MDL Assignment-1

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A1) Exp :- $(p \rightarrow q) \vee (p \rightarrow \sim q)$

To Proove :→ Exp is a Tautology

Proof :→ Tautology refers to universal Truth.

Let's make the truth Table,

p	q	$\sim q$	$p \rightarrow q$	$p \rightarrow \sim q$	$(p\rightarrow q) \vee (p \rightarrow \sim q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	T	T
F	F	T	T	T	T

Here the exp is always True, ∴ It's a Tautology.

A2) Given : ~~$(p \rightarrow q) \vee (p \rightarrow \sim q)$~~

$\psi : (p \wedge \sim q) \longrightarrow (p \wedge q)$

$\phi : \sim p$

To Proove :→ (a) $\phi \Rightarrow \psi$ using Truth Table

(b) $\phi \Rightarrow \psi$ using Valid arguments

(a)

p	q	ϕ $\Uparrow$ $\sim p$	$\sim q$	$p \wedge \sim q$	$p \wedge q$	ψ	$\phi \Rightarrow \psi$
T	T	F	F	F	T	T	T
T	F	F	T	T	F	F	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T

Since $\phi \Rightarrow \psi$ is always true, its a Tautology.

Hence Prooved,,

(b) $\phi \Rightarrow \psi \equiv \sim \phi \vee \psi$

Put the values of ϕ & ψ we get,

$\equiv \sim(\sim p) \vee ((p \wedge \sim q) \rightarrow (p \wedge q))$

$p \vee ((p \wedge \sim q) \longrightarrow (p \wedge q))$

$p \vee (\sim(p \wedge \sim q) \vee (p \wedge q))$

$p \vee ((\sim p \vee q) \vee (p \wedge q))$ [Demorgan]

$p \vee \sim p \vee q \vee (p \wedge q)$ [Associative]

$\text{True} \vee q \vee (p \wedge q)$

$\equiv \text{True}$

$\Rightarrow$ Hence $\phi \Rightarrow \psi$ is always $\therefore$ Tautology

Hence Proved.

A3) To verify whether a 9x9 grid represents a valid Sudoku Solution, the set of axioms that must satisfy are as follows (simultaneously) :→

1.) Cell Uniqueness (Each Cell Contains Exactly One Number)

$$\forall i,j \quad \underbrace{\left(\bigvee_{k=1}^{9} x_{ijk}\right)}_{\text{Atleast one}} \wedge \cancel{\sim x_{ijk}} \underbrace{\bigwedge_{k=1}^{9}\bigwedge_{l=k+1}^{9} \sim(x_{ijk} \wedge x_{ijl})}_{\text{No two}}$$

2.) Row Uniqueness (Each num appears exactly once in each row)

$$\forall i, \cancel{j} \forall k \quad \left(\bigvee_{j=1}^{9} x_{ijk}\right) \wedge \bigwedge_{j=1}^{9}\bigwedge_{l=j+1}^{9} \sim(x_{ijk} \wedge x_{ilk})$$

3.) Column Uniqueness (Each number appears exactly once in each column)

$$\forall j, \forall k \quad \left(\bigvee_{i=1}^{9} x_{ijk}\right) \wedge \bigwedge_{i=1}^{9}\bigwedge_{l=i+1}^{9} \sim(x_{ijk} \wedge x_{ljk})$$

4.) Subgrid Uniqueness (Each number appears exactly once in each 3X3 subgrid)

$\forall k \ \forall (r,s) \in \{(0,0),(0,3),(0,6),(3,0),(3,3),(3,6),$
$(6,0),(6,3),(6,6)\},$

$$\bigwedge_{(i,j)\in Subgrid(r,s)} x_{ijk} \wedge \bigwedge_{(i_1,j_1)\neq(i_2,j_2)} \sim(x_{i_1j_1k} \wedge x_{i_2j_2k})$$

(Valid Sudoku Solution) $\Longleftrightarrow$ Cell Uniqueness $\wedge$ Row Uniqueness $\wedge$ Column Uniqueness $\wedge$ Subgrid Unique

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A4) Given: $((\sim p \to q) \to (q \to \sim r))$

To Convert: CNF and DNF

CNF:→ $(\sim p \to q) \to (q \to \sim r)$

Using $A \to B \equiv \sim A \vee B$]

$\equiv \sim(p \vee q) \vee (\sim q \vee \sim r)$

$\equiv [\sim p \wedge \sim q] \vee (\sim q \vee \sim r)$ [DeMorgan]

$\equiv [\sim p \vee \sim q \vee \sim r] \wedge (\sim q \vee \sim q \vee \sim r)$

CNF $\Rightarrow (\sim p \vee \sim q \vee \sim r) \wedge (\sim q \vee \sim r)$

DNF : $(\sim p \to q) \to (q \to \sim r)$

$\equiv \sim(p \vee q) \vee (\sim q \vee \sim r)$

DNF $\equiv$ $(\sim p \wedge \sim q) \vee (\sim q) \vee (\sim r)$

A5)

(i) R : it rains

U : I carry an umbrella

W : I get wet

T : I drink tea

C : I drink coffee

ⓐ $R \to U \oplus W$

ⓑ $W \to T$

ⓒ $\sim W \to C$

ⓓ $C \wedge R \to U$

No, statement (iv) isn't valid according to (i), (ii), (iii).

Counter Example

Let the following assignment of variables

$R =$ true

$U =$ false

$W =$ true

$T =$ true

$C =$ true

(i) $R \to U \oplus W \equiv$ true $\to (f \oplus t) \equiv t \to t$ ✓

(ii) $W \to T \equiv$ true $\to$ true ✓

(iii) $\sim W \to C \equiv$ false $\to$ true ✓

(iv) $(C \land R) \to U \equiv$ (true $\land$ true) $\to$ false

$=$ true $\to$ false ✗

Hence (iv) isn't in line with (i), (ii) & (iii)