

1951. $\therefore E9(x) = |\sin(\theta * x + 10 / 2 * \pi) * x ^ 10 / 10!|$
 $= |\sin(\theta * x) * x ^ 10 / 10!|$
 $\leq |\theta * x ^ 11 / 10!|$
 当 $\theta * x = 0$ 取“=”
 $\theta \in [0, 1]$
 $\therefore x \in [-1, 1]$
 $\therefore E9(x) < 1 / 10!$
 $\therefore 2.75574e-7 * 10! = 1.0000029312 > 1$
 $\therefore E9(x) < 1 / 10! \leq 2.75574e-7$

1952. $\therefore E8(x) = |\cos(\theta * x + 10 / 2 * \pi) * x ^ 9 / 9!|$
 $= |\cos(\theta * x) * x ^ 9 / 9!|$
 $\leq |x ^ 9 / 9!|$
 当 $\theta * x = 0$ 取“=”
 $\theta \in [0, 1]$
 $\therefore x \in [-1, 1]$
 $\therefore E9(x) < 1 / 9!$
 $\therefore 2.75574e-6 * 9! = 1.0000029312 > 1$
 $\therefore E9(x) < 1 / 9! \leq 2.75574e-6$

2052.

`%Horner.m`

```
function [P] = Horner(a, c)
    b = zeros(size(a));
    b(size(a)) = a(size(a));

    for i = size(a, 1) - 1:-1:1
        b(i) = b(i + 1) * c + a(i);
    end

    P = b(1);
end
```

`%derivative.m`

```
function [d] = derivative(a, c)
    ad = a(1:size(a, 1) - 1) .* (size(a, 1) - 1:-1:1);
    d = Horner(ad, c);
end
```

`%definite_integral.m`

```

function [i] = definite_integral(a, c1, c2)
    ai = [a ./ (size(a, 1):-1:1)'; 1];
    i = Horner(ai, c2) - Horner(ai, c1);
end

%eliminate.m
function [UY] = eliminate(AB)
    for i = 2:size(AB, 1)
        for j = 1:(i - 1)
            AB(i, :) = AB(i, :) - AB(i, j) / AB(j, j) *
AB(j, :);
        end
    end
    UY = AB;
end

%solve.m
function [solution] = solve(UV)
    solution = zeros(size(UV, 1), 1);
    for i = size(UV, 1):-1:1
        UV(i, :) = UV(i, :) / UV(i, i);
        for j = 1:(i - 1)
            UV(j, :) = UV(j, :) - UV(j, i) / UV(i, i) *
UV(i, :);
        end
        solution(i) = UV(i, size(UV, 2));
    end
end

%p205t2.m
a = [-0.04; 0.14; -0.16; 2.08];
P4 = Horner(a, 4);
Pd4 = derivative(a, 4);
Pt4 = definite_integral(a, 0, 3);
P45 = Horner(a, 4.5);
x = [0; 1; 2; 4];
y = [2.08; 2.02; 2.00; 1.12];
AB = [x.^0, x.^1, x.^2, x.^3, y];
coe = solve(eliminate(AB));

(a) P(3) = 55.100000000000010
(b) P'(4) = -3.180000000000000
(c) P(4.5) = 1.868900000000000e+02
(d) coef = 2.080000000000000

```

-0.16000000000000000
0.14000000000000000
-0.04000000000000000

2171.

(a)

$$\therefore f(-1) = -1$$

$$f(0) = 0$$

$$\begin{aligned}\therefore P1(x) &= f(-1) * (x - 0) / (-1 - 0) + \\ &\quad f(0) * (x - (-1)) / (0 - (-1)) \\ &= x\end{aligned}$$

(b)

$$\therefore f(-1) = -1$$

$$f(0) = 0$$

$$f(1) = 1$$

$$\begin{aligned}\therefore P2(x) &= f(-1) * (x-0) * (x-1) / ((-1-0) * (-1-1)) + \\ &\quad f(0) * (x-(-1)) * (x-1) / ((0-(-1)) * (0-1)) + \\ &\quad f(1) * (x-(-1)) * (x-0) / ((1-(-1)) * (1-0)) \\ &= x * (x-1) / (-2) + (x+1) * x / 2 \\ &= x\end{aligned}$$

(c)

$$\therefore f(-1) = -1$$

$$f(0) = 0$$

$$f(1) = 1$$

$$f(2) = 8$$

$$\begin{aligned}\therefore P3(x) &= f(-1) * (x-0) * (x-1) * (x-2) / \\ &\quad ((-1-0) * (-1-1) * (-1-2)) + \\ &\quad f(0) * (x-(-1)) * (x-1) * (x-2) / \\ &\quad ((0-(-1)) * (0-1) * (0-2)) + \\ &\quad f(1) * (x-(-1)) * (x-0) * (x-2) / \\ &\quad ((1-(-1)) * (1-0) * (1-2)) + \\ &\quad f(2) * (x-(-1)) * (x-0) * (x-1) / \\ &\quad ((2-(-1)) * (2-0) * (2-1)) \\ &= x * (x-1) * (x-2) / 6 + \\ &\quad (x+1) * x * (x-2) / (-2) + \\ &\quad 8 * (x+1) * x * (x-1) / 6 \\ &= (x^2 - 2 * x) * ((x-1) / 6 - (x+1) / 2) + 4 / 3 * (x^3 - x) \\ &= (x^2 - 2 * x) * (-x / 3 - 2 / 3) + 4 / 3 * (x^3 - x) \\ &= 1 / 3 * ((2 * x - x^2) * (x+2) + 4 * x^3 - 4 * x) \\ &= x^3\end{aligned}$$

(d)

$$\therefore f(1) = 1$$

$$\begin{aligned}
 f(2) &= 8 \\
 \therefore P_4(x) &= f(1) * (x - 2) / (1 - 2) + \\
 &\quad f(2) * (x - 1) / (2 - 1) \\
 &= -x + 2 + 8 * x - 8 \\
 &= 7 * x - 6
 \end{aligned}$$

(e)

$$\begin{aligned}
 \therefore f(0) &= 0 \\
 f(1) &= 1 \\
 f(2) &= 8 \\
 \therefore P_5(x) &= f(0) * (x-1) * (x-2) / ((0-1) * (0-2)) + \\
 &\quad f(1) * (x-0) * (x-2) / ((1-0) * (1-2)) + \\
 &\quad f(2) * (x-0) * (x-1) / ((2-0) * (2-1)) \\
 &= x * (x-2) / (-1) + 4 * x * (x-1) \\
 &= x * (2-x+4*x-4) \\
 &= 3 * x^2 - 2 * x
 \end{aligned}$$

2172.

(a)

$$\begin{aligned}
 \therefore f(1) &= 3 \\
 f(2) &= 3 \\
 f(2.5) &= 3.3 \\
 \therefore P(x) &= f(1) * (x-2) * (x-2.5) / ((1-2) * (1-2.5)) + \\
 &\quad f(2) * (x-1) * (x-2.5) / ((2-1) * (2-2.5)) + \\
 &\quad f(2.5) * (x-1) * (x-2) / ((2.5-1) * (2.5-2)) \\
 &= 2 * (x-2) * (x-2.5) \\
 &\quad - 6 * (x-1) * (x-2.5) \\
 &\quad + 4.4 * (x-1) * (x-2) \\
 \therefore P(1.5) &= 2 * (-0.5) * (-1) \\
 &\quad - 6 * 0.5 * (-1) \\
 &\quad + 4.4 * 0.5 * (-0.5) \\
 &= 1 + 3 - 1.1 \\
 &= 2.9 \\
 P(1.2) &= 2 * (-0.8) * (-1.3) \\
 &\quad - 6 * 0.2 * (-1.3) \\
 &\quad + 4.4 * 0.2 * (-0.8) \\
 &= 2.936
 \end{aligned}$$

(b)

$$\begin{aligned}
 \therefore f(0.5) &= 4.5 \\
 f(1) &= 3 \\
 f(2) &= 3 \\
 f(2.5) &= 3.3 \\
 \therefore P(x) &= f(0.5) * (x-1) * (x-2) * (x-2.5) /
 \end{aligned}$$

$$\begin{aligned}
& ((0.5-1)*(0.5-2)*(0.5-2.5)) + \\
& f(1)*(x-0.5)*(x-2)*(x-2.5) / \\
& ((1-0.5)*(1-2)*(1-2.5)) + \\
& f(2)*(x-0.5)*(x-1)*(x-2.5) / \\
& ((2-0.5)*(2-1)*(2-2.5)) + \\
& f(2.5)*(x-0.5)*(x-1)*(x-2) / \\
& ((2.5-0.5)*(2.5-1)*(2.5-2)) \\
= & -3*(x-1)*(x-2)*(x-2.5) \\
& +4*(x-0.5)*(x-2)*(x-2.5) \\
& -4*(x-0.5)*(x-1)*(x-2.5) \\
& +2.2*(x-0.5)*(x-1)*(x-2) \\
\therefore P(1.5) = & -3*0.5*(-0.5)*(-1) \\
& +4*(-0.5)*(-1) \\
& -4*0.5*(-1) \\
& +2.2*0.5*(-0.5) \\
= & 2.7 \\
P(1.2) = & -3*0.2*(-0.8)*(-1.3) \\
& +4*0.7*(-0.8)*(-1.3) \\
& -4*0.7*0.2*(-1.3) \\
& +2.2*0.7*0.2*(-0.8) \\
= & 2.7696
\end{aligned}$$

2297.

`%divided_difference_table.m`

```
function [t] = divided_difference_table(f)
    t = [f, zeros(size(f, 1), size(f, 1) - 1)];
    for i = 2:size(f, 1)
        t(i:size(f, 1), i) = (t(i:size(f, 1), i - 1) -
t(i - 1:size(f, 1) - 1, i - 1)) / (i - 1);
    end
end
```

`%p229t7`

```
xk = [0; 1; 2; 3; 4];
f = [0; 0.75; 2.25; 3; 2.25];
t = divided_difference_table(f);
real_t = [xk, t];

x = 1.5;
P1 = zeros(size(f, 1) - 1, 1);
tt = t(1:size(t, 1) + 1:size(t, 1) * size(t, 1));
tt(1, 2:size(tt, 2)) = tt(1, 2:size(tt, 2)) * (x -
xk(1, 1));
```

```

P1(1) = t(1, 1) + tt(1, 2);
for i = 2:size(f, 1) - 1
    tt(1, i + 1:size(tt, 2)) = tt(1, i + 1:size(tt, 2))
    * (x - xk(i, 1));
    P1(i) = P1(i - 1) + tt(1, i + 1);
end
y1 = 3 * (sin(pi * x / 6)) ^ 2;
d1 = abs(P1 - y1);

x = 3.5;
P2 = zeros(size(f, 1) - 1, 1);
tt = t(1:size(t, 1) + 1:size(t, 1) * size(t, 1));
tt(1, 2:size(tt, 2)) = tt(1, 2:size(tt, 2)) * (x -
xk(1, 1));
P2(1) = t(1, 1) + tt(1, 2);
for i = 2:size(f, 1) - 1
    tt(1, i + 1:size(tt, 2)) = tt(1, i + 1:size(tt, 2))
    * (x - xk(i, 1));
    P2(i) = P2(i - 1) + tt(1, i + 1);
end
y2 = 3 * (sin(pi * x / 6)) ^ 2;
d2 = abs(P2 - y2);

```

(a)

xk	f[xk]	1 st	2 nd	3 rd	4 th
0	0	0	0	0	0
1	0.7500	0.7500	0	0	0
2	2.2500	1.5000	0.3750	0	0
3	3.0000	0.7500	-0.3750	-0.2500	0
4	2.2500	-0.7500	-0.7500	-0.1250	0.03125

(b)

```

P1(x) = 0.75*x
P2(x) = 0.75*x+0.375*x*(x-1)
P3(x) = 0.75*x+0.375*x*(x-1)-0.25*x*(x-1)*(x-2)
P4(x) = 0.75*x+0.375*x*(x-1)-0.25*x*(x-1)*(x-2)
      +0.03125*x*(x-1)*(x-2)*(x-3)

```

(c)

x = 1.5:

```

1.1250000000000000
1.4062500000000000
1.5000000000000000
1.5175781250000000

```

$x = 3.5:$

2.625000000000000
5.906250000000000
2.625000000000000
2.830078125000000

(d)
 $f(1.5) = 1.500000000000000$
 $f(3.5) = 2.79903810567666$

设 $d = \text{abs}(f(x) - P(x))$

d:
0.375000000000000
0.0937499999999996
4.44089209850063e-16
0.0175781250000004

d:
0.174038105676658
3.10721189432334
0.174038105676658
0.0310400193233420

除少数上升，总体来说， d 是在下降的