In Exercises 1 and 2, find the least-squares line $y = f(x) = Ax + B$ for the data and calculate $E_2(f)$

1. (a)

x_k	y_k	$f(x_k)$
−2	1	1.2
−1	2	1.9
0	3	2.6
1	3	3.3
2	4	4.0

(b)

x_k	y_k	$f(x_k)$
−6	7	7.0
−2	5	4.6
0	3	3.4
2	2	2.2
6	0	−0.2

分别计算$\sum_{k=1}^{N} x_k^2$, $\sum_{k=1}^{N} x_k$, $\sum_{k=1}^{N} x_k y_k$, N, $\sum_{k=1}^{N} y_k$:

```
%a1.m
X = [-2; -1; 0; 1; 2];
Y = [1; 2; 3; 3; 4];
F = [1.2; 1.9; 2.6; 3.3; 4.0];
sum_X_2 = sum(X.^2)
sum_X = sum(X)
sum_XY = sum(X .* Y)
N = size(X, 1)
sum_Y = sum(Y)
```

得:

```
>> a1

sum_X_2 =

    10


sum_X =

     0


sum_XY =

     7


N =

     5


sum_Y =

    13
```

求解方程组:

$$\begin{cases} (\sum_{k=1}^{N} x_k^2)A + (\sum_{k=1}^{N} x_k)B = \sum_{k=1}^{N} x_k y_k \\ (\sum_{k=1}^{N} x_k)A + NB = \sum_{k=1}^{N} y_k \end{cases}$$

代入

$$\begin{cases} \sum_{k=1}^{N} x_k^2 = sum\_X\_2 \\ \sum_{k=1}^{N} x_k = sum\_X \\ \sum_{k=1}^{N} x_k y_k = sum\_XY \\ N = N \\ \sum_{k=1}^{N} y_k = sum\_Y \end{cases}$$

得:

$$\begin{cases} 10A = 7 \\ 5B = 13 \end{cases}$$

得:

$$\begin{cases} A = 0.7 \\ B = 2.6 \end{cases}$$

求$E_2(f) = (\frac{1}{N}\sum_{k=1}^{N}|f(x_k) - y_k|^2)^{1/2}$:

```
E2 = sqrt(1 / N * sum((F - Y).^2))
```

得:

```
E2 =

    0.2449
```

所以，$E_2(f) = 0.2449$

8. Find the power fits $y = Ax^2$ and $y = Bx^3$ for the following data and use $E_2(f)$ to determine which curve fits best.

(a)

x_k	y_k
2.0	5.1
2.3	7.5
2.6	10.6
2.9	14.4
3.2	19.0

(b)

x_k	y_k
2.0	5.9
2.3	8.3
2.6	10.7
2.9	13.7
3.2	17.0

分别计算$A = (\sum_{k=1}^{N} x_k^2 y_k)/(\sum_{k=1}^{N} x_k^{2\cdot 2})$，$B = (\sum_{k=1}^{N} x_k^3 y_k)/(\sum_{k=1}^{N} x_k^{2\cdot 3})$

```
%a8.m
X = [2.0; 2.3; 2.6; 2.9; 3.2];
Y = [5.1; 7.5; 10.6; 14.4; 19.0];
A = sum(X.^2 .* Y) / sum(X.^(2 * 2))
B = sum(X.^3 .* Y) / sum(X.^(2 * 3))
```

```
>> a8

A =

    1.6866


B =

    0.5902
```

所以，$A = 1.6866$，$B = 0.5902$

计算E2:

```
E2A = sqrt(1 / N * sum((A .* X.^2 - Y).^2))
E2B = sqrt(1 / N * sum((B .* X.^3 - Y).^2))
```

```
E2A =

    1.2971


E2B =

    0.2870
```

所以$M=2$时，$E2=1.2971$，$M=3$时，$E2=0.2870$

2. Find the least-squares parabola $f(x)=Ax^2+Bx+C$ for each set of data.

(a)

x_k	y_k
-2	-5.8
-1	1.1
0	3.8
1	3.3
2	-1.5

(b)

x_k	y_k
-2	2.8
-1	2.1
0	3.25
1	6.0
2	11.5

(c)

x_k	y_k
-2	10
-1	1
0	0
1	2
2	9

分别计算$\sum_{k=1}^{N}x_k^4$，$\sum_{k=1}^{N}x_k^3$，$\sum_{k=1}^{N}x_k^2$，$\sum_{k=1}^{N}y_kx_k^2$，$\sum_{k=1}^{N}x_k$，$\sum_{k=1}^{N}y_kx_k$，N，$\sum_{k=1}^{N}y_k$。

```
%a2.m
X = [-2; -1; 0; 1; 2];
Y = [-5.8; 1.1; 3.8; 3.3; -1.5];
sum_X_4 = sum(X.^4)
sum_X_3 = sum(X.^3)
sum_X_2 = sum(X.^2)
sum_Y_X_2 = sum(Y .* X.^2)
sum_X = sum(X)
sum_Y_X = sum(Y .* X)
N = size(X, 1)
sum_Y = sum(Y)
```

```
>> a2

sum_X_4 =

    34


sum_X_3 =

     0


sum_X_2 =

    10


sum_Y_X_2 =

  -24.8000


sum_X =

     0


sum_Y_X =

   10.8000


N =

     5


sum_Y =

    0.9000
```

高斯消元法解方程组：

```
%solve.m
function [solution] = solve(UV)
    solution = zeros(size(UV, 1), 1);
    for  i = 1:size(UV, 1) - 1
        for j = i + 1:size(UV, 1)
            UV(j, :) = UV(j, :) - UV(j, i) / UV(i, i) * UV(i, :);
        end
    end
    for  i = size(UV, 1):-1:1
        UV(i, :) = UV(i, :) / UV(i, i);
        solution(i) = UV(i, size(UV, 2));
        for j = 1:i - 1
            UV(j, :) = UV(j, :) - UV(j, i) / UV(i, i) * UV(i, :);
        end
    end
end
```

```
solve([sum_X_4, sum_X_3, sum_X_2, sum_Y_X_2;
    sum_X_3, sum_X_2, sum_X, sum_Y_X;
    sum_X_2, sum_X, N, sum_Y])
```

```
ans =

   -1.9000
    1.0800
    3.9800
```

所以$A=-1.9$，$B=1.08$，$C=3.98$

3. For the given set of data, find the least-squares curve:

(a) $f(x)=Ce^{Ax}$, by using the change of variables $X=x$, $Y=\ln(y)$, and $C=e^{B}$, from Table 5.6, to linearize the data points.

(b) $f(x)=Cx^{A}$, by using the change of variables $X=\ln(x)$, $Y=\ln(y)$, and $C=e^{B}$, from Table 5.6, to linearize the data points.

(c) Use $E_2(f)$ to determine which curve gives the best fit.

x_k	y_k
1	0.6
2	1.9
3	4.3
4	7.6
5	12.6

因为$y=Ce^{Ax}$带入$X=x$，$Y=ln(y)$，$C=e^{B}$得，$Y=AX+B$

分别计算$\sum_{k=1}^{N}X_k^2$，$\sum_{k=1}^{N}X_k$，$\sum_{k=1}^{N}X_kY_k$，N，$\sum_{k=1}^{N}Y_k$，使用高斯消元法求解方程组：

```
x = [1; 2; 3; 4; 5];
y = [0.6; 1.9; 4.3; 7.6; 12.6];
X = x;
Y = log(y);
sum_X_2 = sum(X.^2);
sum_X = sum(X);
sum_XY = sum(X .* Y);
N = size(X, 1);
sum_Y = sum(Y);
[A; B] = solve([sum_X_2, sum_X, sum_XY;
            sum_X, N, sum_Y])
```

```
>> a3

A =

    0.7475


B =

   -1.0123
```

所以$A=0.7475$，$B=1.2303$ 计算C:

```
C =

    0.3634
```

所以$f(x)=0.3634e^{0.7475x}$