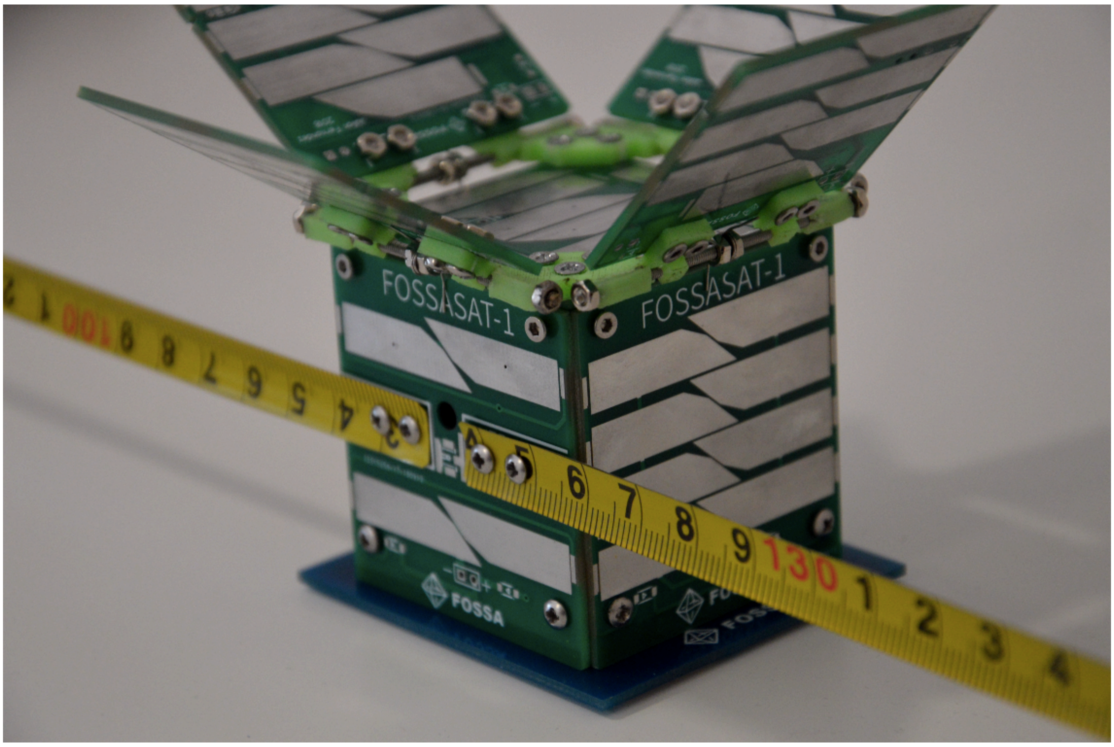
Design of a PocketQube Passive Attitude Control System



FOSSA

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1. Abstract

This paper is intended to document the passive attitude stabilization system design of the FossaSat-1 PocketQube picosatellite developed by Fossa Systems that will launch in Q4 of 2019 creating a global IoT network and testing LoRa communications.

This paper details the various calculations necessary to determine the external disturbances exerted on a satellite while in orbit and thereupon the necessary magnetic and hysteric material to control its attitude in a controlled and predictable manner.

2. Introduction

Recent developments in smartphone electronics in the past decade have hugely facilitated the development of miniaturized satellite components. This overall reduction in satellite size has subsequently hugely lowered the barrier of entry for access to space. It is now cheaper than ever to launch a satellite into space, with prices for nanosatellite launches costing well under EUR 100.000.

The most popular of these is the CubeSat platform, developed in early 2004 by Professor Bob Twiggs at CalPoly University. This nanosatellite form factor is sized at just over 10x10x10cm and usually weighs under 1kg per unit. These small satellites require only a very small investment compared to larger satellites and therefore tend to use simpler and cheaper systems. One system, which is particularly complicated and expensive to develop, is the attitude determination and control system. It is responsible for controlling the orientation of the satellite while in orbit. Typically controlled by a series of magnetorquers and reaction wheels, this system can result in extremely precise pointing accuracy. Its main downside however is its inherent development, electrical and mechanical complexity. For this reason, a new system involving a series of passive magnets was tested for the first time in 1960 aboard the Transit-1B satellite.

The system aligns passively with the electromagnetic field of earth, and thus offers no active control over the satellite during orbit. It can only stabilize 2 axes and performs a full 360º cycle of rotation per polar orbit. Although the system is not active, the position of the magnets can be adjusted to account for the magnetic deviation of earth. A practical example of this would be adjusting the angle of the magnets in an earth imaging satellite so the camera will face a specific point on earth while above it, this however can only be done for one specific point during the orbit and will make the camera spin in an axis.

The system has the advantage of being cheap, light, robust, redundant, simple and consumes no power. Unless a dampener is provided, the satellite will tend to oscillate. For this reason, a damper is usually added to convert the oscillation into heat. Hysteric material tends to be used for Low Earth Orbit satellites.

3. Torque Formulae Analysis

The design of the passive attitude control system is based on the analysis of external disturbances exerted on the satellite. Torque is defined as a quantitative measure of force being exerted on an object causing it to rotate. A satellite is primarily affected by the following torques in order of importance while in a LEO:

1. Aerodynamic torque
2. Magnetic Torque
3. Gravity Gradient Torque
4. Solar Pressure Torque

It is therefore necessary to calculate the maximum exerted torques on our satellite in order to account for them regarding magnet sizing.

All example calculations shown in this paper are intended for an initial orbit of 400km taking in consideration the mass, centre of gravity, centre of aerodynamic pressure and cross sectional area of the FossaSat-1 satellite.

3.1 Aerodynamic Torque Calculation

Aerodynamic torques exerted on the satellite are caused due to the drag of the satellite while travelling through the earth's atmosphere. This force is influenced by the following variables:

- Atmospheric Density
- Spacecraft Geometry
- Centre of gravity

The aerodynamic torque is defined as:

$$T_a = F \cdot (C_p - C_g) = F \cdot L$$

Where, $F = 0.5[\rho C_d A V^2]$

Equation 1 - Aerodynamic Force

And the parameters are defined as:

T_a = aerodynamic torque ($N \cdot m$)
F = aerodynamic force (N)
ρ = atmospheric density ($kg \cdot m^3$)
C_d = drag coefficient
A = cross sectional area (m^2)
V = satellite velocity (m/s)
C_p = center of aerodynamic pressure (m)
C_g = center of gravity (m)

We define the variables as:

ρ = dependent on altitude
$C_d = 2.2$
$A = 0.0065\ m^2$
$V_{circular} = 7670$ m/s

Circular orbital velocity is calculated from:

$$V_{circular} = \sqrt{\frac{G \cdot M}{R}} = \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 5.972 \cdot 10^{24}}{R}}$$

Equation 2 - Orbital Velocity

Where G is the gravitational constant.

$C_p - C_g = 0.03$ m (this is a worst case approximation based on the fact that our deployable solar panels displace the center of aerodynamic torque to be above that of the center of mass)

Therefore from equation 1 we calculate:

$$F = 0.5 \cdot [\ (\rho\ ) \cdot (2.2) \cdot (0.0065\ m^3) \cdot \left(V \frac{m}{s}\right)^2]$$

Since ρ is largely dependent on altitude, it is necessary to plot a graph.

Therefore given the ρ (atmospheric density) at each altitude:

$$T_a = 0.5 \cdot [\ (\rho\ ) \cdot (2.2) \cdot (0.0065\ m^3) \cdot \left(V \frac{m}{s}\right)^2] \cdot (0.03\ m)$$

3.2 Magnetic Torque Calculation

Magnetic torques that act on our satellite will result from the interaction of the spacecraft's residual magnetic field and the geomagnetic field. This torque is defined as:

$$T_m = D \times B$$

Equation 3 - Magnetic Torque

Where,

D = Residual dipole of the satellite ($a \cdot m^2$)

B = $\frac{M}{R^3}$ for an equatorial orbit with 0 inclination, and

B = $\frac{2M}{R^3}$ for a polar orbit with 90º inclination.

Therefore, B = $\frac{2.135 \cdot M}{R^3}$ for a 96º sun synchronous orbit by linear interpolation.

Equation 4 - Magnetic Field

M is the magnetic moment of Earth measured as: M = $7.96 \cdot 10^{15}$ ($tesla \cdot m^2$)
R = the radius from the center of earth to the satellite in m.
D = 1 $a \cdot m^2$ (common value for an uncompensated small-sized satellite)

Therefore given R (orbital radius):

$$T_m = 1\ (a \cdot m^2) \times \frac{2.135 \cdot (7.96 \cdot 10^{15}\ (tesla \cdot m^2))}{R^3}$$

3.3 Gravity Gradient Torque

Earth's gravitational field acting on an object causes gravity gradient torque, this force gets stronger as the distance between earth and the satellite decreases.

This torque is defined as:

$$T_g = \frac{3 \cdot G \cdot M}{2R^3} \left|I_y - I_z\right| \sin 2 \cdot \theta$$

Equation 5 - Gravity Gradient Torque

Where,
T_g = Gravity torque ($N \cdot m$)
R = Orbital Radius (m)
I_z , I_y = Spacecraft moment of inertia
G = Gravitational Constant
M = Mass of Earth
θ = Angle of deviation of the Z axis from local vertical

The moments of inertia for each axis where found to be:

θ is assumed to be 45º in a worst-case scenario.

Therefore given R:

$$T_g = \frac{3 \cdot 6.67 \cdot 10^{-11} \cdot 5.972 \cdot 10^{24} kg}{2 \cdot R^3} \cdot \left| \begin{matrix} 2.604 \cdot 10^{-4} \\ -1.042 \cdot 10^{-4} kg \cdot m^2 \end{matrix} \right| \cdot \sin(90º)$$

3.4 Solar Pressure Torque

Radiation proceeding from the sun exerts a force on the receiving surface. This force depends on the distance from the sun, the surface area of incidence and the specific solar activity.

This force is defined as:

$$T_{sp} = F(C_{ps} - C_g)$$

$$F = \frac{F_s}{C} A_s (1 + q) \cos \alpha$$

Equation 6 - Solar Pressure Torque

Where,

T_{sp} = Solar Pressure Torque
F_s = Solar Constant (1367 W/m^2)
C = Velocity of Light (3 x 10^8 m/s)
q = Surface Reflectance [0-1]
C_{ps} = Centre of Solar Pressure
C_g = Centre of Gravity
α = Angle of incidence to the sun
A_S = Surface Area (m^2)

The parameters are set for worst-case conditions as follows:
A_s = 0.0065 m^2
q = 0.6
α = 0 (Direct incidence)
C_{sp} - $C_g \cong$ 0.02 m

Therefore based on the previous equations:

$$T_{sp} = \frac{1367}{3 \cdot 10^8} \cdot 0.0065 \cdot (1 + 0.6) \cos 0 \cdot 0.02$$

$$T_{sp} = \ 5.924 \cdot 10^{-10} \cdot m$$

This value is negligible compared to other external disturbances and is not affected by orbital altitude, thus we can simply take a constant.

3.5 Total Torque

The total torque exerted on the spacecraft can simply be calculated by adding up all external disturbances for a given altitude

$$T_t = T_{sp} + T_g + T_m + T_a$$

Equation 7 - Total Torque

4. Torque Graphing

All external disturbances besides the solar pressure are dependant on altitude; it is therefore necessary to plot a graph. Based on the various data points at different altitudes, we can determine the total torque required from the magnet/s to overcome external disturbances.

4.1 Aerodynamic Torque

The following calculations are based on the COSPAR INTERNATIONAL REFERENCE ATMOSPHERE (CIRA-2012) taking in consideration mean solar and geomagnetic activities. All results are calculated using equations 1 & 2.

Altitude (km)	Density (kg/m3)	Orbital Velocity (m/s)	Aerodynamic Torque (N*m)
100	5.73E-07	7845.80	9.64E-07
120	2.03E-08	7833.70	3.41E-08
140	3.44E-09	7821.66	5.77E-09
160	1.20E-09	7809.68	2.01E-09
180	5.46E-10	7797.74	9.13E-10
200	2.84E-10	7785.87	4.74E-10
220	1.61E-10	7774.05	2.68E-10
240	9.60E-11	7762.28	1.60E-10
260	5.97E-11	7750.56	9.93E-11
280	3.83E-11	7738.90	6.36E-11
300	2.52E-11	7727.29	4.18E-11
320	1.69E-11	7715.73	2.80E-11
340	1.16E-11	7704.23	1.92E-11
360	7.99E-12	7692.77	1.32E-11
380	5.60E-12	7681.37	9.23E-12
400	3.96E-12	7670.02	6.52E-12

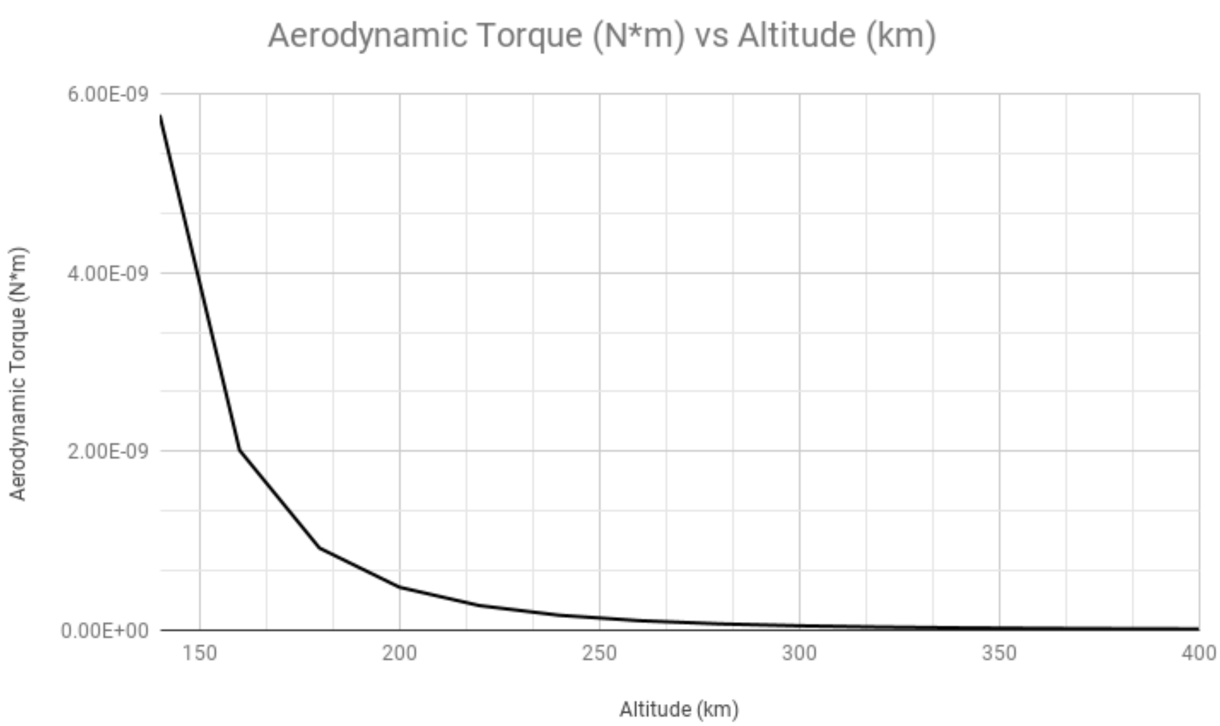

Figure 1 - Aerodynamic Torque vs Altitude

4.2 Magnetic Torque

Taking in consideration a mean magnetic moment of Earth measured as: M = $7.96 \cdot 10^{15}\ tesla\ \cdot m^2$, the following values were calculate using equation 3 & 4.

Altitude (km)	Magnetic Torque (N*m)
100	6.27E-05
120	6.21E-05
140	6.16E-05
160	6.10E-05
180	6.04E-05
200	5.99E-05
220	5.94E-05
240	5.88E-05
260	5.83E-05
280	5.78E-05
300	5.72E-05
320	5.67E-05
340	5.62E-05
360	5.57E-05
380	5.52E-05
400	5.47E-05

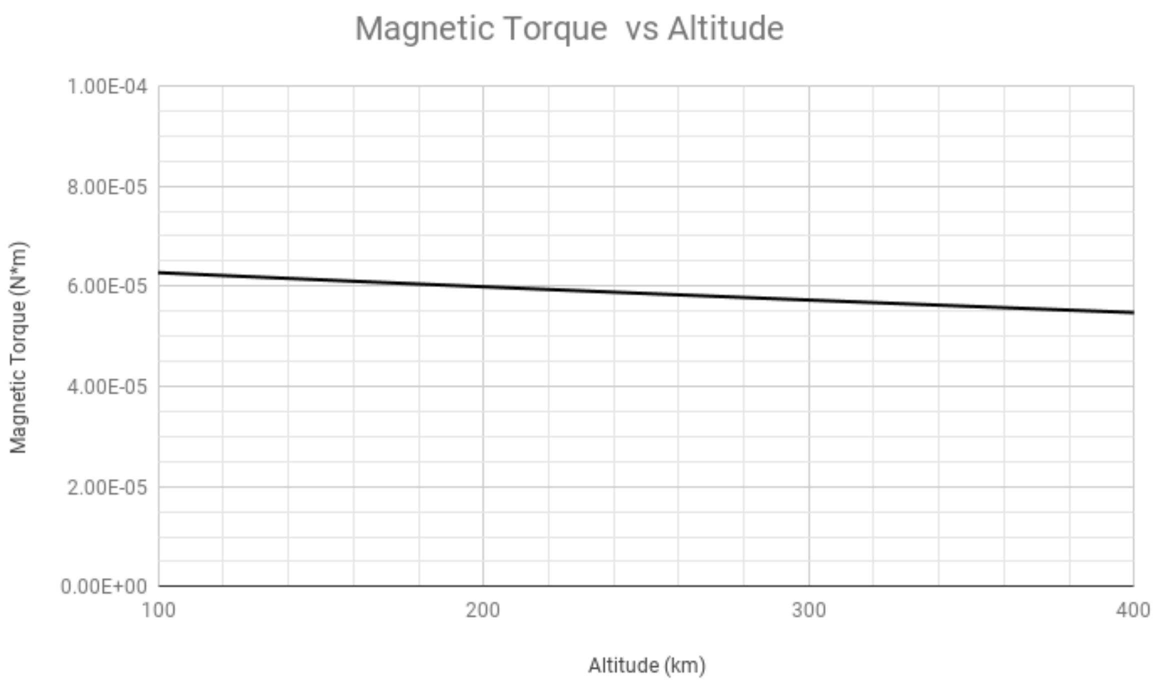

Figure 2 - Magnetic Torque vs Altitude

4.3 Gravity Gradient Torque

Based on equation 5 the following values are calculated.

Altitude (km)	Gravity Torque (N*m)
100	3.44E-10
120	3.41E-10
140	3.38E-10
160	3.35E-10
180	3.32E-10
200	3.29E-10
220	3.26E-10
240	3.23E-10
260	3.20E-10
280	3.17E-10
300	3.14E-10
320	3.12E-10
340	3.09E-10
360	3.06E-10
380	3.03E-10
400	3.01E-10

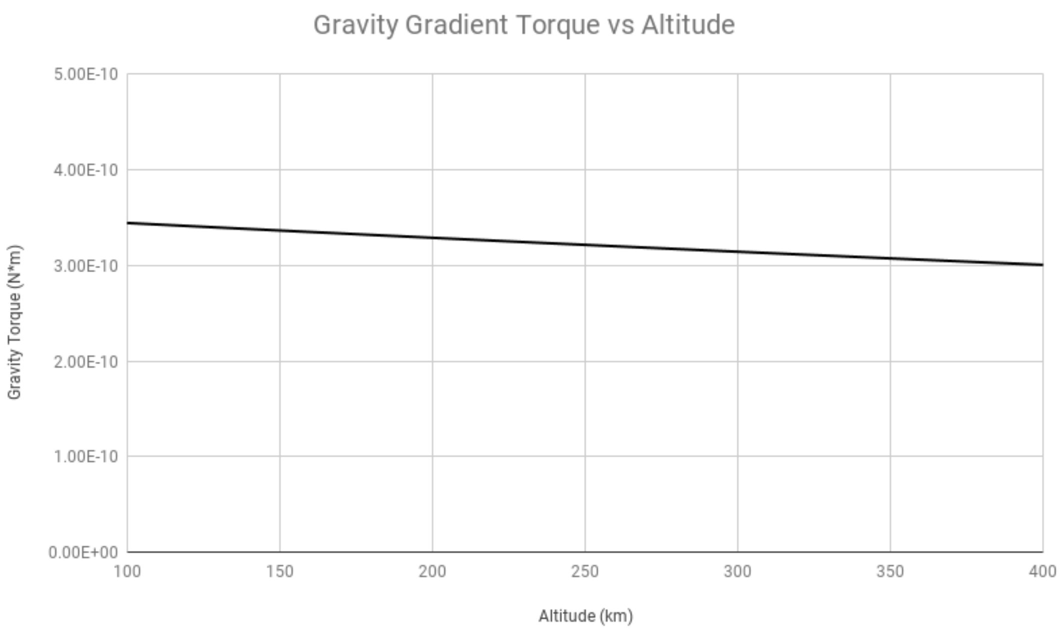

Figure 3 - Gravity Gradient Torque vs Altitude

4.4 Total External Disturbances

Following equation, all external disturbances are added up for each corresponding altitude. One thing we can observe is the insignificance of solar pressure and gravity gradient torque on the satellite in contrast with the magnetic and aerodynamic torque. Magnetic torque evolves in a linear manner in contrast with the exponential growth of aerodynamic torque on the satellite.

Altitude (km)	Solar Pressure Torque (N*m)	Gravity Torque (N*m)	Magnetic Torque (N*m)	Aerodynamic Torque (N*m)	Total Torque (N*m)
100	5.92E-10	3.44E-10	6.27E-05	1.01E-02	1.02E-02
120	5.92E-10	3.41E-10	6.21E-05	3.56E-04	4.18E-04
140	5.92E-10	3.38E-10	6.16E-05	6.02E-05	1.22E-04
160	5.92E-10	3.35E-10	6.10E-05	2.09E-05	8.19E-05
180	5.92E-10	3.32E-10	6.04E-05	9.50E-06	6.99E-05
200	5.92E-10	3.29E-10	5.99E-05	4.92E-06	6.48E-05
220	5.92E-10	3.26E-10	5.94E-05	2.78E-06	6.21E-05
240	5.92E-10	3.23E-10	5.88E-05	1.65E-06	6.05E-05
260	5.92E-10	3.20E-10	5.83E-05	1.03E-06	5.93E-05
280	5.92E-10	3.17E-10	5.78E-05	6.56E-07	5.84E-05
300	5.92E-10	3.14E-10	5.72E-05	4.30E-07	5.77E-05
320	5.92E-10	3.12E-10	5.67E-05	2.88E-07	5.70E-05
340	5.92E-10	3.09E-10	5.62E-05	1.97E-07	5.64E-05
360	5.92E-10	3.06E-10	5.57E-05	1.35E-07	5.59E-05
380	5.92E-10	3.03E-10	5.52E-05	9.45E-08	5.53E-05
400	5.92E-10	3.01E-10	5.47E-05	6.66E-08	5.48E-05

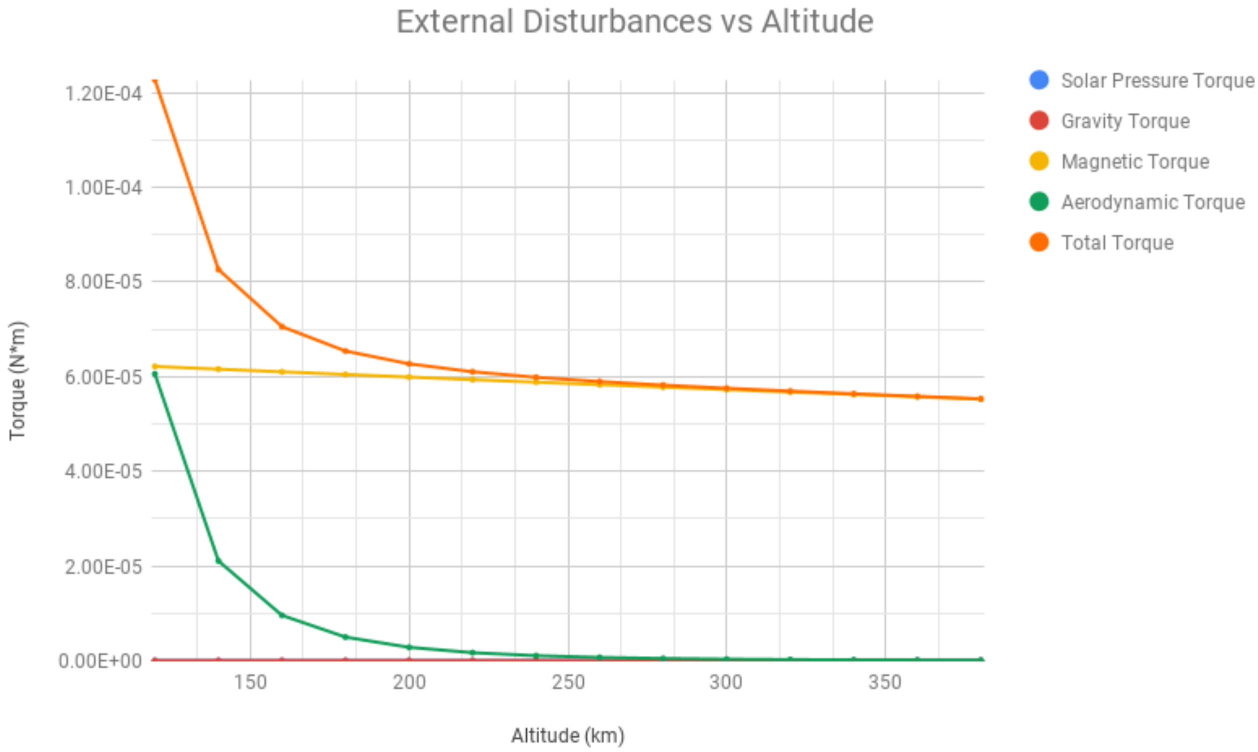

Figure 4 - Total Torque Graph

5. Magnet Size Calculation

Depending on the material and volume of the permanent magnet, its magnetic moment will differ. It is therefore necessary to calculate the specific volume of magnet needed to overcome the external disturbances at a specific altitude.
Since permanent magnets come in a limited amount of shapes and sizes, we will approximate the result and reverse calculate at which altitude the system will be overcome by external disturbances.

Coercivity is defined as the material resistance to becoming demagnetized; remanence is defined as the strength of the magnetic field.

The magnetization of a magnet is given by: $M = \frac{Br}{\mu o}$

Equation 8 - Magnetization

Where:
Br = Remanence (Tesla)
$\mu_o = 1.257 \cdot 10^{-6}$

The magnetic moment of a magnet is defined as: $m = M \cdot V$

Equation 9 - Magnetic Moment

Where:
V = Volume (m^3)

We will be using N35 rare earth neodymium magnets, which have the following properties:

Density (g/cm^3)	Coercivity (A/m)	Remanence (Tesla)
7.3	1.500 E6	1.3

Therefore the magnetism of one magnet is given by:

$$m = \frac{Br\,V}{\mu o} \Rightarrow m = \frac{1.2 \cdot 1.5 \cdot 10^{-7}}{1.257 \cdot 10^{-6}} \Rightarrow 1.432\ Am^2$$

Equation 10 - Magnetism

Torque is then given by: $T = m \cdot B$

Equation 11 - Torque

Where:

B = Magnetic Flux Density (Tesla)
M = Magnetization

Taking in consideration that the magnetic flux density varies by altitude, it is necessary to plot a graph:

Altitude (km)	Minimum B (T)	Torque (Nm)
200	2.09E-05	5.97E-05
220	2.07E-05	5.93E-05
240	2.05E-05	5.87E-05
260	2.04E-05	5.84E-05
280	2.02E-05	5.79E-05
300	2.01E-05	5.76E-05
320	1.99E-05	5.70E-05
340	1.98E-05	5.67E-05
360	1.97E-05	5.64E-05
380	1.96E-05	5.61E-05
400	1.95E-05	5.58E-05

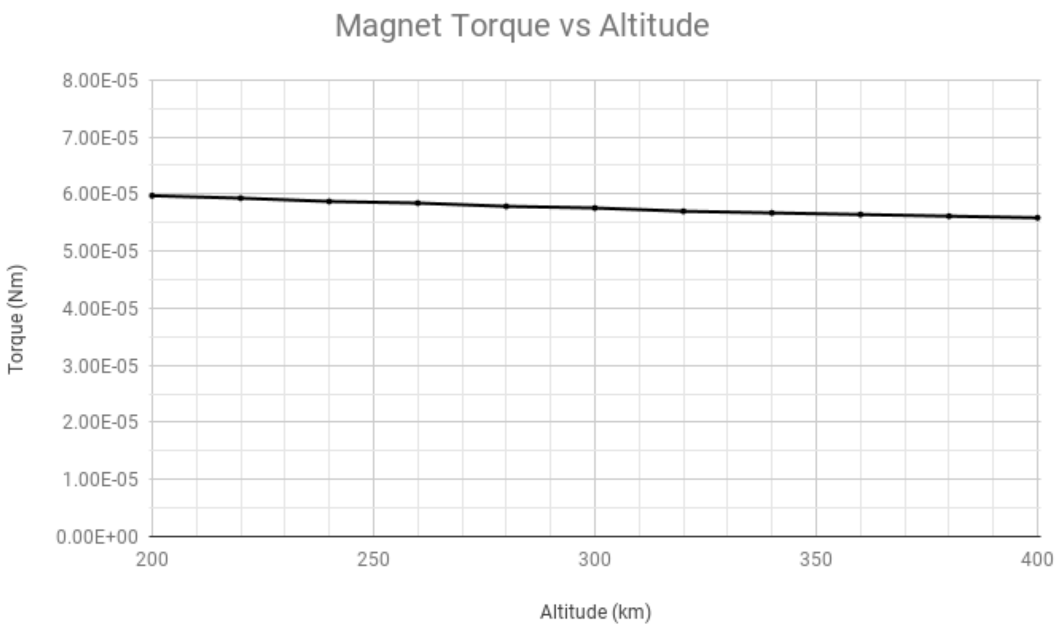

Figure 5 - Magnet Torque vs Altitude

By simply consulting the total external disturbance graph we can determine that 2 magnets would be needed in order for the torque to be greater than the external disturbances down to an altitude of approximately 280 kilometres. It is important not too have too strong of a magnet as this will have to be compensated by the hysteric material to avoid oscillations while in orbit.

In our case we consulted our orbital decay predictions, here we can observe the satellite will be above 280km for 90% of its orbit. Taking this in consideration, we used enough magnetic material to overcome these disturbances down to approximately this altitude.

DOCUMENT NOT FINISHED, HYSTERIC MATERIAL CALCULATION TO BE DEFINED.