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A1)

(a) 
$$A = \begin{pmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & 0 \end{pmatrix}$$

 $R_1 \leftarrow R_1/a$  $R_2 \leftarrow R_2/b$ 

$$\begin{array}{c} \Rightarrow & \begin{pmatrix} 0 & 1 & 0 & | 1/a & 0 & 0 \\ 1 & 0 & | 1/b & | 0 & | 1/b & 0 \\ 0 & | 0 & | 0 & | 0 & | 1 \end{pmatrix} \end{array}$$

 $R_1 \longleftrightarrow R_2$  ,  $R_3 \longleftrightarrow R_3/d$ 

 $R_3 \leftarrow R_3 - R_2$ 

$$\Rightarrow \begin{pmatrix} 1 & 0 & c/b & 0 & 1/b & 0 \\ 0 & 1 & 0 & 1/a & 0 & 0 \\ 0 & 0 & 0 & -1/a & 0 & 1/d \end{pmatrix}$$

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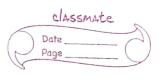
As Matrix A can not be row reduced to form an Identity Matrix as one of the rows of A is converted to zero and we already know the fact that an Identity matrix can not be row reduced to a matrix with zero row.

Therefore, Matrix A does not have an Inverse

 $R_1 \leftarrow R_1/\alpha$   $R_2 \leftarrow R_2/\alpha$ 

 $R_3 \leftarrow R_3/a$ 

 $R_2 \leftarrow R_2 - \frac{1}{\alpha} \cdot R_1$  $R_3 \leftarrow R_3 - \frac{1}{\alpha} \cdot R_2$ 



Hence inverse exist and A'is its Inverse

A2

To RTP:- If a Symmetric Matrix is invertible, then prove its inverse is also symmetric

Definition: If AT is transpose of Matrix A then A is symmetric if and only if A = AT

Proof: 8- Let Po be a symmetric Matrix (AT=A) and AT exist and be inverse of A

By definition of Inverse,  $A \cdot A^{-1} = I \quad --0$ 

$$a \cdot a^{-1} = \tau$$
 — (1)

Abo  $I^T = I$ 

Taking transpose on both sides.

$$(A \cdot A^{-1})^{T} = I^{T}$$

 $(A \cdot A^{-1})^{T} = I^{T}$   $(A \cdot A^{-1})^{T} = A \cdot A^{-1}$ 

Also we know that,  $(AB)^T = B^T \cdot A^T$ 

commutative

Now as A and A-1 one commutable i.e. A. A-1 = A-1. A

$$(A^{-1})^{\mathsf{T}} \cdot A^{\mathsf{T}} = A^{-1} \cdot A$$

Os 
$$A^{T} = A$$
.  
 $(A^{-1})^{T}$ .  $A = A^{-1}$ .  $A = A^{-1$ 

By definition, AT is a Symmetric Matrix.

Hence Prooved.

$$\begin{array}{c}
A = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

$$H_{5} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A_A = A \cdot B_3 = A \cdot (-1) = -A = \begin{pmatrix} 0 - 1 \\ -1 & -1 \end{pmatrix}$$

$$A5 = A^3 \cdot A^2 = -A^2 = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$$A^{6} = (A^{3})^{2} = (-I)^{2} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



$$A^{7} = A \cdot A^{6} = A \cdot J = A = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

By Observing we can say that matrix repeats after every 6 powers of A

$$A^{2015} = (A^6)^{350} \cdot A^5$$

$$= I^{350} \cdot A^5 = A^5$$

$$= (1 - 1)$$

$$= (1 0) A_0$$

A4) RTP:- There are no Square matriaces X and Y with the property that XXX- XX = I.

Proof:As we know trace of a square matrix A

(represented by tr(A)) is the sum of diagonal
entries of Aire SAii where Ais DXD.

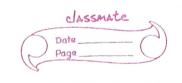
Now lets there are few properties of trace which are as follows:
(i) tr(A+B) = tr(A) + tr(B)

(ii) tr (CA) = (.tr(A)

(iii) tr (AB) = tr(BA)

Now for the sake of contradiction, lets assume there exist matrices X and Y such that XY-YX=I.

Taking Trace on both sides,



$$\Rightarrow$$
 0 =  $\eta$ 

This contradicts our assumption that there exist matrices X and Y such that XY-YX=I.

Therefore Hence Prooved.

Proof of the properties trans = tr (BA)

$$t_{\Gamma}(AB) = \sum_{i=1}^{N} (AB)ii$$

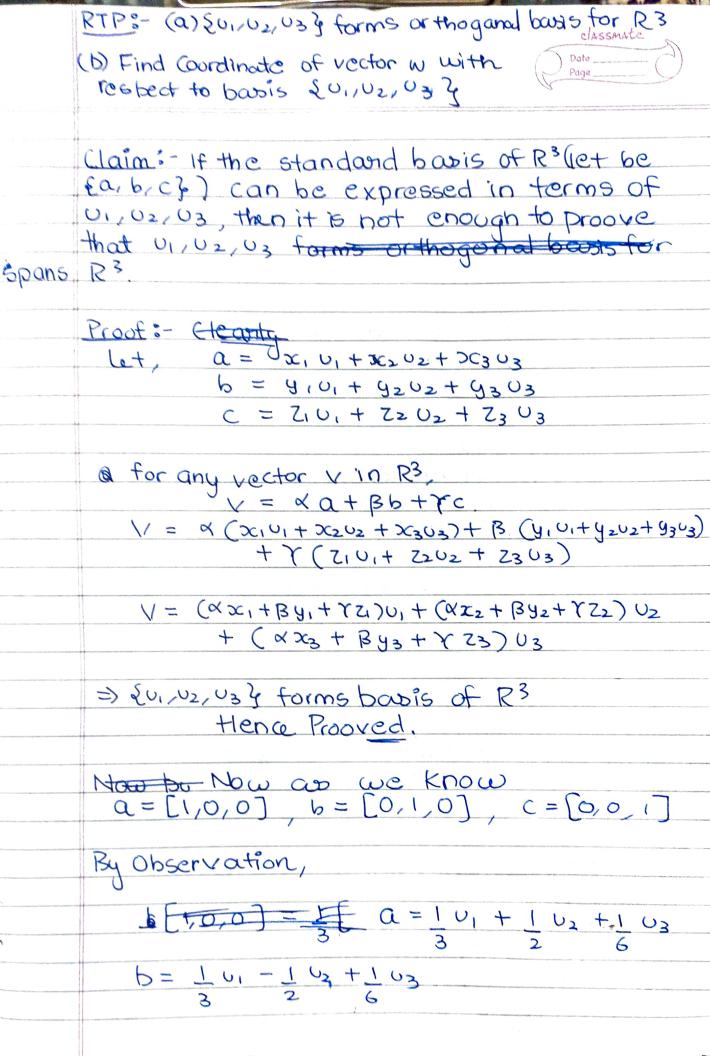
$$= \sum_{i=1}^{N} Aik Bki$$

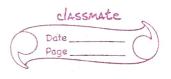
$$= \sum_{k=1}^{\infty} \mathbb{E}(BA)^{kk}$$

A5) Given: 
$$0 = 1$$
  $0 = 1$   $0 = 4$   $0 = 4$ 

also (a,b) = a,b, + a2b2 + a3b3 for

$$a = \begin{bmatrix} a_1 \\ 4_2 \\ a_3 \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$





$$C = 10, + 002 + -103$$

Gnd;

C = 10, + 002 to -103

3 spans

Therefore, &u, u2, u3 & forms or thougand boosis

for R3.

Now,

 $\langle 0_2, 0_3 \rangle = 1.1 + (-1).1 + 0.(-2) = 0$ 

 $\langle 0_3, 0_1 \rangle = 1.1 + 1.1 + 1. (-2) = 0$ 

How This implies vectors U1, U2, U3 are mutually perpendicular to each other Also, u, u2, u3 would be linearly

Independent.

Therefore, &v1, v2, v3 & forms orthogonal basis for R3 Hence Prooved (a)

As wer3, (P) W = QU1 + BU2+ YU3

Now by definition, Now by definitions,

 $\alpha = 4 + 5 + 6 = 5$ 

B = 4-5+0 = -1/2 : Coordinates are

55, - 4, - 1/2 7 Y = 4+5-12 = -1/2



A6)

## Prooving it true

RIP:- Product of two upper triangular matrices is upper triangular.

Proof: - Let A and B be two upper triangular matrices.

By definition, A matrix is called upper triangular if all the elements below the main diagonal are zero.

i.e Aij = 0 \ i7j

Now, (AB) ij = SAIKBKj

Now by def., for AB to Upper Triangward

(AB)ij = 0 + i > j.

Since A and B are upper triangular Aik = 0 Y K< i & B kj = 0 Y j < K

Now, K<i p j < K

=> i > j

ie for i > j any one of Aikor Bkj must be true and,

(AB) ij = 0 + i > j .. AB is upper triangular Matrix

Hence Prooved