

Big Problems for **Small Networks**: Statistical Analysis of Small Networks and Team Performance

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SONIC Speaker Series
Northwestern University, IL
March 20, 2019

Acknowledgements



This material is based upon work support by, or in part by, the U.S. Army Research Laboratory and the U.S. Army Research Office under grant number W911NF-15-1-0577

Computation for the work described in this paper was supported by the University of Southern California's Center for High-Performance Computing (<https://hpcc.usc.edu>).



We thank members of our MURI research team, USC's Center for Applied Network Analysis, Andrew Slaughter, and attendees of the NASN 2018 conference for their comments.



Network Science of Teams

a Multidisciplinary University Research Initiative

UC SANTA BARBARA



Research Problem

In the context of network science of small teams:

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What characterizes the social networks that emerge from small teams?

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Is there any association between how team networks are structured and their performance?

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 - ▶ Social Networks: Advice Seeking, Leadership, Influence (among others).

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Part I: Network Structure

Part II: Association between network structure and team performance

Part I: Network Structure

Exponential Random Graph Models (ERGMs)



Figure 1: Friendship network of a UK university faculty. Source: **igraphdata** R package (Csardi, 2015). Figure drawn using the R package **netplot** (yours truly, <https://github.com/usccana/netplot>)

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How can we explain what we see here?

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- ▶ See Wasserman, Pattison, Robins, Snijders, Handcock and others.

Structures




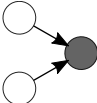

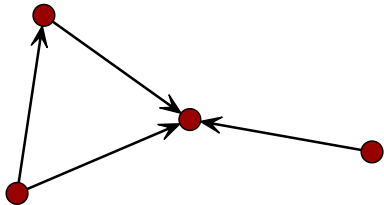
Representation	Description
	Mutual Ties (Reciprocity) $\sum_{i \neq j} y_{ij} y_{ji}$
	Transitive Triad (Balance) $\sum_{i \neq j \neq k} y_{ij} y_{jk} y_{ik}$
	Homophily $\sum_{i \neq j} y_{ij} \mathbf{1}(x_i = x_j)$
	Covariate Effect for Incoming Ties $\sum_{i \neq j} y_{ij} x_j$
	Four Cycle $\sum_{i \neq j \neq k \neq l} y_{ij} y_{jk} y_{kl} y_{li}$

Figure 2: Besides of the common edge count statistic (number of ties in a graph), ERGMs allow measuring other more complex structures that can be captured as sufficient statistics.

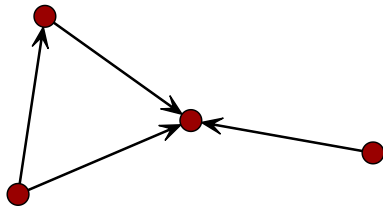
Example of model

In this network



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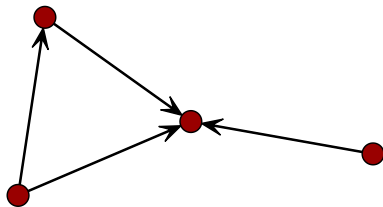
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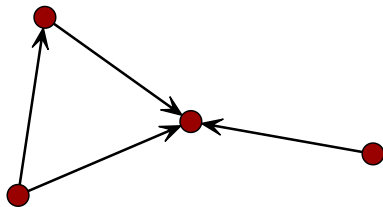
The probability function of this model would be

$$\Pr(\mathbf{G} = \mathbf{g} \mid \theta) = \frac{\exp \{ 4\theta_{edges} + \theta_{ttriads} + 0\theta_{mutual} \}}{\sum_{\mathbf{g}' \in \mathcal{G}} \exp \{ \theta^t s(\mathbf{g}') \}}$$

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This model has **MLE parameter estimates** of -0.19 (low density), 0.27 (high chance of ttriads), and -9.75 (low chance of mutuality) for the parameters edges, ttriads, and mutual respectively.

Estimation of ERGMs

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- ▶ For this reason, statistical methods have focused on avoiding the direct calculation of κ ; most modern methods for estimating ERGMs rely on MCMC.

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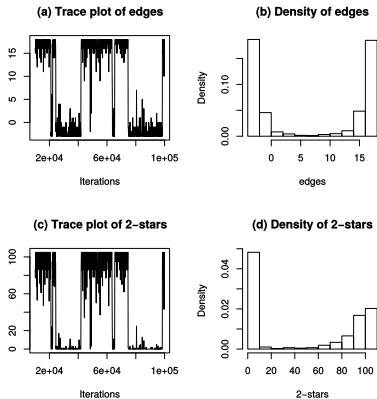


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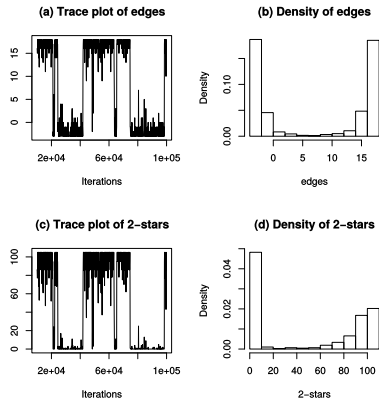
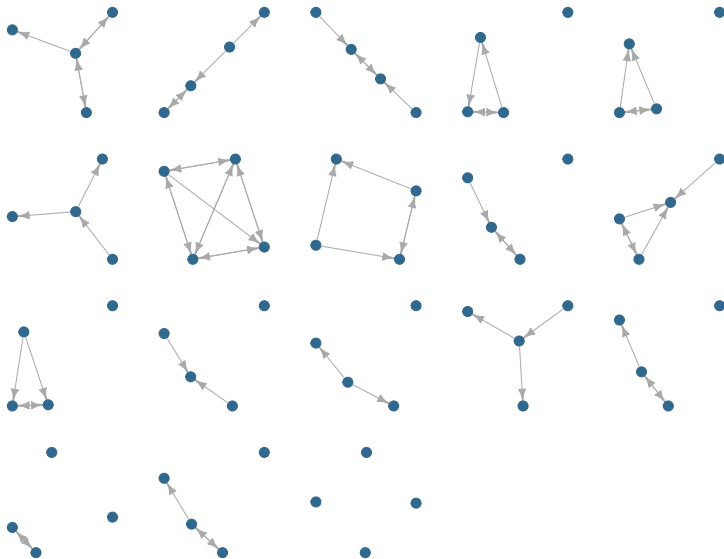


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- ▶ Model degeneracy is particularly problematic with small networks. . . (says anyone who has tried to fit one).

ERGMs for Small Networks



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How different is this from the “normal” way to fit ERGMs?

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We have implemented this and more in the `ergmito` R package

Sidetrack...

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Special thanks to George Barnett who proposed the name during the 2018 NASN!

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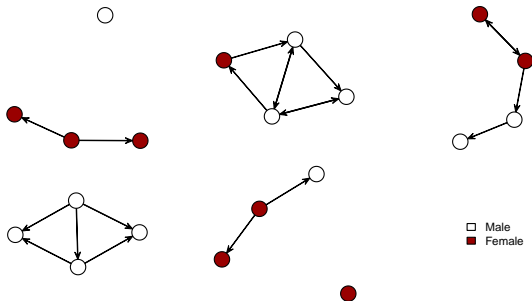
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- ▶ Implements pooled ERGM models.
- ▶ Includes a simulation function for efficiently drawing samples of small networks, and by **efficiently** we mean **fast**.

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ergmito example

```
library(ergmito)  
data(fivenets, package = "ergmito")
```



```
# Looking at one of the five networks  
fivenets[[1]]
```

```
## Network attributes:  
##   vertices = 4  
##   directed = TRUE  
##   hyper = FALSE  
##   loops = FALSE  
##   multiple = FALSE  
##   bipartite = FALSE  
##   total edges= 2  
##     missing edges= 0  
##     non-missing edges= 2  
##  
## Vertex attribute names:  
##   female name  
##  
## No edge attributes
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So how can we fit this model?

ergmito example (cont'd)

The same as you would do with the `ergm` package:

```
(model1 <- ergmito(fivenets ~ edges + nodematch("female")))
```

```
##  
## ERGMito estimates  
##  
## Coefficients:  
##           edges  nodematch.female  
##          -1.705             1.587
```

	Model 1
edges	-1.70** (0.54)
nodematch.female	1.59* (0.64)
AIC	73.34
BIC	77.53
Log Likelihood	-34.67
Num. networks	5
*** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$	

Table 1: Statistical models

Simulation Study

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We simulated 100,000 samples, each one composed of an average of 30 networks.

Simulation Study (cont'd)

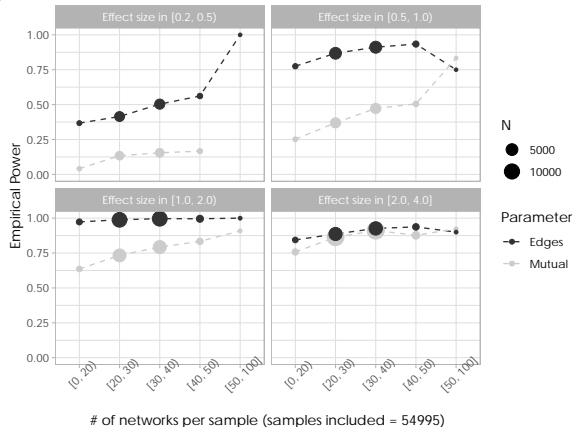


Figure 4: Empirical power of Pooled-ERGM estimates at various levels of effect size. As expected, power increases significantly with sample size (# of networks per sample). Interestingly, the discovery rate of an effect size within [1, 2) is very high even with a sample size of 20-30 networks. More extreme points have higher volatility due to small number of samples included.

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... what can this tell us about our 42 teams?

Preliminary results

	Advice	Influence	Leadership
Edges	-1.87*** (0.30)	-0.78*** (0.13)	-0.57*** (0.14)
Transitive Triads	0.24*** (0.06)	0.21** (0.08)	
Indeg. RME	0.35*** (0.08)		
Outdeg. Female	0.43* (0.19)		
Outdeg. Social Accomodation	0.11 (0.08)		
Indeg. Female			-0.38* (0.19)
AIC	693.18	760.40	655.78
BIC	714.50	769.12	664.32
Log Likelihood	-341.59	-378.20	-325.89
Num. networks	38	41	38

*** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$

Table 2: The two statistics that showed to be the most robust were **Indeg. RME** and **Outdeg. Female**. These two effects can be described as (1) individuals with high levels of RME receive more ties, and (2) female subjects were more likely of seeking advice than male. Other statistics such as GPA, religiousness, age, and ethnicity were not significant.

Part II: Association between network structure and team performance

Testing effects of social network structure on group performance

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BTW Degree sequence \mapsto Scale-free networks

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The **structural diversity of real-world networks** uncovered here presents both a puzzle and an opportunity. The strong focus in the scientific literature on **explaining and exploiting scale-free patterns** has meant **relatively less is known about mechanisms that produce non-scale-free structural patterns**, e.g., those with degree distributions better fitted by a log-normal. Two important directions of future work will be the **development and validation of novel mechanisms for generating more realistic degree structure in networks**, and novel statistical

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techniques for identifying or untangling them given empirical data

– p. 8, Broido and Clauset (2019)

See Holme (2019) for a recent reference on the Scale-free issue.

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In principle, this would be equivalent to a revised rewiring test. . .

Algorithm

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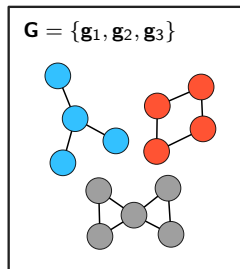
Note An important distinction to make is that structures that gave origin to the graph need not to be relevant for the team's performance per se.

Illustrated example

Suppose that we have a 3 networks of sizes 4, 4, and 5 respectively. The

Step 1:

Fit the ERGMito



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This will give us $\mathcal{D}(\hat{\theta}, X_j)$

Step 2:

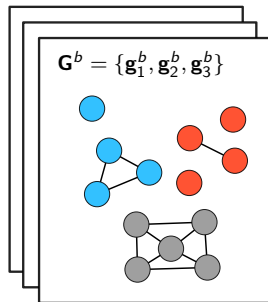
Calculate $t_0 =$

$$t \left(\begin{bmatrix} \text{blue graph} \\ \text{red graph} \\ \text{grey graph} \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right)$$

Throughout the simulations
the only part that changes is
the networks, not Y

Step 3:

For $b \in 1, \dots, B$ do



3.1) For $j \in \{1, 2, 3\}$ draw a
new network from \mathcal{D}

3.2) Use the new sample to
calculate $t_b = t(\mathbf{G}^b, Y)$

We can use the distribution of the sequence $\{t_1, \dots, t_B\}$ as null to compare against t_0

Simulation

Going back to our `fivenets` example:

- ▶ Imagine that $y_{\mathbf{g}} = \text{nodeicov}(\text{"female"})_{\mathbf{g}} + \varepsilon$, $\varepsilon \sim N(0, 1)$
- ▶ Our test statistic t is the correlation between y and `nodeicov("female")`

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3. Working on a more formal statistical framework (when is it a good/bad idea to use this kind of method).

Thanks!



George G. Vega Yon

Let's chat!

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@gvegayon

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References I

Allaire, JJ, Yihui Xie, Jonathan McPherson, Javier Luraschi, Kevin Ushey, Aron Atkins, Hadley Wickham, Joe Cheng, Winston Chang, and Richard Iannone. 2018. Rmarkdown: Dynamic Documents for R. <https://rmarkdown.rstudio.com>.

Broido, Anna D, and Aaron Clauset. 2019. "Scale-Free Networks Are Rare." Nature Communications 10 (1): 1017.

Csardi, Gabor. 2015. Igraphdata: A Collection of Network Data Sets for the 'Igraph' Package. <https://CRAN.R-project.org/package=igraphdata>.

Geyer, Charles J., and Elizabeth A. Thompson. 1992. "Constrained Monte Carlo Maximum Likelihood for Dependent Data." Journal of the Royal Statistical Society. Series B (Methodological) 54 (3): 657–99. <http://www.jstor.org/stable/2345852>.

Handcock, Mark S. 2003. "Assessing Degeneracy in Statistical Models of Social Networks." Working Paper No. 39 76 (39): 33–50. <https://doi.org/10.1.1.81.5086>.

References II

Handcock, Mark S., David R. Hunter, Carter T. Butts, Steven M. Goodreau, Pavel N. Krivitsky, and Martina Morris. 2018. Ergm: Fit, Simulate and Diagnose Exponential-Family Models for Networks. The Statnet Project (<http://www.statnet.org>).
<https://CRAN.R-project.org/package=ergm>.

Handcock, Mark, Peng Wang, Garry Robins, Tom Snijders, and Philippa Pattison. 2006. "Recent developments in exponential random graph (p^*) models for social networks." Social Networks 29 (2): 192–215. <https://doi.org/10.1016/j.socnet.2006.08.003>.

Holme, Petter. 2019. "Rare and everywhere: Perspectives on scale-free networks." Nature Communications 10 (1): 1016. <https://doi.org/10.1038/s41467-019-09038-8>.

Hunter, David R., Mark S. Handcock, Carter T. Butts, Steven M. Goodreau, and Martina Morris. 2008. "Ergm: A Package to Fit, Simulate and Diagnose Exponential-Family Models for Networks." Journal of Statistical Software 24 (3): 1–29.

References III

Kim, Young Ji, David Engel, Anita Williams Woolley, Jeffrey Yu-Ting Lin, Naomi McArthur, and Thomas W. Malone. 2017. “What Makes a Strong Team?: Using Collective Intelligence to Predict Team Performance in League of Legends.” In Proceedings of the 2017 Acm Conference on Computer Supported Cooperative Work and Social Computing, 2316–29. CSCW '17. New York, NY, USA: ACM. <https://doi.org/10.1145/2998181.2998185>.

Milo, R, N Kashtan, S Itzkovitz, M E J Newman, and U Alon. 2004. “On the uniform generation of random graphs with prescribed degree sequences.” Arxiv Preprint Condmat0312028 cond-mat/0: 1–4. <http://arxiv.org/abs/cond-mat/0312028>.

R Core Team. 2018. R: A Language and Environment for Statistical Computing. Vienna, Austria: R Foundation for Statistical Computing. <https://www.R-project.org/>.

Wasserman, Stanley, and Philippa Pattison. 1996. “Logit models and logistic regressions for social networks: I. An introduction to Markov graphs andp.” Psychometrika 61 (3): 401–25. <https://doi.org/10.1007/BF02294547>.

Xie, Yihui. 2018. Knitr: A General-Purpose Package for Dynamic Report Generation in R. <https://yihui.name/knitr/>.