
Some Terminology
A sentence is valid if it is True under all possible assignments of True/False to its propositional variables (e.g. P ∨ ¬P)
Valid sentences are also referred to as tautologies
SAT: A sentence is satisfiable if and only if there is some assignment of True/False to its propositional variables for which the sentence is True
A sentence is unsatisfiable if and only if it is not satisfiable (e.g. P ∧ ¬P)


Towards Formal Proof
• α ⇒ β — whenever all the formulae in the set α are True, β is True
• This is a *semantic* notion; it concerns the notion of *Truth*
• To determine if α ⇒ β construct a truth table for α, β
α ⇒ β if, in any row of the truth table where all formulae of α are true, β is also true


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Modus Ponens
• Write Truth Table for P, P → Q
• (and what can you conclude about Q)
• Therefore, P, P→Q ⇒ Q
• α : P ∧ (P→Q), β: Q


Intend to formally capture the notion of proof that is commonly applied in other fields (e.g. mathematics)
A proof of a formula from a set of premises is a sequence of steps in which any step of the proof is:
An axiom or premise
A formula deduced from previous steps of the proof
using some rule of inference
The last step of the proof should deduce the formula we wish to prove.
We say that S follows from (premises) α to denote that the set of formulae α "prove" the formula S.

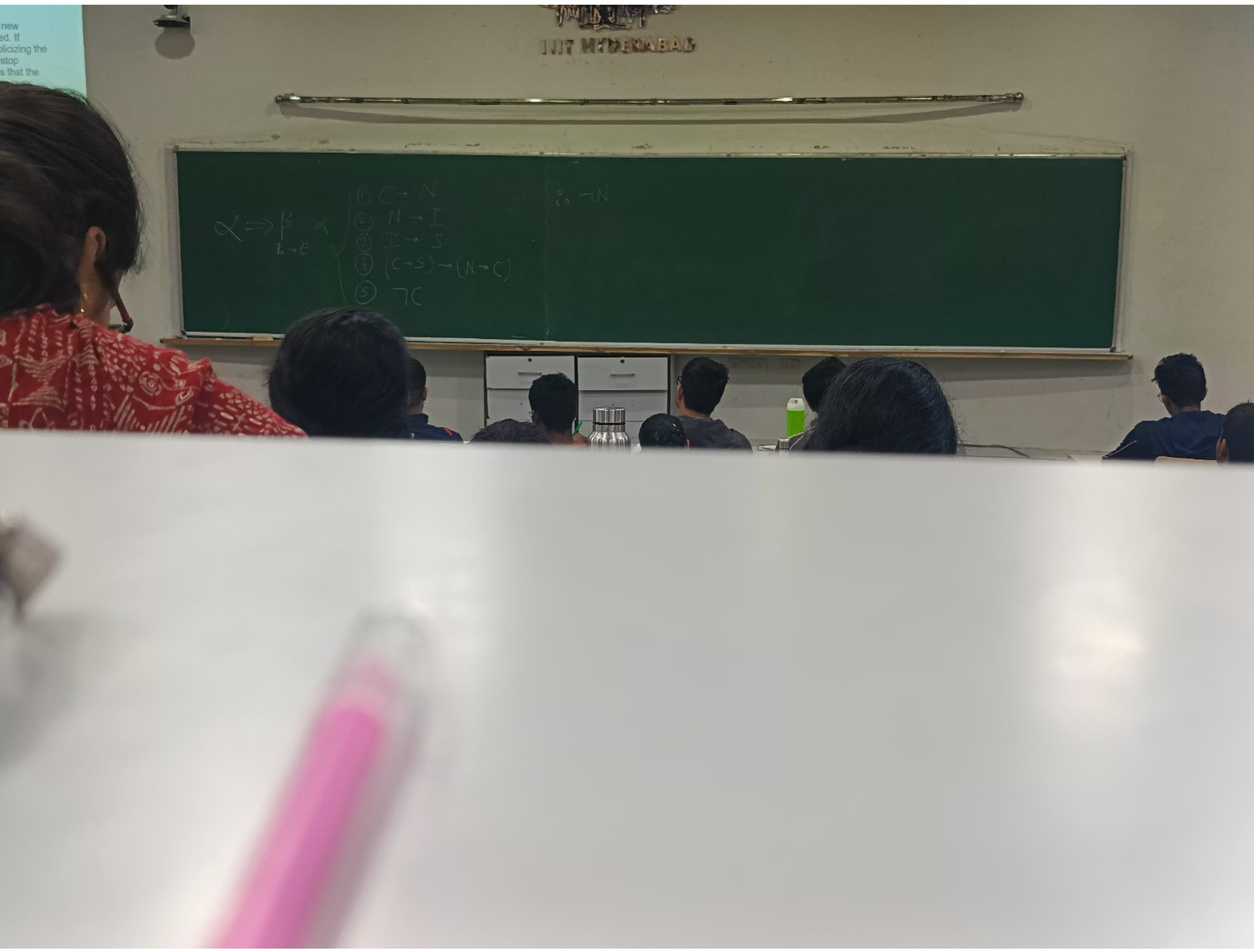

Soundness and Completeness
• A logic is sound if it preserves truth (i.e. if a set of premises are all true, any conclusion drawn from those premises must also be true)
• A logic is complete if it is capable of proving any valid consequence
• (Syntactic) Completeness requires that every sentence, or its negation, is provable (i.e., a theorem)
• A logic is decidable if there is a mechanical procedure (computer program) to prove any given consequence. in the theory.
• Decidability: there is a decision procedure (finite) for theorem


Inference rules
1.Modus Ponens: $P, P \to Q \Rightarrow Q$
2.Modus Tollens: $P \to Q, \neg Q \Rightarrow \neg P$
3.Hypothetical Syllogism: $P \to Q, Q \to R \Rightarrow P \to R$
4.And-Elimination: $P_1 \wedge P_2 \wedge \ldots \wedge P_n \Rightarrow P_i$
5.And-Introduction: $P_1, P_2, \ldots, P_n \Rightarrow P1 \wedge P2 \wedge \ldots \wedge Pn$
6.Or-Introduction: $P_i \Rightarrow P_1 \vee P_2 \vee \ldots \vee P_n$
7.Double-Negation Elimination: $\neg\neg P \Rightarrow P$
8.Unit Resolution: $P \vee Q, \neg Q \Rightarrow P$
9.Resolution: $P \vee Q, \neg Q \vee R \Rightarrow P \vee R$


Example of a Formal Proof
1.A ∨(B → D)
2.¬C→(D→E)
3.A→C
4.¬C
∴B→E
5.¬A
6.B → D
7.D → E
8.B → E
• 3,4 (Modus Tollens)
• 1,5, (Unit resolution)
• 2,4 (Modus Ponens)
• 6,7 (Hypothetical Syllogism)


Exercise
• Construct formal proof of validity for:
If the investigation continues, then new evidence is brought to light. If new evidence is brought to light, then several leading citizens are implicated. If several leading citizens are implicated, then the newspapers stop publicizing the case. If continuation of the investigation implies that the newspapers stop publicizing the case, then the bringing to light of new evidence implies that the investigation continues. The investigation does not continue. Therefore, new evidence is not brought to light.


Exercise
• Construct formal proof of validity for:
If the investigation continues, then new evidence is brought to light. If new evidence is brought to light, then several leading citizens are implicated. If several leading citizens are implicated, then the newspapers stop publicizing the case. If continuation of the investigation implies that the newspapers stop publicizing the case, then the bringing to light of new evidence implies that the investigation continues. The investigation does not continue. Therefore, new evidence is not brought to light.
• C: The investigation continues. N: New evidence is brought to light. I: Several leading citizens are implicated. S: The newspapers stop publicizing the case.
BOSE


IIIT HYDERABAD
α ⟹ β
B → E
α
1 C → N
2 N → I
3 I → S
4 (C → S) → (N → C)
5 ¬C
∴ ¬N


Solution
1. C→N
2. N→I
3. I→S
4. (C→S)→(N→C)
5. ¬C. ∴¬N
6. C → I 1,2 (Hypothetical Syllogism)
7. C → S 6,3(Hypothetical Syllogism)
8. N → C
9. ¬N 8,5 (Modus Tollens)


Exercise
• If I study, I make good grades. If I do not study, I enjoy myself. Therefore, either I make good grades or I enjoy myself.
• S, G, E


IIIT HYDERABAD
• S, G, E
∴ ¬N
α ① S → G
② ¬S → E
∴ G ∨ E
β
→ ¬S ∨ G
¬(¬S) ∨ E
S ∨ E
G ∨ E
cello


Solution
1. S→G
2. ¬S→E ∴GVE
3. ¬SVG
1
4. ¬¬SVE
2
5. S VE
4, (DoubleNegationEliminate)
6. G VE
3,5 (Resolution)


Conjunctive Normal Form (CNF)
• A literal is a propositional letter or the negation of a propositional letter
• A clause is a disjunction of literals
• Conjunctive Normal Form (CNF) – a conjunction of clauses
• e.g. $(P \vee Q \vee \neg R) \wedge (\neg S \vee \neg R)$
• Disjunctive Normal Form (DNF) – a disjunction of conjunctions of literals
• e.g. $(P \wedge Q \wedge \neg R) \vee (\neg S \wedge \neg R)$
• Every propositional logic formula can be converted to CNF and DNF

Converting to CNF
• Eliminate $\leftrightarrow$ rewriting $P \leftrightarrow Q$ as $(P \rightarrow Q) \wedge (Q \rightarrow P)$
• Eliminate $\rightarrow$ rewriting $P \rightarrow Q$ as $\neg P \vee Q$
• Use *De Morgan's laws* to push $\neg$ inwards:
• Rewrite $\neg(P \wedge Q)$ as $\neg P \vee \neg Q$
• Rewrite $\neg(P \vee Q)$ as $\neg P \wedge \neg Q$
• Eliminate double negations: rewrite $\neg\neg P$ as P
• Use the *distributive laws* to get CNF:
• Rewrite $(P \wedge Q) \vee R$ as $(P \vee R) \wedge (Q \vee R)$
• Rewrite $(P \vee Q) \wedge R$ as $(P \wedge R) \vee (Q \wedge R)$
Exercise: Convert $\neg(P \rightarrow (Q \wedge R))$ to CNF


IIIT HYDERABAD
Convert ¬(P→(Q∧R))
¬(P → (Q∧R))
¬(¬P ∨ (Q∧R))
¬¬P ∧ ¬(Q∧R)
P ∧ (¬Q∨¬R)