

Breadth First Search-

Connected graph: for any pair of vertices $u, v \in V$, there is a path from u to v
(undirected)

(Can be checked using BFS)

BFS(s):

Discovered [s] = True

For all $v \in V \setminus \{s\}$

Discovered [v] = False

$L[0] \leftarrow \{s\}$

$i \leftarrow 0$

$T \leftarrow \emptyset$

While $L[i]$ is not empty:

$L[i+1] \leftarrow []$ } init

for each $u \in L[i]$:

for each edge $(u, v) \in E$ (incident on u):

if Discovered [v] == false:

Discovered [v] \leftarrow True

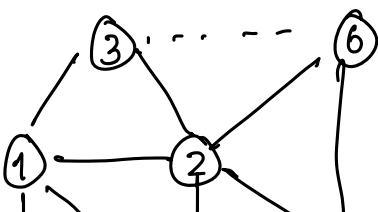
$T \leftarrow T \cup \{u, v\}$

$L[i+1].append(v)$

} Init

Qn: Compute Connected components

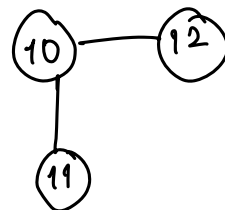
BFS(1)

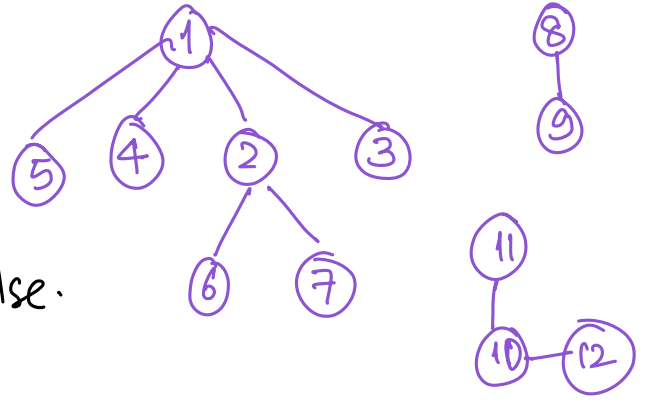
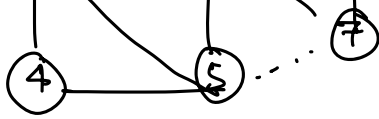


BFS(8)



BFS(11)





Init: For all $v \in V$:
 $\text{Discovered}[v] \leftarrow \text{false}.$

Pick a vertex: (say u)

Run $\text{BFS}[u]$ \leftarrow Connected component containing u .

Testing bipartiteness

Bipartite graphs

|||

2-colourable graphs.

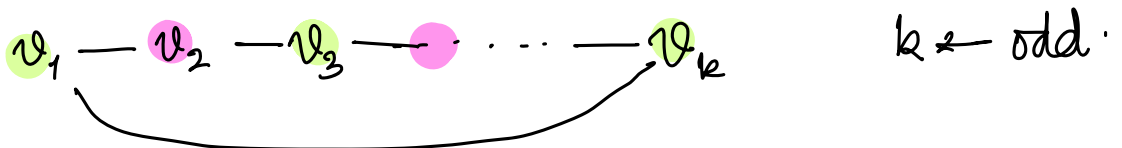
(Undirected)

Lemma: A graph is bipartite if and only if it has no odd cycles.

\nwarrow Cycles with odd no. of edges.

\rightarrow Bipartite \Rightarrow no odd cycles.

For the sake of contradiction: Say there is an odd cycle

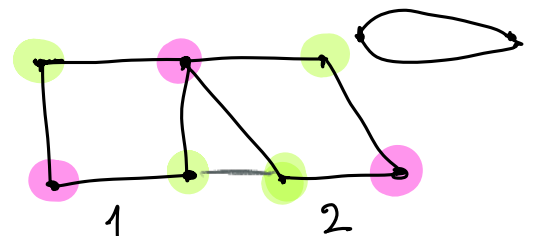
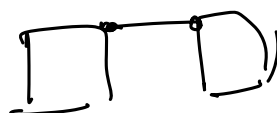


But (v_1, v_k) edge is monochromatic contradicting the 2-colourability of the entire graph.

no Odd cycles \Rightarrow Bipartite

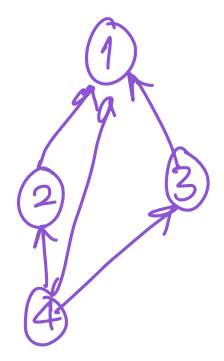
Start w/ a vertex.

\rightarrow Even cycles

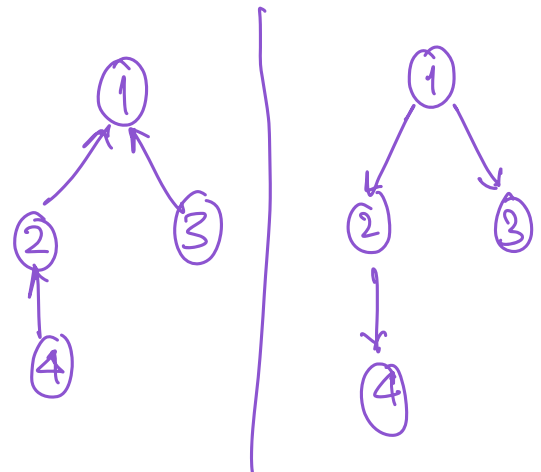


→ Acyclic.
undirected
acyclic
graph is a
tree.

(directed acyclic
graph need not be
a directed tree)

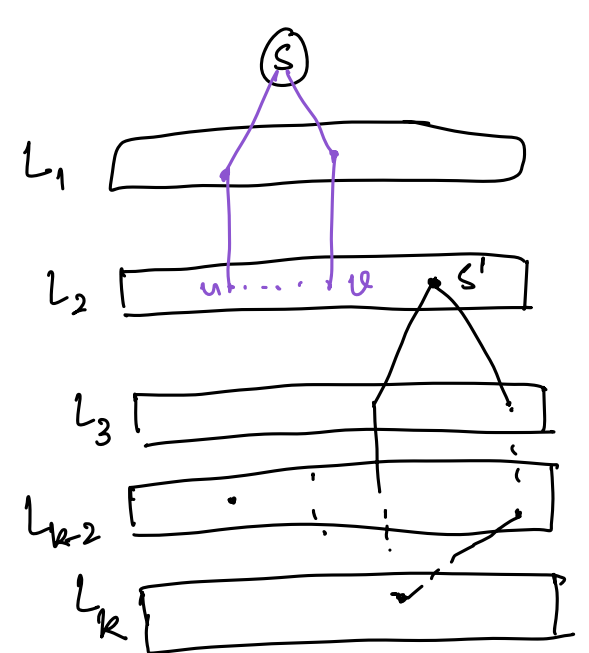


Qn: Suppose we have BFS tree
and Layer computed for a given
connected graph. Using this, can
we infer bipartiteness?



Cross edges in the same layer

↳ s -



BFS(s)

$L_0 \ L_1 \ L_2 \ \dots \ L_k$

Part 1: $L_0 \cup L_2 \cup \dots \cup L_k$

Part 2: $L_1 \cup L_3 \cup \dots \cup L_{k-1}$

$\text{dist}(s', u)$
 $= \text{dist}(s', u')$

