

CS 5000: Theory of Computation

Assignment 11: Partially Computable & Computable Functions

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Learning Objectives

1. Bounded Existential & Universal Quantification
2. Primitive Recursive Functions
3. Minimalization

Problem 1 (5 points)

Let $sum_of_2_squares(n)$ be the n -th natural number which is the sum of two squares. For example, 0 is the 0-th such number, 2 is the 1st such number, because $1^2 + 1^2 = 2$, 4 is the 2nd such number, because $0^2 + 2^2 = 4$, 5 is the 3rd such number, because $1^2 + 2^2 = 5$, etc. Show that $sum_of_2_squares(n)$ is primitive recursive.

Problem 2 (4 points)

Let $gcd(x, y)$ and $lcm(x, y)$ be the greatest common divisor and the least common multiple, respectively, of x and y . Show that both are primitive recursive.

Problem 3 (3 points)

Let $h(x)$ be the natural number n such that $n \leq (1 + \sqrt{2})x < n + 1$. Show that $h(x)$ is computable.

Problem 4 (3 points)

Show that there exists a computable predicate $P(t, x_1, \dots, x_n)$ such that $\min_t P(t, x_1, \dots, x_n)$ is not computable.