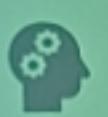




### Natural languages exhibit ambiguity

The boy saw a girl with a telescope  
Our shoes are guaranteed to give you a fit  
Ambiguity makes reasoning difficult / incomplete



### Why formal languages

#### Formal languages

promote rigour and thereby reduce possibility of human error  
help reduce implicit / unstated assumptions by removing familiarity with subject matter  
help achieve generality due to possibility of finding alternative interpretations for sentences and arguments.



## Formal logic

- Formal logic: Uses syllogisms to make inferences, and examines how conclusions follow from premises based on the structure of arguments
  - Symbolic logic: Uses symbols to accurately map out valid and invalid arguments.
  - Mathematical logic: Uses mathematical symbols to prove theoretical arguments
  - Propositional Logic

## Why is it important?

- Core of AI
  - Possibility of *automating* reasoning  
Reasoning: draw inferences from knowledge
  - answer queries
  - discover facts that follow from the knowledge base decide what to do etc.
- In AI, propositional logic is essential for knowledge representation, reasoning, and decision-making processes

## Logical Arguments

- (A) All humans have 2 eyes.
- (B) Sujit is a human.
  - Therefore (P) Sujit has 2 eyes.
- (C) All humans have 4 eyes.
- (B) Sujit is a human.
  - Therefore (Q) Sujit has 4 eyes.
- Which of P and Q are true / false ?
- Is deducing P from A and B correct? Q from B and C?

- All humans have 2 eyes.
- Kishore has 2 eyes.
  - Therefore (P) Kishore is a human.
- No human has 4 eyes.
- Kishore has 2 eyes.
  - Therefore (Q) Kishore is not human.
- Which of P and Q are true / false ?

- All humans have 2 eyes.
  - Kishore has 2 eyes.
    - Therefore (P) Kishore is a human.
  - No human has 4 eyes.
  - Kishore has 2 eyes.
    - Therefore (Q) Kishore is not human.
  - Which of P and Q are true / false ?
  - Is deducing P correct? Q?
- ...fallacy conclusion may be correct. The reasoning is incorrect

## Propositional Logic

Deals with propositions which are true or false

Also known as propositional calculus

Zero-order logic

Foundations for first order and higher order logics

## Order of 'Logic'

- The order of a logic refers to the degree of quantification that can be performed over sets:
  - First-order logic: Quantifies only over individuals. It is also known as predicate logic, predicate calculus, or quantificational logic.
  - Second-order logic: Quantifies over sets.
  - Third-order logic: Quantifies over sets of sets.
  - Higher-order logic: The union of first-, second-, third-, and higher-order logic. It allows quantification over sets that are nested arbitrarily deeply.

## Propositional Logic

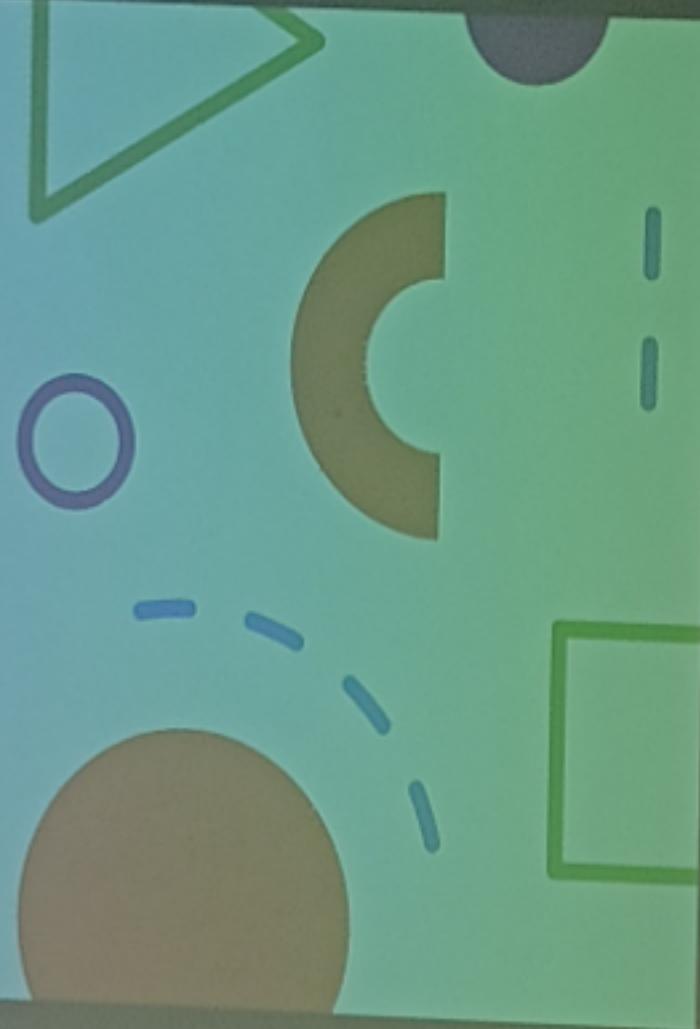
- Propositions are Declarative Statements
- Atomic Propositions:
  - Simple, indivisible statements, cannot be broken down further
  - Each atomic proposition represents a basic fact or condition
  - Example: "The door is closed."
- Compound Propositions:
  - Multiple atomic propositions can be combined using logical connectives (like AND, OR, NOT) to create compound propositions.
  - Example: "The door is closed AND the heater is on."



## Excercise

- Which of the following are propositions

- (P) Today is Wednesday
- (Q) It is raining today
- (R) It will be raining tomorrow
- (S) Close the door
- (S)  $2 + 7 = 9$
- (T)  $3+9 = 10$
- (U)  $X+2 = 1$



## Logical connections

AND ( $\wedge$ ) conjunction	$S: P \text{ AND } Q$ $S: P \wedge Q$	$S$ is true if both $P$ and $Q$ are true
OR ( $\vee$ ) disjunction	$S: P \text{ OR } Q$ $S: P \vee Q$	$S$ is true if any of $P, Q$ is true
NOT ( $\neg$ ): Negation.	$S: \neg P$	$S$ is true only if $P$ is false
IMPLIES ( $\rightarrow$ )	$S: P \rightarrow Q$ $S: \neg P \vee Q$	$S$ is true if $P$ implies $Q$
IFF ( $\leftrightarrow$ )	$S: P \leftrightarrow Q$ $S: \neg P \text{ XOR } Q$	$S$ is true if $P$ and $Q$ are true or false together

$P \rightarrow Q$

The only time  $P \rightarrow Q$  evaluates to False is when

- P is True and Q is False

If  $P \rightarrow Q$  is True, then:

- P is a sufficient condition for Q
- Q is a necessary condition for P



$P \leftrightarrow Q$

$P \rightarrow Q$

$Q \rightarrow P$

If  $P \leftrightarrow Q$  is true

- P and Q are equivalent
- P is necessary and Sufficient for Q
- Q is necessary and Sufficient for P



BOSE

## Truth Table

- A truth table is a breakdown of all the possible truth values returned by a logical expression
- Write down truth tables for,  $P$ ,  $Q$ ,  $\neg P$ ,  $P \vee Q$ ,  $P \rightarrow Q$ ,  $P \leftrightarrow Q$ ,  $\neg P \vee Q$ ,  
 $\neg P \text{ XOR } Q$ 
  - One row for each possible assignment of True/False to propositional variables
  - Important: Above  $P$  and  $Q$  can be any sentence, including complex sentences

$S: P \rightarrow Q$



The above  $P \& Q$  can be any sentence, including complex sentences

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$P$	$Q$	$P \vee Q$	$P \wedge Q$	$P \rightarrow Q$	$P \leftarrow Q$
0	0	0	0	1	1
0	1	1	0	1	0
1	0	1	0	0	0
1	1	1	1	1	1

## Exercise

Given: A and B are true; X and Y are false, determine truth values of:

$$\neg(A \vee X)$$

$$A \vee (X \wedge Y)$$

$$A \wedge (X \vee (B \wedge Y))$$

$$[(A \wedge X) \vee \neg B] \wedge \neg[(A \vee X) \vee \neg B]$$

$$(P \wedge Q) \wedge (\neg A \vee X)$$

$$[(X \wedge Y) \rightarrow A] \rightarrow [X \rightarrow (Y \rightarrow A)]$$

$$\therefore P \rightarrow Q$$

