Big Problems for Small Networks: Statistical Analysis of Small Networks and Team Performance

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Contents

Acknowledgements

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Network Science of Teams
a Multidisciplinary University Research Initiative
UC SANTA BARBARA
USC University of Southern California
Massachusetts Institute of Technology
NORTHWESTERN UNIVERSITY

Part I: Exponential Random Graph Models for Small Networks

Exponential Random Graph Models (ERGMs)

Figure 1: Friendship network of a UK university faculty. Source: **igraphdata** R package (Gabor Csardi, 2015). Figure drawn using the R package **netplot** ((https://github.com/usccana/netplot)

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How can we explain what we see here?

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Where $\kappa\left(\theta, \mathbf{X}\right)$ is the normalizing constant and equals $\sum_{\mathbf{g}' \in \mathcal{G}} \exp\left\{\theta^{\mathbf{t}} s\left(\mathbf{g}', \mathbf{X}\right)\right\}$. Figure 2 shows some examples of values in $s\left(\right)$.

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- See Wasserman, Pattison, Robins, Snijders, Handcock and others.

Structures

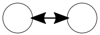
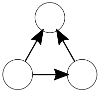
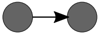
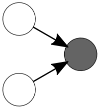
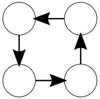
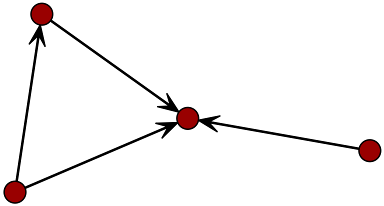
Representation	Description
	Mutual Ties (Reciprocity) $\sum_{i \neq j} y_{ij} y_{ji}$
	Transitive Triad (Balance) $\sum_{i \neq j \neq k} y_{ij} y_{jk} y_{ik}$
	Homophily $\sum_{i \neq j} y_{ij} \mathbf{1}\left(x_i = x_j\right)$
	Covariate Effect for Incoming Ties $\sum_{i \neq j} y_{ij} x_j$
	Four Cycle $\sum_{i \neq j \neq k \neq l} y_{ij} y_{jk} y_{kl} y_{li}$

Figure 2: Besides of the common edge count statistic (number of ties in a graph), ERGMs allow measuring other more complex structures that can be captured as sufficient statistics.

Example of model

In this network



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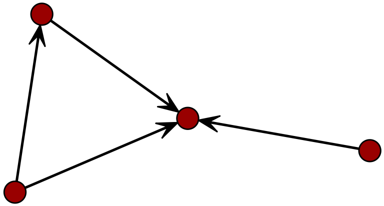

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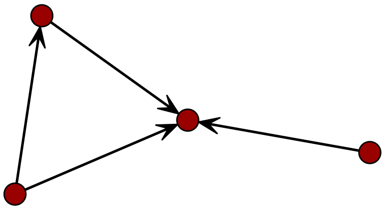

We see 4 **edges**, 1 **transitive triad** and **no mutual ties**.

The probability function of this model would be

$$\Pr\left(\mathbf{G}=\mathbf{g}\mid\theta\right)=\frac{\exp\left\{4\theta_{edges}+\theta_{ttriads}+0\theta_{mutual}\right\}}{\sum_{\mathbf{g}'\in\mathcal{G}}\exp\left\{\theta^{\mathbf{t}}s\left(\mathbf{g}'\right)\right\}}$$

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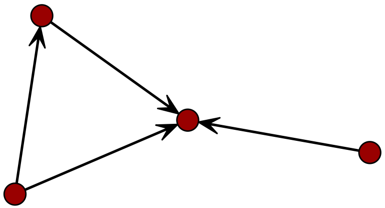

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This model has **MLE parameter estimates** of -0.19 (low density), 0.27 (high chance of ttriads), and -9.75 (low chance of mutuality) for the parameters `edges`, `ttriads`, and `mutual` respectively.

Estimation of ERGMs

- Calculating of the normalizing constant in (1), $\kappa = \sum_{\mathbf{g}' \in \mathcal{G}} \exp\left\{\theta^{\mathbf{t}} s\left(\mathbf{g}', \mathbf{X}\right)\right\}$, makes ERGMs difficult to estimate.

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- For this reason, statistical methods have focused on avoiding the direct calculation of κ; most modern methods for estimating ERGMs rely on MCMC.

Estimation of ERGMs (cont'd)

Description of the MCMC-MLE algorithm (one of the approaches)

1. Make an initial guess of the model parameters using MPL (maximum pseudo likelihood), $\hat{\theta}_0$
2. While the algorithm doesn't converge, do:
   a. Generate a large sample of graphs from $\Pr\left(\mathbf{G} = \mathbf{g} \,\middle|\, \hat{\theta}_0, \mathbf{X}\right)$ using MCMC
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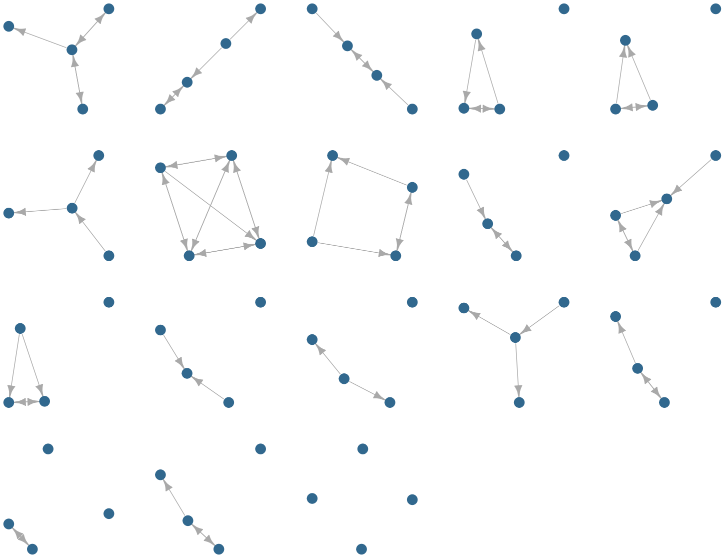
- Model degeneracy is particularly problematic with small networks.

Figure 3: Model generacy. Figure

ERGMs for Small Networks



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How different is this from the "normal" way to fit ERGMs?

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We have implemented this and more in the `ergmito` R package

An important pause...

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e.g.:

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Special thanks to George Barnett who proposed the name during the 2018 NASN!

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- Implements pooled ERGM models.
- In the same spirit of the exhaustive enumeration, includes a simulation function for small networks sampling from the true distribution.

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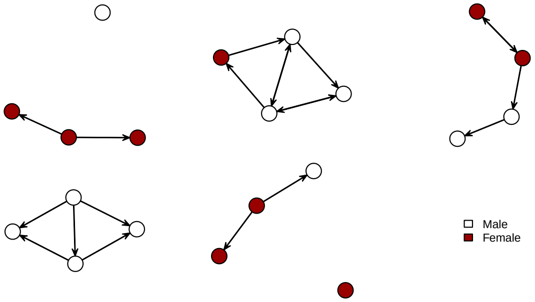
`ergmito` example

```r
library(ergmito)
data(fivenets, package = "ergmito")
```



```r
# Looking at one of the five networks
fivenets[[1]]
```

```
##  Network attributes:
##   vertices = 4
##   directed = TRUE
##   hyper = FALSE
##   loops = FALSE
##   multiple = FALSE
##   bipartite = FALSE
##   total edges= 2
##     missing edges= 0
##     non-missing edges= 2
##
##  Vertex attribute names:
##     female name
##
## No edge attributes
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So how can we fit this model?

ergmito example (cont'd)

The same as you would do with the ergm package:

```r
(model1 <- ergmito(fivenets ~ edges + nodematch("female")))
```

```
##
## ERGMito estimates
##
##  Coefficients:
##            edges  nodematch.female
##           -1.705             1.587
```

	Model 1
edges	-1.70^{**}
	(0.54)
nodematch.female	1.59^{*}
	(0.64)
AIC	73.34
BIC	77.53
Log Likelihood	-34.67
# Networks	5

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 1: Statistical models

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We simulated 100,000 samples, each one composed of an average of 30 networks.

Simulation Study (cont'd)

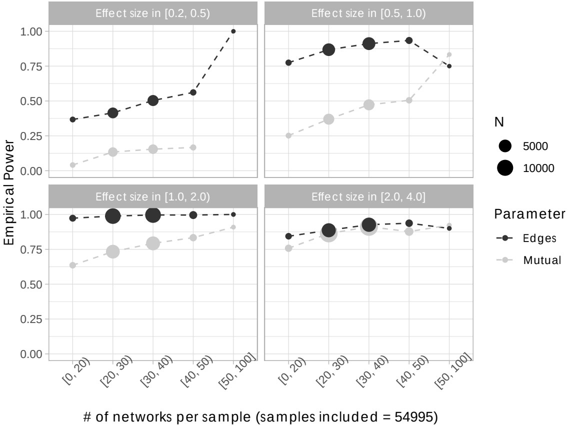

Figure 4: Empirical power of Pooled-ERGM estimates at various levels of effect size. As expected, power increases significantly with sample size (# of networks per sample). Interestingly, the discovery rate of an effect size within $[1, 2)$ is very high even with a sample size of 20-30 networks. More extreme points have higher volatility due to small number of samples included.

Part II: Association between group structure and team performance

Testing effects of social network structure on group performance

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- Permutation tests:
  - Common approach: sample from graphs with the same degree sequence–the observed sequence of in/out degree
  - This is oversimplifying/constraining
  - And worse, in a network of size 4, how many different networks can be observed **holding the degree sequence fixed**?

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- Sure, but what about ERGMs?Ultimately these models describe a distribution of graphs that have <u>on average</u> the same set of network statistics
- This overcomes the problem observed in permutation tests.

A semiparametric test

Notation

- $\mathbf{G} = \{\mathbf{g}_j\}$ is a sequence of J graphs that share a common data-generating-process, e.g. teams formed in a lab.
- Each network has node-level attributes $x \in \mathcal{X}$.
- A group(graph) level outcome variable, such as team performance, Y.
- Under the null, network structure and group performance are not associated, this is $Y \perp \mathbf{G}$.

Algorithm

1. Estimate an ERGM (estimates can come from a single graph or pooled estimates). We denote the data-generating-process of this model as $\mathcal{D} : \Theta \times \mathcal{X} \mapsto \mathcal{G}$.

2. Calculate the value $s_0 = s(\mathbf{G}, Y)$.

3. Now, for $b \in \{1, \dots, B\}$ do:

  3.1 For each group j in $\{1, \dots, J\}$, draw a new network $\mathbf{g}_j^b \sim \mathcal{D}(\hat{\theta}, X_j)$, this new sequence is denoted $\mathbf{G}^b$

  3.2 Using $\mathbf{G}^b$ and Y, calculate $s_b = s(\mathbf{G}^b, Y)$

  3.3 Next b.

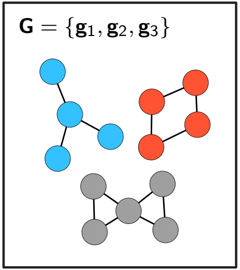
This will generate a null distribution for the statistic s, which we can use to compare against the observed statistic, s_0.

Note An important distinction to make is that structures that gave origin to the graph need not to be relevant for the team's performance per se.

Illustrated example

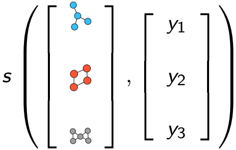
Suppose that we have a 3 networks of sizes 4, 4, and 5 respectively. The

Step 1:
Fit the ERGMito



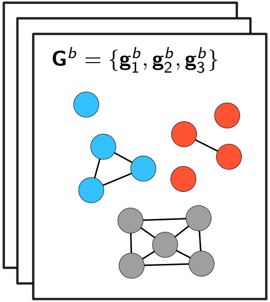
Fit the ERGMito,
This will give us $\mathcal{D}(\hat{\theta}, X_j)$

Step 2:
Calculate $s_0 =$



Throughout the simulations the only part that changes is the networks, not Y

Step 3:
For $b \in 1, \ldots, B$ do



3.1) For $j \in \{1, 2, 3\}$ draw a new network from $\mathcal{D}$
3.2) Use the new sample to calculate $s_b = s(\mathbf{G}^b, Y)$

We can use the distribution of the sequence $\{s_1, \ldots, s_B\}$ as null to compare against s_0

Part III: An empirical example

Small teams performance

Data

Results

Concluding remarks

Exponential Random Graph Models for Small Networks:

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Test for Association between graph level outcomes and graph structures:

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3. What about other properties such as type I error?

Thanks!

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Let's chat!

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