School of Data and Computer Science, Sun Yat-sen University

Summary of Generative Adversarial Self-Imitation Learning

18308045 Zhengyang Gu

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1 Theories

1.1 Policy Gradient

1.1.1 On-policy

The purpose of policy gradient is to maximize the expected reward as follows:

$$\overline{R}_\theta = \sum_\tau R(\tau)p_\theta(\tau) = E_{\tau\sim p_\theta}[R(\tau)]$$

where $R(\tau)$ is the final reward of trajectory τ and p_θ is the probability of trajectory τ.Therefore, we need to calculate its gradient to do gradient ascent. The gradient of it can be calculated by:

$$\begin{aligned}\nabla\overline{R}_\theta =& \sum_\tau R(\tau)\nabla p_\theta(\tau)\\ =& \sum_\tau R(\tau)p_\theta(\tau)\frac{\nabla p_\theta(\tau)}{p_\theta(\tau)}\\ =& \sum_\tau R(\tau)p_\theta(\tau)\nabla\log(p_\theta(\tau))\\ =& \mathbb{E}_{\tau\sim p_\theta}R(\tau)\nabla\log(p_\theta(\tau))\end{aligned}$$

We can estimate the expectation using sampling, so we have:

$$\begin{aligned}\nabla\overline{R}_\theta \approx& \frac{1}{N}\sum_{n=1}^{N} R(\tau^n)\nabla\log(p_\theta(\tau^n))\\ =& \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} R(\tau^n)\nabla\log(p_\theta(a_t^n|s_t^n))\end{aligned}$$

where N is the total number of samples, and τ^n is the nth sampled trajectory. Since the $R(\tau^n)$ is always positive, a biased sampling may cause unexpected ascent of some bad trajectories whose rewards are lower than average reward. Therefore, we can add a baseline to it to penalize those bad trajectories, as follows:

$$\nabla\overline{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n}(R(\tau^n)-b)\nabla\log(p_\theta(a_t^n|s_t^n))$$

where b can be easily assigned $\frac{1}{N}\sum_{n=1}^{N} R(\tau^n)$. Additionally, we can separately evaluate each action given a state in a trajectory, so we have:

$$\nabla\overline{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} A^\theta(s_t,a_t)\nabla\log(p_\theta(a_t^n|s_t^n))$$

where $A^\theta(s_t,a_t)$ is the advantage function to estimate the $\sum_{t'=t}^{T_n}\gamma^{t'-t}r_{t'}^n - b$.

1.1.2 Off-policy

Using on-policy policy gradient, we have to sample trajectories every time we update the policy to calculate the expectation, which is not data efficient. However, we can exploit off-policy policy gradient to improve the data efficiency as follows:

$$\begin{aligned}\mathbb{E}_{x\sim p}[f(x)] =& \sum_x p(x)f(x)\\ =& \sum_x q(x)\frac{p(x)}{q(x)}f(x)\\ =& \mathbb{E}_{x\sim q}[\frac{p(x)}{q(x)}f(x)]\end{aligned}$$

The expectation of off-policy policy gradient, thouth, is the same as on-policy gradient, the variance is very different. To solve the problem, there are several algorithm such as PPO, TRPO and PPO2 etc.

1.2 GAN

The objective of GAN is to acquire an implicit generative model which is a generative probability distribution that is similar to the actual probability distribution. The idea behind it is exploiting a confrontation between a discriminator and a generator, where the discriminator's job is to identify whether a set of samples are from actual or generative probability distribution, while the generator's job is to generate a set of vivid samples to fool the discriminator. GAN's loss function is actually a binary cross entropy loss for the discriminator, which has the form as follows:

$$\begin{aligned}L =& \frac{1}{2N}\sum_{i=1}^{2N} -(y_i\log(D(x_i)) + (1-y_i)\log(1-D(x_i)))\\ =& \frac{1}{2N}\sum_{i=1}^{2N}\begin{cases}-\log(D(x_i)), & y_i = 1\\ -\log(1-D(G(z_i))), & y_i = 0\end{cases}\end{aligned}$$

where $2N$ is the number of samples, (x_i, y_1) is the $(feature, classification)$ pair, D is the the possibility predicted by discriminator that the x_i is sampled from actual probability distribution, and G is the generator network which is a part of implicit generative model. Assuming that half of the samples are actual while the other half of the samples are generated, we have:

$$L = \frac{1}{2N}\sum_{j=1}^{N} -\log(D(x_j)) + \frac{1}{2N}\sum_{k=1}^{N} -\log(1-D(G(z_k)))$$

Then we define that x p is the actual possibility distribution while z q is the generative possibility distribution. Therefore, about $Np(x)$ x_js are equal to x and about $Nq(z)$ z_ks are equal to z, so the

L can be written as the following form:

$$
\begin{aligned}
L \approx & \frac{1}{2N}\sum_{x} Np(x)(-\log(D(x))) + \frac{1}{2N}\sum_{z} Nq(z)(-\log(1-D(G(z)))) \\
= & \frac{1}{2}\mathbb{E}_{x\sim p}[-\log(D(x))] + \frac{1}{2}\mathbb{E}_{z\sim q}[-\log(1-D(G(z)))]
\end{aligned}
$$

The higher the binary cross entropy is, the better the classification between actual samples and generated samples are. Therefore, the discriminator wants to maxizmize the loss, while the generator wants to minimize the loss.

1.3 GASIL

GASIL is actually a kind of GAN. The only difference between it and other GANs is that the actual samples here are historical good trajectories and the generative probability distribution here is trained policy. And it also exploit entropy regularization to prevent the loss function dropping into locally optimal point. Therefore, the loss function of it has the following form:

$$
L_{GASIL}(\theta, \phi) = \mathbb{E}_{\pi_\theta}[\log(D_\phi(s,a))] + E_{\pi_E}[\log(1-D_\phi(s,a))] - \lambda H(\pi_\theta)
$$

The training algorithm can be written as follows:

Algorithm 1 Generative Adversarial Self-Imitation Learning

Initialize policy parameter θ
Initialize discriminator parameter ϕ
Initialize good trajectory buffer $\mathcal{B} \leftarrow \emptyset$
for each iteration **do**
  Sample policy trajectories $\tau_\pi \sim \pi_\theta$
  Update good trajectory buffer $\mathcal{B}$ using τ_π
  Sample good trajectories $\tau_E \sim \mathcal{B}$
  Update the discriminator parameter ϕ via gradient ascent with:

$$
\nabla_\phi \mathcal{L}_{\text{GASIL}} = \mathbb{E}_{\tau_\pi}\left[\nabla_\phi \log D_\phi(s,a)\right] + \mathbb{E}_{\tau_E}\left[\nabla_\phi \log(1-D_\phi(s,a))\right] \tag{8}
$$

  Update the policy parameter θ via gradient descent with:

$$
\begin{aligned}
&\nabla_\theta \mathcal{L}_{\text{GASIL}} = \mathbb{E}_{\tau_\pi}\left[\nabla_\theta \log \pi_\theta(a|s) Q(s,a)\right] - \lambda \nabla_\theta \mathcal{H}(\pi_\theta), \\
&\text{where } Q(s,a) = \mathbb{E}_{\tau_\pi}\left[\log D_\phi(s,a) | s_0 = s, a_0 = a\right]
\end{aligned} \tag{9}
$$

end for

where the good trajectories here are defined as historical top-K trajectories, and the gradient with respect to θ is actually to calculate the expectation's gradient, which is discussed in the policy gradient section. Since the similarity between the policy's training and policy gradient, the $\log(D_\phi(s,a))$ here can be regarded as a reward of an action a given a state s. Therefore, GASIL can be easily combined with some policy gradient algorithms. So we can update the policy parameter θ via gradient ascent with:

$$\nabla_\theta J_{PG} - \alpha \nabla_\theta L_{GASIL} = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log(\pi_\theta(a|s)\hat{A}_t^\alpha)]$$

where $\hat{A}_t^\alpha$ is an advantage estimation using a modified reward function $r^\alpha(s,a) = r(s,a) - \alpha \log(D_\phi(s,a))$.

2 Experiments

The GASIL+PPO performs better than PPO, PPO+BC and PPO+SIL most of the time, and expecially performs well in the delayed environment:

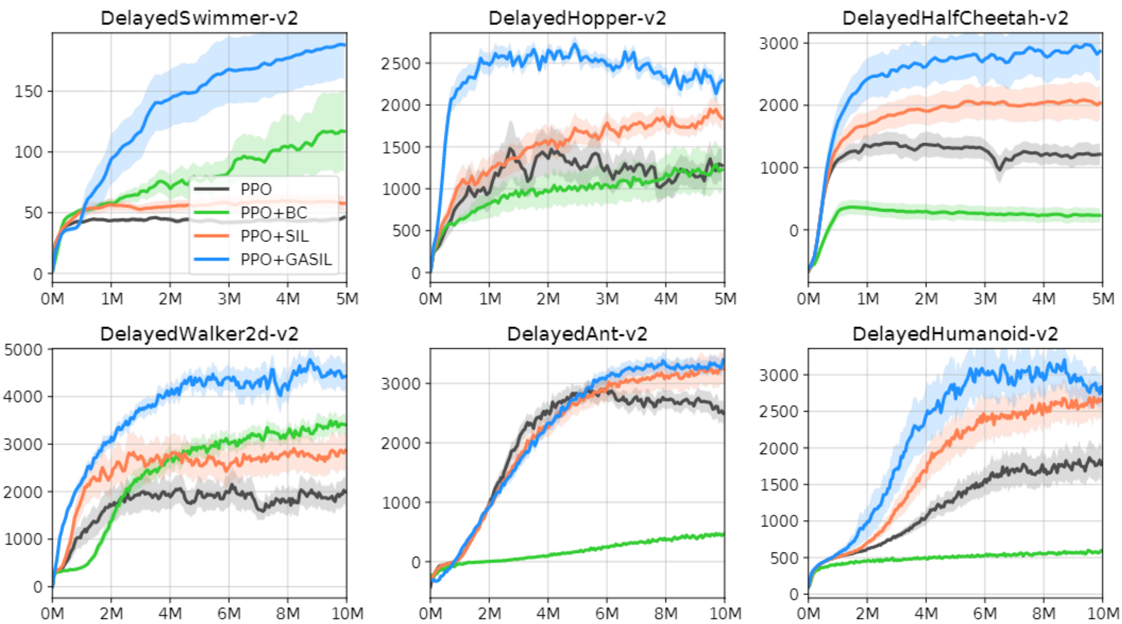
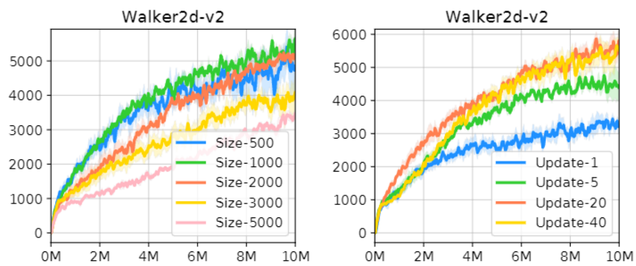


Figure 6: Learning curves on delayed-reward versions of OpenAI Gym MuJoCo tasks averaged over 10 independent runs. x-axis and y-axis correspond to the number of steps and average reward.

since although the updating of accumulated reward is delayed, the updating of $\log(D_\phi(s,a))$ which can be also seen as a reward is immediate. Additionally, the effect of hyperparameters is similar with GANs:



(a) Buffer size (b) Discriminator updates

Figure 7: Effect of GASIL hyperparameters.

3 Shortcomings And Future works

1. The current approach to training the discriminator is to simply discriminate top-K trajectories and generated trajectories, but the approch can be more theoretical.
2. Exploit other policy instead of Gaussian policy to deal with the multi-modal trajectories problem.
3. Introduce model-based methods such as MGAIL to make it more sample-efficient.