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E344 Assignment 1

Gary Allen
23905093

Report submitted in partial fulfilment of the requirements of the module Design (E) 344 for the degree Baccalaureus in Engineering in the Department of Electrical and Electronic Engineering at Stellenbosch University.

August 5, 2022

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Nomenclature

Variables and functions

V_{ss}	Voltage positive.
V_{dd}	Voltage negative.

Acronyms and abbreviations

CMRR	Common Mode Rejection Ratio
RC	Resistor-Capacitor
PWM	Pulse Width Modulation
LPF	Low Pass Filter
HPF	High Pass Filter
TTL	Transistor-Transistor Logic

Chapter 1

Literature review

The following study presents an overview of the current configurations and design techniques for both operational amplifier and current-sensing circuits. Operational amplifier limitations are considered, as well as the system requirements. These findings are then discussed to ensure that there is enough information to aid the design process.

1.1. Operational amplifiers

Limitations

Often, it is applicable to design an amplifier using the ideal operational amplifier model, however this may be inadequate in low voltage, high current or high frequency environments. The following are common limitations of non-ideal op-amps [1]:

- Voltage supply saturation. For given k, output cannot go above $V_{ss} - k$ or below $V_{dd} + k$.
- Finite bandwidth. Output signal magnitude reduces at high frequencies. This effect can be catered for using the gain-bandwidth product equation.
- Offset voltage/bias current. Even with no input, there exists a small "offset voltage" and "bias current" into the amplifier. This results in unwanted voltage at the output.
- Finite slew rate. The output cannot change quicker than a specific rate. This is different to the high-cutoff, but has a similar limiting effect.
- Finite common-mode rejection ratio (CMRR). An op-amp should only amplify $V_+ - V_-$, but also amplifies the unwanted common signal (e.g. noise) on both inputs.

Considerations

System requirements should first be considered in order to attain an overview of the specifications:

- The output range of the circuit should be from 0.1 to 3 V. Since a low-side current sense resistor will be used, this means initial measurements will be at a lower voltage initially.
- The step response of the circuit should reach 90% of its output in less than 100 ms. The op-amp's slew rate should be analyzed to ensure this condition is met.

- Noise above 1 kHz should be filtered out such that a 10 mV, 1 kHz signal at the input is less than 250 mV at the output. Filter options should therefore be researched.

It is clear that voltage supply saturation, slew rate and CMRR are the main parameters to be considered. A filter circuit should also be designed for noise specifications.

MCP6242

The op-amp used in this project is the MCP6242. Listed below are its notable parameters [2]:

- Typical CMMR of 75 dB (DC) to 65 dB (1 kHz).
- Ability to output between 0.035 and 5.465 V if $V_{ss} = 6$ V and $V_{dd} = 0$ V
- Slew rate of 0.3 V/uS.

Configurations

A number of well-known op-amp configurations exist that all achieve slightly different amplification goals. The following list compares common configurations [3] which may achieve this project's goal of amplifying a small-signal voltage across a resistor. The figures below also include a modified non-inverting amplifier to allow for input signal filtering:

Type	Advantages	Disadvantages
Non-inverting	- Simple to design and build - High input impedance	- Large input bias currents - Amplifies noise from input
Differential	- Good noise rejection - Flexible	- Complex design - Low input impedance
Instrumentation	- Same as differential - Very high input impedance	- Complex and expensive design

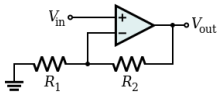

Figure 1.1: Non-inverting amplifier [4]

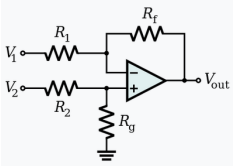

Figure 1.2: Differential amplifier [4]

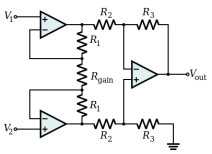

Figure 1.3: Instrumentation amplifier [4]

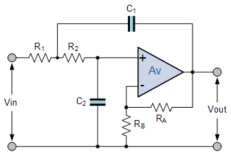

Figure 1.4: Modified non-inverting amplifier [5]

1.2. Current sensing

Sensing Techniques

Measurement techniques are usually either ”invasive” or ”non-invasive”. Invasive techniques tap into the circuit directly and often have a significant affect on its operation, whereas non-invasive techniques might use e.g. the circuit's magnetic flux to determine the strength of the current. A list of a few of these techniques [6] include:

- A current-sensing resistor in series, which uses Ohm's law with a voltage measurement to calculate current.
- Hall element sensors, which determine the strength of current based on how much its magnetic field ”bends” another current left and right (which creates a potential difference).
- Coil techniques, which make using of Faraday's law.

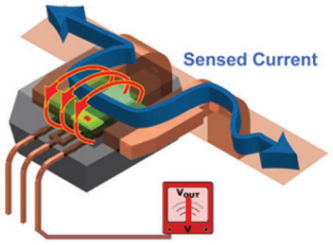

Figure 1.5: Illustration of Hall Effect Sensors

High-side vs low-side sensing

These names of these two techniques refer to the placement of a current-sense resistor relative to the load. For circuits which draw higher currents, high-side sensing can be used (placing the resistor closer to the positive of the voltage source) and is often more convenient. Low-side sensing, on the other hand, can potentially cause ground loop issues [7], but has the ability to detect faults earlier.

AC, DC, and Power Requirements

As mentioned, there are various non-invasive and even wireless techniques used to measure current. Faraday's law is used to make measurements using induction, which requires AC. The Hall effect and sense resistors, on the other hand, are often used in the DC case. Wireless techniques have benefits over resistors in that they can be configured to draw much less power, whereas sense resistors usually need high power handling capabilities as they pass through all current drawn by the actual load.

1.3. Ultrasonic Sensor Interface

The HC-SR04 ultrasonic ranging module will be used. It has 4 pins, namely V_{cc}, *GND, Trig (Trigger)* and *Echo*. It should be powered with 5 V_{dc} and requires at least 15 mA to function, meaning it will dissipate 75 mW of power. First, *Trig* should be pulled high to indicate that the device should send a burst of ultrasonic sound out. Then, if the sound wave is received back, the device will output a distance-proportional pulse on the *Echo* pin.

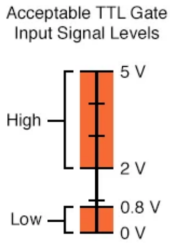

Figure 1.6: TTL Input/Output Levels [8]

The following detailed procedure should be followed to make a distance measurement [3], as also indicated in Figure 1.7:

1. A pulse of at least 10 µs should be output onto *Trig*. This pulse must be TTL compliant as indicated in Figure 1.6 i.e. between 2 V and 5 V.

2. If the sound wave is received back by the sensor, *Echo* will go high (TTL) for t_{high} seconds, where t_{high} is the time it took the signal to return to the sensor. This signal should therefore be conditioned if a circuit which requires 3.3 V is used.

3. The distance can then be calculated. One of the following methods may be used:

   (a) The length of time of the *Echo* output signal can be digitally measured and the distance calculated using $Distance = \frac{t_{high} * v_{sound}}{2}$ (1.1) [3].

   (b) The output pulse can be converted to an analog signal using filtering. This will also be proportional to t_{high}, however a new transfer function should be determined.

4. A minimum of 60 ms should be present in between each pulse. This allows for a theoretical range of $\frac{60\,\text{ms} * 340\,\text{m}\,\text{s}^{-1}}{2} = 10.2\,\text{m}$. In practice, the device has a range of $\approx 4m$.

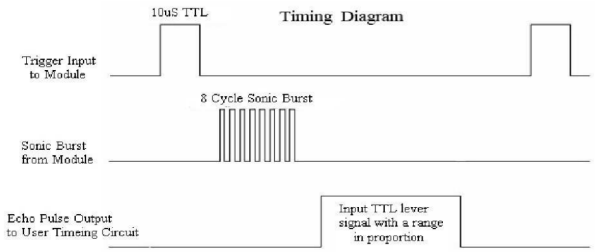

Figure 1.7: Sensor timing diagram [3]

1.4. PWM to Analog Conversion

PWM (Pulse Width Modulation) is useful in that it allows analog information to be conveyed in digital signals. PWM waves are similar to square waves, however differ in that $t_{high} \neq t_{low}$. A PWM signal is defined by 3 properties:

- Frequency: The rate at which the signal oscillates. $f = \frac{1}{t_{high}+t_{low}}$ Hz.
- Amplitude: The difference between "high" and "low" voltages. $A = V_{high} - V_{low}$ V.
- Duty cycle: The ratio of the signal's "high time" to its oscillation period. $\tau = \frac{t_{high}}{t_{high}+t_{low}}$ s.

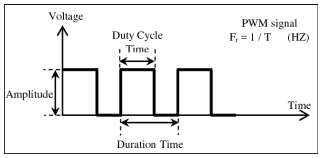

Figure 1.8: PWM Signal Voltage vs Time [9]

Often, this analog information is conveyed in a varying duty cycle (τ). To convert this varying value to analog, a LPF can be used. Since the signal's average increases with τ, filtering out all "harmonics" of the signal will result in $V_{out} = A \times \tau$ (1.2) [10]. The cutoff frequency (f_c) of the filter should be as low as possible, while maintaining an acceptable rise time, in order to minimize ripple on the filter output. Then, since $t_r \propto \frac{1}{f_c}$, $t_{r(max)}$ will determine $f_{c(min)}$. This will be based on the filter used e.g. $f_{c(min)} = \frac{3.3}{2\pi \cdot t_{r(max)}}$ for a 2nd order filter [C.1.1].

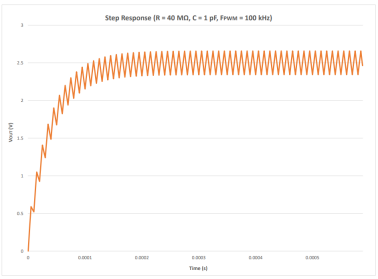

Figure 1.9: Filtered PWM signal

Assuming $f_{c(min)}$ is fixed, $f_{c(max)}$ should be determined by the maximum acceptable noise on the output of the filter. Since filtering will leave a ripple of frequency f_{PWM}, this will be the largest noise component. Given the following:

- A: Amplitude of the PWM signal (V)
- τ_{max}: Maximum duty cycle of the PWM signal (%)
- N_{max}: Maximum acceptable noise of the filtered output (V)

Then the required attenuation at f_{PWM} is given by $A_{dB} = 20 \log \left[\frac{A \cdot \tau_{max}}{N_{max}}\right]$ (1.3). This will determine $f_{c(max)}$ based on the type of filter used.

1.5. Ultrasonic Sensor Fundamental Principles

Chapter 2

Detail design

2.1. Current sensor

Configuration

The differential amplifier configuration will be used for this project for the following reasons:

- The configuration is specifically designed to reject common-mode noise, which the non-inverting amp suffers from.
- Only 1 op-amp is needed.
- The design is relatively simple compared to the instrumentation amplifier.

However, a modified version with two extra capacitors will be used. Two capacitors will be added in parallel with R_f and R_g called C_f and C_g respectively. It is important that these capacitors have the same value in order to preserve CMRR [4]. It is likely, however, that one of these will be dominant due to differing resistor values, and therefore this design will only consider the filter to be first-order. This is acceptable, however, due to the amplifier's noise-rejecting capabilities.

Gain and Cutoff

Before calculating component values, both the cutoff frequency of the RC filter, and the DC gain of the amplifier, need to be determined. The gain of the amplifier should be determined such that, when the maximum current flows through the sense resistor, the maximum voltage is output.

First, the maximum current of the motor should be measured. This can be done by connecting the motor to a power supply (through an ammeter), stalling the motor, and reading the output current. With a supply voltage of 6.4 V connected, the motor for this project read 740 mA, however different motors may read between 1 and 1.5 A. The average running current, however, is between 200 mA and 300 mA. A value of $I_{max} = 1A$ will therefore be chosen to allow for more headroom and a slightly lower gain. Since a sense resistor of $10\,\mathrm{m}\Omega$ is to be used, the maximum voltage through this resistor will therefore be 8 mV. A gain of $\frac{3V}{10mV} = 300$ is therefore chosen. Although this voltage level is small, it is expected, given that the resistance value is so small. It is also desirable, as it shows that the circuit is not "stealing" too much power from the motor itself.

The cutoff frequency f_H should be selected such that the slew rate requirements are not affected, while still adequately filtering a 1 kHz noise signal. In order for the noise requirement to be satisfied, the input signal must be attenuated by $20 \log \frac{300*10mV}{250mV} \approx 22dB$. This means the cutoff should be around 100 Hz (a decade below 1 kHz). Since the specification requires a change from 0.1 to $3 \times 0.9 = 2.7V$ in less than 100 ms, the output should be capable of oscillating at $\frac{1}{50ms} = 20Hz$, which is below the designed cutoff.

Since input current of the circuit needs to be limited below 150 uA, high resistor values should be chosen:

- Assuming $i_n = 0$, choose $\frac{V_{out(max)}}{R_1+R_f} << 150uA \therefore R_1 + R_f >> 20\,\text{k}\Omega$. Choose $R_f = 100\,\text{k}\Omega$.
- $G = \frac{R_f}{R_1} = 300 \therefore R_1 = 330\,\Omega$
- Choose $R_2 = R_1 = 330\,\Omega$ and $R_g = R_f = 100\,\text{k}\Omega$ for differential symmetry.
- Since R_{eq} of C_f is R_f and of C_g is $R_g||R_2$, it is clear that C_f is dominant, $\therefore C_f = \frac{1}{2\pi \cdot R_f \cdot 100Hz} \approx 15nF$. Also set $C_g = 15nF$ for symmetry. It should be noted, however, that this technique (using time constants) is not always valid - especially in complex circuits such as these - and so these results should be checked using simulations.

Circuit diagram

The following figure details the final circuit design for the sensor and amplifier system.

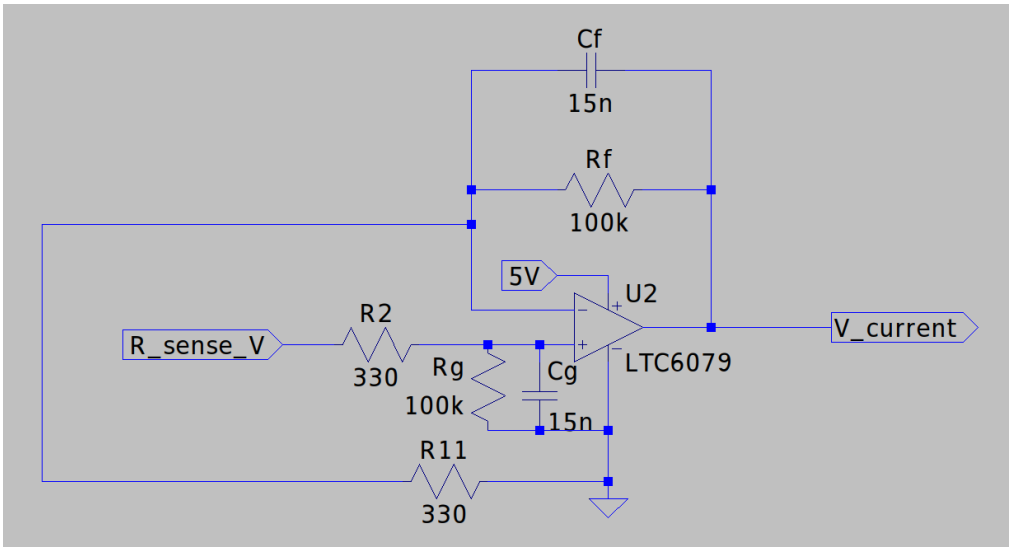

Figure 2.1: Circuit diagram of final configuration

A single 5 V supply will be used. Although there are benefits to a dual rail supply, that would prove impractical in the context of the larger system.

2.2. Range sensor

2.2.1. Sensor power & output

The range sensor requires $5\,V_{dc}$ power and will draw around 15 mA. Since the LD1117V provides a stable 5.0 V output and is capable of providing up to 800 mA of current, the sensor will be powered directly from this regulator.

The sensor's output is a PWM wave of varying duty cycle τ. Since a 16 Hz trigger will be used, the output signal will have $f_{PWM} = 16\,\text{Hz}$. This signal's amplitude could be as high as 5 V (TTL level). The maximum output A_{max} *after* unity filtering, however, will be determined by the maximum duty cycle, t_{max}. Since a distance of $D_{max} = 1\,\text{m}$ and $D_{min} = 5\,\text{cm}$, Equations 1.1 and 1.2 can be used to calculate:

- At D_{max}, $t_{max} \approx 5.9\,\text{ms}$, $\tau_{max} = \frac{5.9\,\text{ms}}{1/16} \approx 9.5\%$ and $A_{max} = (5\,\text{V}) \cdot (9.5\%) = 475\,\text{mV}$.
- At D_{min}, $t_{min} \approx 0.29\,\text{ms}$, $\tau_{min} = \frac{0.29\,\text{ms}}{1/16} \approx 0.47\%$ and $A_{min} = (5\,\text{V}) \cdot (0.47\%) = 23\,\text{mV}$.

2.2.2. Filter selection

To determine the filter's order, rise time and noise requirements should be considered:

- After filtering, since gain $= \frac{3\,\text{V}}{475\,\text{mV}} \approx 7\,\text{V/V}$, filter ripple must be under $\frac{70\,\text{mV}}{7} = 7\,\text{mV}$.
- A 10% to 90% rise time (t_r) of 1.5 s should be adhered to.

These specifications result in the requirement that A_{dB} at $f_{PWM} = 20\log\left[\frac{5\,\text{V}}{7\,\text{mV}}\right] \approx 60\,\text{dB}$.

Filter type	**Design Spec.**	**Cutoff f_c**	**Rise time t_r**	**Attenuation A_{dB}**
1st order	Rise Time	0.233 Hz	1.5 s	37 dB
	Attenuation	0.016 Hz	22 s	60 dB
2nd order	Rise Time	0.350 Hz	1.5 s	66 dB
	Attenuation	0.505 Hz	1.038 s	60 dB
3rd order (cascade)	Rise Time	0.420 Hz	1.5 s	95 dB
	Attenuation	1.600 Hz	0.394 s	60 dB

Table 2.1: Filter type vs Rise Time and Attenuation using [C.1.1]

Referring to 2.1, the 2nd order provides results that will theoretically work, but may be very sensitive to component tolerances. A 3rd order filter will therefore be used in this design.

2.2.3. Circuit configuration

A "two-and-a-half" stage configuration will be used. The first stage will be a 2nd order filter to remove PWM harmonics. The second stage will be a gain and 1st order filter stage. A Sallen-Key topology will be used for stage 1, as in Figure 2.2, and a non-inverting amplifier with filtering will be used for stage 2, as in Figure 1.4.

2.2.4. Filter design

A cutoff frequency of $f_c = 0.6\,\text{Hz}$ will be used, providing $t_r \approx 1.1s$ and $A_{dB} \approx 86dB$ for the combined 3rd order filter [C.1.1]. This results in the 2nd order stage transfer function $H(s) = \frac{1}{1+0.3751s+0.07036s^2} = \frac{1}{1+a_1 s+b_1 s^2}$. Now, component values can be chosen. A number of formulae from [15] will be used:

- The idle current of this stage is very low, since both capacitors are open-circuit during DC. A maximum step input will result in a surge of current from the input through C_2 to ground. With $V_{in(max)} = 5\,\text{V}$, choose $R_1 + R_2 >> \frac{5\,\text{V}}{250\,\mu\text{A}} = 20\,\text{k}\Omega$.
- Since $C_1 = \frac{a_1}{2\pi f_c(R_1+R_2)}$ [15], choose $C_1 = 220\,\text{nF}$ so that $R_1 + R_2 = 450\,\text{k}\Omega$.
- To meet $C_2 \geq C_1 \cdot \frac{4 \cdot b_1}{a_1^2}$ [15] $\approx 2C_1$, choose $C_2 = 470\,\text{nF}$.
- Since $C_1 \cdot C_2 = \frac{b_1}{((2\pi f_c)^2 R_1 R_2)}$, and $R_1 + R_2 = 450\,\text{k}\Omega$ solve to obtain $R_1 = 170\,\text{k}\Omega$ and $R_2 = 280\,\text{k}\Omega$. Choose $R_1 = 180\,\text{k}\Omega$ and $R_2 = 270\,\text{k}\Omega$ as practical values.

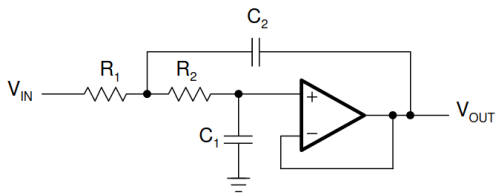

Figure 2.2: 2nd order unity-gain Sallen-Key LPF [15]

2.2.5. Amplifier design

The amplifier circuit will

Chapter 3

Results

3.1. Current sensor

3.1.1. Simulation

After running initial simulations, it was clear that the constant assumed in the design stage for the dominant capacitor C_f was too high. The capacitor value was then experimentally increased to 100nF where a satisfactory response which was reasonably below 250 mV (10-20%) was obtained. This section details the rest of the results of these simulations.

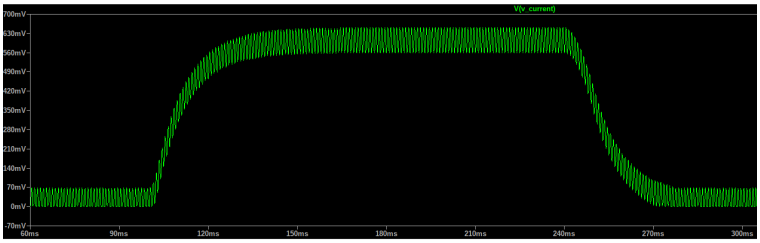

Figure 3.1: Amplifier output in response to step input

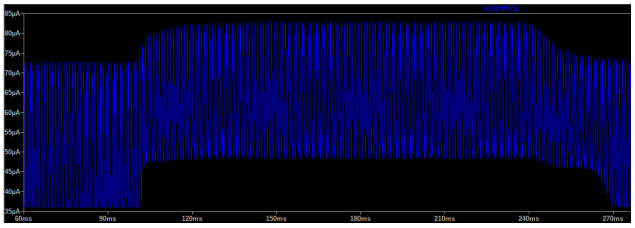

Figure 3.2: Current draw of op-amp

As can be seen, all specifications were adhered to:

- The noise level at idle is well below the 250 mV requirement.
- The step response input changes 20-25 ms, which is below the 100 ms requirement.
- The power draw of the circuit (as measure at the positive terminal of the op-amp) is less than 85 uA, which is much less than the specified 150 uA.

Lastly, the input simulation parameters were modified during testing to analyze the various output voltages for different input currents. For input currents of 400mA, 500mA, 800mA and 1A, the output voltages were 1.22V, 1.52, 2.43 and 3.024 V respectively. This matches perfectly with the initial design.

3.1.2. Implementation

Tests were conducted with a function generator to ensure the amplifier met specifications.

As can be seen in Figure 3.3, the amplifier noise requirement was satisfied. The input signal (channel 1, bottom) is a $10\,\mathrm{mV_{pp}}$ 1 kHz sine wave with 6 mV offset. It is required that a sinusoidal "noise" signal greater than this frequency should not be amplified to more than $250\,\mathrm{mV_{pp}}$. An output of 1.13 V - 0.949 V = 181 mV was obtained - within the threshold.

In Figure 3.4, the amplifier's step response time was recorded as 4.957 ms, which is much below the 100 ms requirement. This time is 4x better than the designed/simulated circuit, presumably due to the increased resistance value and other tolerances.

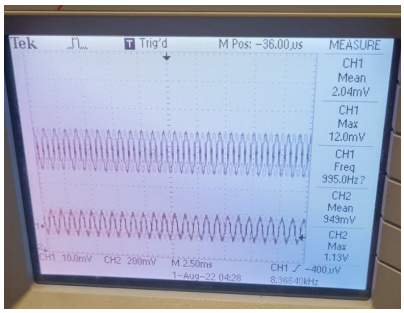

Figure 3.3: Amplifier noise rejection

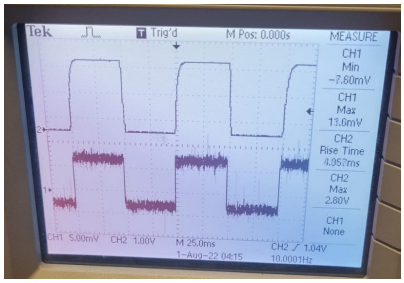

Figure 3.4: Amplifier response time

Current measurements were then made with the motor connected, as visible in Table 3.1. As can be analyzed, there is an offset voltage of 0.42 V at the output. The actual gain can be calculated as e.g. $\frac{1.42\,\mathrm{V}-0.42\,\mathrm{V}}{210\,\mathrm{mA}*10\,\mathrm{m\Omega}} = 476V/V$. With the 120 kΩ resistor, an ideal gain of $\frac{120\,\mathrm{k\Omega}}{330\,\Omega} = 363.6V/V$ was expected, however due to the nature of the circuit (which contains 4 resistors) a tolerance of 10% * 4 should be accounted for, which explains the higher gain.

Motor Condition	PSU Current (mA)	Output Voltage (V)
Stall	1200	3.31
Slight Load	305	1.96
Free Running	210	1.42
I = 150 mA	150	1.13
I = 100 mA	100	0.91
I = 50 mA	50	0.63
I = 0 mA	0	0.42

Table 3.1: Measurements of Motor Current and Voltage

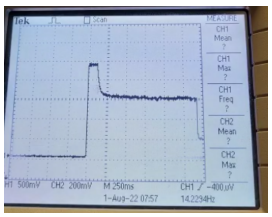

Figure 3.5: 0 A to free running

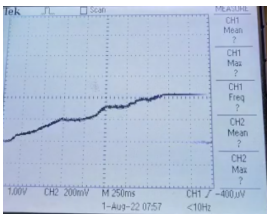

Figure 3.6: Increasing load

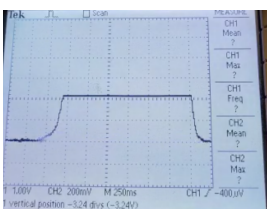

Figure 3.7: Free running to stall

Chapter 4

Physical implementation

4.1. Current sensor

The implementation of the circuit was different in two ways compared to the original design:

- 120 kΩ resistors were used instead of 100 kΩ, as they were easier to obtain, and would only result in a slightly larger gain and lower cutoff.
- The amplifier circuit was powered with regulated 3.3 V. This was done to protect the ESP's input pins by saturating the output when it would have been too large.

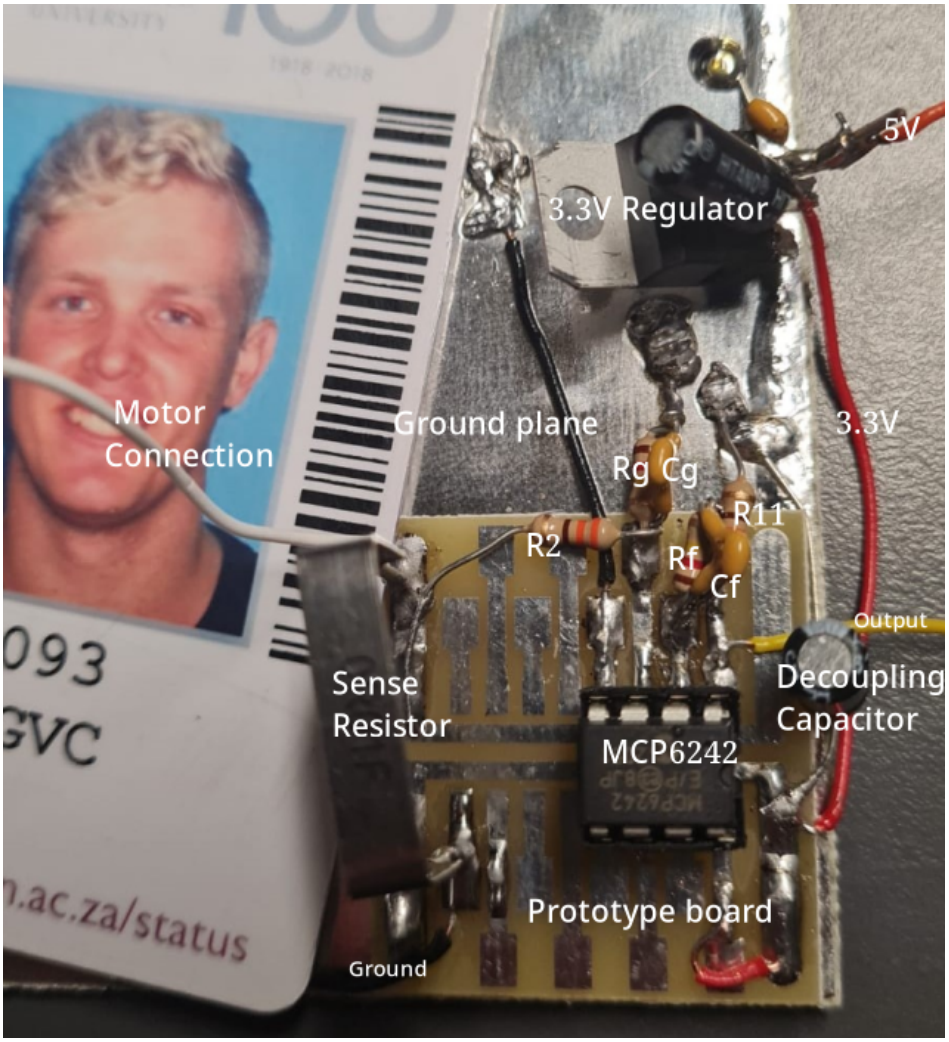

Figure 4.1: Final Circuit Implementation

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Appendix A

Social contract

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E-design 344 Social Contract

2022

The purpose of this document is to establish commitment between the student and the organisers of E344. Beyond the commitment made here, it is not binding.

In the months preceeding the term, the lecturer (Thinus Booysen) and a few paid helpers (Rita van der Walt, Keegan Hull, and Michael Ritchie) spent countless hours to prepare for E344 to ensure that you get your money's worth, that you are enabled to learn from the module, and demonstrate and be assessed on your skills. We commit to prepare the assignments, to set the assessments fairly, to be reasonably available, and to provide feedback and support as best and fast we can. We will work hard to give you the best opportunity to learn from and pass analogue electronic design E344.

I, Gary Allen have registered for E344 of my own volition with the intention to learn of and be assessed on the principals of analogue electronic design. Despite the potential publication online of supplementary videos on specific topics, I acknowledge that I am expected to attend the scheduled lectures to make the most of these appointments and learning opportunities. Moreover, I realise I am expected to spend the additional requisite number of hours on E344 as specified in the yearbook.

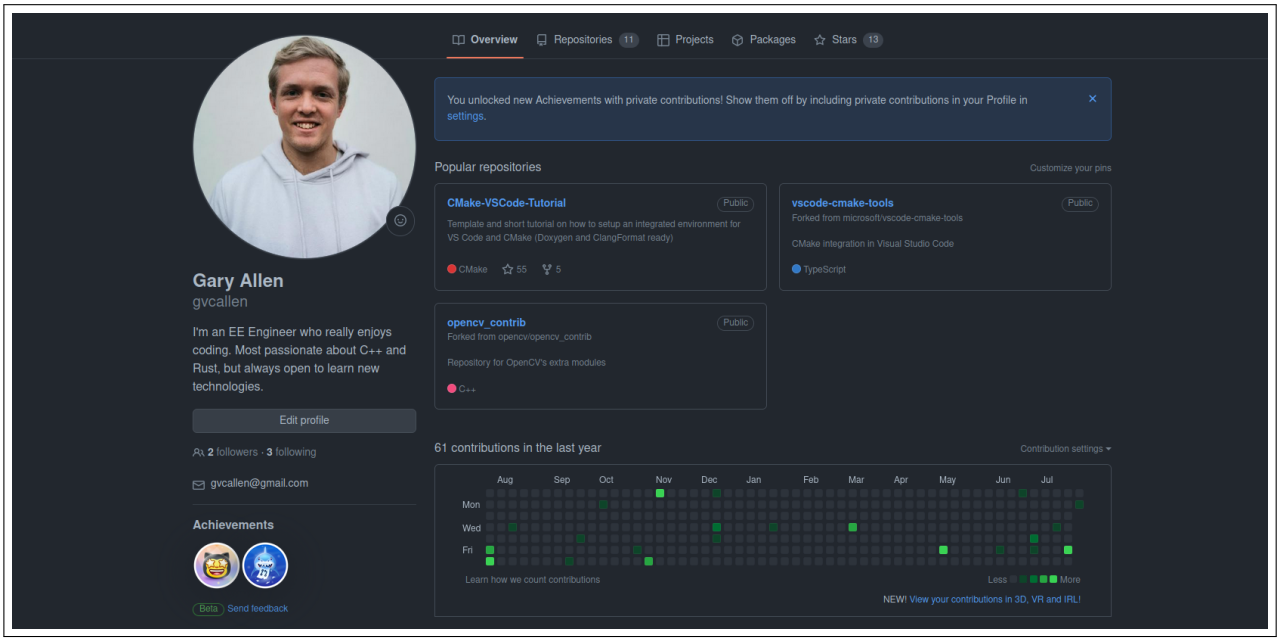
I acknowledge that E344 is an important part of my journey to becoming a professional engineer, and that my conduct should be reflective thereof. This includes doing and submitting my own work, working hard, starting on time, and assimilating as much information as possible. It also includes showing respect towards the University's equipment, staff, and their time.

Prof. MJ (Thinus) Booysen	Student number: 23905093
Signature: MJ Booysen Digitally signed by MJ Booysen Date: 2022.07.02 13:22:09 +02'00'	Signature:
Date: 1 July 2022	Date: 25 / 07 / 2022

1

Appendix B

GitHub Activity Heatmap



Appendix C

Formulae and Derivations

C.1. Filters

C.1.1. Common Filter Formulae

In the following formulae, f_c is the filter's cutoff frequency, $w_c = 2\pi f_c$, and $H(s)$ is the Laplace transfer function of the filter.

For a 1st order filter:

- $H(s) = \frac{w_c}{s+w_c}$
- Attenuation (dB) $= -10\log[1 + (\frac{f}{f_c})^2]$ [10]
- 10% to 90% Rise time, $t_r \approx \frac{2.2}{w_c}$ [11]

For a 2nd order filter:

- $H(s) = \frac{{w_c}^2}{s^2+2\zeta w_c s+{w_c}^2}$
- Attenuation (dB) $= -10\log[1 + (\frac{f}{f_c})^4]$ with $\zeta = 0.707$ [C.2]
- 10% to 90% Rise time, $t_r \approx \frac{3.3}{w_c}$ with $\zeta = 0.707$ [12]

For a 3rd order filter (formed by cascading a 1st and 2nd order):

- $H(s) = \left(\frac{w_c}{s+w_c}\right)\left(\frac{{w_c}^2}{s^2+2\zeta w_c s+{w_c}^2}\right)$
- Attenuation (dB) $= -10\log[1 + (\frac{f}{f_c})^2 + (\frac{f}{f_c})^4 + (\frac{f}{f_c})^6] \approx -60\log\left[\frac{f}{f_c}\right]$ with $\zeta = 0.707$ and $f >> f_c$.
- 10% to 90% Rise time, $t_r \approx \frac{3.97}{w_c}$ with $\zeta = 0.707$, using $t_{ro} = \sqrt{t_{r1}^2 + t_{r2}^2}$ from [14]

C.1.2. 2nd Order Filter Attenuation

For a 2nd order filter with the following transfer function:

$$H(s) = \frac{{w_c}^2}{s^2 + 2\zeta w_c s + {w_c}^2}$$

The attenuation A_{dB} at frequency w can be calculated as follows:

$$\begin{aligned} A_{dB} &= 20\log\left|\frac{w_c^2}{(jw)^2 + 2\zeta w_c(jw) + w_c^2}\right| \\ &= -20\log\left|\frac{(w_c^2 - w^2) + j(w2\zeta w_c)}{w_c^2}\right| \\ &= -20\log\sqrt{\frac{w_c^4 - 2w_c^2w^2 + w^4 + 4\zeta^2 w_c^2 w^2}{w_c^2}} \\ &= -10\log\left[\frac{w_c^4 + w^4}{w_c^4} + \frac{w_c^2 w^2(4\zeta^2 - 2)}{w_c^4}\right] \\ &= -10\log\left[1 + \left(\frac{w}{w_c}\right)^4 + \left(\frac{w}{w_c}\right)^2(4\zeta^2 - 2)\right] \end{aligned} \tag{C.1}$$

For the case when $\zeta = 0.707$ (i.e. when optimally damped), then C.1 simplifies to:

$$A = -10\log\left[1 + \left(\frac{w}{w_c}\right)^4\right] \tag{C.2}$$

C.1.3. 1st and 2nd Order Filter Cutoff vs Attenuation

Given:

- $f_n = f_{\text{cutoff }(n^{th}\text{ order})}$.
- For 1st order, attenuation (dB) @ $f = -10\log[1 + (\frac{f}{f_c})^2]$ [10]
- For 2nd order, attenuation (dB) @ $f = -10\log[1 + (\frac{f}{f_c})^4]$ with $\zeta = 0.707$ [C.2]

A 1st order filter will therefore always attenuate a given frequency f less than a 2nd order filter with $f_1 = f_2$. The cutoff frequency of a 2nd order filter that will attenuate f by the same amount as a 1st order filter with a given cutoff frequency f_1 can be calculated as follows:

$$\begin{aligned} -10\log\left[1 + \left(\frac{f}{f_1}\right)^2\right] &= -10\log\left[1 + \left(\frac{f}{f_2}\right)^4\right] \\ \left(\frac{f}{f_1}\right)^2 &= \left(\frac{f}{f_2}\right)^4 \\ \therefore f_2^4 &= \frac{f^4}{f^2}f_1^2 \\ \therefore f_2^4 &= f^2 f_1^2 \\ \therefore f_2 &= \sqrt{f \cdot f_1} \end{aligned} \tag{C.3}$$