Fibonacci Series

September 23, 2016

1 Fibonacci Number

Fibonacci number are generated by the below equation given $F_0 = 0$, $F_1 = 1$

$$F_{n+2} = F_{n+1} + F_n \tag{1}$$

Instead of doing it recursively, a easy way is use to use Binet's formula

$$\phi = \frac{1+\sqrt{5}}{2} \tag{2}$$

$$\begin{aligned} \tau &= \phi^{-1} && (3)\\ &= \frac{1-\sqrt{5}}{2} && (4) \end{aligned}$$

$$\begin{aligned} F_n &= \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}} && (5)\\ &= \frac{\phi^n - \tau^n}{\sqrt{5}} && (6) \end{aligned}$$

In order to avoid the numerical errors for large values of n, one has to use equation 6 instead of equation 5 for better accuracies.

2 Fibonacci Index

For a large n, τ^n is negligible and hence,

$$n = log_\phi(\sqrt{5}F_n + \epsilon) \tag{7}$$

We have to add ϵ, a very small number to avoid logarithm error when fibnonacci number is zero and also, since we are ignoring the tau term, we have to take the round of the value n to get the index. This logic wont work when Fibonacci number is 0 or 1