





What do you see?





# Interactions



# Interactions

cow eats grass





(co)Evolution



# (co)Evolution

cow's stomach digest grass



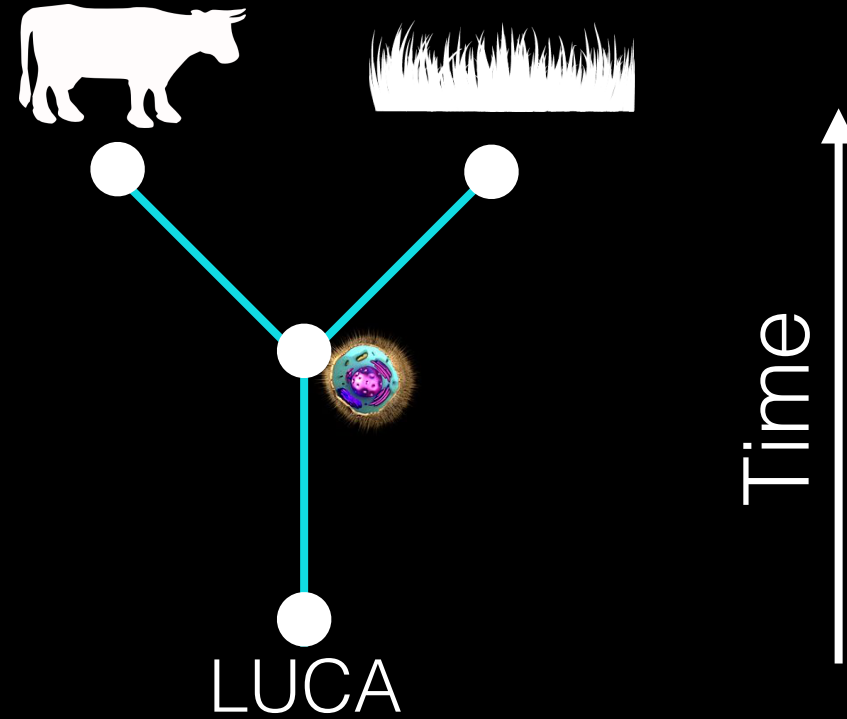


How do you write it?

(if you are a mathematician)

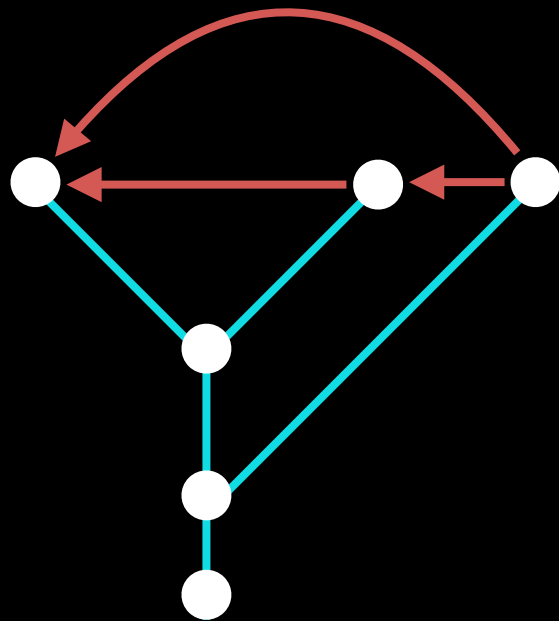




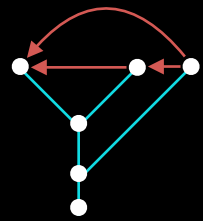


# the Web and the Tree

on the interplay between ecological processes and  
evolutionary histories







# the Web and the Tree

on the interplay between ecological  
processes and evolutionary histories

Giulio Valentino Dalla Riva

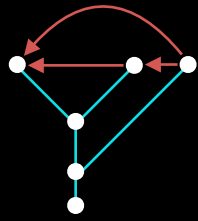
Supervisors:

Mike Steel

Charles Semple

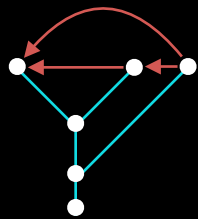
Daniel Stouffer

University of Canterbury 31 March 2016



1. Random Dot Product Graphs (Chap. 4)
2. Centrality & Uniqueness (Chaps. 5 & 7)
3. Niche Evolution and Diversity (Chap. 6)





# RDPG





SYNTHESISING  
ECOLOGY

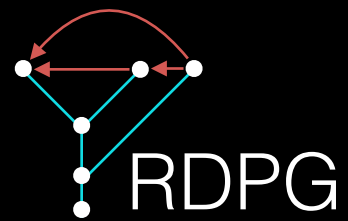
Research

**Exploring the evolutionary signature of food webs' backbones using functional traits**

Giulio V. Dalla Riva<sup>1</sup> and Daniel B. Stouffer

Issue



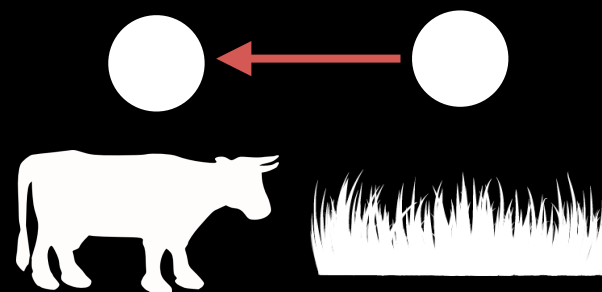
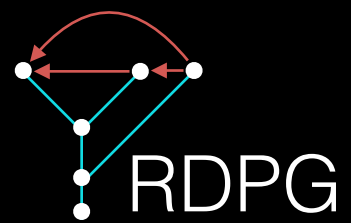


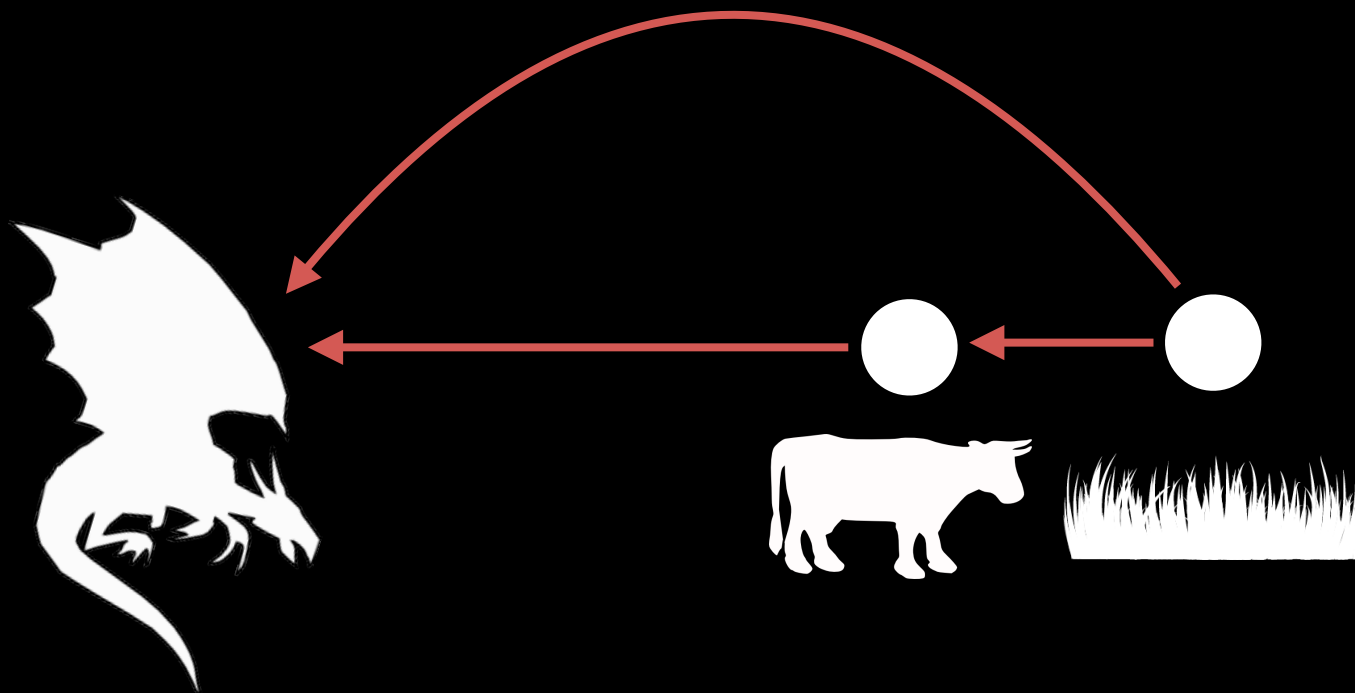
Did evolution leave a trace  
in the structure  
of ecological networks?



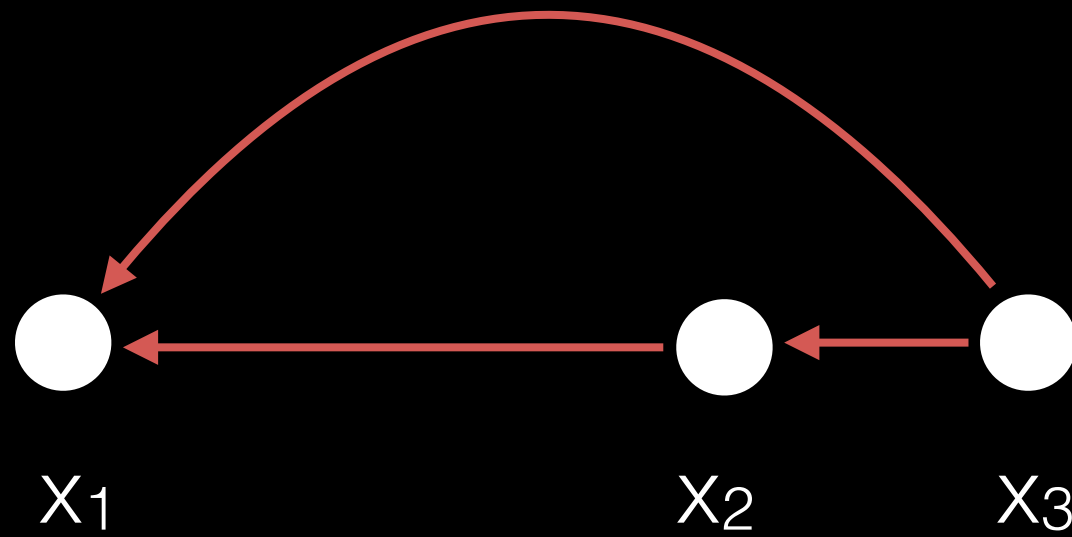
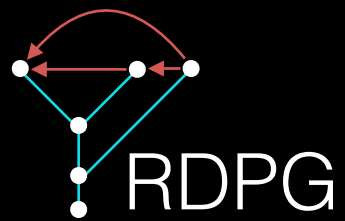
How do we do  
phylogenetic comparative analysis  
with food webs?

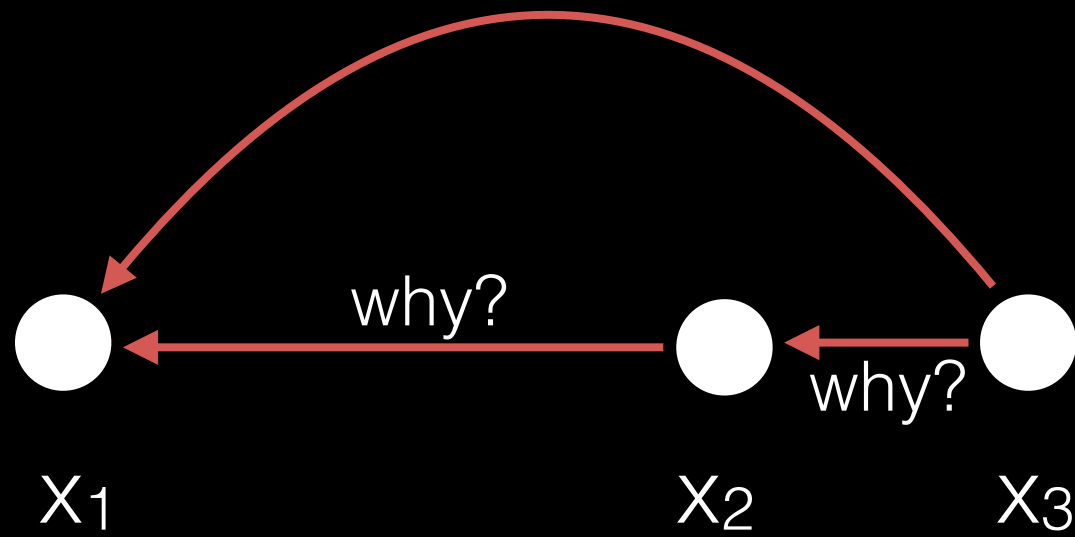


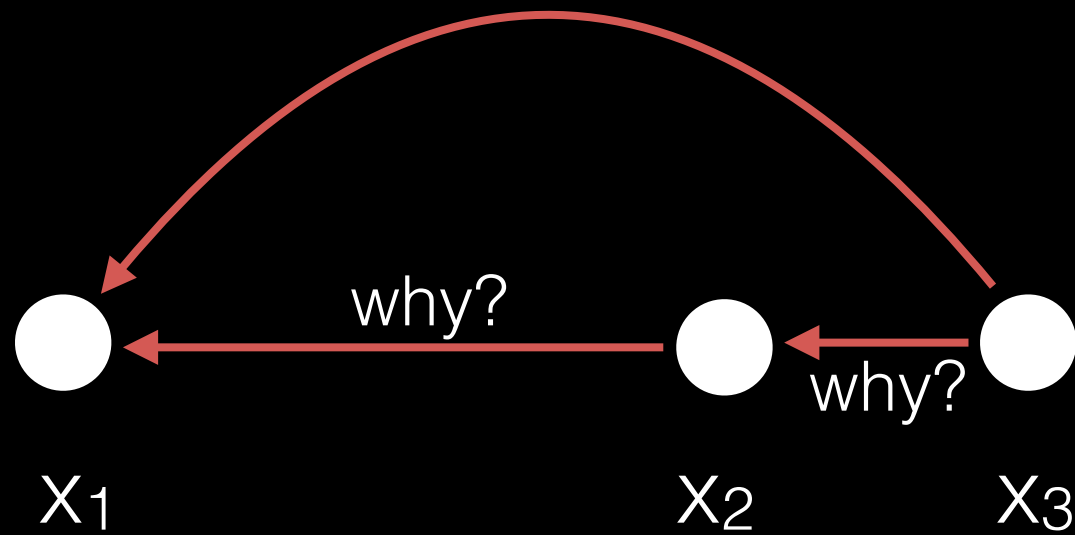
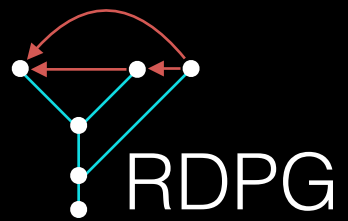






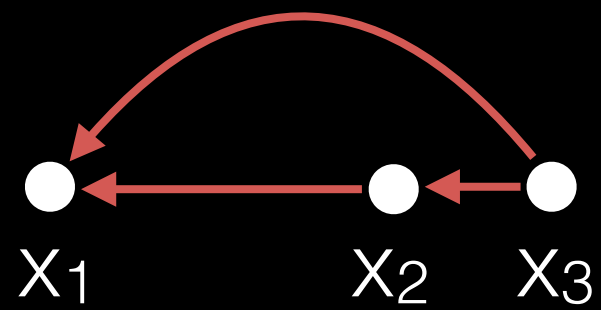
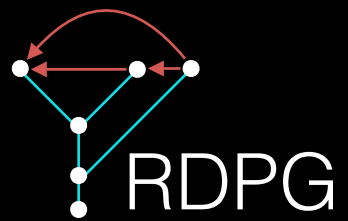






Because *TRAITS*

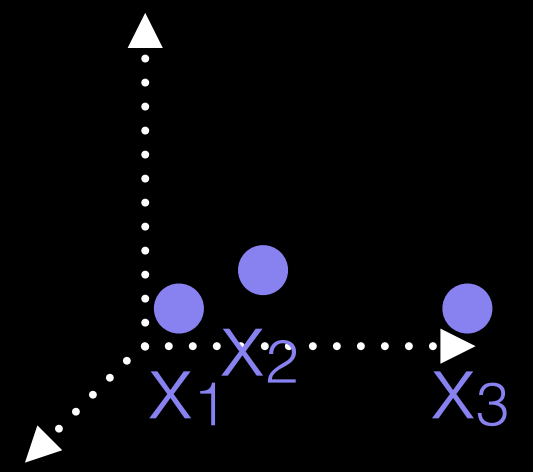
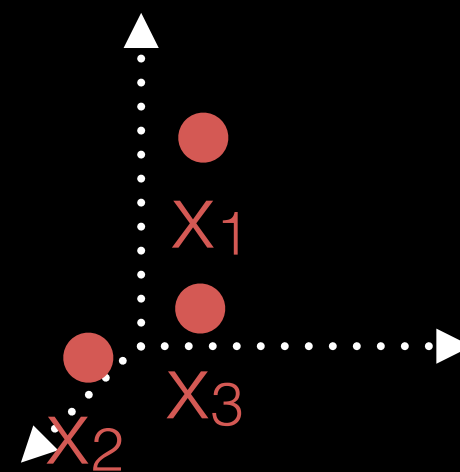


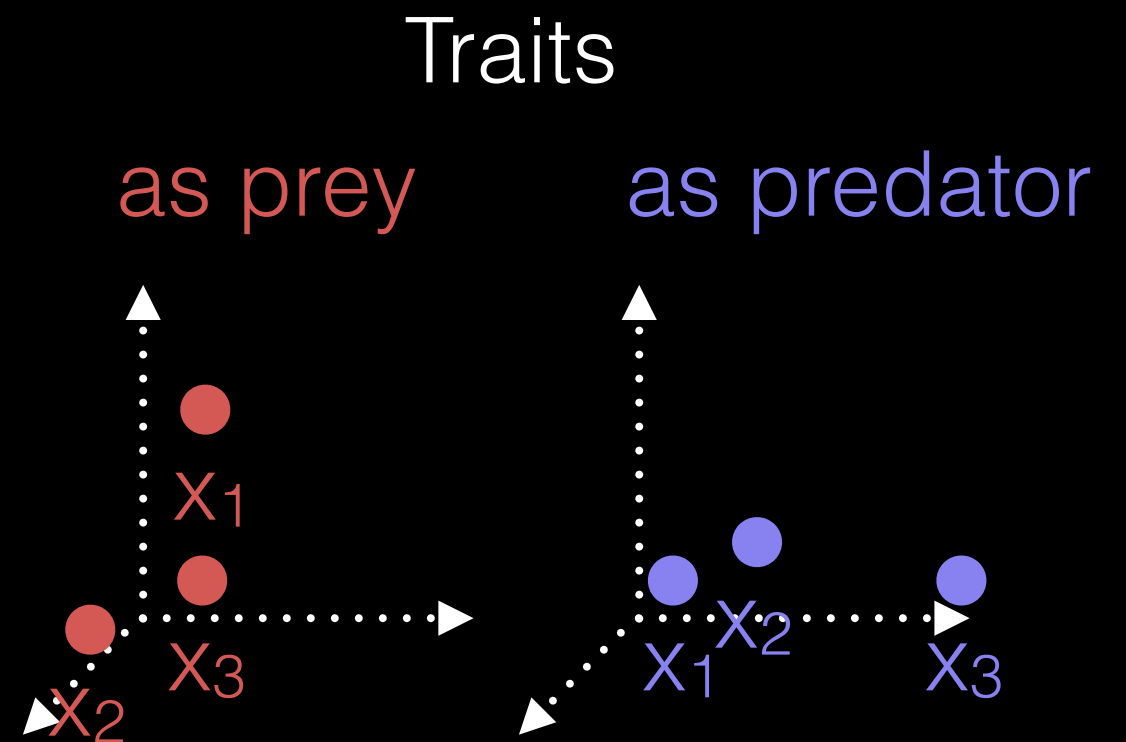
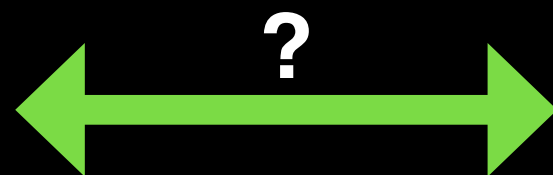
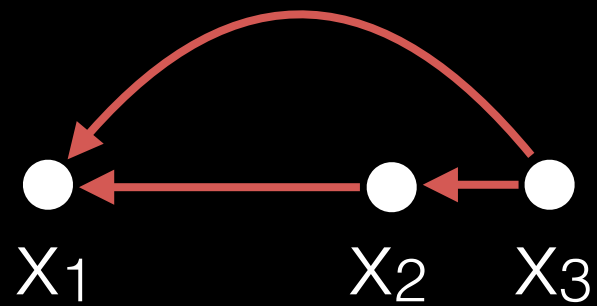
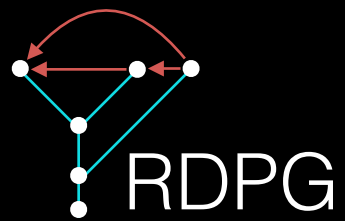


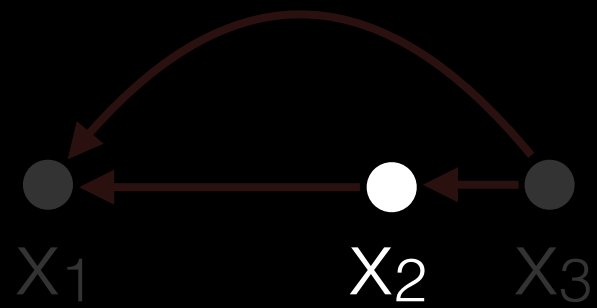
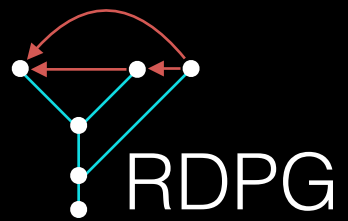
Traits

as prey

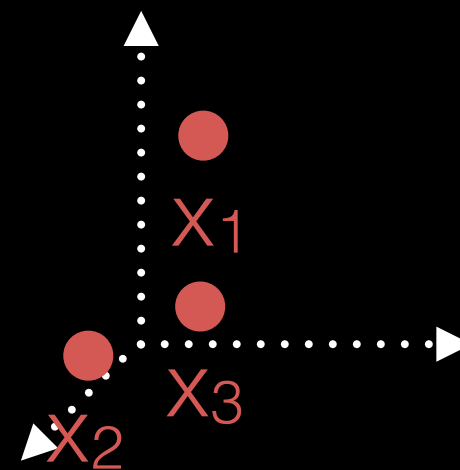
as predator



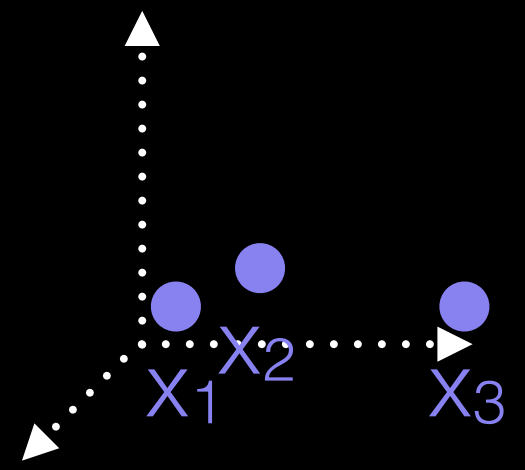




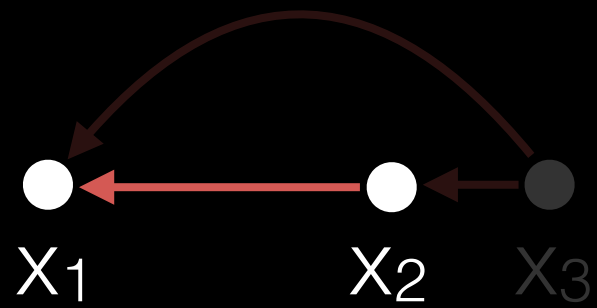
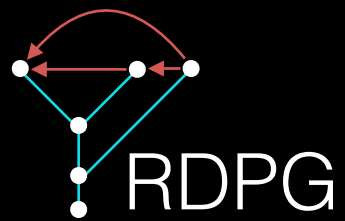
as prey



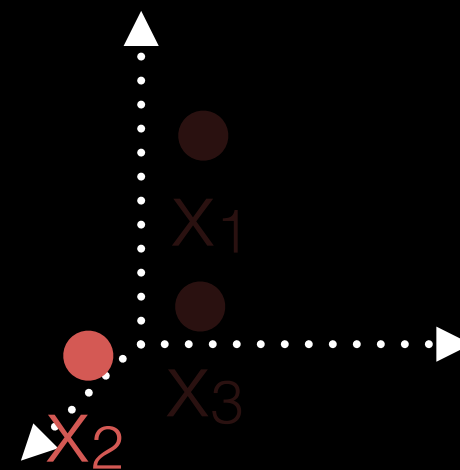
as predator



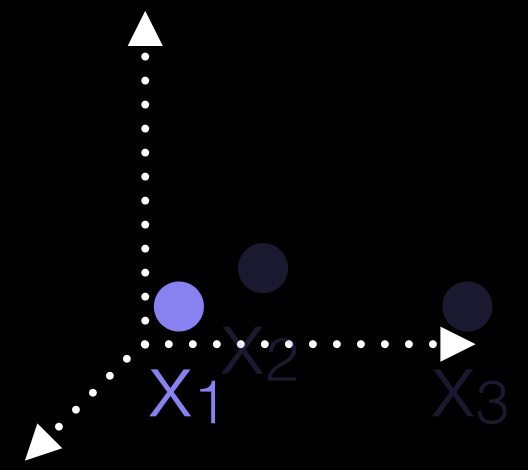




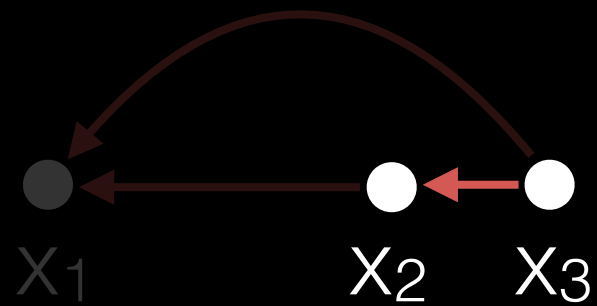
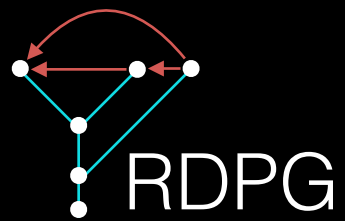
as prey



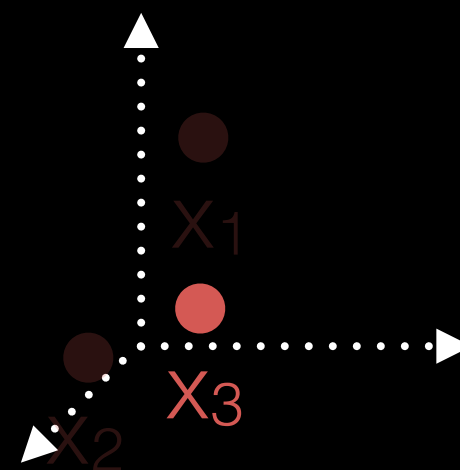
as predator



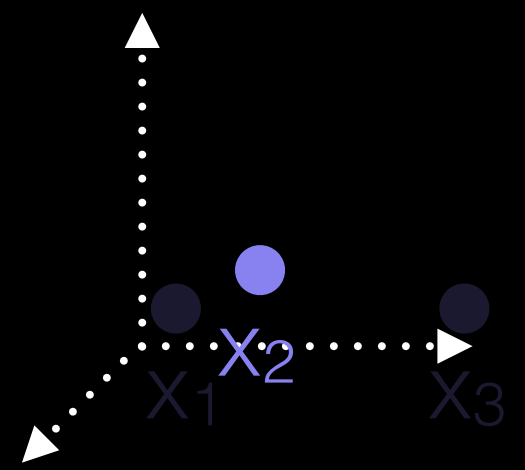
$$\mathbf{P}(X_2 \rightarrow X_1) = X_2 \cdot X_1$$



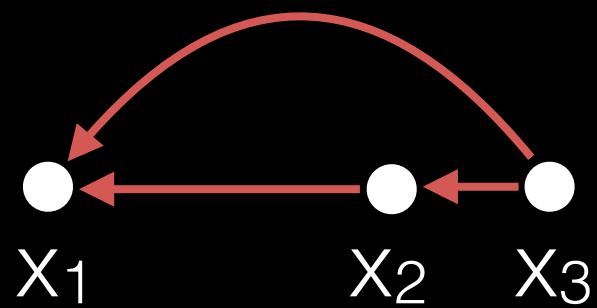
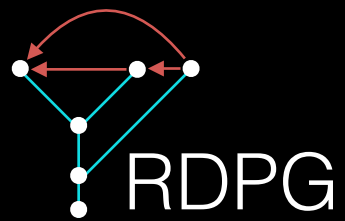
as prey



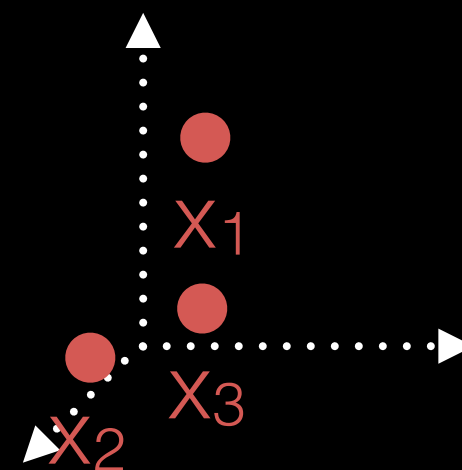
as predator



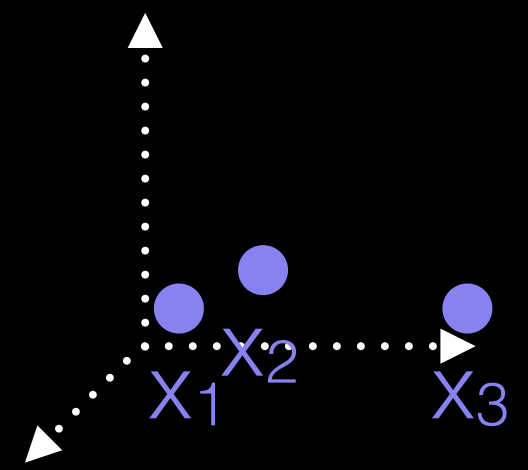
$$\mathbf{P}(X_2 \leftarrow X_3) = X_3 \cdot X_2$$



as prey



as predator



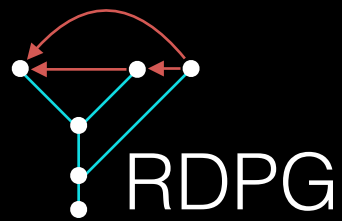
$$\mathbf{M} := \mathbf{X} \cdot \mathbf{X}^t$$

$$\mathbf{M}_{ij} = \mathbf{P}(x_i \rightarrow x_j)$$

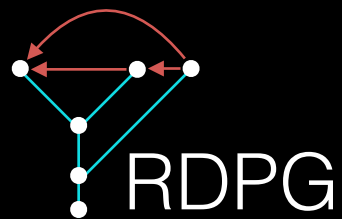


$$P\left( \begin{array}{c} \text{Diagram 1} \mid \text{Diagram 2} \end{array} \right) =$$



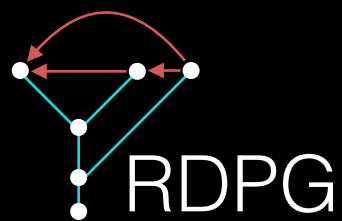


$$P\left( \begin{array}{c} \text{Diagram 1} \end{array} \mid \begin{array}{c} \text{Diagram 2} \end{array}, \begin{array}{c} \text{Diagram 3} \end{array} \right) =$$



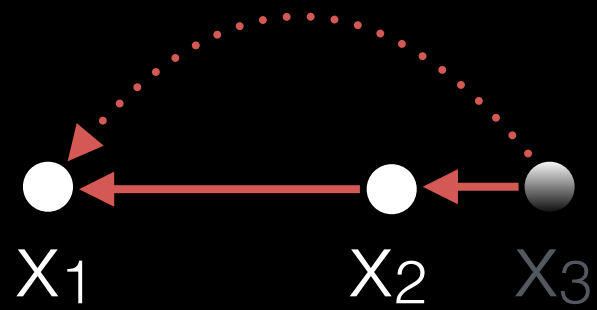
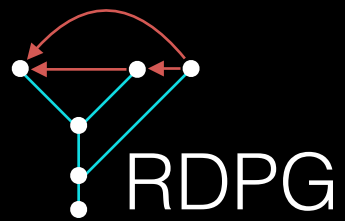
$$P\left( \begin{array}{c} \text{Diagram 1} \end{array} \mid \begin{array}{c} \text{Diagram 2} \end{array}, \begin{array}{c} \text{Diagram 3} \end{array} \right) =$$

$$B(A, M)$$

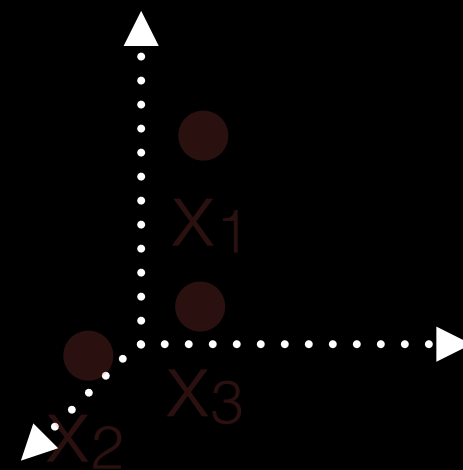


$$P\left( \begin{array}{c} \text{Diagram 1} \end{array} \middle| \begin{array}{c} \text{Diagram 2} \end{array} \right) =$$

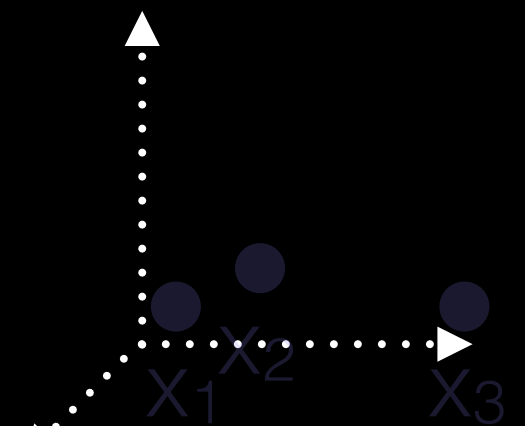
$$P\left( \begin{array}{c} \text{Diagram 1} \end{array} \middle| \begin{array}{c} \text{Diagram 3} \end{array} \right)$$



as prey

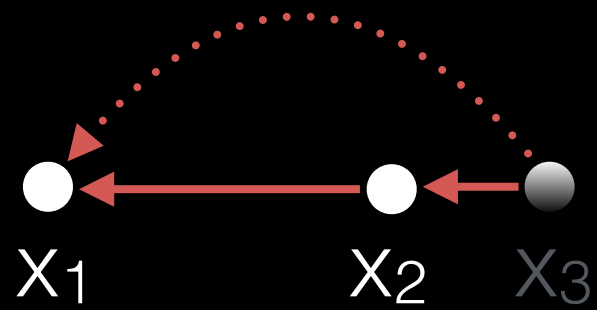
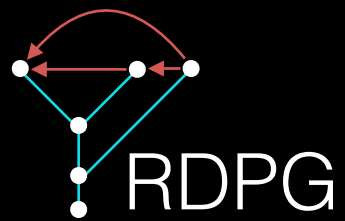


as predator

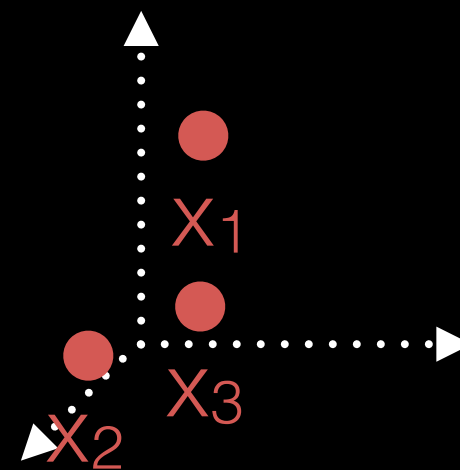


**Alas!**

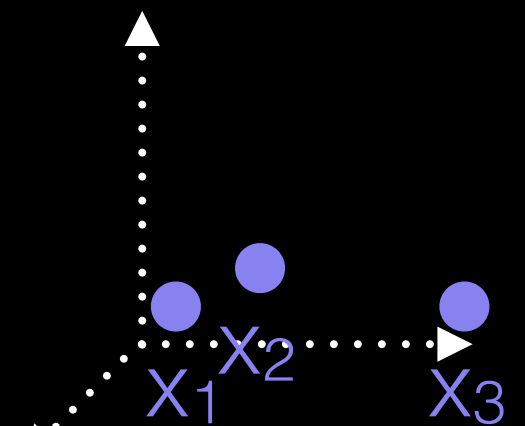




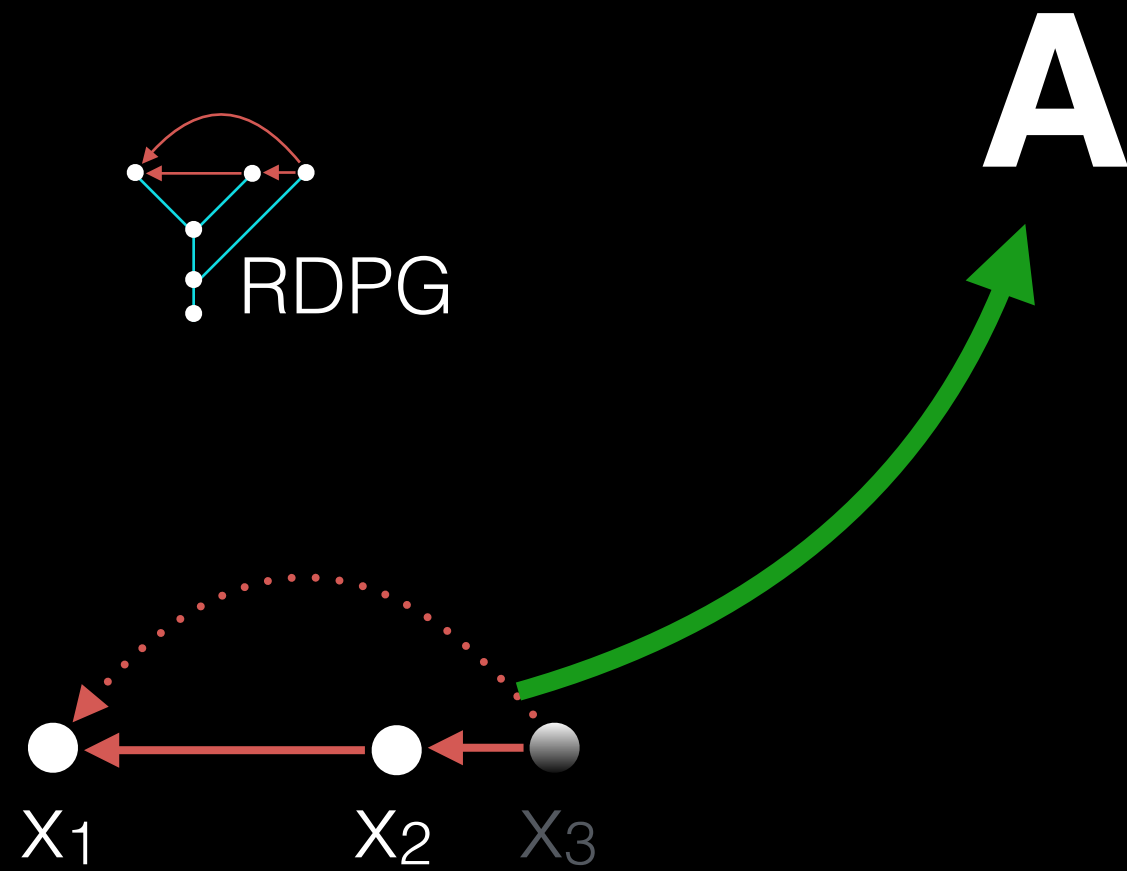
as prey

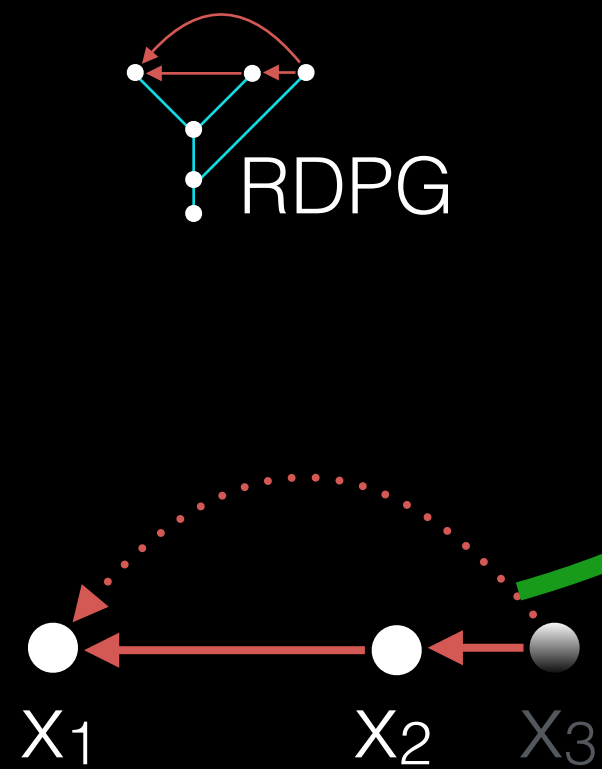


as predator



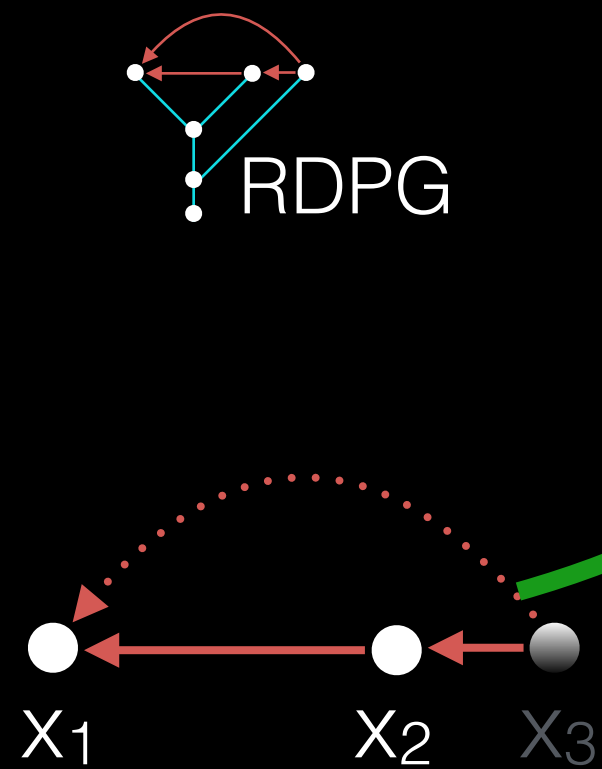
**SVD!**





A

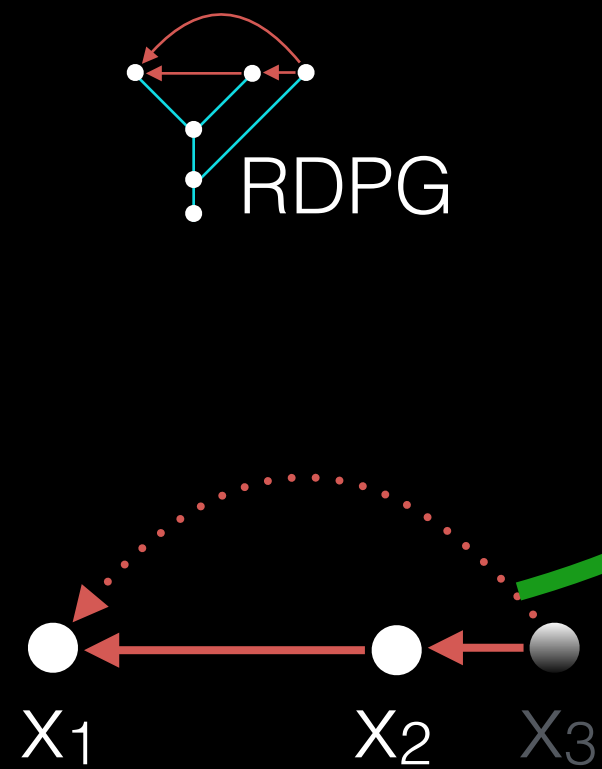
LSR



A

LSR

$L\sqrt{S} \sqrt{S}R$



**A**

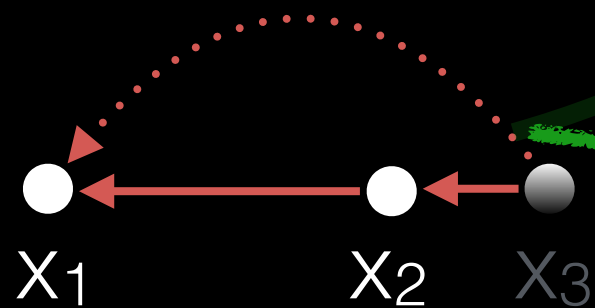
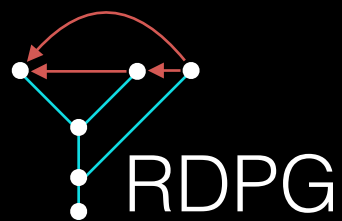
**LSR**

**$L\sqrt{S}\sqrt{S}R$**

**d**

**$L_d\sqrt{S_d}$   
 $\sqrt{S_d}R_d$**





$A$

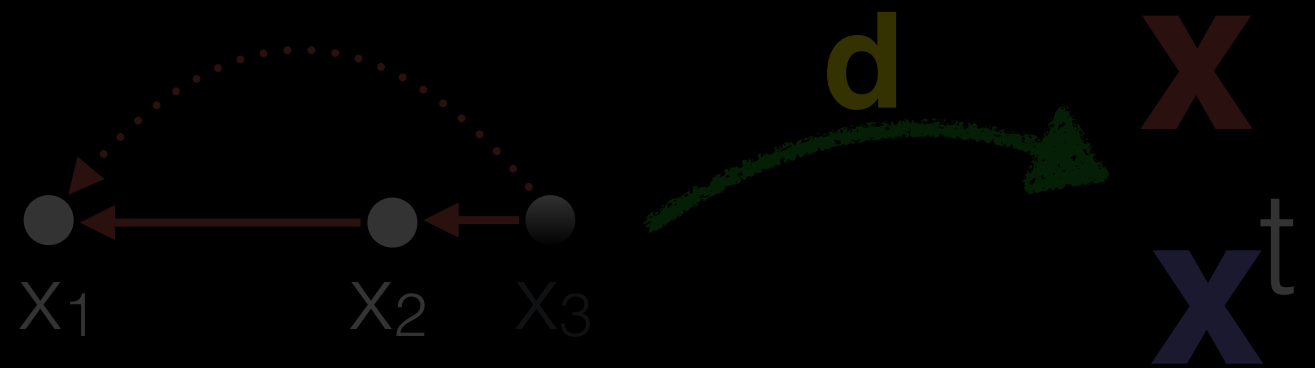
$LSR$

$L\sqrt{S} \sqrt{S}R$

$d$

$$L_d \sqrt{S_d} = \mathbf{x}$$

$$\sqrt{S_d} R_d = \mathbf{x}^t$$



we need to choose **d**



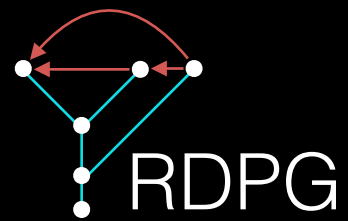
$$P\left(\begin{array}{c} \text{Graph} \\ \text{X}_1 \text{ X}_2 \text{ X}_3 \end{array} \middle| \begin{array}{c} \text{Point Cloud 1} \\ \text{X}_1 \text{ X}_2 \text{ X}_3 \end{array}, \begin{array}{c} \text{Point Cloud 2} \\ \text{X}_1 \text{ X}_2 \text{ X}_3 \end{array} \right) =$$

$$P\left(\begin{array}{c} \text{Graph} \\ \text{X}_1 \text{ X}_2 \text{ X}_3 \end{array} \middle| \begin{array}{c} \text{Point Cloud 1} \\ \text{X}_1 \text{ X}_2 \text{ X}_3 \end{array}, \begin{array}{c} \text{Point Cloud 2} \\ \text{X}_1 \text{ X}_2 \text{ X}_3 \end{array} \right)$$



**X**, **X**

unique up to an orthogonal transformation

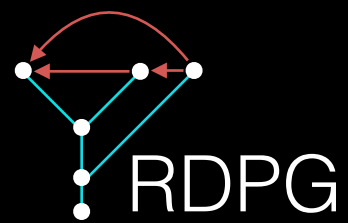


Did evolution leave a trace  
in the structure  
of ecological networks?

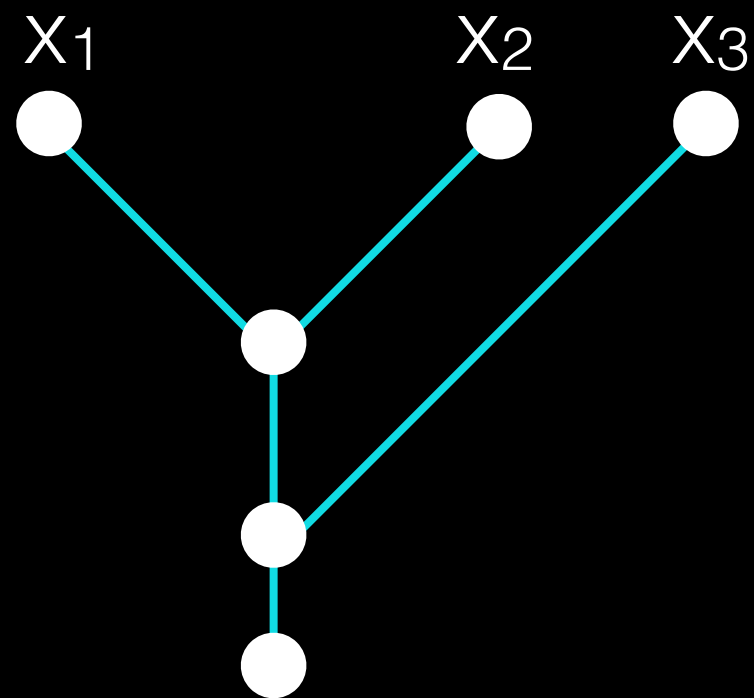


Did evolution leave a trace  
in the structure  
of ecological networks?

Are  *predicted by*  ?



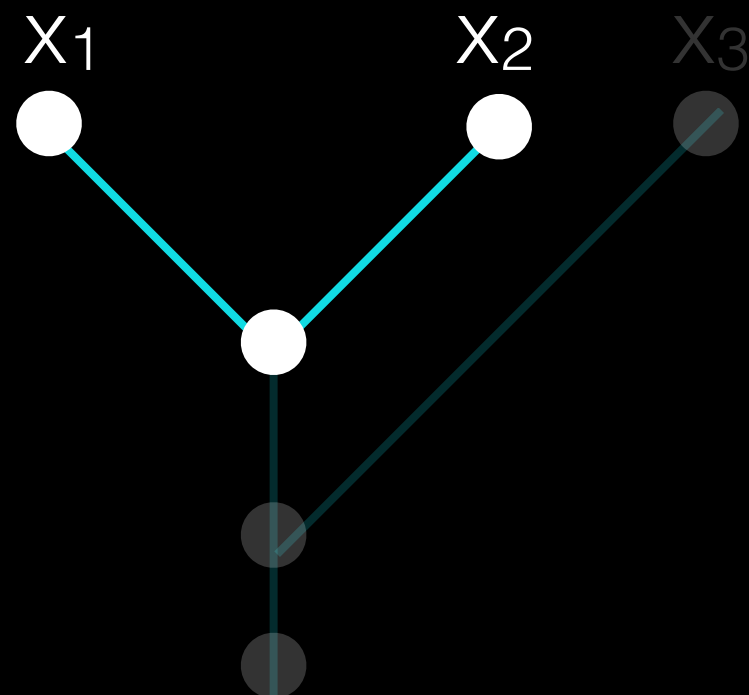
Did evolution leave a trace  
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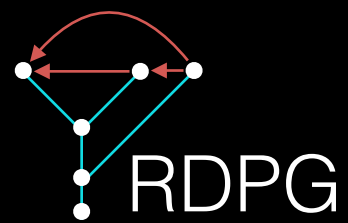




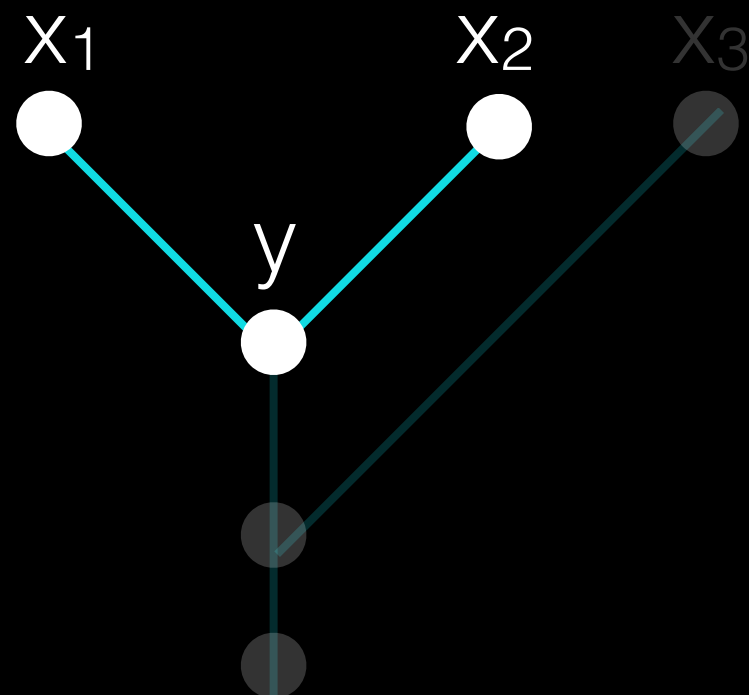


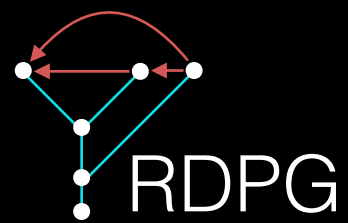
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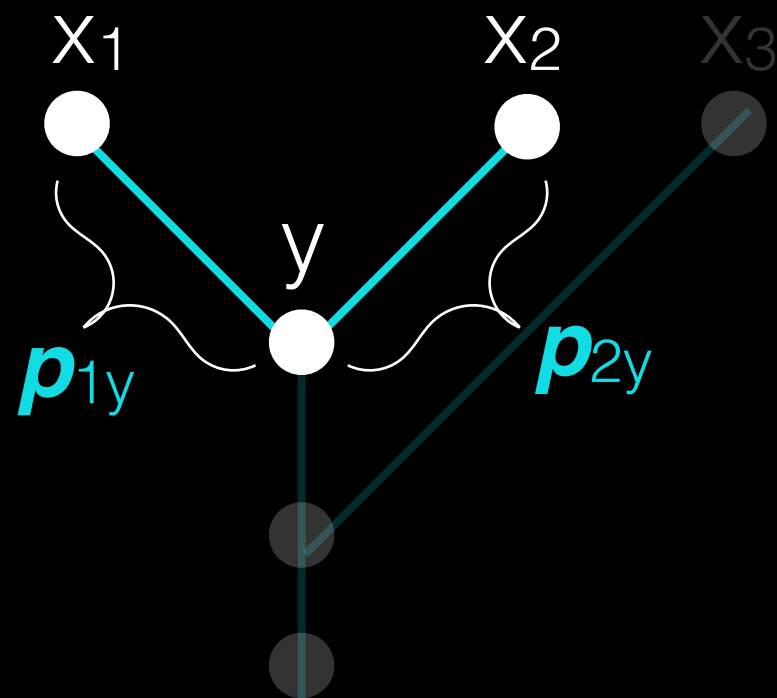


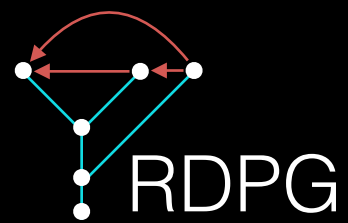
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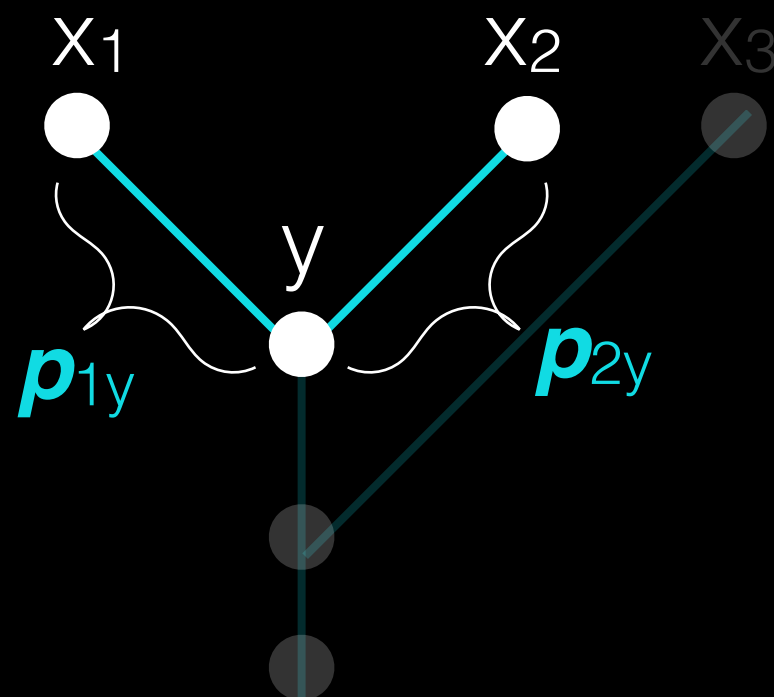


Did evolution leave a trace  
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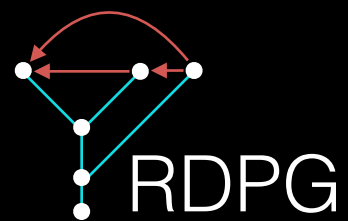




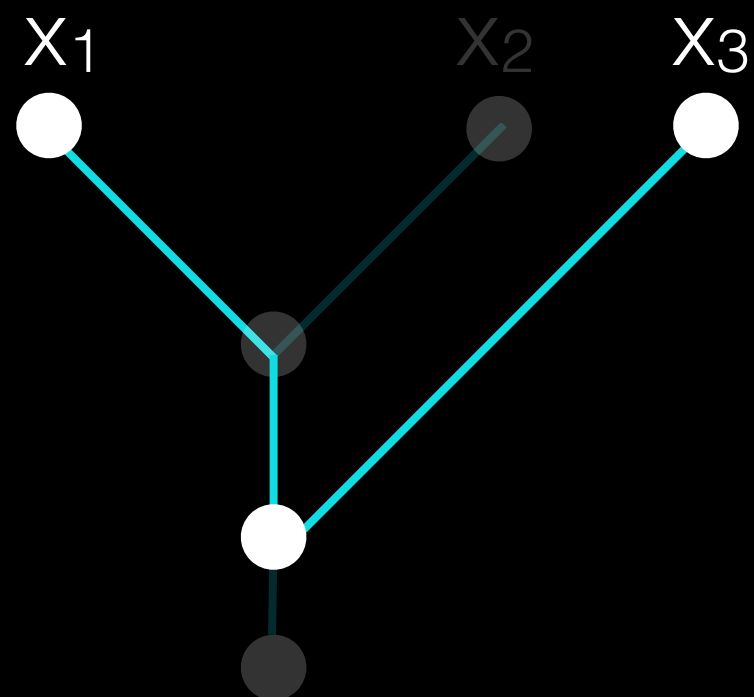
Did evolution leave a trace  
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$$p_{12} = p_{1y} + p_{2y}$$

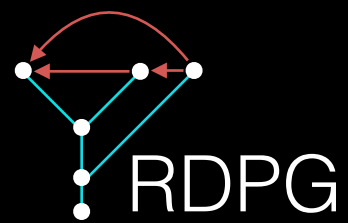


Did evolution leave a trace  
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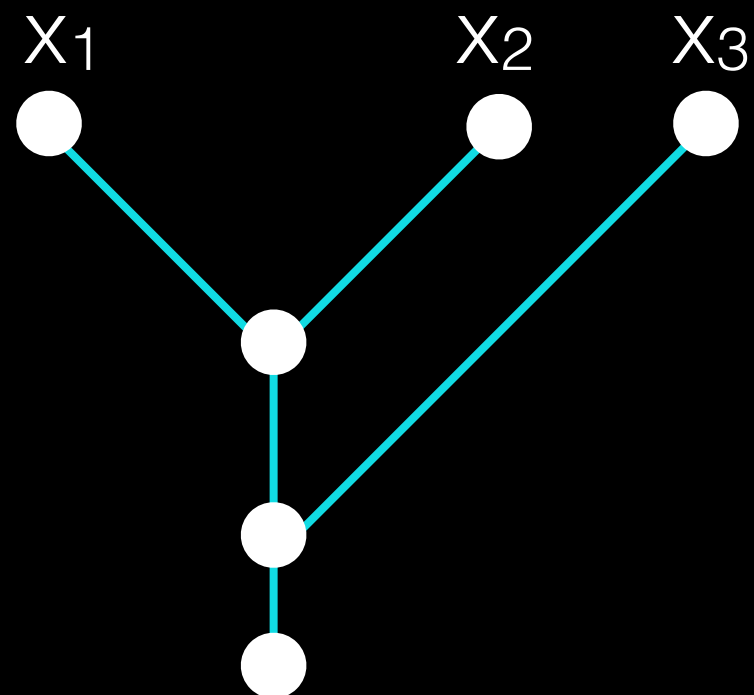


$$p_{12} = p_{1y} + p_{2y}$$

$$p_{13} = \dots$$



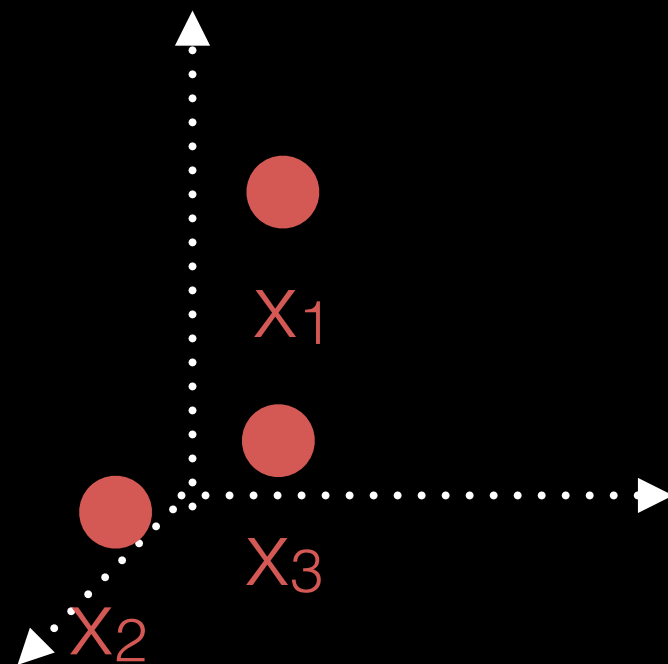
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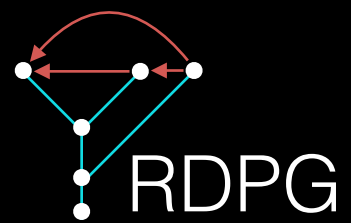
$d_p$



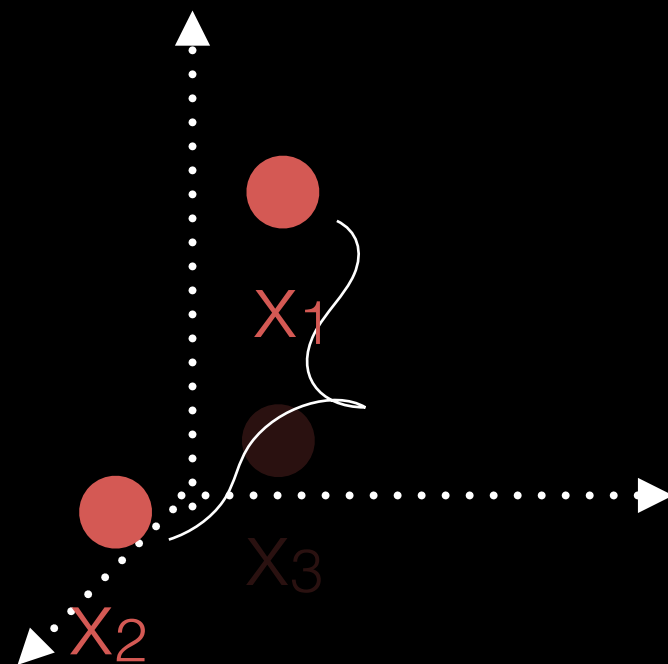
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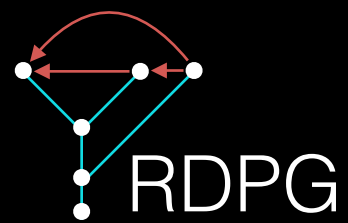




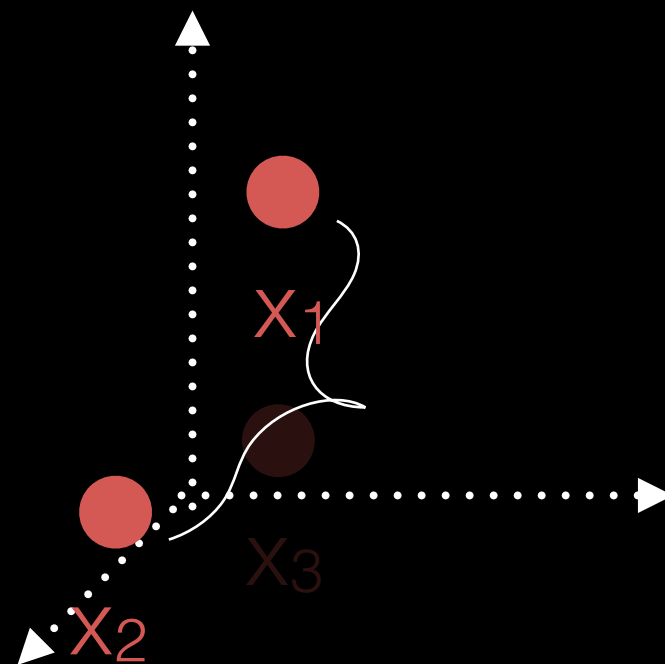


Did evolution leave a trace  
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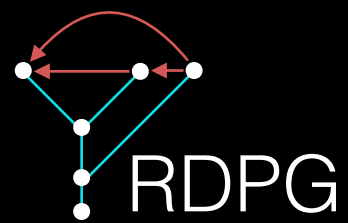




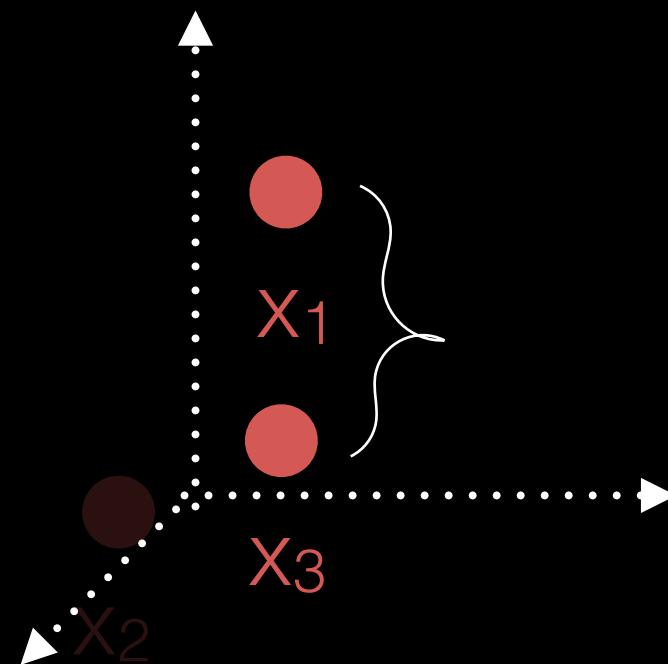
Did evolution leave a trace  
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of ecological networks?



$$x_{12} = d(x_1, x_2)$$

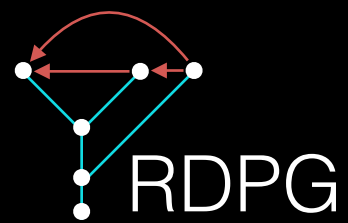


Did evolution leave a trace  
in the topological structure  
of ecological networks?

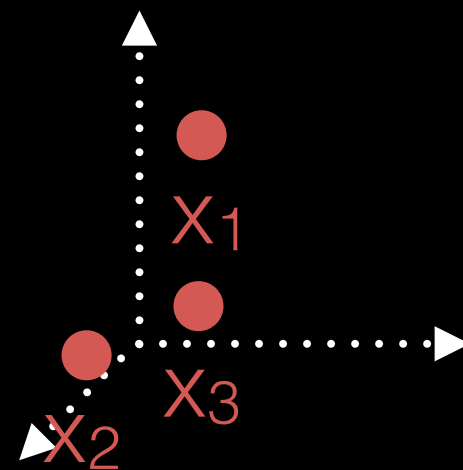


$$\mathbf{x}_{12} = d(x_1, x_2)$$

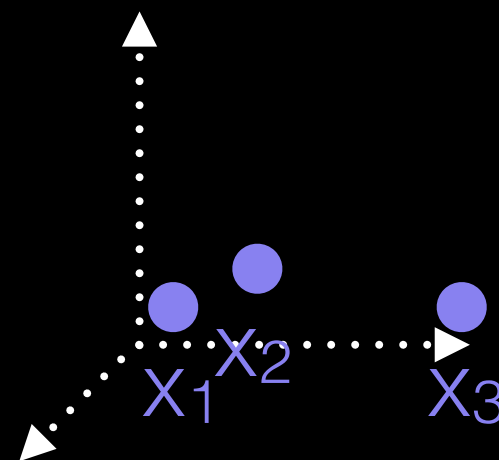
$$\mathbf{x}_{13} = d(x_1, x_3)$$



Did evolution leave a trace  
in the topological structure  
of ecological networks?



$d_x$



$d_x$



Did evolution leave a trace  
in the structure  
of ecological networks?

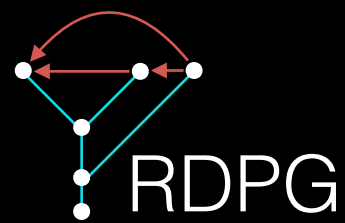
Are  *predicted by*  ?

Are  $\mathbf{d}_x$   $\mathbf{d}_x$  *predicted by*  $\mathbf{d}_p$  ?

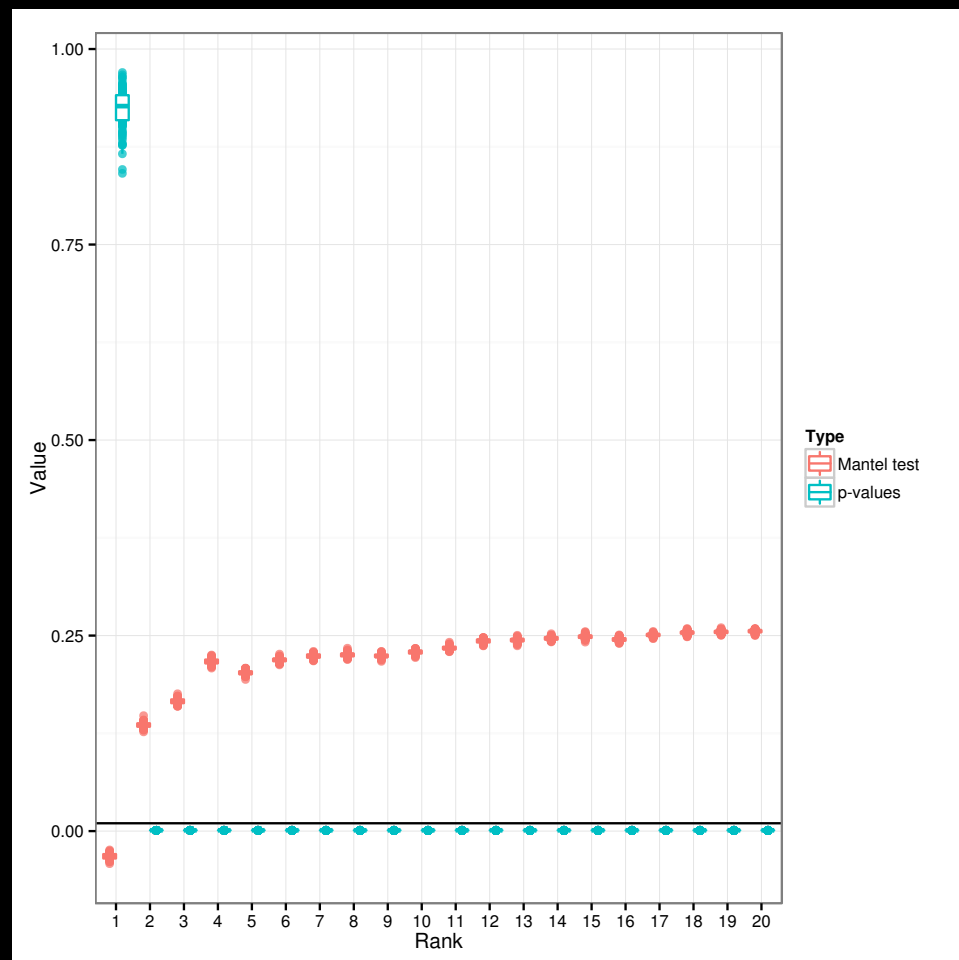


Did evolution leave a trace  
in the structure  
of ecological networks?

yes



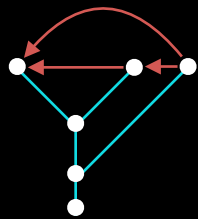
Did evolution leave a trace  
in the structure  
of ecological networks?



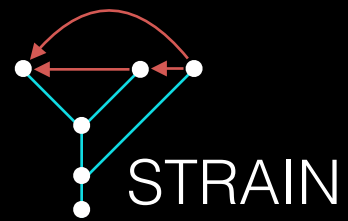
yes, and starting from **d**

Dalla Riva, G. V. and Stouffer, D. B. (2015), Exploring the evolutionary signature of food webs' backbones using functional traits. *Oikos*. doi: 10.1111/oik.02305

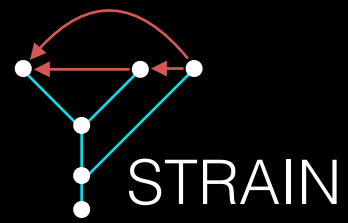




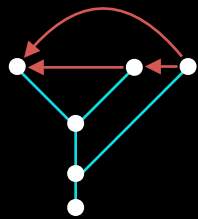
# CENTRALITY



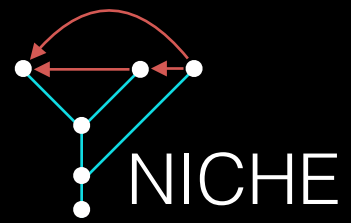
How do we measure  
the importance of nodes  
in a RDPG framework?



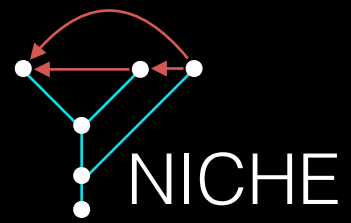
Are ecological unique species  
evolutionary distinctive?



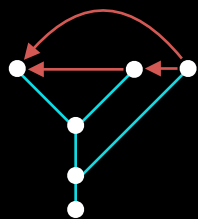
# NICHE



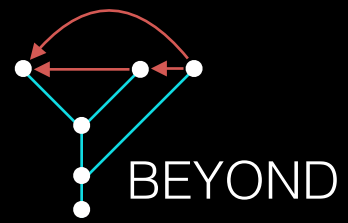
How do species' niches evolve?

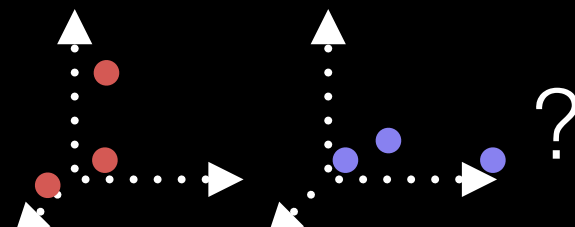


How do we detect the effect  
of interactions  
in species' evolution?

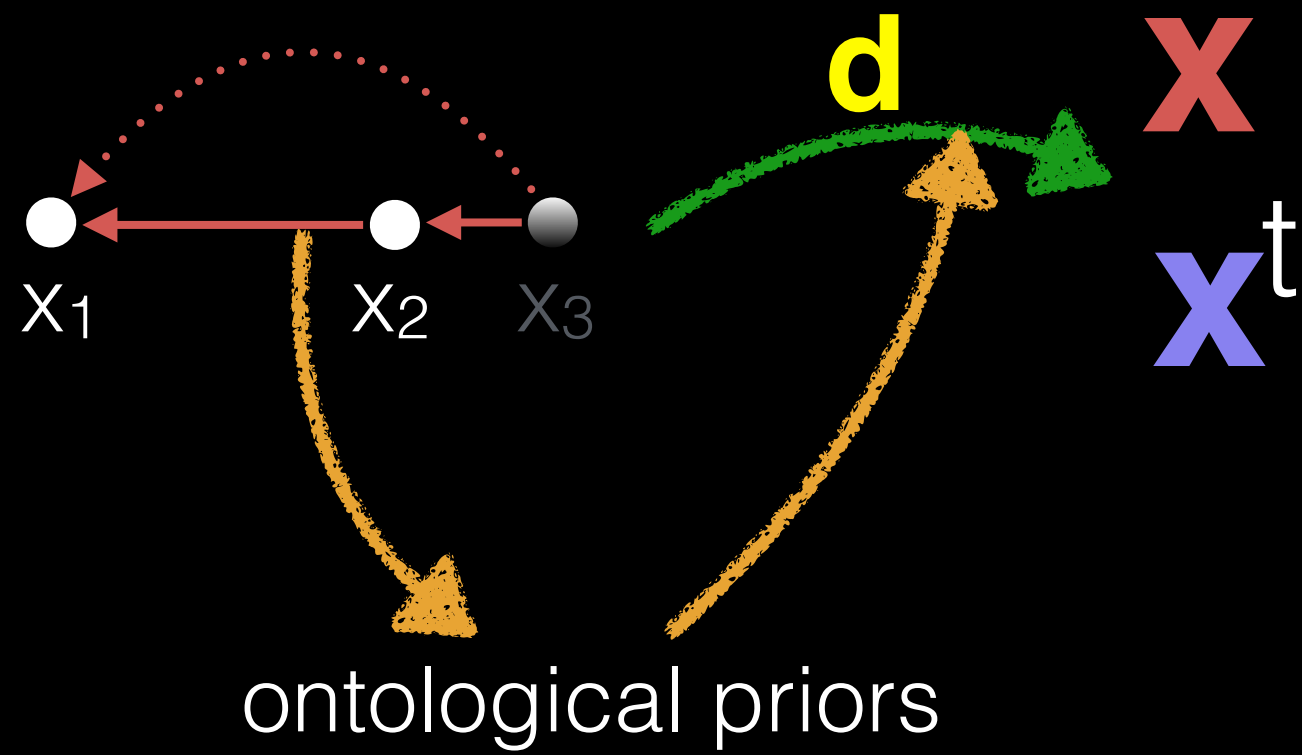
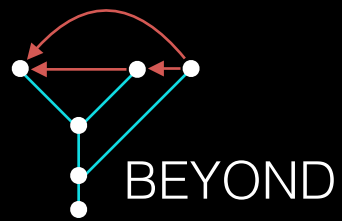


BEYOND

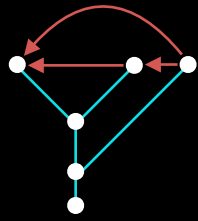


How do species' evolve in ?





?



*Because we are all responsible for all  
[...] I go for all [...]*

*–Fyodor Dostoyevsky  
The Brothers Karamazov*