Towards HBAR near causal diamond horizon

Gustavo Valdivia-Mera

Department of Physics, University of Houston

IQSE Summer School

Casper, Wyoming

July 21-25, 2025

Outline

Setup / Atom-field interaction

Setup: 2-level atomic dipole[1] falling freely toward the causal horizon.

This interaction is modeled by a dipole coupling:

$$\hat{H}_{int}(\lambda) = g\hat{\phi}(\mathbf{r}(\lambda), t(\lambda))\hat{\sigma}(\lambda). \tag{1}$$

The field and detector operators are given by

$$\hat{\phi} = \sum_s [\hat{a}_s \phi_s(\mathbf{r}, t) + \hat{a}_s^\dagger \phi_s^*(\mathbf{r}, t)] \quad , \quad \hat{\sigma} = \hat{\sigma}_- e^{-i\nu\lambda} + \hat{\sigma}_+ e^{i\nu\lambda}, \tag{2}$$

where (λ) is the detector's proper time, $\hat{\sigma}_-$ being the lowering operator, while $\hat{\sigma}_+$ being the raising operator.

1. Scully, M.O., Kocharovsky, V.V., Belyanin, A., Fry, E. and Capasso, F., 2003.

As the atom falls, it may become excited ($b \to a$) and emit a (scalar) photon ($0 \to 1$). The probability associated with this process is given by:

$$P_{\rm em} = \left| \int d\lambda \, \langle 1_{\mathbf{s}}, a | \, \hat{H}_{int}(\lambda) \, | 0, b \rangle \right|^2 . \tag{3}$$

Field-detector state:

- $|0, b\rangle$: Field in the vacuum state and detector in the ground state.
- $|1_{\mathbf{s}}, a\rangle$: Field in the 1-particle state and detector in the excited state.

Only the term $\hat{a}^{\dagger}_{\mathbf{s}} \hat{\sigma}_{+}$ contributes, yielding

$$P_{\rm em} = g^2 \left| \int d\lambda \, \phi^{*}_{\omega}(\eta(\lambda), \rho(\lambda)) \, e^{i\nu\lambda} \right|^2 . \tag{4}$$

Field equation of motion and CQM modes

The equation of motion for a real massive scalar field on the geometry of the causal diamond spacetime, described by $g_{\mu\nu}$, is given by:

$$\left[\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right) - m^2\right]\phi = 0. \tag{5}$$

From $g_{\mu\nu}$ (check Nada's poster):

$$\left[-\partial_\eta^2 + \frac{4\rho}{\alpha^2}\partial_\rho + \frac{4\rho^2}{\alpha^2}\partial_\rho^2 - \frac{4\rho^2}{\alpha^2}\Lambda^2(\eta,\rho)m^2\right]\phi(\eta,\rho) = 0. \tag{6}$$

We can reduce the complexity of this equation by taking the near horizon approximation ($\rho \ll 1$):

$$\left[-\partial_\eta^2 + \frac{4\rho}{\alpha^2}\partial_\rho + \frac{4\rho^2}{\alpha^2}\partial_\rho^2 - \frac{4\rho^2}{\alpha^2}(16)m^2\right]\phi(\eta,\rho) = 0. \tag{7}$$

From the ansatz:

$$\phi_\omega(\eta,\rho) \sim e^{-i\omega\eta}\frac{\psi(\rho)}{\sqrt{\rho}}, \tag{8}$$

we obtain the following form of the equation of motion:

$$\psi''(\rho) + \frac{1}{\rho^2}\left(\frac{1}{4} + \Theta^2\right)\psi(\rho) \approx 0, \tag{9}$$

where $\Theta = \alpha\omega/2$

In this approximation ($\rho \ll 1$), the original mass scale from the field theory disappears. However, a new scale appear given by Θ:

$$\frac{\psi(\rho)}{\sqrt{\rho}} = \rho^{\pm i\Theta} \Rightarrow \phi_\omega^{\pm(CQM)}(\eta,\rho) \sim e^{-i\omega\eta}\rho^{\pm i\Theta}. \tag{10}$$

Conformal **Q**uantum **M**echanics (CQM)[2].

2. De Alfaro, V., Fubini, S. and Furlan, G., 1977.

Emission and absorption probabilities

The emission rate is given by

$$R_{\text{em}} = \mathfrak{r} g^2 \left| \int d\lambda\, \phi^*_\omega(\eta(\lambda), \rho(\lambda))\, e^{i\nu\lambda} \right|^2, \tag{11}$$

Then

$$\begin{aligned}
\int d\lambda\, \phi^*_\omega(\eta(\lambda), \rho(\lambda))\, e^{i\nu\lambda} &= \int d\lambda\, e^{i\omega\eta} \rho^{-i\Theta} e^{i\nu\lambda} \\
&= -\frac{16\kappa}{e} \int_0^{\rho_\circ} d\rho\, \rho^{-2i\omega/\kappa+1} e^{-i\nu\frac{8\kappa}{e}\rho^2} \\
&= -\frac{16\kappa}{e} \int_0^{\infty} d\rho\, \rho^{-2i\omega/\kappa+1} e^{-i\nu\frac{8\kappa}{e}\rho^2} \\
&= \frac{i2^{\frac{3i\omega}{\kappa}}}{\nu} \Gamma\left(1 - \frac{i\omega}{\kappa}\right) \left(\frac{i\kappa\nu}{e}\right)^{\frac{i\omega}{\kappa}}.
\end{aligned} \tag{12}$$

The emission rate then becomes

$$R_{\text{em}} = \frac{2\pi\mathfrak{r}g^2\omega}{\kappa\nu^2}\left(\frac{1}{e^{2\pi\omega/\kappa} - 1}\right), \tag{13}$$

which corresponds to a thermal distribution at temperature

$$T = \frac{\kappa}{2\pi} = \frac{1}{\pi\alpha}. \tag{14}$$

The absorption rate R_{ab} can be obtained analogously, or directly from the emission rate by $\omega \to -\omega$, yielding

$$R_{\text{ab}} = \frac{2\pi\mathfrak{r}g^2\omega}{\kappa\nu^2}\left(\frac{1}{1 - e^{-2\pi\omega/\kappa}}\right) = e^{2\pi\omega/\kappa} R_{\text{em}}. \tag{15}$$

Horizon **B**rightened **A**cceleration **R**adiation (HBAR)[3]

3. Scully, M.O., Fulling, S., Lee, D.M., Page, D.N., Schleich, W.P. and Svidzinsky, A.A., 2018.

Conclusion

- The detector traveling in a geodesic to the edge of the causal diamond spacetime described by the approximation $\rho \ll 1$ perceived the vacuum state of the field theory as a thermal bath with temperature $T = 1/(\pi\alpha)$.
- The nature of the observed radiation is due to the (HBAR) effect, given by relative acceleration between the detector following a geodesic and the field.

Acknowledgments

- I gratefully acknowledge the Center for Mexican American and Latino/a Studies at the University of Houston for their generous support through a Lydia Mendoza Fellowship.

Center for Mexican American and Latino/a Studies
College of Liberal Arts and Social Sciences

- This work was partially supported by the U.S. Army Research Office (ARO), Grant No. W911NF-23-1-0202.