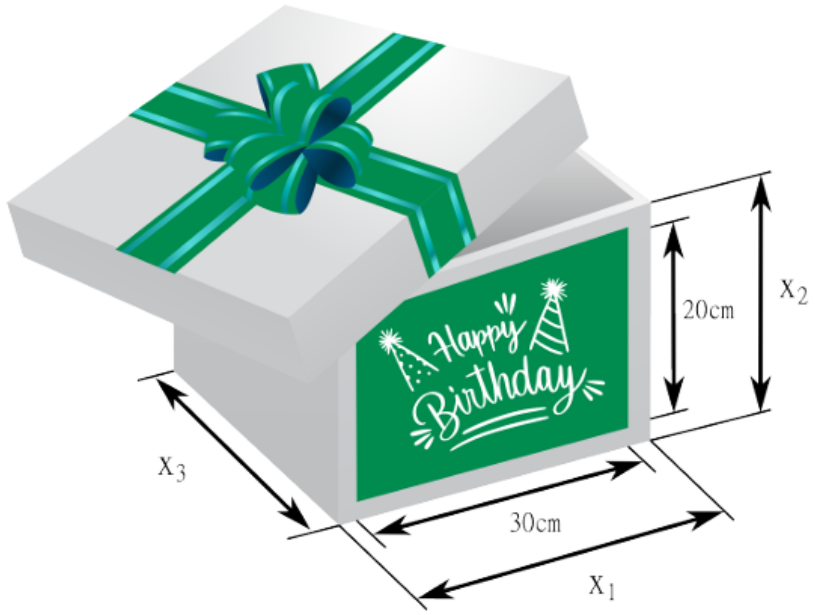

a) $A = 2x_1 \cdot x_2 + 2x_3 \cdot x_2 + x_1 \cdot x_3$

$\min A$

subject to

$x_1 \geq 30$ cm

$x_2 \geq 20$ cm

$x_1 \cdot x_2 \cdot x_3 = 25\,000$ cm^3

$x_3 > 0$ cm

$A \in \mathbb{R}^2$;

$x_1, x_2, x_3 \in \mathbb{R}$.

b) $x_3 = \frac{25\,000}{x_1 \cdot x_2}$

$$\Rightarrow A = 2 \cdot x_1 \cdot x_2 + \frac{2 \cdot 25000}{x_1} + \frac{25000}{x_2}$$

$$A = 2x_1x_2 + 25000\left(\frac{2}{x_1} + \frac{1}{x_2}\right)$$

$$A = 2x_1x_2 + 25000\left(\frac{2x_2 + x_1}{x_1 x_2}\right)$$

$$\rightsquigarrow \frac{25000}{x_1x_2} > 0$$

New problem:

$$\min A \in \mathbb{R}^2$$

subject to.: $x_1 \geq 30\,cm \in \mathbb{R}$

$x_2 \geq 20\,cm \in \mathbb{R}$

$\frac{25000}{x_1 x_2} > 0 \rightsquigarrow$ already satisfied!

c) $F(x) = A \in \mathbb{R}^2$

$g_1(x) = 30 - x_1 \leq 0 \in \mathbb{R}$

$g_2(x) = 20 - x_2 \leq 0 \in \mathbb{R}$

Inserting quadratic penalty functions:

$\tilde{F}(x,\sigma) = F(x) + \sum_{i=1}^{m} \frac{\sigma_{h,i}}{2}((h_i(x))^2 + \sum_{i=1}^{p} \frac{\sigma_{g,i}}{2}(\max\{g_i(x),0\})^2 ; \sigma_i > 0$

$$\tilde{F}(x,\sigma) = F(x) + \frac{\sigma_{g1}}{2}\left(\max\{30-x_1,0\}\right)^2 + \frac{\sigma_{g2}}{2}\left(\max\{20-x_2,0\}\right)^2$$

$\sigma_{g_1}, \sigma_{g_2} > 0.$

New problem:

$$\min \tilde{F}(x) \in \mathbb{R}^2$$

$$\tilde{F}(x) = 2x_1x_2 + 25000\left(\frac{2x_2 + x_1}{x_2 x_1}\right) + \frac{\sigma_{g1}}{2}\left(\max\{30-x_1,0\}\right)^2 + \frac{\sigma_{g2}}{2}\left(\max\{20-x_2,0\}\right)^2$$