Applied SNA with R

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About this book

This book will be build as part of a workshop on Applied Social Network Analysis with R. Its contents will be populated as the sessions take place, and for now there is particular program that we will follow, instead, we have the following workflow:

- 1. Participants will share their data and what they need to do with it.
- 2. Based on their data, I'll be preparing the sessions trying to show attendees how would I approach the problem, and at the same time, teach by example about the R language.
- 3. Materials will be published on this website and, hopefully, video recordings of the sessions.

At least in the first version, the book will be organized by session, this is, one chapter per session.

All the book materials can be downloaded from https://github.com/gvegayon/appliedsnar

In general, we will besides of R itself, we will be using R studio and the following R packages: dplyr for data management, stringr for data cleaning, and of course igraph, netdiffuseR (a bit of a bias here), and statnet for our neat network analysis.¹

¹Some of you may be wondering "what about ggplot2 and friends? What about tidyverse", well, my short answer is I jumped into R before all of that was that popular. When I started plots were all about lattice, and after a couple of years on that, about base R graphics. What I'm saying is that so far I have not find a compelling reason to leave my "old-practices" and embrace all the tidyverse movement (religion?).

Introduction

For this book we need the following

1. Install R from CRAN: https://www.r-project.org/

2. (optional) Install Rstudio: https://rstudio.org

While I find RStudio extreamly useful, it is not necesary to use it with R.

R Basics

3.1 What is R

A good reference book for both new and advanced user is "The Art of R programming" (Matloff 2011)¹

3.2 How to install packages

Nowadays there are two ways of installing R packages (that I'm aware of), either using install.packages, which is a function shipped with R, or use the devtools R package to install a package from some remote repository other than CRAN, here is a couple of examples:

```
# This will install the igraph package from CRAN
> install.packages("netdiffuseR")

# This will install the bleeding-edge version from the project's github repo!
> devtools::install_github("USCCANA/netdiffuseR")
```

The first one, using install.packages, installs the CRAN version of netdiffuseR, whereas the second installs whatever version is plublished on https://github.com/USCCANA/netdiffuseR, which is usually called the development version.

¹Here a free pdf version distributed by the author.

10 CHAPTER 3. R BASICS

In some cases users may want/need to install packages from command line as some packages need extra configuration to be installed. But we won't need to look at it now.

Network Nomination Data

The data can be downloaded from here.

The codebook for the data provided here is in the appendix.

This chapter's goals are:

- 1. Read the data into R,
- 2. Create a network with it,
- 3. Compute descriptive statistics
- 4. Visualize the network

4.1 Data preprocessing

4.1.1 Reading the data into R

R has several ways of reading data in. You data can be Raw plain files like CSV, tab delimited or specified by column width, for which you can use the readr package (Wickham, Hester, and Francois 2017); or it can be binary files like dta (Stata), Octave, SPSS, for which foreign (R Core Team 2017) can be used; or it could be excel files in which case you should be using readxl (Wickham and Bryan 2017). In our case, the data for this session is in Stata format:

```
# Reading the data
dat <- foreign::read.dta("03-sns.dta")

# Taking a look at the data's first 5 columns and 5 rows
dat[1:5, 1:10]</pre>
```

```
photoid school hispanic female1 female2 female3 female4 grades1 grades2
##
            1
## 1
                             1
                                                        0
                                                                0
                 111
                                     NA
                                              NA
                                                                        NA
                                                                                 NA
## 2
            2
                 111
                             1
                                      0
                                              NA
                                                       NA
                                                                 0
                                                                       3.0
                                                                                 NA
## 3
           7
                 111
                             0
                                      1
                                               1
                                                        1
                                                                1
                                                                       5.0
                                                                                4.5
                                               1
                                                        1
## 4
           13
                 111
                             1
                                      1
                                                                1
                                                                       2.5
                                                                                2.5
## 5
           14
                 111
                                      1
                                               1
                                                        1
                                                               NA
                                                                       3.0
                                                                                3.5
     grades3
##
## 1
         3.5
## 2
          NA
## 3
         4.0
         2.5
## 4
## 5
         3.5
```

4.1.2 Creating a unique id for each participant

Now suppose that we want to create a unique id using the school and photo id. In this case, since both variables are numeric, a good way of doing it is to encode the id such that, for example, the last three x numbers are the photoid and the first ones are the school id. To do this we need to take into account the range of the variables. Here, photoid has the following range:

```
(photo_id_ran <- range(dat$photoid))
## [1] 1 2074</pre>
```

As the variable spans up to 2074, we need to set the last 4 units of the variable to store the photoid. We will use dplyr (Wickham et al. 2017) and magrittr (Bache and Wickham 2014)]

(the pipe operator, %>%) to create this variable, and we will call it... id (mind blowing, right?):

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
## filter, lag

## The following objects are masked from 'package:base':
##
## intersect, setdiff, setequal, union

library(magrittr)

(dat %<>% mutate(id = school*10000 + photoid)) %>%
head %>%
select(school, photoid, id)
```

```
school photoid
##
                           id
## 1
        111
                   1 1110001
## 2
        111
                   2 1110002
## 3
                   7 1110007
        111
## 4
        111
                  13 1110013
## 5
                  14 1110014
        111
## 6
        111
                  15 1110015
```

Wow, what happend in the last three lines of code! What is that %>%? Well, that's the piping operator, and it is a very nice way of writing nested function calls. In this case, instead of having write something like

```
dat_filtered$id <- dat_filtered$school*10000 + dat_filtered$photoid
subset(head(dat_filtered), select = c(school, photoid, id))</pre>
```

4.2 Creating a network

- We want to build a social network. For that, we either use an adjacency matrix or an edgelist.
- Each individual of the SNS data nomitated 19 friends from school. We will use those nominations to create the social network.
- In this case, we will create the network by coercing the dataset into an edgelist.

4.2.1 From survey to edgelist

Let's start by loading a couple of handy R packages for this task, tidyr (Wickham and Henry 2017), which we will use to reshape the data, and stringr (Wickham 2017), which we will use to process strings using regular expressions¹.

```
library(tidyr)
library(stringr)
```

Optionally, we can use the tibble type of object which is an alternative to the actual data.frame. This object is claimed to provide *more efficient methods for matrices and data frames*.

```
dat <- as_tibble(dat)</pre>
```

What I like from tibbles is that when you print them on the console these actually look nice:

dat

```
## # A tibble: 2,164 x 100
##
      photoid school hispanic female1 female2 female3 female4 grades1 grades2
        <int> <int>
                          <dbl>
                                                                     <dbl>
                                                                              <dbl>
##
                                   <int>
                                           <int>
                                                    <int>
                                                             <int>
    1
             1
                  111
                           1.00
                                      NA
                                              NA
                                                        0
                                                                     NA
                                                                              NA
##
             2
##
    2
                  111
                           1.00
                                       0
                                              NA
                                                       NA
                                                                 0
                                                                       3.00
                                                                              NA
##
    3
             7
                  111
                                       1
                                                1
                                                        1
                                                                 1
                                                                       5.00
                                                                               4.50
##
            13
                  111
                           1.00
                                       1
                                                1
                                                        1
                                                                 1
                                                                       2.50
                                                                               2.50
```

¹Please refer to the help file?'regular expression' in R. The R package rex (Ushey, Hester, and Krzyzanowski 2017) is a very nice companion for writing regular expressions. There's also a neat (but experimental) RStudio addin that can be very helpful for understanding how regular expressions work, the regexplain addin.

```
##
   5
           14
                 111
                         1.00
                                     1
                                             1
                                                     1
                                                            NA
                                                                  3.00
                                                                           3.50
   6
           15
                 111
                         1.00
                                     0
                                             0
                                                     0
                                                             0
                                                                  2.50
                                                                           2.50
##
                                             1
##
   7
           20
                 111
                         1.00
                                     1
                                                     1
                                                             1
                                                                  2.50
                                                                           2.50
##
   8
           22
                 111
                         1.00
                                    NA
                                            NA
                                                     0
                                                             0
                                                                 NA
                                                                          NA
##
   9
           25
                 111
                                     1
                                             1
                                                    NA
                                                             1
                                                                  4.50
                                                                           3.50
           27
                         1.00
                                     0
                                            NA
                                                     0
                                                             0
## 10
                 111
                                                                  3.50
                                                                          NA
## # ... with 2,154 more rows, and 91 more variables: grades3 <dbl>,
       grades4 <dbl>, eversmk1 <int>, eversmk2 <int>, eversmk3 <int>,
## #
       eversmk4 <int>, everdrk1 <int>, everdrk2 <int>, everdrk3 <int>,
## #
       everdrk4 <int>, home1 <int>, home2 <int>, home3 <int>, home4 <int>,
## #
       sch_friend11 <int>, sch_friend12 <int>, sch_friend13 <int>,
## #
       sch_friend14 <int>, sch_friend15 <int>, sch_friend16 <int>,
## #
## #
       sch_friend17 <int>, sch_friend18 <int>, sch_friend19 <int>,
       sch_friend110 <int>, sch_friend111 <int>, sch_friend112 <int>,
## #
       sch_friend113 <int>, sch_friend114 <int>, sch_friend115 <int>,
## #
       sch_friend116 <int>, sch_friend117 <int>, sch_friend118 <int>,
## #
## #
       sch_friend119 <int>, sch_friend21 <int>, sch_friend22 <int>,
       sch_friend23 <int>, sch_friend24 <int>, sch_friend25 <int>,
## #
       sch_friend26 <int>, sch_friend27 <int>, sch_friend28 <int>,
## #
## #
       sch_friend29 <int>, sch_friend210 <int>, sch_friend211 <int>,
## #
       sch_friend212 <int>, sch_friend213 <int>, sch_friend214 <int>,
## #
       sch_friend215 <int>, sch_friend216 <int>, sch_friend217 <int>,
       sch_friend218 <int>, sch_friend219 <int>, sch_friend31 <int>,
## #
## #
       sch_friend32 <int>, sch_friend33 <int>, sch_friend34 <int>,
       sch_friend35 <int>, sch_friend36 <int>, sch_friend37 <int>,
## #
       sch_friend38 <int>, sch_friend39 <int>, sch_friend310 <int>,
## #
## #
       sch_friend311 <int>, sch_friend312 <int>, sch_friend313 <int>,
## #
       sch_friend314 <int>, sch_friend315 <int>, sch_friend316 <int>,
## #
       sch_friend317 <int>, sch_friend318 <int>, sch_friend319 <int>,
## #
       sch_friend41 <int>, sch_friend42 <int>, sch_friend43 <int>,
       sch_friend44 <int>, sch_friend45 <int>, sch_friend46 <int>,
## #
## #
       sch_friend47 <int>, sch_friend48 <int>, sch_friend49 <int>,
## #
       sch_friend410 <int>, sch_friend411 <int>, sch_friend412 <int>,
```

```
## #
       sch_friend413 <int>, sch_friend414 <int>, sch_friend415 <int>,
## #
       sch_friend416 <int>, sch_friend417 <int>, sch_friend418 <int>,
       sch_friend419 <int>, id <dbl>
## #
# Maybe too much piping... but its cool!
net <- dat %>%
  select(id, school, starts_with("sch_friend")) %>%
  gather(key = "varname", value = "content", -id, -school) %>%
  filter(!is.na(content)) %>%
  mutate(
    friendid = school*10000 + content,
             = as.integer(str_extract(varname, "(?<=[a-z])[0-9]")),
    year
             = as.integer(str_extract(varname, "(?<=[a-z][0-9])[0-9]+"))</pre>
    nnom
```

Let's take a look at this step by step:

First, we subset the data: We want to keep id, school, sch_friend*. For the later we
use the function starts_with (from the tidyselect package). This allows us to select
all variables that starts with the word "sch_friend", which means that sch_friend11,
sch_friend12, ... will all be selected.

```
dat %>%
select(id, school, starts_with("sch_friend"))
```

```
## # A tibble: 2,164 x 78
##
           id school sch_friend11 sch_friend12 sch_friend13 sch_friend14
##
        <dbl> <int>
                             <int>
                                          <int>
                                                       <int>
                                                                     <int>
##
    1 1110001
                 111
                                NA
                                             NA
                                                           NA
                                                                        NA
    2 1110002
                 111
                               424
                                            423
                                                          426
                                                                       289
##
## 3 1110007
                 111
                               629
                                            505
                                                           NA
                                                                        NA
## 4 1110013
                 111
                               232
                                            569
                                                           NA
                                                                        NA
## 5 1110014
                 111
                                                                       592
                               582
                                            134
                                                           41
## 6 1110015
                 111
                                26
                                            488
                                                           81
                                                                       138
## 7 1110020
                 111
                               528
                                             NA
                                                          492
                                                                       395
## 8 1110022
                 111
                                NA
                                             NA
                                                           NA
                                                                        NA
```

```
## 9 1110025
                 111
                              135
                                           185
                                                         553
                                                                       84
                                                                        5
## 10 1110027
                 111
                              346
                                           168
                                                         559
## # ... with 2,154 more rows, and 72 more variables: sch_friend15 <int>,
## #
       sch_friend16 <int>, sch_friend17 <int>, sch_friend18 <int>,
## #
       sch_friend19 <int>, sch_friend110 <int>, sch_friend111 <int>,
## #
       sch_friend112 <int>, sch_friend113 <int>, sch_friend114 <int>,
       sch_friend115 <int>, sch_friend116 <int>, sch_friend117 <int>,
## #
       sch_friend118 <int>, sch_friend119 <int>, sch_friend21 <int>,
## #
## #
       sch_friend22 <int>, sch_friend23 <int>, sch_friend24 <int>,
       sch_friend25 <int>, sch_friend26 <int>, sch_friend27 <int>,
## #
       sch_friend28 <int>, sch_friend29 <int>, sch_friend210 <int>,
## #
## #
       sch_friend211 <int>, sch_friend212 <int>, sch_friend213 <int>,
       sch_friend214 <int>, sch_friend215 <int>, sch_friend216 <int>,
## #
## #
       sch_friend217 <int>, sch_friend218 <int>, sch_friend219 <int>,
## #
       sch_friend31 <int>, sch_friend32 <int>, sch_friend33 <int>,
       sch_friend34 <int>, sch_friend35 <int>, sch_friend36 <int>,
## #
## #
       sch_friend37 <int>, sch_friend38 <int>, sch_friend39 <int>,
## #
       sch_friend310 <int>, sch_friend311 <int>, sch_friend312 <int>,
## #
       sch_friend313 <int>, sch_friend314 <int>, sch_friend315 <int>,
## #
       sch_friend316 <int>, sch_friend317 <int>, sch_friend318 <int>,
## #
       sch_friend319 <int>, sch_friend41 <int>, sch_friend42 <int>,
       sch_friend43 <int>, sch_friend44 <int>, sch_friend45 <int>,
## #
## #
       sch_friend46 <int>, sch_friend47 <int>, sch_friend48 <int>,
## #
       sch_friend49 <int>, sch_friend410 <int>, sch_friend411 <int>,
       sch_friend412 <int>, sch_friend413 <int>, sch_friend414 <int>,
## #
       sch_friend415 <int>, sch_friend416 <int>, sch_friend417 <int>,
## #
       sch_friend418 <int>, sch_friend419 <int>
## #
```

2. Then, we reshape it to *long* format: By transposing all the sch_friend* to long. We do this by means of the function gather (from the tidyr package). This is an alternative to the reshape function, and I personally find it easier to use. Let's see how it works:

```
dat %>%
  select(id, school, starts_with("sch_friend")) %>%
```

```
gather(key = "varname", value = "content", -id, -school)
```

```
## # A tibble: 164,464 x 4
##
           id school varname
                                 content
##
        <dbl> <int> <chr>
                                   <int>
                111 sch_friend11
##
   1 1110001
                                      NA
##
   2 1110002
                111 sch_friend11
                                     424
## 3 1110007
                111 sch_friend11
                                     629
## 4 1110013
                111 sch_friend11
                                     232
                111 sch_friend11
## 5 1110014
                                     582
## 6 1110015
                111 sch_friend11
                                      26
## 7 1110020
                111 sch_friend11
                                     528
## 8 1110022
                111 sch_friend11
                                      NA
## 9 1110025
                111 sch_friend11
                                     135
## 10 1110027
                111 sch_friend11
                                     346
## # ... with 164,454 more rows
```

In this case the key parameter sets the name of the variable that will contain the name of the variable that was reshaped, while value is the name of the variable that will hold the content of the data (that's why I named those like that). The -id, -school bit tells the function to "drop" those variables before reshaping, in other words, "reshape everything but id and school".

Also, notice that we passed from 2164 rows to 19 (nominations) * 2164 (subjects) * 4 (waves) = 164464 rows, as expected.

3. As the nomination data can be empty for some cells, we need to take care of those cases, the NAs, so we filter the data:

```
dat %>%
  select(id, school, starts_with("sch_friend")) %>%
  gather(key = "varname", value = "content", -id, -school) %>%
  filter(!is.na(content))
```

```
## # A tibble: 39,561 x 4
## id school varname content
```

```
##
        <dbl> <int> <chr>
                                    <int>
    1 1110002
                 111 sch_friend11
##
                                      424
                 111 sch_friend11
   2 1110007
                                      629
##
   3 1110013
                 111 sch_friend11
##
                                      232
                 111 sch_friend11
## 4 1110014
                                      582
## 5 1110015
                 111 sch_friend11
                                       26
## 6 1110020
                 111 sch_friend11
                                      528
## 7 1110025
                 111 sch_friend11
                                      135
## 8 1110027
                 111 sch_friend11
                                      346
## 9 1110029
                 111 sch_friend11
                                      369
## 10 1110030
                 111 sch_friend11
                                      462
## # ... with 39,551 more rows
```

4. And finally, we create three new variables from this dataset: friendid, year, and nom_num (nomination number). All this using regular expressions:

```
dat %>%
  select(id, school, starts_with("sch_friend")) %>%
  gather(key = "varname", value = "content", -id, -school) %>%
  filter(!is.na(content)) %>%
  mutate(
    friendid = school*10000 + content,
    year = as.integer(str_extract(varname, "(?<=[a-z])[0-9]")),
    nnom = as.integer(str_extract(varname, "(?<=[a-z][0-9])[0-9]+"))
    )
}</pre>
```

```
## # A tibble: 39,561 x 7
##
          id school varname
                                 content friendid year nnom
       <dbl> <int> <chr>
##
                                   <int>
                                            <dbl> <int> <int>
##
   1 1110002
                111 sch_friend11
                                     424 1110424
                                                      1
   2 1110007
                111 sch_friend11
                                     629 1110629
                                                      1
##
                                                            1
  3 1110013
                111 sch_friend11
##
                                     232 1110232
                                                      1
                                                            1
                111 sch_friend11
## 4 1110014
                                     582 1110582
                                                      1
                                                            1
## 5 1110015
                111 sch_friend11
                                      26 1110026
                                                      1
                                                            1
```

```
## 6 1110020
                                      528 1110528
                 111 sch_friend11
                                                       1
                                                             1
## 7 1110025
                 111 sch_friend11
                                      135 1110135
                                                       1
                                                             1
## 8 1110027
                 111 sch_friend11
                                      346 1110346
                                                             1
                                                       1
## 9 1110029
                                                             1
                 111 sch_friend11
                                      369 1110369
                                                       1
## 10 1110030
                 111 sch_friend11
                                      462 1110462
                                                       1
                                                             1
## # ... with 39,551 more rows
```

The regular expression (?<=[a-z]) matches a string that is preceded by any letter from a to z, whereas the expression [0-9] matches a single number. Hence, from the string "sch_friend12", the regular expression will only match the 1, as it is the only number followed by a letter. On the other hand, the expression (?<=[a-z][0-9]) matches a string that is preceded by a letter from a to z and a number from a to a0; and the expression [0-9]+ matches a string of numbers—so it could be more than one. Hence, from the string "sch_friend12", we will get 2. We can actually se this

```
str_extract("sch_friend12", "(?<=[a-z])[0-9]")

## [1] "1"

str_extract("sch_friend12", "(?<=[a-z][0-9])[0-9]+")

## [1] "2"</pre>
```

And finally, the as.integer function coerces the returning value from the str_extract function from character to integer. Now that we have this edgelist, we can create an igraph object

4.2.2 igraph network

For coercing the edgelist into an igraph object, we will be using the graph_from_data_frame function in igraph (Csardi and Nepusz 2006). This function receives a data frame where the two first columns are sorce(ego) and target(alter), whether is it directed or not, and an optional data frame with vertices, in which's first column should contain the vertex ids.

Using the optional vertices argument is a good practice since by doing so you are telling the function what is the set of vertex ids that you are expecting to find. Using the original dataset, we will create a data frame name vertices:

```
vertex_attrs <- dat %>%
select(id, school, hispanic, female1, starts_with("eversmk"))
```

Now, let's now use the function graph_from_data_frame to create an igraph object:

```
library(igraph)

ig_year1 <- net %>%
  filter(year == "1") %>%
  select(id, friendid, nnom) %>%
  graph_from_data_frame(
    vertices = vertex_attrs
)
```

Error in graph_from_data_frame(., vertices = vertex_attrs): Some vertex names in edge l

Ups! It seems that individuals are making nominations to other students that were not included on the survery. How to solve that? Well, it all depends on what you need to do! In this case, we will go for the *quietly-remove-em'-and-don't-tell* strategy:

```
ig_year1 <- net %>%
filter(year == "1") %>%

# Extra line, all nominations must be in ego too.
filter(friendid %in% id) %>%

select(id, friendid, nnom) %>%
graph_from_data_frame(
   vertices = vertex_attrs
   )

ig_year1
```

```
## IGRAPH 2c1e2b6 DN-- 2164 9514 --
## + attr: name (v/c), school (v/n), hispanic (v/n), female1 (v/n),
## | eversmk1 (v/n), eversmk2 (v/n), eversmk3 (v/n), eversmk4 (v/n),
```

```
## | nnom (e/n)
## + edges from 2c1e2b6 (vertex names):
## [1] 1110007->1110629 1110013->1110232 1110014->1110582 1110015->1110026
## [5] 1110025->1110135 1110027->1110346 1110029->1110369 1110035->1110034
## [9] 1110040->1110390 1110041->1110557 1110044->1110027 1110046->1110030
## [13] 1110050->1110086 1110057->1110263 1110069->1110544 1110071->1110167
## [17] 1110072->1110289 1110073->1110014 1110075->1110352 1110084->1110305
## [21] 1110086->1110206 1110093->1110040 1110094->1110483 1110095->1110043
## + ... omitted several edges
```

So there we have, our network with 2164 nodes and 9514 edges. The next steps: get some descriptive stats and visualize our network.

4.3 Network descriptive stats

While we could do all networks at once, in this part we will focus on computing some network statistics for one of the schools only. We start by school 111. The first question that you should be asking your self now is, "how can I get that information from the igraph object?." Well, vertex attributes and edges attributes can be accessed via the V and E functions respectively; moreover, we can list what vertex/edge attributes are available:

```
list.vertex.attributes(ig_year1)

## [1] "name"     "school"     "hispanic" "female1"     "eversmk1" "eversmk2"

## [7] "eversmk3" "eversmk4"

list.edge.attributes(ig_year1)

## [1] "nnom"
```

Just like we would do with data frames, accessing vertex attributes is done via the dollar sign operator \$ together with the V function, for example, accessing the first 10 elements of the variable hispanic can be done as follows:

```
V(ig_year1)$hispanic[1:10]
```

```
## [1] 1 1 0 1 1 1 1 1 0 1
```

Now that you know how to access vertex attributes, we can get the network corresponding to school 111 by identifying which vertices are part of it and pass that information to the induced_subgraph function:

```
# Which ids are from school 111?
school11lids <- which(V(ig_year1)$school == 111)

# Creating a subgraph
ig_year1_111 <- induced_subgraph(
    graph = ig_year1,
    vids = school11lids
)</pre>
```

The which function in R returns a vector of indices indicating which elements are true. In our case it will return a vector of indices of the vertices which have the attribute school equal to 111. Now that we have our subgraph, we can compute different centrality measures² for each vertex and store them in the igraph object itself:

```
# Computing centrality measures for each vertex

V(ig_yearl_111)$indegree <- degree(ig_yearl_111, mode = "in")

V(ig_yearl_111)$outdegree <- degree(ig_yearl_111, mode = "out")

V(ig_yearl_111)$closeness <- closeness(ig_yearl_111, mode = "total")

V(ig_yearl_111)$betweeness <- betweenness(ig_yearl_111, normalized = TRUE)</pre>
```

From here, we can *go back* to our old habits and get the set of vertex attributes as a data frame so we can compute some summary statistics on the centrality measurements that we just got

```
# Extracting each vectex features as a data.frame
stats <- as_data_frame(ig_year1_111, what = "vertices")

# Computing quantiles for each variable
stats_degree <- with(stats, {
    cbind(
    indegree = quantile(indegree, c(.025, .5, .975)),</pre>
```

²For more information about the different centrality measurements, please take a look at the "Centrality" article on Wikipedia.

```
outdegree = quantile(outdegree, c(.025, .5, .975)),
  closeness = quantile(closeness, c(.025, .5, .975)),
  betweeness = quantile(betweeness, c(.025, .5, .975))
)
})
stats_degree
```

```
## indegree outdegree closeness betweeness
## 2.5% 0 0 3.526640e-06 0.000000000
## 50% 4 4 1.595431e-05 0.001879006
## 97.5% 16 16 1.601822e-05 0.016591048
```

The with function is somewhat similar to what dplyr allows us to do when we want to work with the dataset but without mentioning its name everytime that we ask for a variable. Without using the with function, the previous could have been done as follows:

```
stats_degree <-
cbind(
  indegree = quantile(stats$indegree, c(.025, .5, .975)),
  outdegree = quantile(stats$outdegree, c(.025, .5, .975)),
  closeness = quantile(stats$closeness, c(.025, .5, .975)),
  betweeness = quantile(stats$betweeness, c(.025, .5, .975))
)</pre>
```

Now we will compute some statistics at the graph level:

```
cbind(
    size = vcount(ig_year1_111),
    nedges = ecount(ig_year1_111),
    density = edge_density(ig_year1_111),
    recip = reciprocity(ig_year1_111),
    centr = centr_betw(ig_year1_111)$centralization,
    pathLen = mean_distance(ig_year1_111)
)
```

```
## size nedges density recip centr pathLen
## [1,] 533 2638 0.009303277 0.3731513 0.02179154 4.23678
```

Triadic census

```
triadic <- triad_census(ig_year1_111)
triadic</pre>
```

```
[1] 24059676
                  724389
                           290849
                                       3619
                                                3383
                                                         4401
                                                                  3219
            2997
                      407
                                33
                                       836
                                                 235
                                                          163
                                                                  137
##
   [8]
## [15]
            277
                       85
```

To get a nicer view of this, we can use a table that I retrieved from ?triad_census. Moreover, instead of looking a the raw counts, we can normalize the triadic object by its sum so we get proportions instead³

```
knitr::kable(cbind(
   Pcent = triadic/sum(triadic)*100,
   read.csv("triadic_census.csv")
), digits = 2)
```

³During our workshop, Prof. De la Haye suggested using $\binom{n}{3}$ as a normalizing constant. It turns out that sum(triadic) = choose(n, 3)! So either approach is correct.

Pcent	code	description
95.88	003	A,B,C, the empty graph.
2.89	012	A->B, C, the graph with a single directed edge.
1.16	102	A<->B, C, the graph with a mutual connection between two vertices.
0.01	021D	A<-B->C, the out-star.
0.01	021U	A->B<-C, the in-star.
0.02	021C	A->B->C, directed line.
0.01	111D	A<->B<-C.
0.01	111U	A<->B->C.
0.00	030T	A->B<-C, A->C.
0.00	030C	A<-B<-C, A->C.
0.00	201	A<->B<->C.
0.00	120D	A<-B->C, A<->C.
0.00	120U	A->B<-C, A<->C.
0.00	120C	A->B->C, A<->C.
0.00	210	A->B<->C, A<->C.
0.00	300	A<->B<->C, A<->C, the complete graph.

4.4 Plotting the network in igraph

4.4.1 Single plot

Let's take a look at how does our network looks like when we use the default parameters in the plot method of the igraph object:

```
plot(ig_year1)
```

Not very nice, right? A couple of things with this plot:

- 1. We are looking at all schools simultaneously, which does not make sense. So, instead of plotting ig_year1, we will focus on ig_year1_111.
- 2. All the vertices have the same size, and more over, are overalaping. So, instead of using the default size, we will size the vertices by indegree using the degree function, and

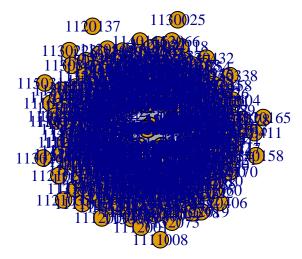


Figure 4.1: A not very nice network plot. This is what we get with the default parameters in igraph.

passing the vector of degrees to vertex.size.4

- 3. Given the number of vertices in these networks, the labels are not useful here. So we will remove them by setting vertex.label = NA. Moreover, we will reduce the size of the arrows' tip by setting edge.arrow.size = 0.25.
- 4. And finally, we will set the color of each vertex to be a function of whether the individual is hispanic or not. For this last bit we need to go a bit more of programming:

```
col_hispanic <- V(ig_year1_111)$hispanic + 1
col_hispanic <- coalesce(col_hispanic, 3)
col_hispanic <- c("steelblue", "tomato", "white")[col_hispanic]</pre>
```

Line by line, we did the following:

- 1. The first line added one to all no NA values, so that the 0s (non-hispanic) turned to 1s and the 1s (hispanic) turned to 2s.
- 2. The second line replaced all NAs with the number 3, so that our vector col_hispanic now ranges from 1 to 3 with no NAs in it.
- 3. In the last line we created a vector of colors. Essentially, what we are doing here is telling R to create a vector of length length(col_hispanic) by selecting elements by index

⁴Figuring out what is the optimal vertex size is a bit tricky. Without getting too technical, there's no other way of getting *nice* vertex size other than just playing with different values of it. A nice solution to this is using netdiffuseR::igraph_vertex_rescale which rescales the vertices so that these keep their aspect ratio to a predefined proportion of the screen.

from the vector c("steelblue", "tomato", "white"). This way, if, for example, the first element of the vector col_hispanic was a 3, our new vector of colors would have a "white" in it.

To make sure we know we are right, let's print the first 10 elements of our new vector of colors together with the original hispanic column:

```
cbind(
  original = V(ig_yearl_111)$hispanic[1:10],
  colors = col_hispanic[1:10]
)
```

```
original colors
##
##
    [1,] "1"
                  "tomato"
    [2,] "1"
                  "tomato"
##
## [3,] "0"
                  "steelblue"
                  "tomato"
## [4,] "1"
## [5,] "1"
                  "tomato"
## [6,] "1"
                  "tomato"
## [7,] "1"
                  "tomato"
                  "tomato"
## [8,] "1"
## [9,] "0"
                  "steelblue"
## [10,] "1"
                  "tomato"
```

With our nice vector of colors, now we can pass it to plot.igraph (which we call implicitly by just calling plot), via the vertex.color argument:

```
# Fancy graph
set.seed(1)
plot(
    ig_year1_111,
    vertex.size = degree(ig_year1_111)/10 +1,
    vertex.label = NA,
    edge.arrow.size = .25,
    vertex.color = col_hispanic
    )
```

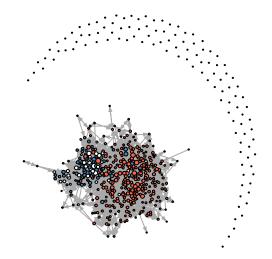


Figure 4.2: Friends network in time 1 for school 111.

Nice! So it does look better. The only problem is that we have a lot of isolates. Let's try again by drawing the same plot without isolates. To do so we need to filter the graph, for which we will use the function induced_subgraph

```
# Which vertices are not isolates?
which_ids <- which(degree(ig_year1_111, mode = "total") > 0)

# Getting the subgraph
ig_year1_111_sub <- induced_subgraph(ig_year1_111, which_ids)

# We need to get the same subset in col_hispanic
col_hispanic <- col_hispanic[which_ids]</pre>
```

```
# Fancy graph
set.seed(1)
plot(
    ig_year1_111_sub,
    vertex.size = degree(ig_year1_111_sub)/5 +1,
    vertex.label = NA,
    edge.arrow.size = .25,
    vertex.color = col_hispanic
    )
```

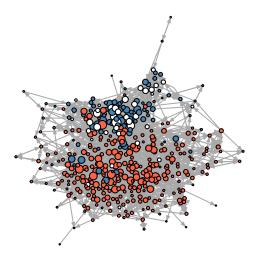


Figure 4.3: Friends network in time 1 for school 111. The graph excludes isolates.

Now that's better! An interesting pattern that shows up is that individuals seem to cluster by whether they are hispanic or not.

We can actually write this as a function so that, instead of us copying and pasting the code n times (supposing that we want to crate a plot similar to this n times). The next subsection does that.

4.4.2 Multiple plots

When you are repeating yourself over and over again, it is a good idea to write down a sequence of commands as a function. In this case, since we will be running the same type of plot for all schools/waves, we write a function in which the only things that changes are: (a) the school id, and (b) the color of the nodes.

```
myplot <- function(
  net,
  schoolid,
  mindgr = 1,
  vcol = "tomato",
  ...) {

# Creating a subgraph
  subnet <- induced_subgraph(</pre>
```

```
net,
    which(degree(net, mode = "all") >= mindgr & V(net)$school == schoolid)
  )
  # Fancy graph
  set.seed(1)
  plot(
    subnet,
    vertex.size
                   = degree(subnet)/5,
    vertex.label
                    = NA
    edge.arrow.size = .25,
    vertex.color
                  = vcol,
    . . .
    )
}
```

The function definition:

- The myplot <- function([arguments]) {[body of the function]} tells R that we
 are going to create a function called myplot.
- 2. In the arguments part, we are declaring 4 specific arguments: net, schoolid, mindgr, and vcol. These are an igraph object, the school id, the minimum degree that a vertex must have to be included in the plot, and the color of the vertices. Notice that, as a difference from other programming languages, in R we don't need to declare the types that these objects are.
- 3. The elipsis object, ..., is a special object in R that allows us passing other arguments without us specifying which. In our case, if you take a look at the plot bit of the body of the function, you will see that we also added ...; this means that whatever other arguments (different from the ones that we explicitly defined) are passed to the function, these will be passed to the function plot, moreover, to the plot.gexf function (since the subnet object is actually an igraph object). In practice, this implies that we can, for example, set the argument edge.arrow.size when calling myplot, even though we did not included it in the function definition! (See ?dotsMethods in R for more details).

In the following lines of code, using our new function, we will plot each schools' network in the same plotting device (window) with the help of the par function, and add legend with the legend:

```
# Plotting all together
oldpar <- par(no.readonly = TRUE)</pre>
par(mfrow = c(2, 3), mai = rep(0, 4), oma = c(1, 0, 0, 0))
myplot(ig_year1, 111, vcol = "tomato")
myplot(ig_year1, 112, vcol = "steelblue")
myplot(ig_year1, 113, vcol = "black")
myplot(ig_year1, 114, vcol = "gold")
myplot(ig_year1, 115, vcol = "white")
par(oldpar)
# A fancy legend
legend(
  "bottomright",
  legend = c(111, 112, 113, 114, 115),
  pt.bg = c("tomato", "steelblue", "black", "gold", "white"),
  pch
        = 21,
  cex
        = 1,
  bty = "n",
  title = "School"
```

So what happend here?

- oldpar <- par(no.readonly = TRUE) This line stores the current parameters for plotting. Since we are going to be changing them, we better make sure we are able to go back!.
- par(mfrow = c(2, 3), mai = rep(0, 4), oma=rep(0, 4)) Here we are setting various things at the same time. mfrow specifies how many *figures* will be drawn and in what order, in particular, we are asking the plotting device to allow for 2*3 = 6 plots organized in 2 rows and 3 columns, and these will be drawn by row.

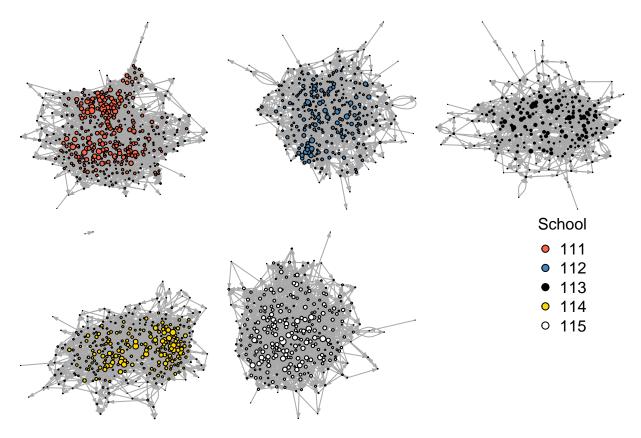


Figure 4.4: All 5 schools in time 1. Again, the graphs exclude isolates.

mai specifies the size of the margins in inches. Setting all margins equal to zero (which is what we are doing now) gives more space to the network itself. The same is true for oma. See ?par for more info.

- myplot(ig_year1, ...) This is simply calling our plotting function. The neat part of this
 is that, since we set mfrow = c(2, 3), R takes care of distributing the plots in the device.
- par(oldpar) This line allows us to restore the plotting parameters.

4.5 Statistical tests

4.5.1 Is nomination number correlated with indegree?

Hypothesis: Individuals that on average are among the first nominations of their peers are more popular

5 6.00 1110015

7 9.00 1110025

6 7.00 1110020

8 10.0 1110027

9 11.0 1110029

10 12.0 1110030

... with 1,551 more rows

111

111

111

111

111

111

3

6

6

13

14

6

0.150

0.154

0.214

0.220

0.131

0.222

##

##

```
# Getting all the data in long format
edgelist <- as_long_data_frame(ig_year1) %>%
  as_tibble
# Computing indegree (again) and average nomination number
# Include "On a scale from one to five how close do you feel"
# Also for egocentric friends (A. Friends)
indeg_nom_cor <- group_by(edgelist, to, to_name, to_school) %>%
  summarise(
    indeg = \mathbf{n}(),
    nom_avg = 1/mean(nnom)
  ) %>%
  rename(
    school = to_school
  )
indeg_nom_cor
## # A tibble: 1,561 x 5
## # Groups: to, to_name [1,561]
##
         to to_name school indeg nom_avg
      <dbl> <chr>
##
                     <int> <int>
                                   <dbl>
## 1 2.00 1110002
                       111
                              22
                                  0.222
##
  2 3.00 1110007
                       111
                              7
                                  0.175
## 3 4.00 1110013
                       111
                               6
                                  0.171
## 4 5.00 1110014
                                  0.134
                       111
                              19
```

35

```
# Using pearson's correlation
with(indeg_nom_cor, cor.test(indeg, nom_avg))
##
## Pearson's product-moment correlation
##
## data: indeg and nom_avg
## t = -12.254, df = 1559, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## -0.3409964 -0.2504653
## sample estimates:
## cor
## -0.2963965
save.image("03.rda")</pre>
```

Chapter 5

Exponential Random Graph Models

I strongly suggest reading the vignette included in the ergm R package

```
vignette("ergm", package="ergm")
```

So what are ERGMs anyway...

The purpose of ERGMs, in a nutshell, is to describe parsimoniously the local selection forces that shape the global structure of a network. To this end, a network dataset, like those depicted in Figure 1, may be considered like the response in a regression model, where the predictors are things like "propensity for individuals of the same sex to form partnerships" or "propensity for individuals to form triangles of partnerships". In Figure 1(b), for example, it is evident that the individual nodes appear to cluster in groups of the same numerical labels (which turn out to be students' grades, 7 through 12); thus, an ERGM can help us quantify the strength of this intra-group effect.

— (Hunter et al. 2008)

The distribution of **Y** can be parameterized in the form

$$\Pr(\mathbf{Y} = \mathbf{y} | \theta, \mathcal{Y}) = \frac{\exp\left\{\theta^{\mathsf{T}} \mathbf{g}(\mathbf{y})\right\}}{\kappa(\theta, \mathcal{Y})}, \quad \mathbf{y} \in \mathcal{Y}$$

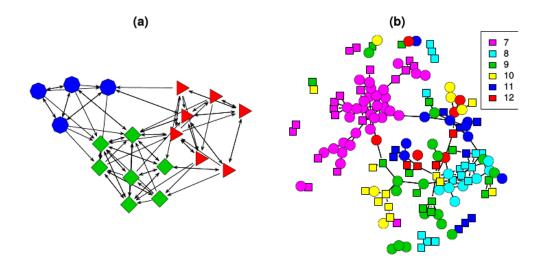


Figure 1: The (a) samplike and (b) faux.mesa.high networks described in Section 2. The values of nodal covariates may be indicated using various colors, shapes, and labels of nodes.

Figure 5.1: Source: Hunter et al. (2008)

Where $\theta \in \Omega \subset \mathbb{R}^q$ is the vector of model coefficients and $\mathbf{g}(\mathbf{y})$ is a q-vector of statistics based on the adjacency matrix \mathbf{y} .

Model (5) may be expanded by replacing g(y) with g(y, X) to allow for additional covariate information X about the network. The denominator,

$$\kappa(\theta, y) = \sum_{\mathbf{z} \in \mathcal{Y}} \exp\{\theta^{\mathsf{T}} \mathbf{g}(\mathbf{z})\}$$

Is the normalizing factor that ensures that equation (5) is a legitimate probability distribution. Even after fixing \mathcal{Y} to be all the networks that have size n, the size of \mathcal{Y} makes this type of models hard to estimate as there are $N = 2^{n(n-1)}$ possible networks! (Hunter et al. 2008)

5.1 The ergm package

The ergm R package (Handcock et al. 2017)

From the previous section:¹

¹You can download the 03.rda file from this link.

```
library(igraph)
library(magrittr)
library(dplyr)

load("03.rda")
```

In this section we will use the ergm package (from the statnet suit of packages (Handcock et al. 2016)) suit, and the intergraph (Bojanowski 2015) package. The latter provides functions to go back and forth between igraph and network objects from the igraph and network packages respectively²

```
library(ergm)
library(intergraph)
```

As a rather important side note, the order in which R packages are loaded matters. Why is this important to mention now? Well, it turns out that at least a couple of functions in the network package have the same name of some functions in the igraph package. When the ergm package is loaded, since it depends on network, it will load the network package first, which will *mask* some functions in igraph. This becomes evident once you load ergm after loading igraph:

The following objects are masked from 'package:igraph':

```
add.edges, add.vertices, %c%, delete.edges, delete.vertices, get.edge.attribute, get.edg get.vertex.attribute, is.bipartite, is.directed, list.edge.attributes, list.vertex.attribute set.edge.attribute, set.vertex.attribute
```

What are the implications of this? If you call the function list.edge.attributes for an object of class igraph R will return an error as the first function that matches that name comes from the network package! To avoid this you can use the double colon notation:

```
igraph::list.edge.attributes(my_igraph_object)
network::list.edge.attributes(my_network_object)
```

Anyway... Using the asNetwork function, we can coerce the igraph object into a network object so we can use it with the ergm function:

²Yes, the classes have the same name as the packages.

```
# Creating the new network
network_111 <- intergraph::asNetwork(ig_yearl_111)

# Running a simple ergm (only fitting edge count)
ergm(network_111 ~ edges)

## [1] "Warning: This network contains loops"
## [1] "Warning: This network contains loops"
## [1] "Warning: This network contains loops"

## Evaluating log-likelihood at the estimate.

##
## MLE Coefficients:
## edges
## -4.732</pre>
```

So what happened here! We got a warning. It turns out that our network has loops (didn't thought about it before!). Let's take a look on that with the which_loop function

```
E(ig_year1_111)[which_loop(ig_year1_111)]
## + 1/2638 edge from dd829a5 (vertex names):
## [1] 1110111->1110111
```

We can get rid of these using the igraph::-.igraph. Moreover, just to illustrate how it can be done, let's get rid of the isolates using the same operator

```
# Creating the new network
network_111 <- ig_year1_111

# Removing loops
network_111 <- network_111 - E(network_111)[which(which_loop(network_111))]

# Removing isolates
network_111 <- network_111 - which(degree(network_111, mode = "all") == 0)</pre>
```

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```
# Converting the network
network_111 <- intergraph::asNetwork(network_111)</pre>
```

5.2 Running ERGMs

Proposed workflow:

- 1. Estimate the simplest model, adding one variable at a time.
- 2. After each estimation, run the mcmc.diagnostics function to see how good/bad behaved are the chains.
- 3. Run the gof function to see how good is the model at matching the network's structural statistics.

What to use:

- 1. control.ergms: Maximum number of iteration, seed for Pseudo-RNG, how many cores
- 2. ergm.constraints: Where to sample the network from. Gives stability and (in some cases) faster convergence as by constraining the model you are reducing the sample size.

Here is an example of a couple of models that we could compare³

```
ans0 <- ergm(
  network_111 ~
    edges +
    nodematch("hispanic") +
    nodematch("female1") +
    nodematch("eversmk1") +
    mutual
    ,
    constraints = ~bd(maxout = 19),
    control = control.ergm(
    seed = 1,
    MCMLE.maxit = 10,</pre>
```

³Notice that this document may not include the usual messages that the ergm command generates during the estimation procedure. This is just to make it more printable-friendly.

```
parallel = 4,
CD.maxit = 10
)
)
```

So what are we doing here: 1. The model is controlling for: a. edges Number of edges in the network (as opposed to its density) b. nodematch ("some-variable-name-here") Includes a term that controls for homophily/heterophily c. mutual Number of mutual connections between i and j. This can be related to, for example, triadic closure.

```
ans1 <- ergm(
  network_111 ~
    edges +
    nodematch("hispanic") +
    nodematch("female1") +
    nodematch("eversmk1")
    ,
    constraints = ~bd(maxout = 19),
    control = control.ergm(
    seed = 1,
    MCMLE.maxit = 10,
    parallel = 4,
    CD.maxit = 10
    )
}</pre>
```

This example takes longer to compute

```
ans2 <- ergm(
  network_111 ~
    edges +
    nodematch("hispanic") +
    nodematch("female1") +
    nodematch("eversmk1") +</pre>
```

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Now, a nice trick to see all regressions in the same table, we can use the texreg package (Leifeld 2013) which supports ergm ouputs!

```
library(texreg)

## Version: 1.36.23

## Date: 2017-03-03

## Author: Philip Leifeld (University of Glasgow)

##

## Please cite the JSS article in your publications -- see citation("texreg").

##

## Attaching package: 'texreg'

## The following object is masked from 'package:magrittr':

##

## extract

screenreg(list(ans0, ans1, ans2))
```

```
## Note: The constraint on the sample space is not dyad-independent. Null model likelihood
## Note: The constraint on the sample space is not dyad-independent. Null model likelihood
## Note: The constraint on the sample space is not dyad-independent. Null model likelihood
##
```

##		Model 1	Model 2	Model 3	
##					
##	edges	-5.63 ***	-5.53 ***	-5.56 ***	
##		(0.06)	(0.06)	(0.05)	
##	nodematch.hispanic	0.37 ***	0.51 ***	0.38 ***	
##		(0.04)	(0.04)	(0.04)	
##	nodematch.female1	0.82 ***	1.10 ***	0.83 ***	
##		(0.04)	(0.05)	(0.04)	
##	<pre>nodematch.eversmk1</pre>	0.33 ***	0.47 ***	0.35 ***	
##		(0.04)	(0.04)	(0.04)	
##	mutual	4.09 ***		-4.76 ***	
##		(0.07)		(0.25)	
##	balance			0.02 ***	
##				(0.00)	
##					
##	AIC	-37835.55	-35862.16	-37902.94	
##	BIC	-37785.21	-35821.88	-37842.53	
##	Log Likelihood	18922.78	17935.08	18957.47	
##		=========			
##	## *** p < 0.001, ** p < 0.01, * p < 0.05				

Or, if you are using rmarkdown, you can export the results using LaTeX or html, let's try the latter to see how it looks like here:

```
library(texreg)
texreg(list(ans0, ans1, ans2))
```

```
## Note: The constraint on the sample space is not dyad-independent. Null model likelihood
## Note: The constraint on the sample space is not dyad-independent. Null model likelihood
## Note: The constraint on the sample space is not dyad-independent. Null model likelihood
```

	Model 1	Model 2	Model 3
edges	-5.63***	-5.53***	-5.56***
	(0.06)	(0.06)	(0.05)
nodematch.hispanic	0.37***	0.51***	0.38***
	(0.04)	(0.04)	(0.04)
nodematch.female1	0.82***	1.10 * * *	0.83***
	(0.04)	(0.05)	(0.04)
nodematch.eversmk1	0.33***	0.47***	0.35 * * *
	(0.04)	(0.04)	(0.04)
mutual	4.09 * * *		-4.76***
	(0.07)		(0.25)
balance			0.02 * * *
			(0.00)
AIC	-37835.55	-35862.16	-37902.94
BIC	-37785.21	-35821.88	-37842.53
Log Likelihood	18922.78	17935.08	18957.47

^{***}p < 0.001, **p < 0.01, *p < 0.05

Table 5.1: Statistical models

5.3 Model Goodness-of-Fit

In raw terms, once each chain has reach stationary distribution, we can say that there are no problems with autocorrelation and that each sample point is iid. This implies that, since we are running the model with more than 1 chain, we can use all the samples (chains) as a single dataset.

Recent changes in the ergm estimation algorithm mean that these plots can no longer be used to ensure that the mean statistics from the model match the observed network statistics. For that functionality, please use the GOF command: gof(object, GOF=~model).

—?ergm::mcmc.diagnostics

Since ans0 is the one model which did best, let's take a look at it's GOF statistics. First, lets see how the MCMC did. For this we can use the mcmc.diagnostics function including in the package. This function is actually a wrapper of a couple of functions from the coda package (Plummer et al. 2006). This is what is called under the hood:

1. Empirical means and sd, and quantiles:

summary(ans0\$sample)

```
## Iterations = 16384:1063936
## Thinning interval = 1024
## Number of chains = 4
## Sample size per chain = 1024
##
## 1. Empirical mean and standard deviation for each variable,
      plus standard error of the mean:
##
##
                                SD Naive SE Time-series SE
##
                        Mean
## edges
                      -32.32 51.14
                                     0.7990
                                                     3.557
## nodematch.hispanic -26.81 39.08 0.6106
                                                     2.879
## nodematch.female1 -28.03 44.92
                                     0.7018
                                                     3.678
## nodematch.eversmk1 -30.99 45.59
                                     0.7123
                                                     3.420
## mutual
                      -14.35 20.47
                                     0.3199
                                                     3.120
##
## 2. Ouantiles for each variable:
##
                      2.5% 25% 50% 75% 97.5%
##
                      -128 -68 -35
## edges
                                     1
                                          75
## nodematch.hispanic -100 -54 -27
                                          53
## nodematch.female1 -115 -59 -29
                                          65
## nodematch.eversmk1 -116 -63 -33
                                          59
                       -55 -27 -16 -1
                                          29
## mutual
```

2. Cross correlation:

coda::crosscorr(ans0\$sample)

```
##
                          edges nodematch.hispanic nodematch.female1
## edges
                      1.0000000
                                         0.7842851
                                                            0.8454275
## nodematch.hispanic 0.7842851
                                         1.0000000
                                                            0.6875136
## nodematch.female1 0.8454275
                                         0.6875136
                                                            1.0000000
## nodematch.eversmk1 0.8278077
                                                            0.6802329
                                         0.6009145
## mutual
                      0.6761018
                                         0.5362456
                                                            0.6581912
```

```
## edges 0.8278077 0.6761018
## nodematch.hispanic 0.6009145 0.5362456
## nodematch.female1 0.6802329 0.6581912
## nodematch.eversmk1 1.0000000 0.6111560
## mutual 0.6111560 1.0000000
```

3. Autocorrelation: Just for now, we will only take a look at autocorrelation for chain 1 only. Autocorrelation should be rather low (in a general MCMC setting). If autocorrelation is high, then it means that your sample is not idd (no markov property). A way out to solve this is thinning the sample.

```
coda::autocorr(ans0$sample)[[1]]
```

```
## , , edges
##
##
                 edges nodematch.hispanic nodematch.female1
## Lag 0
             1.0000000
                                0.8219687
                                                   0.8705328
## Lag 1024 0.8897580
                                0.7344981
                                                   0.7849176
## Lag 5120 0.6125344
                                0.5093458
                                                   0.5843733
## Lag 10240 0.4944537
                                0.4263616
                                                   0.5010519
## Lag 51200 0.2553223
                                0.2028499
                                                   0.2945153
##
             nodematch.eversmk1
                                   mutual
## Lag 0
                      0.8679762 0.6494130
## Lag 1024
                      0.7854503 0.6389852
## Lag 5120
                      0.5671411 0.5952745
## Lag 10240
                      0.4775676 0.5432361
## Lag 51200
                      0.2399202 0.3539473
##
## , , nodematch.hispanic
##
##
                 edges nodematch.hispanic nodematch.female1
## Lag 0
             0.8219687
                                1.0000000
                                                   0.7213715
## Lag 1024 0.7273184
                                0.8790036
                                                   0.6501736
```

```
## Lag 5120 0.4679330
                          0.5328138
                                            0.4685746
## Lag 10240 0.3548131 0.3719475
                                           0.3735284
                      0.2027639
## Lag 51200 0.2275837
                                             0.2450665
##
          nodematch.eversmk1 mutual
## Lag 0
                  0.6933735 0.5332499
                  0.6242548 0.5214353
## Lag 1024
## Lag 5120
                  0.4278511 0.4686949
            0.3486120 0.4199291
## Lag 10240
## Lag 51200
            0.2193803 0.2047705
##
## , , nodematch.female1
##
               edges nodematch.hispanic nodematch.female1
##
## Lag 0 0.8705328
                           0.7213715
                                            1.0000000
## Lag 1024 0.7882288
                           0.6530082
                                            0.9067042
                         0.4701031
## Lag 5120 0.5848867
                                            0.6960912
## Lag 10240 0.5143759 0.4415248
                                           0.6162009
## Lag 51200 0.2943576 0.2717081
                                      0.3756573
##
        nodematch.eversmk1 mutual
## Lag 0
                   0.7737254 0.6670677
                  0.7116355 0.6624944
## Lag 1024
## Lag 5120
                  0.5435839 0.6422426
                  0.4804301 0.6076076
## Lag 10240
                 0.2807997 0.3770320
## Lag 51200
##
## , , nodematch.eversmk1
##
##
               edges nodematch.hispanic nodematch.female1
## Lag 0 0.8679762
                            0.6933735
                                            0.7737254
## Lag 1024 0.7848037
                           0.6274016
                                            0.7117444
## Lag 5120 0.5581929
                           0.4628434
                                            0.5535105
## Lag 10240 0.4779814
                           0.3987335
                                            0.4923237
                           0.2285842
## Lag 51200 0.2846086
                                            0.3260756
```

```
##
            nodematch.eversmk1
                                  mutual
## Lag 0
                      1.0000000 0.5893853
## Lag 1024
                      0.9073188 0.5852995
## Lag 5120
                      0.6557095 0.5659845
                     0.5476628 0.5493667
## Lag 10240
## Lag 51200
                     0.2846972 0.4000402
##
## , , mutual
##
##
                 edges nodematch.hispanic nodematch.female1
## Lag 0
           0.6494130
                                0.5332499
                                                  0.6670677
## Lag 1024 0.6513036
                                0.5378887
                                                  0.6639583
## Lag 5120 0.6380464
                                0.5446367
                                                  0.6412776
## Lag 10240 0.6214549
                               0.5353477
                                                  0.6214122
## Lag 51200 0.3647324
                                0.2297713
                                                  0.4240353
##
             nodematch.eversmk1
                                   mutual
## Lag 0
                      0.5893853 1.0000000
## Lag 1024
                      0.5878446 0.9884291
## Lag 5120
                      0.5696277 0.9396082
## Lag 10240
                    0.5458346 0.8806506
                     0.2897837 0.3673476
## Lag 51200
```

4. Geweke Diagnostic:

coda::geweke.diag(ans0\$sample)

```
## [[1]]
##

## Fraction in 1st window = 0.1

## Fraction in 2nd window = 0.5

##

## edges nodematch.hispanic nodematch.female1

## 1.2158 1.9078 1.0548

## nodematch.eversmk1 mutual
```

```
##
                                   0.4969
               1.6781
##
##
## [[2]]
##
## Fraction in 1st window = 0.1
## Fraction in 2nd window = 0.5
##
                edges nodematch.hispanic nodematch.female1
##
##
                1.451
                                    4.109
                                                        2.268
## nodematch.eversmk1
                                   mutual
                1.108
                                    1.478
##
##
##
## [[3]]
##
## Fraction in 1st window = 0.1
## Fraction in 2nd window = 0.5
##
##
                edges nodematch.hispanic nodematch.female1
##
               1.1445
                                   0.3823
                                                      -1.4859
## nodematch.eversmk1
                                   mutual
##
               1.9638
                                   0.3500
##
##
## [[4]]
##
## Fraction in 1st window = 0.1
## Fraction in 2nd window = 0.5
##
##
                edges nodematch.hispanic nodematch.female1
##
             -0.07832
                                                      0.62034
                                  1.71294
## nodematch.eversmk1
                                   mutual
```

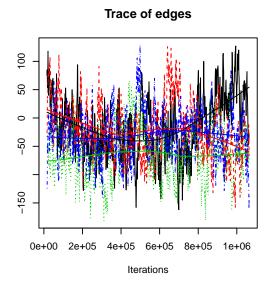
```
## -1.35149 -1.05875
```

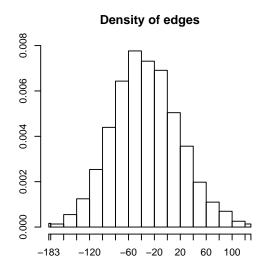
5. (not included) Gelman Diagnostic:

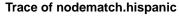
```
coda::gelman.diag(ans0$sample)
```

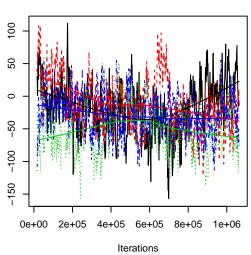
```
## Potential scale reduction factors:
##
                      Point est. Upper C.I.
##
## edges
                            1.23
                                       1.60
## nodematch.hispanic
                            1.13
                                       1.36
## nodematch.female1
                            1.13
                                       1.35
## nodematch.eversmk1
                            1.21
                                       1.58
## mutual
                            1.42
                                       2.08
##
## Multivariate psrf
##
## 1.41
```

plot(ans0\$sample)

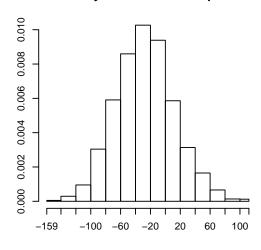




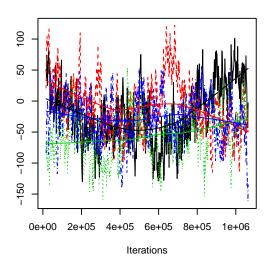




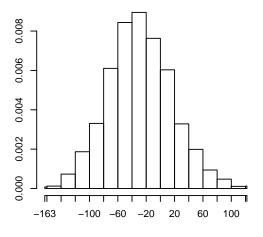
Density of nodematch.hispanic



Trace of nodematch.female1



Density of nodematch.female1

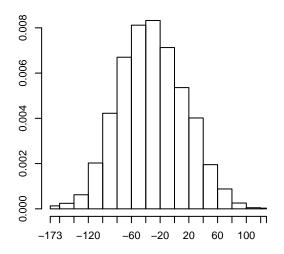




Trace of nodematch.eversmk1

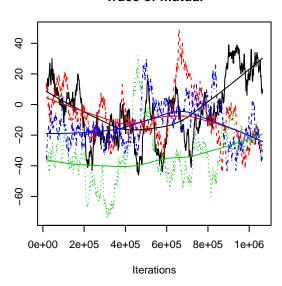
100 20 0 -50 -150 0e+00 2e+05 4e+05 6e+05 8e+05 1e+06

Density of nodematch.eversmk1

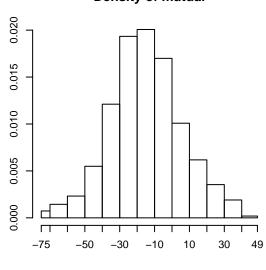


Trace of mutual

Iterations



Density of mutual



If we called the function mcmc.diagnostics this message appears at the end:

MCMC diagnostics shown here are from the last round of simulation, prior to computation of final parameter estimates. Because the final estimates are refinements of those used for this simulation run, these diagnostics may understate model performance. To directly assess the performance of the final model on in-model statistics, please use the GOF command: gof(ergmFitObject, GOF=~model).

—-mcmc.diagnostics(ans0)

Not that bad! First, observe that in the plot we see 4 different lines, why is that? Well, since we

were running in parallel using 4 cores the algorithm actually ran 4 different chains of the MCMC algorithm. An eyeball test is to see if all the chains moved at about the same place, if we have that we can start thinking about model convergence from the mcmc perspective.

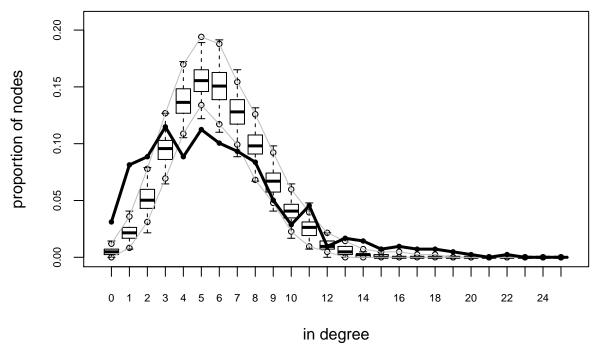
What would be an indicator of no-convergence? Well, if you see something like this:

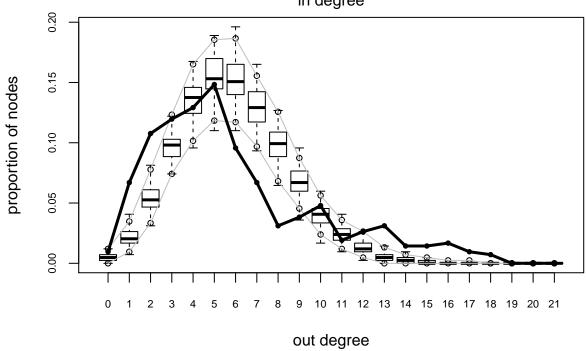
```
# Computing and printing GOF estatistics
ans_gof <- gof(ans0)
ans_gof</pre>
```

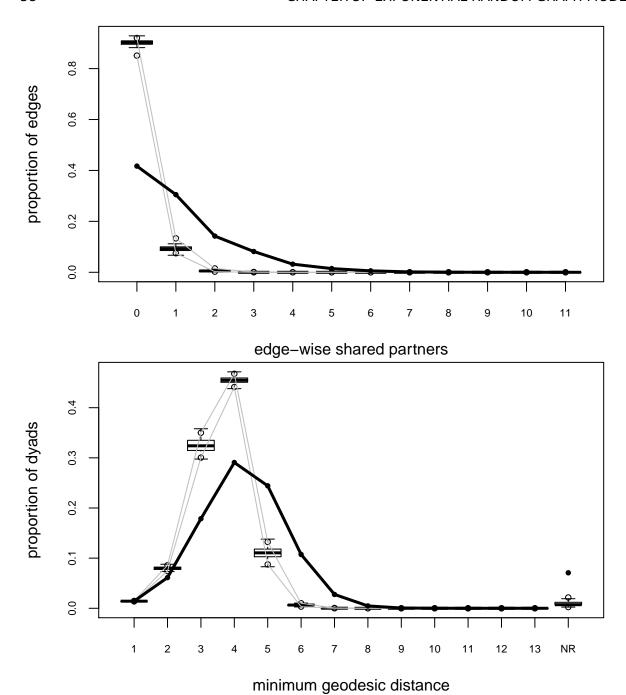
```
##
## Goodness-of-fit for in-degree
##
##
      obs min mean max MC p-value
## 0
       13
            0 1.88
                       6
                                0.00
## 1
       34
            3 9.13
                      17
                                0.00
## 2
       37
            9 21.57
                      33
                                0.00
## 3
       48
           27 40.08
                      58
                                0.22
## 4
       37
           39 57.04
                                0.00
                      72
## 5
       47
           51 65.97
                                0.00
                      87
## 6
       42
           46 62.97
                      86
                                0.00
## 7
       39
           37 53.25
                      69
                                0.04
## 8
           25 41.38
                                0.24
       35
                      55
## 9
       21
           17 27.95
                      41
                                0.20
                                0.24
## 10
       12
            7 16.99
                      27
## 11
                                0.02
       19
            3 10.43
                      20
## 12
        4
               4.89
                                1.00
                      11
## 13
        7
               2.54
                       9
                                0.04
                                0.00
## 14
        6
               1.05
                       4
            0
## 15
        3
               0.50
                       3
                                0.04
## 16
        4
               0.20
                       3
                                0.00
                                0.00
## 17
        3
            0
               0.12
                       1
## 18
        3
                0.04
                       1
                                0.00
## 19
        2
            0
                0.01
                       1
                                0.00
```

```
## 20
                0.01
                        1
                                0.02
## 22
         1
                0.00
                        0
                                 0.00
##
## Goodness-of-fit for out-degree
##
##
      obs min mean max MC p-value
## 0
        4
             0
                1.95
                        5
                                0.24
## 1
       28
             3
               8.74
                                0.00
                       17
## 2
       45
            10 22.41
                                0.00
                       36
## 3
       50
            26 40.35
                       56
                                0.08
## 4
       54
           40 56.78
                       76
                                0.70
## 5
           43 64.59
                       79
                                0.88
       62
## 6
       40
            46 63.32
                       82
                                0.00
## 7
       28
            39 53.87
                       69
                                0.00
## 8
            27 41.03
                                0.00
       13
                       53
## 9
       16
            15 28.27
                       40
                                0.04
## 10
       20
             7 16.87
                       27
                                0.42
## 11
        8
                9.87
                       17
                                0.68
## 12
       11
                5.59
                                0.08
                       11
## 13
       13
                2.43
                                0.00
                        6
## 14
        6
                                0.00
             0
                1.13
                        4
## 15
        6
                0.48
                        3
                                0.00
             0
## 16
        7
                0.22
                        3
                                0.00
## 17
                0.09
                                0.00
        4
             0
                        1
## 18
                0.00
                                0.00
         3
## 19
         0
                0.01
                        1
                                 1.00
##
## Goodness-of-fit for edgewise shared partner
##
##
          obs
              min
                             max MC p-value
                       mean
## esp0 1032 1956 2220.50 2328
                                            0
## esp1 755
               166
                    232.46
                             397
                                            0
## esp2 352
                 4
                      14.80
                              86
                                            0
```

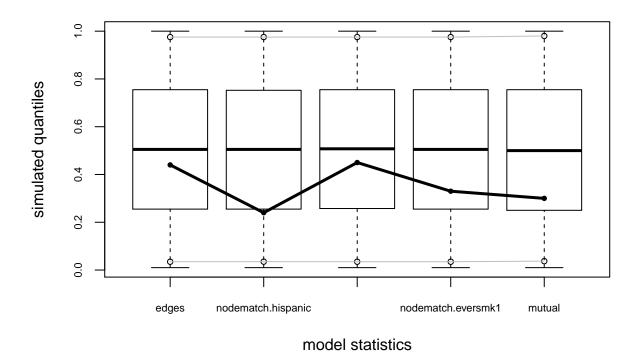
```
0
## esp3 202
                0
                     0.75
                             16
## esp4
          79
                     0.05
                             4
                                         0
                0
## esp5
                     0.00
          36
                0
                                         0
## esp6
                     0.00
                                         0
          14
                0
                             0
## esp7
                                         0
           4
                0
                     0.00
                             0
## esp8
                     0.00
                                         0
           1
                0
                             0
##
## Goodness-of-fit for minimum geodesic distance
##
##
         obs
                               max MC p-value
               min
                       mean
## 1
        2475 2348 2468.56 2598
                                         0.88
## 2
       10672 12763 13933.53 15304
                                         0.00
## 3
       31134 51869 56724.28 62405
                                         0.00
## 4
       50673 76195 79277.10 82187
                                         0.00
       42563 14434 19124.93 24041
## 5
                                         0.00
       18719
               466 1125.17 1964
                                         0.00
## 6
## 7
        4808
                 2
                      33.53
                               192
                                         0.00
## 8
         822
                       0.58
                                12
                                         0.00
         100
                       0.01
                                 1
                                         0.00
## 9
                 0
                       0.00
## 10
           7
                                 0
                                         0.00
## Inf 12333
                    1618.31 4147
                                         0.00
##
## Goodness-of-fit for model statistics
##
##
                       obs min
                                    mean max MC p-value
## edges
                      2475 2348 2468.56 2598
                                                    0.88
## nodematch.hispanic 1615 1504 1588.54 1686
                                                    0.48
## nodematch.female1 1814 1727 1805.55 1905
                                                    0.90
## nodematch.eversmk1 1738 1622 1715.58 1808
                                                    0.66
## mutual
                       486 436 477.46 522
                                                    0.60
# Plotting GOF statistics
plot(ans_gof)
```







Goodness-of-fit diagnostics



5.4 More on MCMC convergence

For more on this issue, I recommend reviewing chapter 1 and chapter 6 from the Handbook of MCMC (Brooks et al. 2011). Both chapters are free to download from the book's website.

Chapter 6

(Separable) Temporal Exponential Family Random Graph Models

This tutorial is great! https://statnet.org/trac/raw-attachment/wiki/Sunbelt2016/tergm_tutorial.pdf

Appendix A

Datasets

A.1 SNS data

A.1.1 About the data

- This data is part of the NIH Challenge grant # RC 1RC1AA019239 "Social Networks and Networking That Puts Adolescents at High Risk".
- In general terms, the SNS's goal was(is) "Understand the network effects on risk behaviors such as smoking initiation and substance use".

A.1.2 Variables

The data has a *wide* structure, which means that there is one row per individual, and that dynamic attributes are represented as one column per time.

- photoid Photo id at the school level (can be repeated across schools).
- school School id.
- hispanic Indicator variable that equals 1 if the indivual ever reported himself as hispanic.
- female1, ..., female4 Indicator variable that equals 1 if the individual reported to be female at the particular wave.

• grades1,..., grades4 Academic grades by wave. Values from 1 to 5, with 5 been the best.

- eversmk1, ..., eversmk4 Indicator variable of ever smoking by wave. A one indicated that the individual had smoked at the time of the survey.
- everdrk1, ..., everdrk4 Indicator variable of ever drinking by wave. A one indicated that the individual had drink at the time of the survey.
- home1, ..., home4 Factor variable for home status by wave. A one indicates home ownership, a 2 rent, and a 3 a "I don't know".

During the survey, participants were asked to name up to 19 of their school friends:

- sch_friend11, ..., sch_friend119 School friends nominations (19 in total) for wave 1.

 The codes are mapped to the variable photoid.
- sch_friend21, ..., sch_friend219 School friends nominations (19 in total) for wave 2.

 The codes are mapped to the variable photoid.
- sch_friend31, ..., sch_friend319 School friends nominations (19 in total) for wave 3.
 The codes are mapped to the variable photoid.
- sch_friend41, ..., sch_friend419 School friends nominations (19 in total) for wave 4.
 The codes are mapped to the variable photoid.

References

Bache, Stefan Milton, and Hadley Wickham. 2014. *Magrittr: A Forward-Pipe Operator for R*. https://CRAN.R-project.org/package=magrittr.

Bojanowski, Michal. 2015. *Intergraph: Coercion Routines for Network Data Objects*. http://mbojan.github.io/intergraph.

Brooks, Steve, Andrew Gelman, Galin Jones, and Xiao-Li Meng. 2011. *Handbook of Markov Chain Monte Carlo*. CRC press.

Csardi, Gabor, and Tamas Nepusz. 2006. "The Igraph Software Package for Complex Network Research." *InterJournal* Complex Systems: 1695. http://igraph.org.

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