	network	n	r
real-world networks	physics coauthorship[a]	52 909	0.363
	biology coauthorship[a]	1 520 251	0.127
	mathematics coauthorship[b]	253 339	0.120
	film actor collaborations[c]	449 913	0.208
	company directors[d]	7 673	0.276
	Internet[e]	10 697	−0.189
	World-Wide Web[f]	269 504	−0.065
	protein interactions[g]	2 115	−0.156
	neural network[h]	307	−0.163
	food web[i]	92	−0.276
models	random graph[u]		0
	Callaway *et al.*[v]		$\delta/(1+2\delta)$
	Barabási and Albert[w]		0

TABLE I: Size n and assortativity coefficient r for a number of different networks: collaboration networks of (a) scientists in physics and biology [16], (b) mathematicians [17], (c) film actors [4], and (d) businesspeople [18]; (e) connections between autonomous systems on the Internet [19]; (f) undirected hyperlinks between Web pages in a single domain [6]; (g) protein-protein interaction network in yeast [20]; (h) undirected (and unweighted) synaptic connections in the neural network of the nematode *C. Elegans* [4]; (i) undirected trophic relations in the food web of Little Rock Lake, Wisconsin [21]. The last three lines give analytic results for model networks in the limit of large network size: (u) the random graph of Erdős and Rényi [22]; (v) the grown graph model of Callaway *et al.* [15]; (w) the preferential attachment model of Barabási and Albert [6].

where j_i, k_i are the degrees of the vertices at the ends of the ith edge, with $i = 1 \dots M$ [36].

In Table I we show values of r for a variety of real-world networks. As the table shows, of the social networks studied (the top five entries in the table) all have significant assortative mixing, which accords with accepted wisdom within the sociological community. By contrast, the technological and biological networks studied (the middle five entries) all have disassortative mixing—high degree vertices preferentially connect with low degree ones and *vice versa*. Various explanations for this observation suggest themselves. In the case of the Internet, for example, it appears that the high degree vertices mostly represent connectivity providers—telephone companies and other communications carriers—who typically have a large number of connections to clients who themselves have only a single connection [19]. Thus the high-degree vertices do indeed tend to be connected to the low-degree ones.

We have also calculated r analytically for three models of networks: (1) the random graph of Erdős and Rényi [22], in which edges are placed at random between a fixed set of vertices; (2) the grown graph model of Callaway *et al.* [15], in which both edges and vertices are added at random at constant but possibly different rates, the ratio of the rates being denoted δ; (3) the grown graph model of Barabási and Albert [6], in which both edges and vertices are added, and one end of each edge is added with linear preferential attachment.

For the random graph, since edges are placed at random without regard to vertex degree it follows trivially that $r = 0$ in the limit of large graph size. The model of Callaway *et al.* however, although apparently similar in construction, gives a markedly different result. From Eq. (21) of Ref. 15, e_{jk} for this model satisfies the recurrence relation

$$(1+4\delta)e_{jk} = 2\delta(e_{j-1,k} + e_{j,k-1}) + p_j p_k, \tag{5}$$

and the degree distribution is $p_k = (2\delta)^k/(1+2\delta)^{k+1}$. Substituting into Eq. (3) and making use of Eq. (2), we then find that $r = \delta/(1+2\delta)$. Thus the model shows significant assortative mixing, with a maximum value of $r = \frac{1}{2}$ in the limit of large δ. This agrees with intuition [15]: in the grown graph the older vertices have higher degree and also tend to have higher probability of being connected to one another, simply by virtue of being around for longer. Thus one would expect positive assortative mixing.

The model of Barabási and Albert [6] provides an interesting counter-example to this intuition. Although this is a grown graph model, in which again older vertices have higher degree [23], it shows no assortative mixing at all. Making use of Eq. (42) of Ref. 24 we can show that e_{jk} for the model of Barabási and Albert goes asymptotically as $1/(j^2k^2) - 6/(j+k)^4$ in the limit of large j and k, which implies that $r \to 0$ as $(\log^2 N)/N$ as N becomes large. The model of Barabási and Albert has been used as a model of the structure of the Internet and the World-Wide Web. Since these networks show significant disassortative mixing however (Table I), it is clear that the model is incomplete. It is an interesting open question what type of network evolution processes could explain the values of r observed in real-world networks.

Turning now to theoretical developments, we propose a simple model of an assortatively mixed network, which is exactly solvable for many of its properties in the limit of large graph size. Consider the ensemble of graphs in which the distribution e_{jk} takes a specified value. This defines a random graph model similar in concept to the random graphs with specified degree sequence [5, 25, 26], except for the added element of assortative mixing.

Consider a typical member of this ensemble in the limit of large graph size, and consider a randomly chosen edge in that graph, one end of which is attached to a vertex of degree j. We ask what the probability distribution is of the number of other vertices reachable by following that edge. Let this probability distribution be generated by a generating function $G_j(x)$, which depends in general on the degree j of the starting vertex. By arguments similar to those of Ref. 5, we can show that $G_j(x)$ must satisfy a self-consistency condition of the form

$$G_j(x) = x \frac{\sum_k e_{jk} \left[G_k(x)\right]^k}{\sum_k e_{jk}}, \tag{6}$$

while the number of vertices reachable from a randomly