# Applied HPC with R

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# **Preface**

The R programming language (R Core Team 2023) can be fantastic for most daily tasks. But as soon as you start dealing with more complicated problems, you may face the for-loop bottleneck. If you ever encounter such a problem, this book is for you. Applied HPC with R is a collection of talks and lectures I have given about speeding up your R code using parallel computing and other resources such as C++. The contents have been primarily developed during my time at USC and  $UofU^1$ 

<sup>&</sup>lt;sup>1</sup>With many to thank, including Paul Marjoram, Zhi Zhang, Emil Hvitfeldt, Malcolm Barrett, Garrett Weaver, USC's IMAGE P01 research group, and my students both at USC and UoU.

# Part I Parallel computing

# 1 Introduction

While most people see R as a slow programming language, it has some powerful features that accelerate your code dramatically<sup>1</sup>. Although R wasn't necessarily built for speed, there are some tools and ways in which we can accelerate R. This chapter introduces what we will understand as High-performance computing in R.

## 1.1 High-Performance Computing: An overview

Loosely, from R's perspective, we can think of HPC in terms of two, maybe three things:

- 1. Big data: How to work with data that doesn't fit your computer
- 2. Parallel computing: How to take advantage of multiple core systems
- 3. Compiled code: Write your low-level code (if R doesn't have it yet...)

(Checkout CRAN Task View on HPC)

# 1.2 Big Data

- Buy a bigger computer/RAM (not the best solution!)
- Use out-of-memory storage, i.e., don't load all your data in the RAM. e.g. The bigmemory, data.table, HadoopStreaming R packages
- Efficient algorithms for big data, e.g.: biglm, biglasso
- Store it more efficiently, e.g.: Sparse Matrices (take a look at the dgCMatrix objects from the Matrix R package)

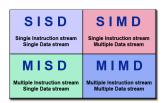


Figure 1.1: Flynn's Classical Taxonomy (Blaise Barney, Introduction to Parallel Computing, Lawrence Livermore National Laboratory)

## 1.3 Parallel computing

We will be focusing on the Single Instruction stream Multiple Data stream

## 1.4 Parallel computing

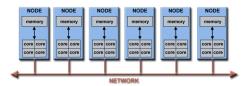
In general terms, a parallel computing program is one in which we use two or more *computational threads* simultaneously. Although computational thread usually means core, there are multiple levels at which a computer program can be parallelized. To understand this, we first need to see what composes a modern computer:

Streaming SIMD Extensions [SSE] and Advanced Vector Extensions [AVX]

#### 1.4.1 Serial vs Parallel

source: Blaise Barney, Introduction to Parallel Computing, Lawrence Livermore National Laboratory

# 1.5 Parallel computing



source: Blaise Barney, Introduction to Parallel Computing, Lawrence Livermore National Laboratory

<sup>&</sup>lt;sup>1</sup>Nonetheless this claim can be said about almost any programming language, there are notable like the R package data.table (Dowle and Srinivasan 2021) which has been demonstrated to out-perform most data wrangling tools.

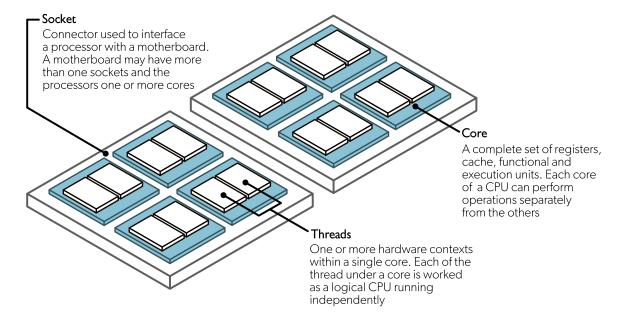


Figure 1.2: Source: Original figure from LUMI consortium documentation (LUMI consortium 2023)

## 1.6 Some vocabulary for HPC

In raw terms

- Supercomputer: A single big machine with thousands of cores/GPGPUs.
- High-Performance Computing (HPC): Multiple machines within a single network.
- High Throughput Computing (HTC): Multiple machines across multiple networks.

You may not have access to a supercomputer, but certainly, HPC/HTC clusters are more accessible these days, e.g. AWS provides a service to create HPC clusters at a low cost (allegedly, since nobody understands how pricing works)

## 1.7 GPU vs CPU

• Why use OpenMP if GPU is *suited to compute-intensive operations*? Well, mostly because OpenMP is **VERY** easy to implement (easier than CUDA, which is the easiest way to use GPU).<sup>2</sup>

<sup>&</sup>lt;sup>2</sup>Sadia National Laboratories started the Kokkos project, which provides a one-fits-all C++ library for parallel programming. More information on the Kokkos's wiki site.

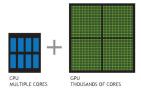


Figure 1.3: NVIDIA Blog

# 1.8 When is it a good idea?

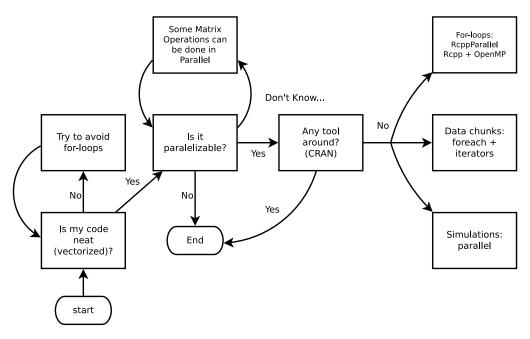


Figure 1.4: Ask yourself these questions before jumping into HPC!

# 1.9 Parallel computing in R

While there are several alternatives (just take a look at the High-Performance Computing Task View), we'll focus on the following R-packages for **explicit parallelism**:

- parallel: R package that provides '[s]upport for parallel computation, including random-number generation'.
- **future**: '[A] lightweight and unified Future API for sequential and parallel processing of R expression via futures.'

• Rcpp + OpenMP: Rcpp is an R package for integrating R with C++ and OpenMP is a library for high-level parallelism for C/C++ and FORTRAN.

Others but not used here

- foreach for iterating through lists in parallel.
- Rmpi for creating MPI clusters.

And tools for implicit parallelism (out-of-the-box tools that allow the programmer not to worry about parallelization):

- gpuR for Matrix manipulation using GPU
- tensorflow an R interface to TensorFlow.

A ton of other types of resources, notably the tools for working with batch schedulers such as Slurm, and HTCondor.

# 2 The parallel R package

#### 2.1 Parallel workflow

(Usually) We do the following:

- Create a PSOCK/FORK (or other) cluster using makePSOCKCluster/makeForkCluster (or makeCluster)
- 2. Copy/prepare each R session (if you are using a PSOCK cluster):
  - a. Copy objects with clusterExport
  - b. Pass expressions with clusterEvalQ
  - c. Set a seed
- 3. Do your call: parApply, parLapply, etc.
- 4. Stop the cluster with clusterStop

# 2.2 Types of clusters: PSOCK

- Can be created with makePSOCKCluster
- Creates brand new R Sessions (so nothing is inherited from the master), e.g.

```
# This creates a cluster with 4 R sessions
cl <- makePSOCKCluster(4)</pre>
```

- Child sessions are connected to the master session via Socket connections
- Can be created outside of the current computer, i.e. across multiple computers!

# 2.3 Types of clusters: Fork

- Fork Cluster makeForkCluster:
- Uses OS Forking,
- Copies the current R session locally (so everything is inherited from the master up to that point).
- Data is only duplicated if it is altered (need to double check when this happens!)
- Not available on Windows.

Other makeCluster: passed to **snow** (Simple Network of Workstations)

## 2.4 Ex 1: Parallel RNG with makePSOCKCluster

```
# 1. CREATING A CLUSTER
  library(parallel)
  nnodes <- 4L
        <- makePSOCKcluster(nnodes)
  # 2. PREPARING THE CLUSTER
  clusterSetRNGStream(cl, 123) # Equivalent to `set.seed(123)`
  # 3. DO YOUR CALL
  ans <- parSapply(cl, 1:nnodes, function(x) runif(1e3))</pre>
  (ans0 <- var(ans))</pre>
            [,1]
                         [,2]
                                      [,3]
                                                   [,4]
[1,] 0.0861888293 -0.0001633431 5.939143e-04 -3.672845e-04
[3,] 0.0005939143 0.0023907904 8.114219e-02 -4.714618e-06
[4,] -0.0003672845 -0.0001462154 -4.714618e-06 8.467722e-02
```

Making sure is reproducible

```
# I want to get the same!
clusterSetRNGStream(cl, 123)
ans1 <- var(parSapply(cl, 1:nnodes, function(x) runif(1e3)))
# 4. STOP THE CLUSTER</pre>
```

```
stopCluster(cl)
all.equal(ans0, ans1) # All equal!
```

[1] TRUE

#### 2.5 Ex 2: Parallel RNG with makeForkCluster

In the case of makeForkCluster

```
# 1. CREATING A CLUSTER
  library(parallel)
  # The fork cluster will copy the -nsims- object
  nsims <- 1e3
  nnodes <- 4L
        <- makeForkCluster(nnodes)
  # 2. PREPARING THE CLUSTER
  clusterSetRNGStream(cl, 123)
  # 3. DO YOUR CALL
  ans <- do.call(cbind, parLapply(cl, 1:nnodes, function(x) {</pre>
   runif(nsims) # Look! we use the nsims object!
               # This would have fail in makePSOCKCluster
                # if we didn't copy -nsims- first.
   }))
  (ans0 <- var(ans))</pre>
                          [,2]
             [,1]
                                       [,3]
[1,] 0.0861888293 -0.0001633431 5.939143e-04 -3.672845e-04
[3,] 0.0005939143 0.0023907904 8.114219e-02 -4.714618e-06
[4,] -0.0003672845 -0.0001462154 -4.714618e-06 8.467722e-02
```

Again, we want to make sure this is reproducible

```
# Same sequence with same seed
clusterSetRNGStream(cl, 123)
ans1 <- var(do.call(cbind, parLapply(cl, 1:nnodes, function(x) runif(nsims))))
ans0 - ans1 # A matrix of zeros</pre>
```

```
[,1] [,2] [,3] [,4]
[1,] 0 0 0 0
[2,] 0 0 0 0
[3,] 0 0 0 0
[4,] 0 0 0 0
```

```
# 4. STOP THE CLUSTER stopCluster(cl)
```

Well, if you are a Mac-OS/Linux user, there's a simpler way of doing this...

# 2.6 Ex 3: Parallel RNG with mclapply (Forking on the fly)

In the case of mclapply, the forking (cluster creation) is done on the fly!

```
# 1. CREATING A CLUSTER
  library(parallel)
  # The fork cluster will copy the -nsims- object
  nsims <- 1e3
  nnodes <- 4L
           <- makeForkCluster(nnodes) # mclapply does it on the fly</pre>
  # 2. PREPARING THE CLUSTER
  set.seed(123)
  # 3. DO YOUR CALL
  ans <- do.call(cbind, mclapply(1:nnodes, function(x) runif(nsims)))
  (ans0 <- var(ans))</pre>
             [,1]
                         [,2]
                                       [,3]
                                                    [,4]
[1,] 0.085384184 0.002390790 0.006576204 -0.003998278
     0.002390790 0.081142190
                               0.001846963 0.001476244
[3,] 0.006576204 0.001846963 0.085175347 -0.002807348
[4,] -0.003998278 0.001476244 -0.002807348 0.082425477
```

Once more, we want to make sure this is reproducible

```
# Same sequence with same seed
  set.seed(123)
  ans1 <- var(do.call(cbind, mclapply(1:nnodes, function(x) runif(nsims))))</pre>
  ans0 - ans1 # A matrix of zeros
    [,1] [,2] [,3] [,4]
[1,]
                0
            0
[2,]
            0
                   0
[3,]
       0
          0
              0
                   0
[4,]
    0
         0
                0
                   0
```

```
# 4. STOP THE CLUSTER
```

<sup>#</sup> stopCluster(cl) no need of doing this anymore

# Part II Working with a Cluster

# 3 What is Slurm

## Note

Most of this section was extracted from the slurm RR package's vignette "Working with Slurm."

Nowadays, high-performance-computing (HPC) clusters are commonly available tools for either in or out of cloud settings. Slurm Work Manager (formerly Simple Linux Utility for Resource Manager) is a program written in C that is used to efficiently manage resources in HPC clusters. The slurmR R package—which we will be using in this book—provides tools for using R in HPC settings that work with Slurm. It provides wrappers and functions that allow the user to seamlessly integrate their analysis pipeline with HPC clusters, emphasizing on providing the user with a family of functions similar to those that the parallel R package provides.

## 3.1 Definitions

First, some important discussion points within the context of Slurm+R that users, in general, will find useful. Most of the points have to do with options available for Slurm, and in particular, with the sbatch command with is used to submit batch jobs to Slurm. Users who have used Slurm in the past may wish to skip this and continue reading the following section.

• **Node** A single computer in the HPC: A lot of times jobs will be submitted to a single node. The simplest way of using R+Slurm is submitting a single job and requesting multiple CPUs to use, for example, parallel::parLapply or parallel::mclapply. Usually, users do not need to request a specific number of nodes to be used as Slurm will allocate the resources as needed.

A common mistake of R users is to specify the number of nodes and expect that their script will be parallelized. This won't happen unless the user explicitly writes a parallel computing script.

The relevant flag for sbatch is --nodes.

• Partition A group of nodes in HPC. Generally large nodes may have multiple partitions, meaning that nodes may be grouped in various ways. For example, nodes belonging to a single group of users may be in a single partition, and nodes dedicated to working with large data may be in another partition. Usually, partitions are associated with account privileges, so users may need to specify which account are they using when telling Slurm what partition they plan to use.

The relevant flag for sbatch is --partition.

• Account Accounts may be associated with partitions. Accounts can have privileges to use a partition or set of nodes. Often, users need to specify the account when submitting jobs to a particular partition.

The relevant flag for sbatch is --account.

• Task A step within a job. A particular job can have multiple tasks. tasks may span multiple nodes, so if the user wants to submit a multicore job, this option may not be the right one.

The relevant flag for sbatch is --ntasks

• **CPU** generally this refers to core or thread (which may be different in systems supporting multithreaded cores). Users may want to specify how many CPUs they want to use for a task. And this is the relevant option when using things like OpenMP or functions that allow creating cluster objects in R (e.g. makePSOCKcluster, makeForkCluster).

The relevant option in sbatch is --cpus-per-task. More information regarding CPUs in Slurm can be found here. Information regarding how Slurm counts CPUs/cores/threads can be found here.

• Job Array Slurm supports job arrays. A job array is in simple terms a job that is repeated multiple times by Slurm, this is, replicates a single job as requested per the user. In the case of R, when using this option, a single R script is spanned in multiple jobs, so the user can take advantage of this and parallelize jobs across multiple nodes. Besides from the fact that jobs within a Job Array may be spanned across multiple nodes, each job in that array has a unique ID that is available to the user via environment variables, in particular SLURM\_ARRAY\_TASK\_ID.

Within R, and hence the Rscript submitted to Slurm, users can access this environment variable with Sys.getenv("SLURM\_ARRAY\_TASK\_ID"). Some of the functionalities of slurmR rely on Job Arrays.

More information on Job Arrays can be found here. The relevant option for this in sbatch is --array.

More information about Slurm can be found their official website here. A tutorial about how to use Slurm with R can be found here.

# 4 A brief intro to Slurm

For a quick-n-dirty intro to Slurm (Yoo, Jette, and Grondona 2003), we will start with a simple "Hello world" using Slurm + R. For this, we need to go through the following steps:

- 1. Copy a Slurm script to HPC,
- 2. Logging to HPC, and
- 3. Submit the job using sbatch.

## 4.1 Step 1: Copy the Slurm script to HPC

We need to copy the following Slurm script to HPC (00-hello-world.slurm):

```
#!/bin/sh
#SBATCH --output=00-hello-world.out
module load R/4.2.2
Rscript -e "paste('Hello from node', Sys.getenv('SLURMD_NODENAME'))"
```

Which has four lines:

- 1. #!/bin/sh: The shebang (shewhat?)
- 2. #SBATCH --output=00-hello-world.out: An option to be passed to sbatch, in this case, the name of the output file to which stdout and stderr will go.
- 3. module load R/4.2.2: Uses Lmod to load the R module.
- 4. Rscript ...: A call to R to evaluate the expression paste(...). This will get the environment variable SLURMD NODENAME (which sbatch creates) and print it on a message.

To do so, we will use **Secure copy protocol (scp)**, which allows us to copy data to and fro computers. In this case, we should do something like the following

```
scp 00-hello-world.slurm [userid]@notchpeak.chpc.utah.edu:/path/to/a/place/you/can/access
```

In words, "Using the username [userid], connect to notchpeak.chpc.utah.edu, take the file 00-hello-world.slurm and copy it to /path/to/a/place/you/can/access. With the file now available in the cluster, we can submit this job using Slurm.

## 4.2 Step 2: Logging to HPC

- 1. Log in using ssh. In the case of Windows users, download the **Putty** client.
- 2. To log in, you will need to use your organization ID. Usually, if your email is something like myemailuser@school.edu, your ID is myemailuser. Then:

```
ssh myemailuser@notchpeak.chpc.utah.edu
```

## 4.3 Step 3: Submitting the job

Overall, there are two ways to use the compute nodes: interactively (salloc) and in batch mode (sbatch). In this case, since we have a Slurm script, we will use the latter.

To submit the job, we can type the following:

```
sbatch 00-hello-world.slurm
```

And that's it! That said, it is often required to specify the account and partition the user will be submitting the job. For example, if you have the account my-account and partition my-partition associated with your user, you can incorporate that information as follows:

```
sbatch 00-hello-world.slurm --acount=my-account --partition=my-partition
```

In the case of interactive sessions, You can start one using the salloc command. For example, if you wanted to run R with 8 cores, using 16 Gigs of memory in total, you would need to do the following:

```
salloc -n1 --cpus-per-task=8 --mem-per-cpu=2G --time=01:00:00
```

Once your request is submitted, you will get access to a compute node. Within it, you can load the required modules and start R:

```
module load R/4.2.2 R
```

Interactive sessions are not recommended for long jobs. Instead, use this resource if you need to inspect some large dataset, debug your code, etc.

# 5 Simulating pi (once more)

The following is an example many people (including me) have used to illustrate parallel computing with R. The example is straightforward: we want to approximate pi by doing some Monte Carlo simulations.

We know that the area of a circle is  $A = \pi r^2$ , which is equivalent to  $\pi = A/r^2$ , so if we can approximate the Area of a circle, then we can approximate  $\pi$ . How do we do this?

Using Monte Carlo experiments, we can approximate the probability of a random point x falling within the unit circle using the following formula:

$$\hat{p} = \frac{1}{n} \sum_{i} \mathbf{1}(x \in \text{Circle})$$

This approximation,  $\hat{p}$ , multiplied by the area of the square containing the circle, which has an area equal to  $(2 \times r)^2$ , thus, we can finally write

$$\hat{\pi} = \hat{p} \times (2 \times r)^2 / r^2 = 4\hat{p}$$

# 5.1 Submitting jobs to Slurm

We will primarily work by submitting jobs using the sbatch function. This function takes as its main argument a bash file with the program to execute. In the case of R, a regular bash file looks something like this:

```
#!/bin/sh
#SBATCH --job-name=sapply
#SBATCH --time=00:10:00

module load R/4.2.2
Rscript --vanilla 01-sapply.R
```

This file has three components:

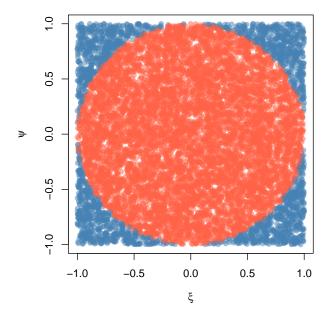


Figure 5.1: 10,000 random points drawn within the unit circle.

- The Slurm flags #SBATCH: These comment-like entries pass Slurm options to the job. In this example, we only specify the options job-name and time. Other common options would include account and partition.
- Loading R module load R/4.2.2: Depending on your system's configuration, you may or may not need to load modules or run bash scripts before being able to run R. In this example, we are loading R version 4.2.2 using LMod (see previous section).
- Executing the R script: After specifying Slurm options and loading whatever needs to be loaded before executing R, we are using RScript to execute the program we wrote.

Submission is then made as follows:

```
sbatch 01-sapply.slurm
```

The following examples have two files, a bash script and an R script, to be called by Slurm.

#### 5.1.1 Case 1: Single job, single core job

The most basic way is submitting a job using the sbatch command. In this case, you must have two files: (1) An R script and (2) a bash script. e.g.

The contents of the R script (01-sapply.R) are:

```
# Model parameters
nsims <- 1e3
n     <- 1e4

# Function to simulate pi
simpi <- function(i) {

   p <- matrix(runif(n*2, -1, 1), ncol = 2)
   mean(sqrt(rowSums(p^2)) <= 1) * 4

}

# Approximation
set.seed(12322)
ans <- sapply(1:nsims, simpi)

message("Pi: ", mean(ans))
saveRDS(ans, "01-sapply.rds")</pre>
```

The contents of the bashfile (01-sapply.slurm) are:

```
#!/bin/sh
#SBATCH --job-name=sapply
#SBATCH --time=00:10:00

module load R/4.2.2
Rscript --vanilla 01-sapply.R
```

## 5.1.2 Case 2: Single job, multicore job

Imagine that we would like to use more than one processor for this job, using the parallel::mclapply function from the parallel package.<sup>1</sup> Then, besides adapting the code, we need to tell Slurm that we are using more than one core per task, as in the following example:

R script (02-mclapply.R):

<sup>&</sup>lt;sup>1</sup>This function is sort of a wrapper of makeForkcluster. Forking provides a way to duplicate a process in the OS without replicating the memory, which is both faster and efficient.

```
# Model parameters
  nsims <- 1e3
          <- 1e4
  ncores <- 4L
  # Function to simulate pi
  simpi <- function(i) {</pre>
    p \leftarrow matrix(runif(n*2, -1, 1), ncol = 2)
    mean(sqrt(rowSums(p^2)) \le 1) * 4
  }
  # Approximation
  set.seed(12322)
  ans <- parallel::mclapply(1:nsims, simpi, mc.cores = ncores)</pre>
  ans <- unlist(ans)</pre>
  message("Pi: ", mean(ans))
  saveRDS(ans, "02-mclpply.rds")
Bashfile (02-mclapply.slurm):
  #!/bin/sh
  #SBATCH --job-name=mclapply
  #SBATCH --time=00:10:00
  #SBATCH --cpus-per-task=4
  module load R/4.2.2
  Rscript --vanilla 02-mclapply.R
```

# 5.2 Jobs with the slurmR package

The slurm R package (Vega Yon and Marjoram 2019, 2022) is a lightweight wrapper of Slurm. The package's primary functions are the \*apply family-mainly through Slurm job arrays-and the makeSlurmCluster()-which is a wrapper of makePSOCKcluster.

This section will illustrate how to submit jobs using the makeSlurmCluster() function and Slurm\_sapply. Furthermore, the last example demonstrates how we can skip writing Slurm

scripts entirely using the sourceSlurm() function included in the package.

## 5.2.1 Case 3: Single job, multinode job

In this case, there is no simple way to submit a multinodal job to Slurm; unless you use the **slurmR** package.<sup>2</sup> In this example, we will combine **slurmR** with the parallel package's parSapply function to submit a multinodal job using the function makeSlurmCluster(). With it, slurmR will submit a job requesting njobs tasks (processors) that could span multiple nodes,<sup>3</sup> and create a Socket cluster out of it (like using makePSOCKcluster.) One thing to keep in mind is that Socket clusters are limited in the number of connections a single R session can span. You can read more about it here and here.

R script (03-parsapply-slurmr.R):

```
# Model parameters
nsims <- 1e3
       <- 1e4
ncores <- 4L
# Function to simulate pi
simpi <- function(i) {</pre>
  p \leftarrow matrix(runif(n*2, -1, 1), ncol = 2)
  mean(sqrt(rowSums(p^2)) \le 1) * 4
}
# Setting up slurmR
library(slurmR) # This also loads the parallel package
# Making the cluster, and exporting the variables
cl <- makeSlurmCluster(ncores)</pre>
# Approximation
clusterExport(cl, c("n", "simpi"))
ans <- parSapply(cl, 1:nsims, simpi)</pre>
# Closing connection
stopCluster(cl)
```

<sup>&</sup>lt;sup>2</sup>See installation instructions **here** 

<sup>&</sup>lt;sup>3</sup>Although possible, most multinode jobs will be allocated if insufficient threads are within a single node. Remember Slurm does not run the jobs but rather reserves computational resources for you to run it.

```
message("Pi: ", mean(ans))
saveRDS(ans, "03-parsapply-slurmr.rds")
Bashfile (03-parsapply-slurmr.slurm):
#!/bin/sh
```

```
#!/bin/sh
#SBATCH --job-name=parsapply
#SBATCH --time=00:10:00

module load R/4.2.2
Rscript --vanilla 03-parsapply-slurmr.R
```

## 5.2.2 Case 4: Multi job, single/multi-core

Another way to submit jobs is using **job arrays**. A job array is a job repeated **njobs** times with the same configuration. The main difference between replicates is what you do with the SLURM\_ARRAY\_TASK\_ID environment variable. This variable is defined within each replicate and can be used to make the "subjob" depending on that.

Here is a quick example using R

```
ID <- Sys.getenv("SLURM_ARRAY_TASK_ID")
if (ID == 1) {
   ...[do this]...
} else if (ID == 2) {
   ...[do that]...
}</pre>
```

The slurm R package makes submitting job arrays easy. Again, with the simulation of pi, we can do it in the following way:

R script (04-slurm\_sapply.R):

```
# Model parameters
nsims <- 1e3
n <- 1e4
# ncores <- 4L
njobs <- 4L</pre>
```

```
# Function to simulate pi
  simpi <- function(i, n.) {</pre>
    p \leftarrow matrix(runif(n.*2, -1, 1), ncol = 2)
    mean(sqrt(rowSums(p^2)) \le 1) * 4
  }
  # Setting up slurmR
  library(slurmR) # This also loads the parallel package
  # Approximation
  ans <- Slurm_sapply(</pre>
    1:nsims, simpi,
             = n,
    njobs
             = njobs,
    plan
             = "collect",
    tmp_path = "/scratch/vegayon" # This is where all temp files will be exported
  message("Pi: ", mean(ans))
  saveRDS(ans, "04-slurm_sapply.rds")
Bashfile (04-slurm_sapply.slurm):
  #!/bin/sh
  #SBATCH --job-name=slurm_sapply
  #SBATCH --time=00:10:00
  module load R/4.2.2
  Rscript --vanilla 04-slurm_sapply.R
```

One of the main benefits of using this approach instead of the makeSlurmCluster function (and thus, working with a SOCK cluster) are:

- The number of jobs is not limited here (only by the admin, but not by R).
- If a job fails, then we can re-run it using sbatch once again (see example here).
- You can check the individual logs of each process using the function Slurm\_lob().

• You can submit the job and quit the R session without waiting for it to finalize. You can always read back the job using the function read\_slurm\_job([path-to-the-temp])

## 5.2.3 Case 5: Skipping the .slurm file

The slurmR package has a function named sourceSlurm that can be used to avoid creating the .slurm file. The user can add the SBATCH options to the top of the R script (including the #!/bin/sh line) and submit the job from within R as follows:

R script (05-sapply.R):

```
#!/bin/sh
#SBATCH --job-name=sapply-sourceSlurm
#SBATCH --time=00:10:00
# Model parameters
nsims <- 1e3
      <- 1e4
# Function to simulate pi
simpi <- function(i) {</pre>
  p \leftarrow matrix(runif(n*2, -1, 1), ncol = 2)
  mean(sqrt(rowSums(p^2)) \le 1) * 4
}
# Approximation
set.seed(12322)
ans <- sapply(1:nsims, simpi)
message("Pi: ", mean(ans))
saveRDS(ans, "05-sapply.rds")
```

From the R console (is OK if you are in the Head node)

```
slurmR::sourceSlurm("05-sapply.R")
```

And voilá! A temporary bash file will be generated to submit the R script to the queue. The following video shows a possible output on the University of Utah's CHPC with slurmR version 0.5-3:

https://youtu.be/OasEla5EszI

Part III
Using C++

# 6 Rcpp

What is Rcpp (Eddelbuettel and François 2011; Eddelbuettel 2013; Eddelbuettel and Balamuta 2018)?

#### 6.1 Before we start

1. You need to have Rcpp installed in your system:

```
install.packages("Rcpp")
```

- 2. You need to have a compiler
  - Windows: You can download Rtools from here.
  - MacOS: It is a bit complicated... Here are some options:
    - CRAN's manual to get the clang, clang++, and gfortran compilers here.
    - A great guide by the coatless professor here

And that's it!

# 6.2 R is great, but...

- The problem:
  - As we saw, R is very fast... once vectorized
  - What to do if your model cannot be vectorized?
- The solution: Use C/C++/Fotran! It works with R!
- The problem to the solution: What R user knows any of those!?
- R has had an API (application programming interface) for integrating C/C++ code with R for a long time.
- Unfortunately, it is not very straightforward

# 6.3 Enter Rcpp

- One of the most important R packages on CRAN.
- As of January 22, 2023, about 50% of CRAN packages depend on it (directly or not).
- From the package description:

The 'Rcpp' package provides R functions as well as C++ classes which offer a seamless integration of R and C++

# 6.4 Why bother?

• To draw ten numbers from a normal distribution with sd = 100.0 using R C API:

```
SEXP stats = PROTECT(R_FindNamespace(mkString("stats")));
SEXP rnorm = PROTECT(findVarInFrame(stats, install("rnorm")));
SEXP call = PROTECT(
    LCONS( rnorm, CONS(ScalarInteger(10), CONS(ScalarReal(100.0),
    R_NilValue))));
SET_TAG(CDDR(call),install("sd"));
SEXP res = PROTECT(eval(call, R_GlobalEnv));
UNPROTECT(4);
return res;
```

• Using Rcpp:

```
Environment stats("package:stats");
Function rnorm = stats["rnorm"];
return rnorm(10, Named("sd", 100.0));
```

# 6.5 Example 1: Looping over a vector

```
#include<Rcpp.h>
using namespace Rcpp;
// [[Rcpp::export]]
NumericVector add1(NumericVector x) {
   NumericVector ans(x.size());
   for (int i = 0; i < x.size(); ++i)</pre>
```

```
ans[i] = x[i] + 1;
    return ans;
  add1(1:10)
 [1] 2 3 4 5 6 7 8 9 10 11
Make it sweeter by adding some "sugar" (the Rcpp kind)
  #include<Rcpp.h>
  using namespace Rcpp;
  // [[Rcpp::export]]
  NumericVector add1Cpp(NumericVector x) {
    return x + 1;
  }
  add1Cpp(1:10)
 [1] 2 3 4 5 6 7 8 9 10 11
6.6 How much fast?
```

Compared to this:

```
add1R <- function(x) {</pre>
  for (i in 1:length(x))
    x[i] <- x[i] + 1
  X
}
microbenchmark::microbenchmark(add1R(1:1000), add1Cpp(1:1000))
```

Unit: microseconds expr  $\mathtt{min}$ lq mean median max neval cld uq add1R(1:1000) 34.412 35.0285 53.54009 36.0720 36.6605 1769.485 100 a add1Cpp(1:1000) 2.175 2.5490 8.33909 2.8625 5.1835 424.318 100

## 6.7 Main differences between R and C++

- 1. One is compiled, and the other interpreted
- 2. Indexing objects: In C++ the indices range from 0 to (n 1), whereas in R is from 1 to n.
- 3. All expressions end with a; (optional in R).
- 4. In C++ object need to be declared, in R not (dynamic).

# 6.8 C++/Rcpp fundamentals: Types

Besides C-like data types (double, int, char, and bool), we can use the following types of objects with Rcpp:

- Matrices: NumericMatrix, IntegerMatrix, LogicalMatrix, CharacterMatrix
- Vectors: NumericVector, IntegerVector, LogicalVector, CharacterVector
- And more!: DataFrame, List, Function, Environment

## 6.9 Parts of "an Rcpp program"

```
#include<Rcpp.h>
using namespace Rcpp

// [[Rcpp::export]]

NumericVector add1(NumericVector x) {
    NumericVector ans(x.size());
    for (int i = 0; i < x.size(); ++i)
        ans[i] = x[i] + 1;
    return ans;
}</pre>
```

Line by line, we see the following:

- 1. The #include<Rcpp.h> is similar to library(...) in R, it brings in all that we need to write C++ code for Rcpp.
- 2. using namespace Rcpp is somewhat similar to detach(...). This simplifies syntax. If we don't include this, all calls to Rcpp members need to be explicit, e.g., instead of typing NumericVector, we would need to type Rcpp::NumericVector

- 3. The // starts a comment in C++, in this case, the // [[Rcpp::export]] comment is a flag Rcpp uses to "export" this C++ function to R.
- 4. It is the first part of the function definition. We are creating a function that returns a Numeric Vector, is called add1, has a single input element named x that is also a Numeric Vector.
- 5. Here, we are declaring an object called ans, which is a Numeric Vector with an initial size equal to the size of x. Notice that .size() is called a "member function" of the x object, which is of class Numeric Vector.
- 6. We are declaring a for-loop (three parts):
  - a. int i = 0 We declare the variable i, an integer, and initialize it at 0.
  - b. i < x.size() This loop will end when i's value is at or above the length of x.
  - c. ++i At each iteration, i will increment in one unit.
- 7. ans[i] = x[i] + 1 set the i-th element of ans equal to the i-th element of x plus 1.
- 8. return ans exists the function returning the vector ans.

Now, where to execute/run this?

- You can use the sourceCpp function from the Rcpp package to run .cpp scripts (this is what I do most of the time).
- There's also cppFunction, which allows compiling a single function.
- Write an R package that works with Rcpp.

For now, let's use the first option.

# 6.10 Example running .cpp file

Imagine that we have the following file named norm.cpp

```
#include <Rcpp.h>
using namespace Rcpp;
// [[Rcpp::export]]
double normRcpp(NumericVector x) {
   return sqrt(sum(pow(x, 2.0)));
}
```

We can compile and obtain this function using this line Rcpp::sourceCpp("norm.cpp"). Once compiled, a function called normRcpp will be available in the current R session.

## 6.11 Your turn

### 6.11.1 Problem 1: Adding vectors

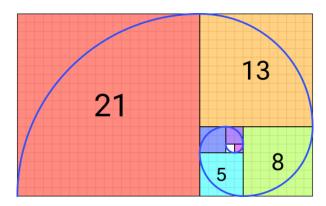
1. Using what you have just learned about Rcpp, write a function to add two vectors of the same length. Use the following template

```
#include <Rcpp.h>
using namespace Rcpp;
// [[Rcpp::export]]
NumericVector add_vectors([declare vector 1], [declare vector 2]) {
    ... magick ...
    return [something];
}
```

2. Now, we have to check for lengths. Use the stop function to make sure lengths match. Add the following lines in your code

```
if ([some condition])
  stop("an arbitrary error message :)");
```

#### 6.11.2 Problem 2: Fibonacci series



Each element of the sequence is determined by the following:

$$F(n) = \left\{ \begin{array}{ll} n, & \text{if } n \leq 1 \\ F(n-1) + F(n-2), & \text{otherwise} \end{array} \right.$$

Using recursions, we can implement this algorithm in R as follows:

```
fibR <- function(n) {
   if (n <= 1)
      return(n)
   fibR(n - 1) + fibR(n - 2)
}
# Is it working?
c(
   fibR(0), fibR(1), fibR(2),
   fibR(3), fibR(4), fibR(5),
   fibR(6)
)</pre>
```

#### [1] 0 1 1 2 3 5 8

Now, let's translate this code into Rcpp and see how much speed boost we get!

## 6.11.3 Problem 2: Fibonacci series (solution)

```
#include <Rcpp.h>
// [[Rcpp::export]]
int fibCpp(int n) {
  if (n <= 1)
    return n;

return fibCpp(n - 1) + fibCpp(n - 2);
}

microbenchmark::microbenchmark(fibR(20), fibCpp(20))</pre>
```

```
Unit: microseconds
       expr
                 min
                             lq
                                      mean
                                               median
                                                                      max neval
                                                             uq
   fibR(20) 5351.777 5802.3785 6513.57586 5943.7375 6295.514 13654.174
                                                                             100
fibCpp(20)
              11.842
                        12.7245
                                              19.2465
                                                        27.513
                                                                  806.393
                                                                             100
                                  28.71172
 cld
  a
   b
```

# 6.12 RcppArmadillo and OpenMP

- Friendlier than **RcppParallel**... at least for 'I-use-Rcpp-but-don't-actually-know-much-about-C++' users (like myself!).
- Must run only 'Thread-safe' calls, so calling R within parallel blocks can cause problems (almost all the time).
- Use arma objects, e.g. arma::mat, arma::vec, etc. Or, if you are used to them std::vector objects as these are thread-safe.
- Pseudo Random Number Generation is not very straightforward... But C++11 has a nice set of functions that can be used together with OpenMP
- Need to think about how processors work, cache memory, etc. Otherwise, you could get into trouble... if your code is slower when run in parallel, then you probably are facing false sharing
- If R crashes... try running R with a debugger (see Section 4.3 in Writing R extensions):

```
~$ R --debugger=valgrind
```

#### 6.12.1 RcppArmadillo and OpenMP workflow

1. Tell Rcpp that you need to include that in the compiler:

```
#include <omp.h>
// [[Rcpp::plugins(openmp)]]
```

2. Within your function, set the number of cores, e.g.

```
// Setting the cores
omp_set_num_threads(cores);
```

3. Tell the compiler that you'll be running a block in parallel with OpenMP

```
#pragma omp [directives] [options]
{
    ...your neat parallel code...
}
```

You'll need to specify how OMP should handle the data:

- shared: Default, all threads access the same copy.
- private: Each thread has its own copy, uninitialized.
- firstprivate Each thread has its own copy, initialized.
- lastprivate Each thread has its own copy. The last value used is returned.

Setting default (none) is a good practice.

4. Compile!

### 6.12.2 Ex 5: RcppArmadillo + OpenMP

Our own version of the dist function... but in parallel!

```
#include <omp.h>
#include <RcppArmadillo.h>
// [[Rcpp::depends(RcppArmadillo)]]
// [[Rcpp::plugins(openmp)]]
using namespace Rcpp;
// [[Rcpp::export]]
arma::mat dist_par(const arma::mat & X, int cores = 1) {
  // Some constants
  int N = (int) X.n_rows;
  int K = (int) X.n_cols;
  // Output
  arma::mat D(N,N);
  D.zeros(); // Filling with zeros
  // Setting the cores
  omp_set_num_threads(cores);
#pragma omp parallel for shared(D, N, K, X) default(none)
```

```
for (int i=0; i<N; ++i)</pre>
      for (int j=0; j<i; ++j) {
        for (int k=0; k<K; k++)
          D.at(i,j) += pow(X.at(i,k) - X.at(j,k), 2.0);
        // Computing square root
        D.at(i,j) = sqrt(D.at(i,j));
        D.at(j,i) = D.at(i,j);
      }
    // My nice distance matrix
    return D;
  }
  # Simulating data
  set.seed(1231)
  K <- 5000
  n <- 500
  x <- matrix(rnorm(n*K), ncol=K)</pre>
  # Are we getting the same?
  table(as.matrix(dist(x)) - dist_par(x, 4)) # Only zeros
250000
  # Benchmarking!
  microbenchmark::microbenchmark(
    dist(x),
                            # stats::dist
    dist_par(x, cores = 1), # 1 core
    dist_par(x, cores = 2), # 2 cores
    dist_par(x, cores = 4), # 4 cores
    times = 1,
    unit = "ms"
  )
Unit: milliseconds
                   expr
                             min
                                      lq
                                              mean median
                                                                   uq
                                                                           max
```

```
dist(x) 2223.023 2223.023 2223.023 2223.023 2223.023 2223.023
dist_par(x, cores = 1) 2414.402 2414.402 2414.402 2414.402 2414.402 2414.402
dist_par(x, cores = 2) 1865.621 1865.621 1865.621 1865.621 1865.621 1865.621
dist_par(x, cores = 4) 1223.261 1223.261 1223.261 1223.261 1223.261
neval
    1
    1
    1
    1
    1
    1
```

#### 6.12.3 Ex 6: The future

- **future** is an R package that was designed "to provide a very simple and uniform way of evaluating R expressions asynchronously using various resources available to the user."
- future class objects are either resolved or unresolved.
- If queried, **Resolved** values are return immediately, and **Unresolved** values will block the process (i.e. wait) until it is resolved.
- Futures can be parallel/serial, in a single (local or remote) computer, or a cluster of them.

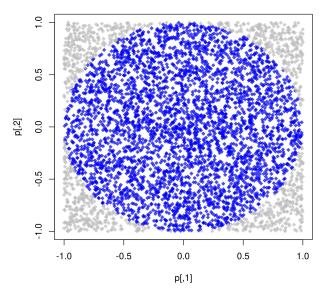
Let's see a brief example

```
library(future)
plan(multicore)
# We are creating a global variable
# Creating the futures has only the overhead (setup) time
system.time({
  x1 \% -\% \{Sys.sleep(3);a^2\}
  x2 \% -\% {Sys.sleep(3);a^3}
})
##
      user system elapsed
     0.023
            0.008
                      0.030
# Let's just wait 5 seconds to make sure all the cores have returned
Sys.sleep(3)
system.time({
  print(x1)
  print(x2)
})
## [1] 4
```

```
## [1] 8
## user system elapsed
## 0.003 0.000 0.003
```

## **6.12.4** Bonus track 1: Simulating $\pi$

- We know that  $\pi = \frac{A}{r^2}$ . We approximate it by randomly adding points x to a square of size 2 centered at the origin.
- So, we approximate  $\pi$  as  $\Pr{\|x\| \le 1\} \times 2^2}$



The R code to do this

```
pisim <- function(i, nsim) { # Notice we don't use the -i-
    # Random points
    ans <- matrix(runif(nsim*2), ncol=2)

# Distance to the origin
    ans <- sqrt(rowSums(ans^2))

# Estimated pi
    (sum(ans <= 1)*4)/nsim
}</pre>
```

```
library(parallel)
  # Setup
  cl <- makePSOCKcluster(4L)</pre>
  clusterSetRNGStream(cl, 123)
  # Number of simulations we want each time to run
  nsim <- 1e5
  # We need to make -nsim- and -pisim- available to the
  # cluster
  clusterExport(cl, c("nsim", "pisim"))
  # Benchmarking: parSapply and sapply will run this simulation
  # a hundred times each, so at the end we have 1e5*100 points
  # to approximate pi
  microbenchmark::microbenchmark(
    parallel = parSapply(cl, 1:100, pisim, nsim=nsim),
             = sapply(1:100, pisim, nsim=nsim),
    times
Unit: milliseconds
     expr
               \mathtt{min}
                         lq
                                mean
                                       median
                                                             max neval
                                                     uq
 parallel 268.7455 268.7455 268.7455 268.7455 268.7455
   serial 329.4686 329.4686 329.4686 329.4686 329.4686
  ans_par <- parSapply(cl, 1:100, pisim, nsim=nsim)</pre>
  ans_ser <- sapply(1:100, pisim, nsim=nsim)</pre>
  stopCluster(cl)
     par
              ser
3.141762 3.141266 3.141593
```

## 6.13 See also

- Package parallel
- Using the iterators package
- Using the foreach package
- 32 OpenMP traps for C++ developers

- The OpenMP API specification for parallel programming
- 'openmp' tag in Rcpp gallery
- OpenMP tutorials and articles

For more, check out the CRAN Task View on HPC

# 7 Misc

### 7.1 General resources

The Center for Advanced Research Computing (formerly HPCC) has tons of resources online. Here are a couple of useful links:

- Center for Advanced Research Computing Website https://carc.usc.edu
- User forum (very useful!) https://hpc-discourse.usc.edu/categories
- Monitor your account https://hpcaccount.usc.edu/
- Slurm Jobs Templates https://carc.usc.edu/user-information/user-guides/high-performance-computing/slurm-templates
- Using R https://carc.usc.edu/user-information/user-guides/software-and-programming/r

## 7.2 Data Pointers

IMHO, these are the most important things to know about data management at USC's HPC:

- 1. Do your data transfer using the transfer nodes (it is faster).
- 2. Never use your home directory as a storage space (use your project's allotted space instead).
- 3. Use the scratch filesystem for temp data only, i.e., never save important files in scratch.
- 4. Finally, besides of **Secure copy protocol (scp)**, if you are like me, try setting up a GUI client for moving your data (see this).

## 7.3 The Slurm options they forgot to tell you about...

First of all, you have to be aware that the only thing Slurm does is allocate resources. If your application uses parallel computing or not, that's another story.

Here some options that you need to be aware of:

- ntasks (default 1) This tells Slurm how many processes you will have running. Notice that processes need not to be in the same node (so Slurm may reserve space in multiple nodes)
- cpus-per-task (defatult 1) This is how many CPUs each task will be using. This is what you need to use if you are using OpenMP (or a package that uses that), or anything you need to keep within the same node.
- nodes the number of nodes you want to use in your job. This is useful mostly if you care about the maximum (I would say) number of nodes you want your job to work. So, for example, if you want to use 8 cores for a single task and force it to be in the same node, you would add the option --nodes=1/1.
- mem-per-cpu (default 1GB) This is the MINIMUM amount of memory you want Slurm to allocate for the task. Not a hard barrier, so your process can go above that.
- time (default 30min) This is a hard limit as well, so if you job takes more than the specified time, Slurm will kill it.
- partition (default "") and account (default"") these two options go along together, this tells Slurm what resources to use. Besides of the private resources we have the following:
  - quick partition: Any job that is small enough (in terms of time and memory) will go this way. This is usually the default if you don't specify any memory or time options.
  - main partition: Jobs that require more resources will go in this line.
  - scavenge partition: If you need a massive number resources, and have a job that shouldn't, in principle, take too long to finalize (less than a couple of hours), and you are OK with someone killing it, then this queue is for you. The Scavenge partition uses all the idle resources of the private partitions, so if any of the owners requests the resources, Slurm will cancel your job, i.e. you have no priority (see more).
  - largemem partition: If you need lots of memory, we have 4 1TB nodes for that.

More information about the partitions here

# 7.4 Good practices (recomendations)

This is what you should use as a minimum:

```
#SBATCH --output=simulation.out

#SBATCH --job-name=simulation

#SBATCH --time=04:00:00

#SBATCH --mail-user=[you]@usc.edu

#SBATCH --mail-type=END,FAIL
```

- output is the name of the logfile to which Slurm will write.
- job-name is that, the name of the job. You can use this to either kill or at least be able to identify what is what you are running when you use myqueue
- time Try always to set a time estimate (plus a little more) for your job.
- mail-user, mail-type so Slurm notifies you when things happen

Also, in your R code

• Any I/O should be done to either Scratch (/scratch/[your usc net id]) or Tmp Sys.getenv("TMPDIR").

## 7.5 Running R interactively

- 1. The HPC has several pre-installed pieces of software. R is one of those.
- 2. To access the pre-installed software, we use the **Lmod module system** (more information **here**)
- 3. It has multiple versions of R installed. Use your favorite one by running

```
module load R/4.2.2/[version number]
```

Where [version number] can be 3.5.6 and up to 4.0.3 (the latest update). The usc module automatically loads gcc/8.3.0, openblas/0.3.8, openmpi/4.0.2, and pmix/3.1.3.

4. It is never a good idea to use your home directory to install R packages, that's why you should try using a **symbolic link instead**, like this

```
cd ~
mkdir -p /path/to/a/project/with/lots/of/space/R
ln -s /path/to/a/project/with/lots/of/space/R R
```

This way, whenever you install your R packages, R will default to that location

5. You can run interactive sessions on HPC, but this recommended to be done using the salloc function in Slurm, in other words, NEVER EVER USE R (OR ANY SOFT-WARE) TO DO DATA ANALYSIS IN THE HEAD NODES! The options passed to salloc are the same options that can be passed to sbatch (see the next section.) For example, if need to do some analyses in the thomas partition (which is private and I have access to), I would type something like

```
salloc --account=lc_pdt --partition=thomas --time=02:00:00 --mem-per-cpu=2G
```

This would put me in a single node allocating 2 gigs of memory for a maximum of 2 hours.

# 7.6 NoNos when using R

- Do computation on the head node (compile stuff is OK)
- Request a number of nodes (unless you know what you are doing)
- Use your home directory for I/O
- Save important information in Staging/Scratch

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