

Stat Mech Bible

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This is my attempt to catalog everything not included on the Statistical Mechanics qualifying exam equation sheets. Here we go.

Let it be hereby known now and forevermore: z will denote fugacity, Z will denote the canonical partition function, and \mathcal{Z} will denote the Grand Canonical partition function. The qual equation sheet does it differently (z for fugacity, \mathcal{Z} for the partition function, and \mathcal{Q} for the Grand Canonical partition function), but this is written to match in-class notes.

Let it also hereby be known now and forevermore: Don't forget to add a factor of $\frac{1}{N!}$ on partition functions when your particles are indistinguishable.

1 General Math

$$df(x, y) = \left(\frac{\partial f}{\partial x} \right) dx + \left(\frac{\partial f}{\partial y} \right) dy \quad (1)$$

$$\left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x + \left(\frac{\partial x}{\partial z} \right)_y = 0 \quad (2)$$

$$\langle a \rangle = \int a P_a da \quad \text{OR} \quad \langle a \rangle = \sum_a a P_a \quad (3)$$

2 Formulas

2.1 Canonical Ensemble: Exchange of energy

2.1.1 Specific to Ideal Gas

$$\Omega(E) = \frac{1}{N!} \alpha^N \frac{E}{N} V^N \quad (4)$$

$$Z_N = \left(\frac{V}{\lambda^3} \right)^N \quad (5)$$

2.1.2 In General

$$\Omega(E) : \text{Number of states for a given energy} \quad (6)$$

$$Z = \sum_n e^{-\beta H_n} : \text{For a quantized phase space} \quad (7)$$

$$Z = \int \cdots \int_{p_N, q_N} e^{-\beta H_N} \frac{dp_N dq_N}{h^{3N}} : \text{For a continuous phase space} \quad (8)$$

$$P(E_i) = \frac{1}{Z} e^{-\beta E_i} \quad (9)$$

$$g(E) = \sum_i \delta(E - E_i) : \text{For discrete energies} \quad (10)$$

$$Z = \int g(E) e^{-\beta E} dE \quad (11)$$

$$F = -kT \ln Z \quad (12)$$

$$U = -\frac{\partial}{\partial \beta} \ln Z \quad (13)$$

$$S = -k \sum_i \rho_i \ln \rho_i = -k \sum_i P_i \ln P_i = k \ln \Omega(E) = -\frac{\partial F}{\partial T} \quad (14)$$

$$p = -\frac{\partial F}{\partial V} \quad (15)$$

$$\mu = \frac{\partial F}{\partial N} \quad (16)$$

$$M = -kT \frac{\partial}{\partial B} \ln Z \quad (17)$$

$$\chi = \frac{\partial M}{\partial B} \approx \frac{C}{T} : \text{where } C \text{ is the Curie constant} \quad (18)$$

$$c_V = \frac{\partial U}{\partial T} = k\beta^2 \frac{\partial^2}{\partial \beta^2} \ln Z \quad (19)$$

2.2 Grand Canonical Ensemble: Exchange of energy and number of particles

$$P(U, N) = \frac{1}{\mathcal{Z}} e^{-\beta(U - \mu N)} \quad (20)$$

$$\mathcal{Z} = \sum_U \sum_N e^{-\beta(U - \mu N)} = \sum_N e^{\beta \mu N} Z_N = \sum_N Z_N z^N \quad (21)$$

$$\Phi_G = -kT \ln \mathcal{Z} \quad (22)$$

$$U = -\frac{\partial}{\partial \beta} \ln \mathcal{Z} \Big|_z \quad (23)$$

$$N = z \frac{\partial}{\partial z} \ln \mathcal{Z} = -\frac{\partial \Phi_G}{\partial \mu} \quad (24)$$

$$p = \frac{\Phi_G}{V} \quad (25)$$

3 Energies

Thermodynamic Potential:

$$U(S, V, N) \quad (26)$$

Helmholtz Free Energy:

$$F(T, V, N) = U - TS \quad (27)$$

Gibbs Free Energy:

$$G(T, p, N) = U - TS - pV \quad (28)$$

Grand Potential:

$$\Phi_G(T, V, \mu) = U - TS - \mu N \quad (29)$$