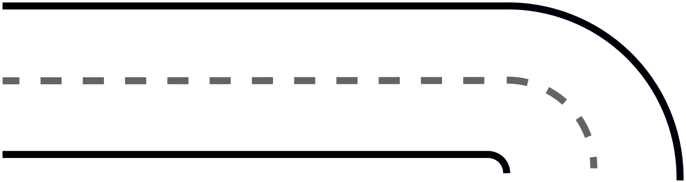

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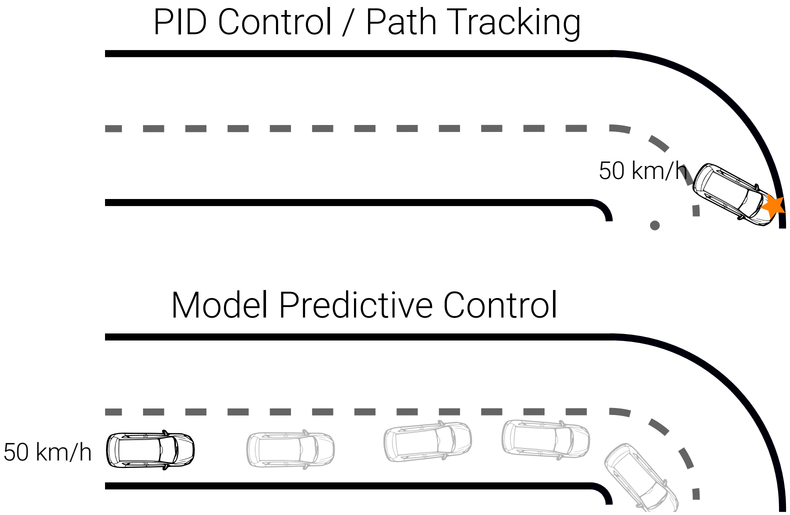
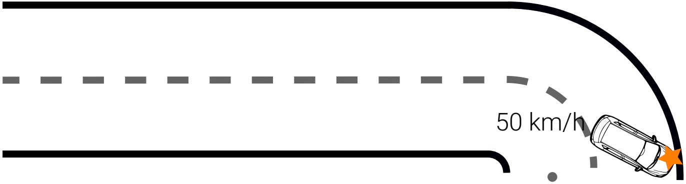


Model Predictive Control

PID Control / Path Tracking



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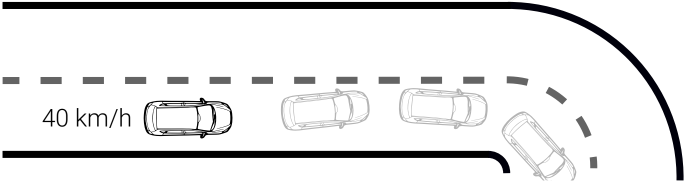

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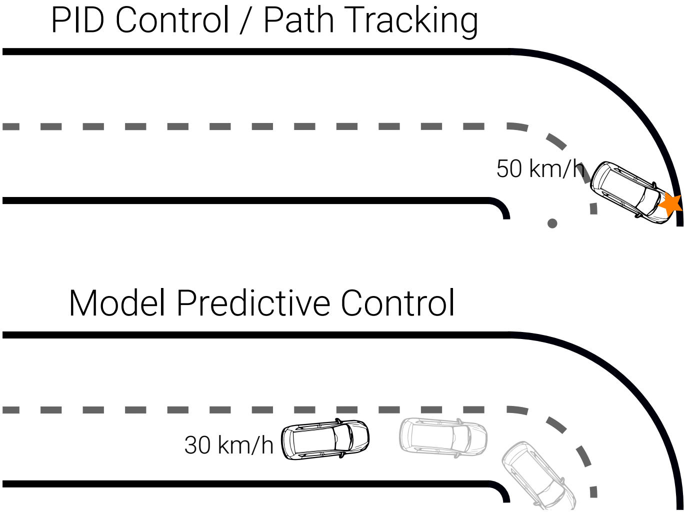

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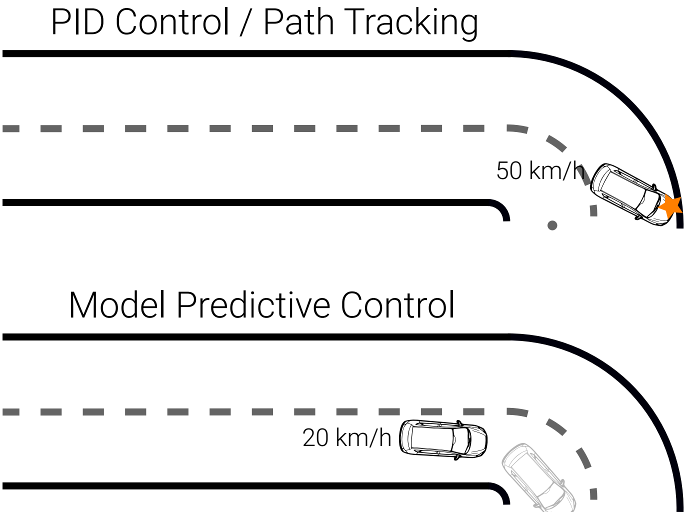
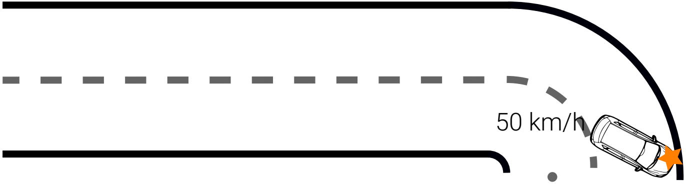
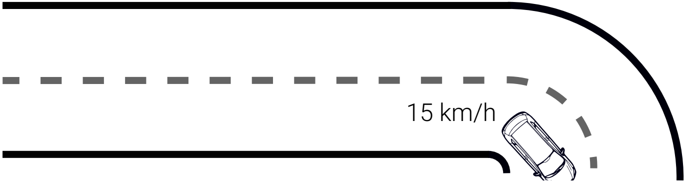


Model Predictive Control

PID Control / Path Tracking



Model Predictive Control



Summary

- Open-loop controllers cannot handle unknown disturbances
- In practice, we thus require closed-loop control with sensor feedback
- Black box controllers don't require knowledge about the process
- Most popular black box controller: PID controller
- Geometric controllers exploit geometric relationships for path tracking
- Optimal controllers use a vehicle model and optimize a cost function
- MPC is the most flexible and powerful approach
- However, MPC requires solving an optimization problem at every time step