1@ Show that regular languages are Closed under Concatenation.

Sol:-

If L_1, L_2 are regular, $L_1 L_2$ is also regular.

There are 2 DFA's.

$M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$

$M_2 = (R, \Sigma, \delta_2, r_0, F_2)$

$L_1 = L(M_1)$ and $L_2 = L(M_2)$

The new DFA is like

$M_3 = (Q \times R, \Sigma, \delta_3, (q_0, r_0), F_1 F_2)$.

where

$\delta_3((q, r), a) = (\delta_1(q, a), \delta_2(r, a))$

$q \in Q, r \in R, a \in \Sigma$.

For any $x \in \Sigma^*$, $x \in L_1 L_2 \Leftrightarrow$

$\delta_1^*(q_0, x) \in F_1$ and $\delta_2^*(r_0, x) \in F_2$

$\Rightarrow (\delta_1^*(q_0, x), q_2^*(r_0, x)) \in F_1 F_2 \Leftrightarrow$

$\delta_3^*((q_0, r_0), x) \in F_1 F_2 . \Leftrightarrow x \in L_1 L_2$

(b) Show that regular languages are closed under Kleen closure.

- If L_1, L_2 & L_3 are regular, $L_1 L_2 L_3$ is also regular.

$M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$
$M_2 = (R, \Sigma, \delta_2, r_0, F_2)$
$M_3 = (S, \Sigma, \delta_3, s_0, F_3)$.

$L_1 = L(M_1)$, $L_2 = L(M_2)$, $L_3 = L(M_3)$

$M_4 = (Q \times R \times S, \Sigma, \delta_{4}, (q_0, r_0, s_0), F_1 F_2 F_3)$.

$\delta_{4}((q, r, s), a) = (\delta_1(q, a), \delta_2(r, a), \delta_3(s, a))$.

$\delta_4^*((q_0, r_0, s_0), x) \in F_1 F_2 F_3$.
$\Leftrightarrow x \in L_1 L_2 L_3$.

(c) Show that regular Languages are closed under difference.

- If L_1 & L_2 are regular Languages, then $L_1 - L_2$ is also regular.

$M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$

$M_2 = (R, \Sigma, \delta_2, r_0, F_2)$

$L_1 = L(M_1)$ $L_2 = L(M_2)$

$M_3 = (Q \times R, \Sigma, \delta_3, (q_0, r_0), F_1 - F_2)$

$\delta_3((q, r), a) = (\delta_1(q, a), \delta_2(r, a))$

$q \in Q$, $r \in R$, $a \in \Sigma$ $x \in \Sigma^*$ $x \in L_1 - L_2$

$(\delta_1^*(q_0, x), \delta_2^*(r_0, x)) \in F_1 - F_2 \Leftrightarrow$

$\delta_3^*((q_0, r_0), x) \in F_1 - F_2 \Rightarrow x \in L_1 - L_2$

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