Performance Modelling for Computer Systems

Assignment

Q1: Consider an $M/M/K/2K$ queue. Model the system as a Markov chain and find its stationary distribution. Does the stationary distribution have a closed form? What is the blocking probability of the system?

Q2: Consider a two server system S_1 and S_2 with no buffer and the same exponential service rate of μ. Arriving jobs are of two types C_1 and C_2. Both arrive according to a Poisson process with rate λ. C_1 and C_2 jobs can be processed by either server at the given service rate. Additionally, if both S_1 and S_2 are free then they together process jobs of type C_2 with rate $\bar{\mu}$ instead. Model the described system as a Markov chain and obtain the stationary distribution for different states.

Q3: Consider a system with two identical servers of unit rate and no buffer. Lets call the two servers S_1 and S_2. Arriving jobs are of two types. Low priority jobs C_1 arrive according to a Poisson process with rate λ_1 and can only use Server S_1. High priority jobs C_2 arrive according to a Poisson process with rate λ_2 and are supposed to use S_2 first if it is idle. But if S_2 is busy while S_1 is idle, they are allowed to use server S_1. Assume both type of jobs have exponential service requirement with rate μ_1 and μ_2 respectively. Model this system as a Markov chain, obtain the stationary distribution for different states.

Q4: Find the probability that a job is declined service (i.e blocking probability) for the previous two systems (Q2 and Q3).

Q5: Consider the following queueing systems:

- A $M/M/1$ system with arrival rate λ and exponential service rate $K\mu$
- A $M/M/K$ system with arrival rate λ and each server having a service rate of μ
- n independent $M/M/1$ systems with arrival rate λ/K and exponential service rate μ

Report the following metrics for each system:

- Expected number of customers in system
- Expected number of customers in the queue
- Expected time spent in the system
- Expected time spent in the queue

Additionally, compare these metrics for all three systems and report which system performs best (according to each metric).

Q6: As we all know, no one comes on time for a class scheduled at 8:30 AM. Fed up with this, the instructor decided that there is a uniform probability that the doors will be closed sometime between 8:30 AM and 8:45 AM. If students always come to class late and as a Poisson process with rate λ, what is the expected number of people who will get attendance?

Q7: Consider a discrete time Markov chain X_n with $X_0 = i$. Let N be the total number of visits made by the chain to a state j. Prove that

$$P(N = n) = \begin{cases} 1 - F_{ij}, & n = 0 \\ F_{ij}F_{jj}^{n-1}(1 - F_{jj}), & n \geq 1 \end{cases}$$

Here F_{ij} denotes the probability of ever coming to state j from state i.

Q8: During *Felicity*, a canteen shop is set up with two service counters to efficiently serve the attendees. The Fast Service Counter (S_1) has a higher service rate μ_1 and is the preferred choice for all customers. If S_1 is busy, incoming customers are directed to the Backup Counter (S_2), which has a lower service rate μ_2. If both counters are busy, the arriving customer leaves without being served, leading to job loss. Customer arrivals follow a Poisson process with rate λ.

As a key organizer of Felicity, you have been given the responsibility to manage the queuing system at the canteen. Your goal is to ensure smooth operations and minimize customer loss. To achieve this, you need to:

(a) Model the canteen system as a Markov Chain and construct the transition rate matrix Q.

(b) Find the stationary distribution of the system and compute the blocking probability (the probability that a customer is unable to buy food).

(c) Compare this system to an M/M/2 queue, where both counters serve at a combined rate $\mu = \frac{\mu_1 + \mu_2}{2}$.

Q9: Consider a Continuous-Time Markov Chain (CTMC) with a probability transition matrix $P(t)$ given by

$$P(t) = \begin{bmatrix} 1 - e^{-t} & \frac{e^{-t}}{2} & \frac{e^{-t}}{2} \\ \frac{e^{-t/2}}{3} & 1 - \frac{2e^{-t/2}}{3} & \frac{e^{-t/2}}{3} \\ e^{-t/4} & \frac{e^{-t/4}}{2} & 1 - \frac{3e^{-t/4}}{2} \end{bmatrix}$$

Find its stationary distribution and discuss whether the system always reaches a steady state. If not, explain why.