TSKS15 Computer Laboratory Exercise 2

Fall 2024

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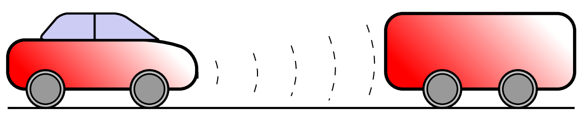
The topic of this computer project is range estimation. The specific task is as follows. A known waveform, $s(t)$, is transmitted, reflected against a target, and received with a delay of T seconds. An example of a situation where this problem arises is the use of radar onboard a car, that measures the distance to a car driving ahead:



The delay, T, is equal to the time of propagation and proportional to the distance to the target. The time-delayed, received waveform is observed in the presence of additive noise, $e(t)$:

$$x(t) = s(t - T) + e(t).$$

The task is to estimate T, based on $x(t)$. We assume that nothing is known a priori about T, except that it is within the range $[-5, 5]$ seconds.

We consider two different choices of waveforms:

$$s_1(t) = \alpha_1 \exp(-0.1t^2)$$

$$s_2(t) = \alpha_2 \exp(-0.1t^2) \cdot \cos(t)$$

where α_1 and α_2 are amplitudes. The noise $e(t)$ is Gaussian, strictly bandlimited to $[-5, 5]$ Hz, and white within that bandwidth.

(a) Plot $s_1(t)$ and $s_2(t)$. Discuss their characteristics. Which one of them should be the most appropriate for range estimation? Why?

(b) For all practical purposes, It is sufficient to consider the received signal, $x(t)$, from $t = -15, \ldots, 15$. Why?

(c) Sample the observed signal, $x(t)$, at $t = n\tau$ where n is an integer and $\tau = 0.1$. Write up a model for the sampled signal. What can be said about the statistics of the noise samples?

(d) Select the amplitudes α_1 and α_2 such that the sampled versions of $s_1(t)$ and $s_2(t)$ have unit energy:

$$\sum_n s_1^2(n\tau) = \sum_n s_2^2(n\tau) = 1$$

A numerical solution is sufficient.

(e) Give the ML estimate, $\hat{T}_{\text{ML}}$, of T.

(f) Denote with SNR the reciprocal noise variance per sample. Determine the CRB for the estimation of T, for $s_1(t)$ and $s_2(t)$, respectively, as function of SNR. A numerical evaluation is sufficient. Plot $\sqrt{\text{CRB}}$ as a function of the SNR in dB for $s_1(t)$ and $s_2(t)$, in the same figure. Use a logarithmic scale on both axes.

(g) Implement on a computer in a language of choice (Python, C++, Matlab/Octave, ...) a Monte-Carlo simulation to determine the RMSE accuracy of $\hat{T}_{\text{ML}}$. For each Monte-Carlo trial, let the true T be uniformly random in the interval $[-5, 5]$ and implement a grid search to find $\hat{T}_{\text{ML}}$. What is an appropriate density of the grid?

Plot the empirical RMSE of the ML estimator for $s_1(t)$ and $s_2(t)$, respectively, versus the SNR, in the same figure as the $\sqrt{\text{CRB}}$. Use an SNR range from 10 to 30 dB, in steps of 1 dB.

(h) Discuss the result. Which signal, $s_1(t)$ or $s_2(t)$, works best for the problem? Why? How much better is it than the other signal, and why? How did you chose the number of Monte Carlo runs?

To get started, the sampled signals can be generated by the following Matlab code:

```
clear all
close all
```

```
Ts = 0.1;
Trange = [-15:Ts:15];

s1 = exp(-0.1*Trange.^2);
s2 = exp(-0.1*Trange.^2).*cos(Trange);

E1 = sum(abs(s1.^2));
E2 = sum(abs(s2.^2));
s1 = s1/sqrt(E1);
s2 = s2/sqrt(E2);
```

Hints

- To access the required software in the computer labs, run the command

  ```
  module initadd courses/TSKS15
  ```

  and then restart. This only needs to be done once.

- Study carefully the slide-set “Introduction to Monte-Carlo Simulation (for TSKS15 students)”.

- How large fraction of the signal energy is contained in the interval $t = -15, \ldots, 15$?

- Suppose white noise is observed through an ideal low-pass filter and then sampled at two time instants t_1 and t_2, such that the resulting samples are uncorrelated. What is the relation between t_1, t_2 and the filter bandwidth?

- You can freely use the lines of Matlab code, or its translation to any language of choice, provided in this document.

- Use the sampled signal for the ML estimation.

- Compute the CRB based on the *samples* of the signal. You can also compute the derivatives numerically (if you wish). No need to evaluate the CRB analytically.

Examination

- Individual oral examination takes place in class (in the computer lab). Please see the course webpage for exact dates, and make sure to sign up.

  To pass the lab examination, you must present a working code that produces relevant the plots. You must also be able to answer questions regarding your implementation and program code, and questions on basic theory related to the CRB and ML estimation.

- Collaboration on this homework in small groups, is encouraged, but each student should write up her/his own program code. Copying of program code from other students, or from previous years' students, or from the Internet is strictly prohibited.