Exercise 1.7

Consider the following table of payoffs $\pi(a|x)$ for action set $\mathbf{A} = \{a_1, a_2, a_3\}$ and states of nature $\mathbf{X} = \{x_1, x_2, x_3, x_4\}$.

	x_1	x_2	x_3	x_4
a_1	3	0	3	0
a_2	0	3	0	3
a_3	1	1	1	1

What are the optimal actions if
(a) $P(X = x_1) = P(X = x_2) = P(X = x_3) = P(X = x_4) = \frac{1}{4}$
(b) $P(X = x_1) = P(X = x_3) = \frac{1}{8}$ and $P(X = x_2) = P(X = x_4) = \frac{3}{8}$?

If X is a continuous random variable, then we use a density function $f(x)$ with $P(x < X \leq x + dx) \equiv f(x)dx$. Then the expected payoff for adopting action a is

$$\pi(a) = \int_{x \in \mathbf{X}} \pi(a|x) f(x)\, dx$$

and an optimal action is

$$a^* \in \underset{a \in A}{\operatorname{argmax}} \int_{x \in \mathbf{X}} \pi(a|x) f(x)\, dx\ .$$

(a)
$\pi(a_1) = \sum_{i=1}^{4} \pi(a_1|x_i) P(X = x_i) = \frac{3}{2}$
$\pi(a_2) = \sum_{i=1}^{4} \pi(a_2|x_i) P(X = x_i) = \frac{3}{2}$
$\pi(a_3) = \sum_{i=1}^{4} \pi(a_3|x_i) P(X = x_i) = 1$
$\therefore a^* = a_1$或a_2
(b)
$\pi(a_1) = \sum_{i=1}^{4} \pi(a_1|x_i) P(X = x_i) = \frac{3}{4}$
$\pi(a_2) = \sum_{i=1}^{4} \pi(a_2|x_i) P(X = x_i) = \frac{9}{4}$
$\pi(a_3) = \sum_{i=1}^{4} \pi(a_3|x_i) P(X = x_i) = 1$
$\therefore a^* = a_2$

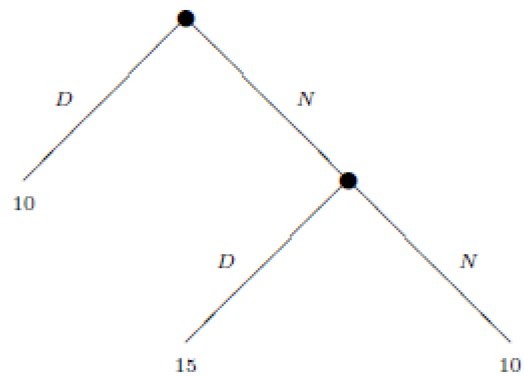


Figure 2.1 Choosing a nickel (N) or a dime (D) on (at most) two occasions. The payoff in cents is given at the end of each branch of the tree.

Example 2.1

Suppose that the adult will offer the "nickel or dime" choice at most twice: if the girl takes the dime on the first occasion, then the choice will be offered only once. The nickel and dime problem can then be represented by the tree shown in Figure 2.1. If she chooses a dime (action D) at the first opportunity, then she receives ten cents and no further offer is made. On the other hand, if she chooses the nickel (action N), she gets five cents and a second choice. It is clear what the girl should do. If she chooses the nickel the first time and then the dime, she gets a payoff of fifteen cents; if she follows any other course of action, she gets only ten cents. Therefore, she should choose the nickel first and then the dime.

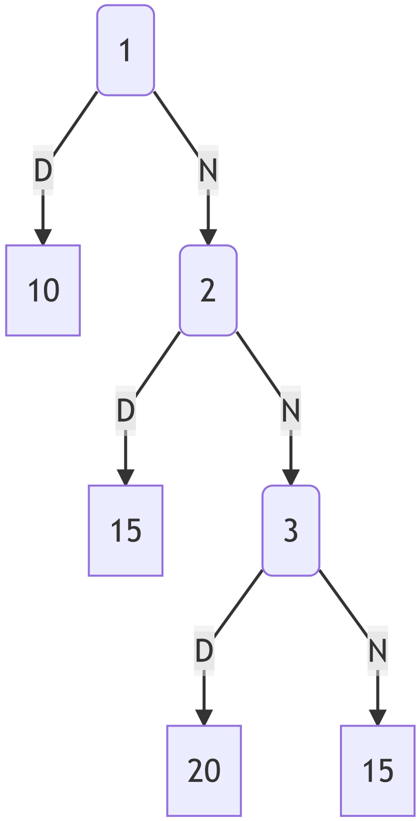
Exercise 2.1

(a) Consider a variant of the "nickel or dime" game from Example 2.1 where the child is offered nickels or dimes on three occasions at most. Draw the tree for this decision problem, determine the pure-strategy set and find the optimal strategy?

(b) Suppose the child is offered the nickel or dime choice on n occasions. What is the optimal strategy?

(c) Suppose the adult offers the child a choice between a nickel or a dime. If the child takes the dime, then the game stops. If the child takes the nickel, then the choice is offered again with probability p. If $p < 1$, then the game will eventually terminate, perhaps because the adult gets bored. What is the child's optimal strategy?

(a)



$S = \{NNN, NND, NDN, NDD, DNN, DND, DDN, DDD\}$

最优是NND

(b)

最优是$N_1 N_2 \cdots N_{n-1} D$

(c)

$\pi(a) = \lim\limits_{i \to \infty} \sum_{i=0}^{n} 5ip^{i-1} * (1-p) = \lim\limits_{i \to \infty} 5(\frac{1-p^n}{1-p} - np^n) = \frac{5}{1-p}$

所以若$p \leq 0.5$时，$\pi(a) \leq 10$，最开始选D

若$p \geq 0.5$时，$\pi(a) \geq 10$，一直选N

Exercise 2.4

Consider a female bird choosing a mate from three displaying males. The attributes of the males are summarised by the following table.

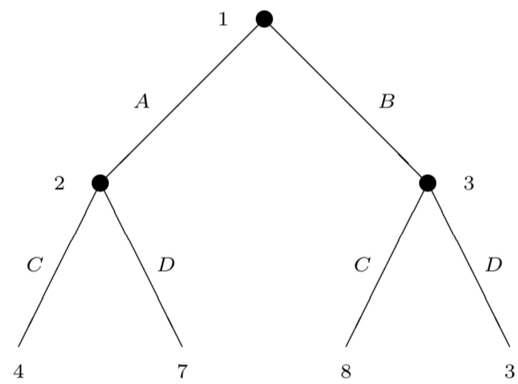
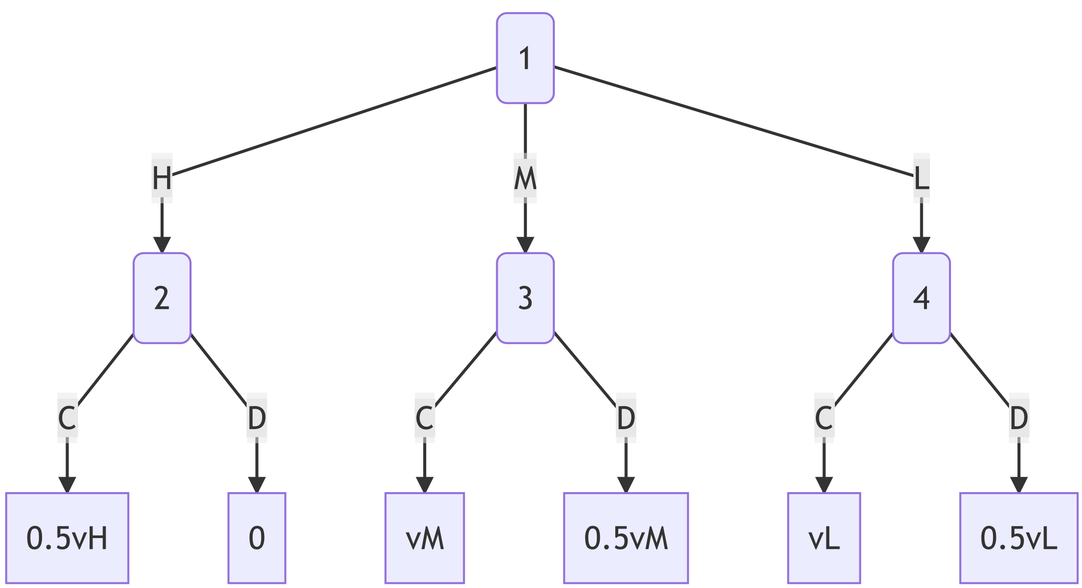


Figure 2.4 Decision tree for Example 2.22.

Male	Genetic quality	Cares for chicks?
1	High	No
2	Medium	Yes
3	Low	Yes

Suppose that the value of offspring depends on the genetic quality of the father. The value of offspring is v_H, v_M, and v_L for the males of high, medium, and low quality, respectively, with $v_H > v_M > v_L$. Once she has mated, the female can choose to care for the chicks or desert them. Chicks that are cared for by both parents will certainly survive; those cared for by only one parent (of either sex) have a 50% chance of survival; and those deserted by both parents will certainly die. Draw the decision tree and find the female's optimal strategy.



$$最优\begin{cases} HC, & 0.5v_H \geq v_M \\ MC, & 0.5v_H \leq v_M \end{cases}$$