IIIT Hyderabad

Spring 2024 - Performance Modelling for Computer Systems
Assignment 1

"The 50-50-90 rule: Anytime you have a 50-50 chance of getting something right, there's a 90% probability you'll get it wrong."
- Andy Rooney

Questions

1. IIIT wants to build a new academic block which will stand for a long time by protecting it against earthquakes. If earthquakes strike Telangana as a Poisson process with rate λ per year and we want the building to last for Y years with probability p. What is the smallest number of earthquakes the new building should be able to endure? Calculate with values $\lambda = 0.1,\ Y = 20,\ p = 0.9$

2. Before starting construction on the academic block, it was decided that simulations should be run to find the earthquake resistance of the planned project. The simulation had 2 types of earthquakes, one of magnitude 4.0 and magnitude 5.0 and rates λ_1 and λ_2 per year respectively. The building fell with probability p_1 if a magnitude 4.0 earthquake was applied and with probability p_2 if a magnitude 5.0 earthquake was applied. If the simulation ran for Y virtual years and Let total number of quakes encountered be N. Find $P\{N = n|Y = y\}$

3. There has been a lot of traffic on the road from the main gate to Bakul Niwas, as a result students have trouble crossing the stress. Sneha wants to cross the street but she will only cross it if there is no traffic for the next T seconds on the road. Cars travel on the road as a poisson process with rate λ. What is the expected amount of time Sneha has to wait

4. As we all know, no one comes on time for a class scheduled at 8:30 AM. Fed up with this, the instructor decided that there is a uniform probability that the doors will be closed sometime between 8:30 AM and 8:45 AM. If students always come to class late and as a poissson process with rate λ, what is the expected number of people who will get attendance?

5. $X(t)$ is known as a birth process with rate $\lambda_n,\ n \in N$ if :

$$\forall t, h > 0, X(t + h) \geqslant X(t)$$

$$P\{X(t + h) - X(t) = 1|X(t) = n\} = \lambda_n h + o(h)$$
$$P\{X(t + h) - X(t) = 0|X(t) = n\} = 1 - \lambda_n h + o(h)$$
$$P\{X(t + h) - X(t) > 1|X(t) = n\} = o(h)$$

   (a) Find the conditions for this to be a poisson process.
   (b) Consider a process where $\lambda_n = \lambda n$. Define the generator matrix for this process.

6. A drunk man tosses a coin at a rate given by λ. If the coin comes up heads, he takes one step towards his home and one step back towards the bar if the coin comes up tails. The main does not do anything if the coin comes up tails and he is already at the bar. Construct the generator matrix for this process and also define the discrete time process embedded in this continuous time process.

7. For the previous question, consider that his home is only 2 steps away. What is the expected time for him to get home?

8. For the previous question, once the man is at home, he has a 0.8 probability of staying at home and a probability 0.2 for going back one step if the coin comes up heads and going back one step if the coin comes up tails. Find the stationary distribution for this process.

9. Consider a CTMC with a probability transition matrix $P(t)$ given by

$$\begin{bmatrix} 1-e^{t} & e^{t} & 0 \\ e^{\frac{t}{2}} & 1-2e^{\frac{t}{2}} & e^{\frac{t}{2}} \\ e^{\frac{t}{4}} & 0 & 1-e^{\frac{t}{4}} \end{bmatrix}$$

Find its stationary distribution

10. Consider a discrete time Markov chain X_n with $X_0 = i$. Let N be the total number of visits made by the chain to a state j. Prove that

$$P(N=n) = \begin{cases} 1-F_{ij} & n=0 \\ F_{ij}F_{jj}^{n-1}(1-F_{jj}) & n \geqslant 1 \end{cases}$$

Here F_{ij} denotes the probability of ever coming to state j from state i.