

TOC - Assignment - 8.

①. $S \rightarrow aSBC$

$S \rightarrow aBC.$

$CB \rightarrow BC.$

$aB \rightarrow ab.$

$bB \rightarrow bb.$

$bC \rightarrow bc.$

$CC \rightarrow CC.$

Ans $L(G) = ?$

$S \rightarrow (aB)c$

$S \rightarrow ab(c).$

$S \rightarrow abc.$

$S \rightarrow aSBC.$

$S \rightarrow a(aB)BC.$

$S \rightarrow aabBCC.$

$S \rightarrow aabbCK.$

$S \rightarrow aabb(c).$

$S \rightarrow aabbcc.$

$\Rightarrow S = \{a^n b^n c^n \mid n \geq 0\}$

Consider $g = uvwxy$ for $S = abc$.

$U = a, V = b, w = c, x = \epsilon, y = \epsilon.$

will satisfy $|VWX| \leq n$; $1 \leq 1 \Rightarrow \text{True}$

It is satisfied when VWX has only

$a's \rightarrow \text{True}$
 $a's \text{ and } b's \rightarrow \text{False}$
 $b's \rightarrow \text{True}$
 $b's \text{ and } c's \rightarrow \text{False}$
 $c's \rightarrow \text{True}$

Here, for $n=1$, 2 rules are Contradicting with Slide 27 of lecture 18. Because having $a's$ and $b's$ (or) $b's$ and $c's$ in VWX will lead to $|VWX| > n$ which should not happen.

②②. $L_1 = \{a^n b^m c^n d^m \mid m > 0, n > 0\}$

for $n=1$,

$L = abcd$.

This Contradicts with one of the five theories of having $|uvwx| \leq 1$ and also having
 (i) only a's (ii) b's and c's (iii) only b's
 (iv) a's and b's (v) only c's.

Hence, $a^n b^m c^n d^m$

③ $L_2 = \{a^n b^{2n} c^{3n}\}$

$L = abbbccccc$

This will satisfy the rules when considering $|uvwx| \leq n$.

It can also justify rules like only a's, only b's, only c's, $a^2 b^4 c^6$ and b's and c's.

Hence, this language is context free.

$$\textcircled{2} L_3 = \{ a^n b^m c^m d^m \mid m, n > 0 \} \cup \{ a^n b^m c^n d^m \mid m, n > 0 \}$$

Union of 2 Context free languages is always Context free

But, here one of them is Context free and the other is not. So, L_3 is not Context free.

$$\textcircled{3} L = L_1, L_2, \dots, L_n.$$

$$L_j = \text{CF} \text{ if } j = \text{Even.}$$

$$L_j = \text{Regular if } j = \text{odd.}$$

Concatenation of a Context free language and regular language is always Context free.

\Rightarrow if ~~n is odd~~ L is a Context free language.