Implementing bi-level matching optimization with propensity score
For PHS 7045

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Introduction

Matching algorithms for causal inference are a fundamental tool in biomedical sciences. Nonetheless problems about optimal matching strategies, balance, and related issues are still open. In this report, I will introduce an implementation of the bi-level matching optimization algorithm published by XYZ and show how it can be used to improve the balance of covariates in a dataset, particularly, looking into the propensity score matching.

Description of the solution

To address the problem, we will be implementing the algorithm using C++ via Rcpp in R. The algorithm will be implemented in the following steps:

1. Compute propensity scores using a logistic regression in R.
2. Use the propensity scores to do a 1:2 matching for generating the counterfactuals.
3. Return the results of the matching a list with: (a) a two column matrix showing the idx of the matched pairs, and (b) the matched estimator.

Results

```
library(Rcpp)

# Simulating some data
n <- 1000
set.seed(3112)
```

```r
group <- sample.int(n, 2, replace = TRUE)
X <- runif(n) # Simulating propensity score
Y <- rnorm(n) # simulating outcome

sourceCpp("match.cpp", cacheDir = "cache")
match_it(
  X = X,
  group = group,
  Y = Y
)
```

```
$X
  [1] 0.490490574 0.106643000 0.126245317 0.616251535 0.620511455 0.198709359
  [7] 0.009153894 0.278994804 0.512462093 0.215286891 0.075820136 0.289247589
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