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ORIGINAL PAPER

A Conditional Likelihood Model of the Relationship Between Officer Features and Rounds Discharged in Police Shootings

Greg Ridgeway[1,2] · **Breanne Cave**[3] · **Julie Grieco**[4] · **Charles Loeffler**[1]

Abstract

Objectives Assess whether the number of rounds fired in an officer-involved shooting is related to police officer features.

Data The data come from 55 member agencies in the Major Cities Chiefs Association. The full dataset describes 2574 officers involved in 1600 shootings between 2010 and 2018 but only incidents involving multiple officers provide information. Our final dataset included 317 shooting incidents involving 849 officers and 5026 rounds.

Methods We match officers on the scene of a shooting incident and develop a conditional truncated Poisson model that eliminates confounding due to time, place, and environment. We use a permutation test to formally assess the strength of the relationship between officer features and shooting rate.

Results We find no officer feature strongly predicts shooting rate. Age at recruitment, age at the time of the shooting, and years of experience all had relative rates nearly equal to 1.0. There was no statistical relationship with an officer being female ($p=0.27$), black ($p=0.64$), or Hispanic ($p=0.39$). Having prior involvement in shootings, prior force complaints, and special assignments appear to elevate the relative rate of shooting, but all confidence intervals included 1.0.

Conclusions Officer features appear to have little or no relationship with shooting rate. These findings correspond with police scholars' supposition that duty assignment may be more responsible for explaining differences in police use of force than individual officer characteristics. It contrasts with some prior research suggesting that officer race, age at recruitment, and prior performance affect shooting risk. In doing so, these results also lend support to theoretical frameworks emphasizing the role of organizational features and other system-level factors over individual-level explanations for police use of force. The proposed methodology addresses bias due to confounding, but demands a large number of shootings. Expanded participation in multi-agency data collections and including data on all non-shooting officers at the scene of the incident can increase precision.

Keywords Police use-of-force · Lethal force · Semi-parametric modeling · Conditional Poisson regression

✉ Greg Ridgeway
gridge@upenn.edu

Extended author information available on the last page of the article

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Introduction

On November 26, 2006 New York City Police Detectives Isnora, Oliver, Cooper, Headley, and Police Officer Carey fired 50 rounds into a Nissan Altima carrying Sean Bell, Trent Benefield, and Joseph Guzman, all unarmed, striking Benefield, permanently injuring Guzman, and killing Bell (Baker 2007). Three of these officers were black and two were white. They ranged in age from 28 to 39 years old. They also varied greatly in the number of rounds fired: Oliver 31, Isnora 11, Cooper 4, Carey 3, and Headley 1. What made one officer shoot so many more rounds than others? Was it just the random alignment of the five officers so that Oliver was positioned in a way to perceive a greater threat and Headley was not? Perhaps officer race played a role, but the two white officers on the scene, Oliver and Carey, differed greatly in the number of rounds discharged. Perhaps officer age played a role, but the two 35-year old officers, Oliver and Headley, also differed greatly in the number of rounds discharged.

From a single police shooting, we cannot link any officer feature to the number of rounds discharged. However, if after studying shooting after shooting we observe that the high-volume shooters consistently have features that tend to differ from the low volume shooters, then this can be a signal of a risk factor for being a high-volume shooter. On the other hand, if the data show no consistent pattern, then this would be evidence that shootings are the result of the system, organization-level features, and environment and that at the scene of a shooting only chance positioning influences the rate of rounds fired.

Zimring (2017) notes that fatalities from police shootings results from four primary decisions, (1) whether police will shoot at all, (2) when police stop shooting and how many rounds they fire, (3) what medical care police provide immediately, and (4) whether police immediately transport to a hospital those they have shot (Zimring 2017). While Ridgeway (2016) focused on Zimring's first decision point, this study focuses on the second. We examine whether officer features are associated with the expected number of rounds fired and the decision to *stop* shooting.

Previous research relating officer features to officer performance often faced confounding, unmeasured variables associated both with officer features and measures of performance. For example, assignment to a high-crime neighborhood may be the cause of an observed correlation between an officer's years of experience and their involvement in a shooting if departments assign younger officers to high-crime neighborhoods. Even if we could account for data on neighborhood crime rates and assignments, perhaps the younger officers are dispatched to the higher risk calls. Properly linking officer features to officer performance requires a methodology that can account rigorously for this kind of confounding.

In this paper we study the link between officer features and the expected number of rounds fired. Specifically, we examine the relationship between an officer's shooting risk and the officer's age at recruitment, years of experience, race, sex, prior involvement in officer-involved shootings, prior complaints of excessive force, rank, and firearm type. The data come from member agencies of the Major Cities Chiefs Association (MCCA). Agencies provided reports for any incident in which an officer discharged a weapon toward or near a subject, whether accidental or intentional. We develop a statistical model, based on a truncated conditional Poisson likelihood, that solves the problem of untangling officer features from the risk of being on the scene of a shooting. By conditioning on the total number of rounds fired, features of the environment, such as time, place, lighting, and suspect features, need not be measured or modeled in order to obtain consistent estimates of officer risk factors. Knowledge of which officers fired their guns at each shooting incident, the number of rounds each officer fired, and the officer features are sufficient, in the statistical sense, for estimating the relationship between officer features and the expected number of rounds fired.

Background

Police shootings draw substantial public attention, have prompted the majority of the incidents of largescale civil unrest in the last century, and strain the relationship between the police and the community. Social scientists have spent considerable effort in trying to extract knowledge from data on police shootings about the environments in which they occur, the suspects involved, and officers who find the need to shoot. The research primarily has tried to tease apart the environmental, organizational, and situational ingredients to understand their role in police shootings, causing them, causing fatal shootings, causing high volume shootings, causing shootings of unarmed subjects, and other characteristics of shootings. Environmental (e.g. neighborhood crime levels, changes in laws) and organizational (e.g. department policy, training, and culture) attributes are critical ingredients (e.g. Geller and Karales 1981; Fyfe 1988; Sklansky 2006; Klinger et al. 2015; Sherman 2018), as are situational (e.g. subject features, officer features) attributes (e.g. White and Klinger 2012; DeGue et al. 2016; Fridell 2017). In this paper we exclusively focus on officer features using a design that controls for all other environmental, organizational, and other situational factors. By doing so, we isolate the contribution of these officer features.

Social scientists have created a substantial body of research on officer features and their links to officer performance and outcomes. Sherman (1980) provides a summary of early research on this topic. Specifically, he reviews research on how officer age, length of service, sex, height, race, education, job satisfaction, and racial attitudes relate to detection, arrest, service (e.g. manner of settling disputes, quality of public interactions), and violence. He also notes research from the 1960s and 1970s found that black officers used unjustified force more often (Reiss 1972) and that black officers used deadly force more often (Fyfe 1978). Sherman's review also cites evidence that more educated officers had fewer allegations of excessive force (Cascio 1977). Advances toward resolving these links between officer features and force would represent progress in answering a key police shooting question, "do police records show some police officers to be predictably more at risk than others to shoot illegally or unnecessarily…?" (Sherman 2018). If officer features do reliably predict the likelihood of officer involved shootings, this will confirm that the officer is the correct unit of analysis for explaining why these shootings occur. However, if officer features only weakly predict the likelihood of officer involved shootings, then this will support a shift in focus towards geographical, organizational, or other system-level explanations for officer-involved shootings.

Fyfe (1981) was one of the earliest researchers to specifically examine the relationship between officer race and the risk of shooting. He found that minority officers were represented disproportionately among the shooting officers, but also argued that minority officers were placed disproportionately in areas with the greatest risk of shooting involvement. More recent research has found mixed relationships between use of force and race (Terrill 2001), but some research suggests that the interaction between officer race and suspect race drives the risk of use of force (Paoline et al. 2018).

Several studies have examined the constellation of officer features and use of force generally and shooting specifically. Riksheim and Chermak (1993) aimed to update the Sherman (1980) review reporting that research in the 1980s found no relationship between force (as well as deadly force) and age, marital status, experience, and education. Ho (1994) concluded that race and sex do not influence the use of deadly force, but officers with less experience are more restrained than veteran officers in some circumstances. McElvain and Kposowa (2008) tracked 648 Riverside (CA) sheriff deputies between 1990 and 2004; 314 of those deputies were involved in shootings by the end of the study period. They found that the shooting hazard rate was higher for white deputies, male deputies, young deputies, deputies with no college degree, and deputies with a prior shooting. Klinger et al. (2015) found that the age, race, sex distribution of St. Louis police officers involved in shootings matched the age, race, sex distribution of the entire department. Studying Philadelphia police officers, Donner et al. (2017) related involvement in a police shooting with measures of officer self-control, as measured by responses on applicants' Personal Data Questionnaire to questions about traffic tickets, car crashes, debts, and divorce. They found that "low self-control" applicants were "quick on the draw" and at greater risk of involvement in police shootings. In addition, they found that male officers, black officers, officers with less education, and officers with police officer parents were more likely to be involved in police shootings.

A frequent refrain in prior research has been the concern that observed relationships between officer features and officer performance are the result of confounding. Sklansky (2006) wrote "If, for example, black officers draw more complaints, is that because they act more aggressively, or because they are assigned to tougher beats…" Even earlier Sherman (1980) noted that younger officers were assaulted more often, "but this may be a spurious result of their more frequent assignment to patrol duties in high-crime areas." Fyfe (1981) showed that "the overrepresentation of minority officers among police shooters closely associated with racially varying pattern of assignment, socialization, and residence." Geller and Karales (1981) likewise note that "black officers are not prominent in the units of the Police Department which see the most shooting action." Fyfe (1988) in discussing New York City policing wrote "blacks were posted to high-risk assignments far more often than whites" and that an age/shooting risk relationship that other researchers found might "be an artifact of age-related variations in assignment and in exposure to potential shooting situations." Paoline and Terrill (2007) observe that "it is quite possible that other factors, such as the extent to which college-educated officers versus non-college-educated officers encounter resistant suspects, may account for why education appears to matter." McElvain and Kposowa (2008) write that "based on an officer's rank, time on the job, age, and gender, he or she may have been less active, assigned to areas with lower crime rates, or working in a position that did not have frequent contact with citizens." Some of these researchers make efforts to adjust for exposure to situations that would put officers at risk of being involved in police shootings, but frequently conclude that they cannot be sure that they have adequately adjusted for exposure.

Beyond the specific bounds of confounding, recent scholarship has also revisited the decision to focus primarily on officer and other situational explanations for police-involved shootings. Scholars have long known that training, deployment, and other organizational factors can have an enormous impact on the probability of a police-involved shooting (Goldstein 1967; Remington 1965). For example, despite often minimal change in the composition of many police forces and dramatic increases in the number of officer/citizen interactions, the absolute numbers of officer involved shootings have dropped dramatically from the 1970s to today due in part to changes in training and policy on approved use of force (Sherman et al. 1986). Likewise, similarly sized police departments with similar crime problems and similar officers can frequently be observed to have rates of officer-involved shooting per citizen interaction that are orders of magnitude different (Nowacki 2015; Morrison 2006). Both examples illustrate the value in contextualizing any individual officer's shoot/no shoot decision within the larger context formed by departmental use-of-force policy, deployment patterns, as well as citizen-police relations. However, theoretical models for making sense of these diverse organizational practices have only recently been fully articulated.

Sherman (2018), borrowing from the work of organizational failure scholars such as Charles Perrow and James Reason, argued that empirical insights like these suggest the value of reconceptualizing individual officer involved shootings in much the same way that other public sector or regulatory failures, such as nuclear disasters, medical errors, or wrongful convictions, have been recast as examples of error occurring within and potentially caused by larger factors embedded in complex social-technical systems (Perrow 1984; Reason 1990). Unlike accounts of problematic organizational outcomes (e.g., a fatal officer-involved shooting) that often focus specifically on the decisions made by a line officer (i.e., was the shooting consistent with departmental policy) (Donner et al. 2017), organizational studies examine contributing factors that explain how the underlying circumstances arose that generated the civilian/police interaction ending in fatal use of force (Klinger 2005).

This renewed focus on system-level factors, alongside traditional concerns of individual officer-level factors, creates the need for empirical tests to assess the relative explanatory power of each of these frameworks. This can be accomplished by inter-jurisdictional comparisons or intra-jurisdictional comparisons, assuming that adequate comparisons free from confounding policy, personnel, or other factors can be identified. While the use of panel datasets and synthetic control methods have simplified the process of making these types of comparisons, confidence in the resulting estimates depends largely on the credibility of the shrinkage-based reweighting of contributing units. As an alternative, different officers operating within the same jurisdictions, on the same beats or patrols, and even responding to the same incidents could be used to form a more transparent counterfactual for isolating the role of officer features. If officer features can readily predict differences between co-responding officers in their likelihood of firearm use, then the historical focus on officer-level factors will be reaffirmed. However, if officer features struggle to distinguish co-responding officers present and participating in shooting incidents, then the renewed attention on organizational factors will be given empirical support.

Isolating officer features from environmental, organizational, and other situational features requires specialized study designs. Ridgeway (2016) gathered data on police shootings in New York City, including information on shooting and non-shooting officers who were at the shooting scene. Using a case–control design and a conditional logistic regression model, he matched officers on the same shooting scene eliminating the need to account for features of the shooting scene environment and removing potential confounding for time, place, and context of the incident. He found an increased risk of being a shooter for black officers, officers who joined the police at a younger age, and officers who rapidly accumulate negative marks in their files. This study most closely resembles the design in the current study. However, a primary complication to the broader application of the Ridgeway (2016) approach is that few departments have readily accessible data on non-shooting officers at the scenes of shootings. The current study remedies this by examining the number of rounds fired, conditional on firing at least one round. Departments nearly universally require documentation of any discharge of a firearm, whether it strikes someone or not, providing a more reliable, comprehensive source of data on police shootings.

Data

In 2015, the Major Cities Chiefs Association (MCCA)[1] and the Police Foundation established a partnership in an effort to help agencies understand descriptive information about the officer-involved shooting incidents occurring within their agencies, as well as whether certain factors lead to differing outcomes of these incidents, such as injury or death to the subject(s) or officer(s) involved. Participation is voluntary, but highly encouraged through MCCA leadership as well as executives in the MCCA. A consortium of 55 MCCA agencies from the United States and Canada agreed to collect and report standardized information on officer-involved shootings to the Police Foundation. The data were collected from 2010 through 2018, though incidents are predominantly from 2015 to 2018. The agencies varied in the number of incidents reported, ranging from one incident in one agency to over four hundred in another. Officer-involved shootings included any incident when a police officer fired a weapon at another individual—excluding training accidents or accidental discharges at other officers—and included both fatal and non-fatal shootings.

Among the officer features of interest are age at recruitment, years of experience, and age at the time of the shooting. Since any two of these determine the third, we include only age at recruitment and years of experience in the model. These two are the least correlated pair. If age at the time of the shooting was important, then coefficients for both age at recruitment and years of experience would be large and significant.

The data also include the officer's race, sex, rank, assignment, and firearm type. We have data on prior involvement in officer-involved shootings and prior complaints of excessive force. For officers with a record of prior OIS or prior force complaint, we know those occurred. However, we cannot be sure that the absence of a record in the data for either of these prior events means that they never happened. Therefore, coefficients for these variables will be biased toward finding null effects.

Importantly, we do not include and do not recommend the inclusion of officer features concerning distance and positioning as these features may be on the causal pathway between officer features and shooting risk. For example, if inexperienced officers have a habit of getting too close to subjects and this results in an increased risk of shooting, then we want to attribute the risk to experience not to distance. A more experienced officer who could better manage risk would maintain distance and cover and could potentially avoid shooting. Including distance or positioning in the analysis would mask the effect of experience. Access to cover and positioning to observe a weapon should sometimes fall to the junior officer and sometimes to the senior officer, unless experience influences their ability to recognize cover or anticipate potential threats. We recognize that there are many steps leading to officer-involved shootings, but we do not want to adjust for those steps that are the result of an officer's experience, special assignment status, firearm type, or other features.

[1] The Major Cities Chiefs Association (MCCA) is a professional association of the sixty-nine largest law enforcement agencies in the United States, and nine largest in Canada. It provides a forum for executives to share ideas and experiences, as well as influence national public policy on law enforcement matters and encourage research.

Table 1 Number of police departments participating in the Police Foundation/MCCA data collection, number of incidents, officers, and rounds fired by year

Year	Number of departments		Number of incidents		Number of officers		Rounds
	All	Reporting number of rounds	All	Reporting number of rounds	All	Reporting number of rounds	
2010	1	0	43	0	48	0	0
2011	1	0	38	0	52	0	0
2012	2	1	58	9	99	14	72
2013	3	2	65	17	110	33	302
2014	19	17	105	63	182	91	372
2015	50	43	383	286	612	428	1998
2016	49	41	479	296	777	481	2463
2017	40	36	288	218	440	319	1567
2018	24	18	135	89	247	144	743
NA	5	1	6	1	7	1	1
Total unique	55	50	1600	979	2574	1511	7518

This dataset offers numerous advantages. First, while we would not claim representativeness, the data do offer a broader cross-section of communities and departments than is typical of use-of-force research. Second, the statistical model described in the next section requires data on shootings involving multiple officers, which are not a common occurrence. Accumulating data from numerous departments is essential to provide a sufficient number of multi-officer shootings to obtain estimates with a reasonable level of precision. Third, the data collection standardized the measurement and reporting of officer features so that merging data from numerous departments provides meaningful results.

The full dataset describes the actions of 2574 officers involved in 1600 shootings. However, several agencies do not report the number of rounds fired, a critical feature to this analysis. Five agencies did not report the number of rounds fired for any of their shootings and another four agencies do not report rounds fired for almost all of their shootings. These nine agencies missing rounds fired for all or most of their incidents represent 88% of the incidents missing the number of rounds. While, technically 55 agencies participated in the data collection, 46 agencies participated in a sufficiently meaningful way for the purposes of this study. Table 1 summarizes the number of departments, incidents, officers, and rounds fired by year. We include counts of all departments, incidents, and officers as well as just those departments, incidents, and officers that had the number of rounds fired recorded.

Dropping those shooting incidents for which we do not have the number of rounds fired results in 979 shootings, involving 1511 officers. Given the distribution of counts shown in Table 1, our study largely will be representative of the shootings of the 46 agencies that substantially reported the number of rounds fired, particularly for the years 2015–2017 when participation was at its highest. The generalizability of our findings will depend on the extent to which the *relationship* between officer features and shooting risk varies across time and departments.

As discussed in more detail in the next section, single officer shootings provide no information for our parameters of interest. That is, we do not drop them, but rather they provide no information. Of the 979 shootings, 32% of them involved multiple officers.[2] Our final dataset included 317 shooting incidents involving 849 officers. These officers fired a total of 5026 rounds, ranging between 1 and 31 rounds per officer in an incident and averaging 5.9 rounds per officer.

Methods

We model the relationship between an officer's features and the rate of shooting with a Poisson regression model. For any officer (not just those involved in shootings) with features $\mathbf{x}$ in any environment (not just those environments involving shootings) with features $\mathbf{z}$, the number of rounds, R, that the officer discharges is distributed Poisson,

$$P(R = r) = \frac{\lambda(\mathbf{x}, \mathbf{z})^r \exp(-\lambda(\mathbf{x}, \mathbf{z}))}{r!} \text{ where} \tag{1}$$

$$\log \lambda(\mathbf{x}, \mathbf{z}) = h(\mathbf{z}) + \beta' \mathbf{x} \tag{2}$$

Equation (1) is the standard Poisson distribution with mean $\lambda(\mathbf{x}, \mathbf{z})$. Equation (2) is the canonical Poisson regression specification, modeling the log of the rate as a linear combination of officer features, but with an intercept term that varies with the features of the officer's environment, $h(\mathbf{z})$. Since almost all environments have minimal risk of an officer-involved shooting, $h(\mathbf{z})$ is a large negative number for almost all values of $\mathbf{z}$. However, certain combinations of environmental features can create a high risk of an officer-involved shooting, such as an environment with an armed offender. While the structure of $h(\mathbf{z})$ is of great interest for theory and policy, we are focused on β, how officer features influence the shooting rate. If we can estimate β then we can tease apart whether officer features explain the differences in rates.

$\mathbf{z}$ is correlated with both $\mathbf{x}$ and shooting risk. This is the source of confounding that concerned all of the previously discussed scholars. Failure to handle $h(\mathbf{z})$ correctly, such as failing to include import features and their interactions, can introduce bias in $\hat{\beta}$. When faced with a statistical problem involving a nuisance parameter, the entire function $h(\mathbf{z})$ in our case, conditional likelihood offers a potential solution (Kalbfleisch and Sprott 1973; Liseo 2015).

Conditional Likelihood

The traditional likelihood approach would sample police officers at random moments in time, record their features $\mathbf{x}$ at that time, exhaustively document the officer's environment $\mathbf{z}$, and tally how many rounds they fired at that instance. Such an approach faces numerous challenges, specifically adequately documenting the environmental factors that might affect shooting risk and the extremely unlikely chance that a sampled police officer at the sampled time actually would fire their weapon.

[2] White and Klinger (2012) found that 25% of Philadelphia police shootings involved multiple shooting officers, Klinger (2004) found that 45% of shootings across 19 departments involved multiple shooters, and Klinger et al. (2015) reported that 22% of shooting in St. Louis involved multiple shooters. In our data, we find the percentage of shootings involving multiple officers to be in the same range.

Generally, if we have a sufficient statistic for a nuisance parameter, then basing inference conditional on the observed value of that sufficient statistic results in a *conditional* likelihood function that does not depend on the nuisance parameter (Kalbfleisch and Sprott 1973). We show that conditioning on the total number of rounds fired in a shooting incident removes the conditional likelihood's dependence on $h(\mathbf{z})$. Indeed, knowing that an incident involved few or many rounds provides a signal for $h(\mathbf{z})$, the baseline rate for that shooting's environmental features.

This leads to a more efficient approach: sample only shooting incidents, record features of officers present at the shooting, tally the number of rounds each officer fired, and adopt a conditional likelihood approach. The conditional likelihood still provides a consistent estimate for β in (2) even if we only analyze officers involved in shootings.

Let $r_1, r_2, \ldots, r_n$ be the number of rounds that each of n officers in a particular incident discharged. The conditional likelihood approach assesses the probability, conditional on the total number of rounds fired equaling $r_1 + r_2 + \ldots + r_n$, that the first officer shot r_1 rounds, the second officer shot r_2 rounds, and so on. That is, conditional on the observed number of rounds fired, what is the probability that the rounds would be distributed among the shooting officers in the configuration actually observed? The two primary benefits of the conditional likelihood approach is that (1) the analyst only needs to collect data on those involved in shootings and (2) the analyst never needs to use or collect data on $\mathbf{z}$, the environmental features, in order to obtain consistent estimates of β.

We first walk through an example for one shooting from our dataset involving two officers, their data shown in Table 2. We selected this particular shooting for demonstration because it involves few rounds and two officers who differed on only one feature. The two officers had the same race, sex, years of experience, assignment, and caliber of weapon. However, the first officer was 24 when he joined the police and the second officer was 25. Since they had the same number of years of experience, this also means that the first officer was one year younger at the time of the shooting. The first officer shot 3 rounds and the second officer shot 4. These two officers were on the same shooting scene and resemble each other in all ways except age (at time of recruitment and time of shooting), but one officer let off one additional round.

Derivation of the Truncated Poisson Conditional Likelihood

The conditional likelihood function is the product of conditional likelihood terms, one for each shooting. The contribution to the conditional likelihood for the one shooting described in Table 2 is the conditional probability that the first officer shot 3 rounds and the second officer shot 4 rounds given that the combined total of rounds fired was 7, the officer features, $\mathbf{x}_1$ and $\mathbf{x}_2$, and the shared environment features, $\mathbf{z}$.

$$P\left(R_1 = 3, R_2 = 4 | R_1 + R_2 = 7, R_1 > 0, R_2 > 0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}\right) \tag{3}$$

Note that, since our data only record information on those officers who fired at least one round at the scene, we also must condition on $R_1 > 0$ and $R_2 > 0$. Applying Bayes Theorem to (3) yields

$$\begin{aligned} &\frac{1}{K} P\left(R_1 + R_2 = 7 | R_1 = 3, R_2 = 4, R_1 > 0, R_2 > 0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}\right) \\ &\quad \times P(R_1 = 3, R_2 = 4 | R_1 > 0, R_2 > 0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) \end{aligned} \tag{4}$$

where K is a normalizing constant that we write out later in (5).

Table 2 Records for an example shooting involving two officers who differ only in age at recruitment

OIS ID	Rounds	Recruit age	Years on job	Sex	Race	Prior OIS #	Force complaints	Rank	Assignment	Gun type	Caliber
2	3	24	4	Male	White	0	0	Officer	Special unit	Pistol	9 mm
2	4	25	4	Male	White	0	0	Officer	Special unit	Pistol	9 mm

The first term in (4) equals 1 since conditioning on $R_1 = 3$ and $R_2 = 4$ forces $R_1 + R_2 = 7$. For the second term in (4), we assume that R_1 and R_2 are conditionally independent in order to factor the expression. In truth, it is unclear whether the numbers of rounds these officers fired are independent. Although we condition on the officers' features and the environment's features, if one officer fires a high volume first then the other officers might determine that they do not need to discharge more than a single round (anti-contagion). On the other hand, if discharging is contagious then the more one officer shoots the more the other officers might shoot, though White and Klinger (2012) present evidence against a contagion effect. Fortunately, even if this independence does not hold, the direction of the coefficient estimates will still be correct. The traditional asymptotic standard errors will not be correct, but we resolve this using a permutation test described later.

Equation (4) simplifies to

$$\frac{P\left(R_1 = 3|R_1 > 0, \mathbf{x}_1, \mathbf{z}\right)P\left(R_2 = 4|R_2 > 0, \mathbf{x}_2, \mathbf{z}\right)}{\sum_{\rho_1+\rho_2=7,\rho_1>0,\rho_2>0} P\left(R_1 = \rho_1|R_1 > 0, \mathbf{x}_1, \mathbf{z}\right)P\left(R_2 = \rho_2|R_2 > 0, \mathbf{x}_2, \mathbf{z}\right)} \tag{5}$$

now with K written out in the denominator. The denominator requires a sum over all pairs of positive integers, ρ_1 and ρ_2, that sum to 7, that is (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1). The denominator is considering all the ways seven rounds could be distributed across two shooters while the numerator considers only the configuration actually observed, (3, 4).

Inserting the Poisson model from (1) and (2) into (5) yields

$$\frac{\frac{\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_1\right)^3 e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_1\right)}}{3!\left(1-e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_1\right)}\right)} \frac{\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_2\right)^4 e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_2\right)}}{4!\left(1-e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_2\right)}\right)}}{\sum_{\rho_1+\rho_2=7,\rho_1>0,\rho_2>0} \frac{\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_1\right)^{\rho_1} e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_1\right)}}{\rho_1!\left(1-e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_1\right)}\right)} \frac{\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_2\right)^{\rho_2} e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_2\right)}}{\rho_2!\left(1-e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_2\right)}\right)}} \tag{6}$$

The two terms in the numerator are truncated Poisson probability distribution functions. The $1 - e^{-\exp\left(h(\mathbf{z})+\beta'\mathbf{x}\right)}$ terms in the denominator, which is $P(R>0)$ for a Poisson, are there specifically to address the problem of only observing the officers who discharged at least one round.[3] Equation (6) simplifies as

$$\frac{\frac{\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_1\right)^3}{3!} \frac{\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_2\right)^4}{4!}}{\sum_{\rho_1+\rho_2=7,\rho_1>0,\rho_2>0} \frac{\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_1\right)^{\rho_1}}{\rho_1!} \frac{\exp\left(h(\mathbf{z})+\beta'\mathbf{x}_2\right)^{\rho_2}}{\rho_2!}} \tag{7}$$

3 We account for the fact that we do not have data on non-shooters at the scene of these incidents. If we had simply assumed that the observed shot count was from a Poisson distribution, then computation would be trivial but would produce biased estimates. Instead, this methodology assumes the observed shot counts come from a *truncated* Poisson distribution, $P(Y = y|Y > 0) = \frac{e^{-\theta}\theta^y}{y!(1-e^{-\theta})}$. This greatly complicates the computational effort needed to obtain estimates, but allows us to obtain consistent and asymptotically unbiased estimates. The absence of data on non-shooters decreases precision, but does not introduce bias when using a truncated Poisson.

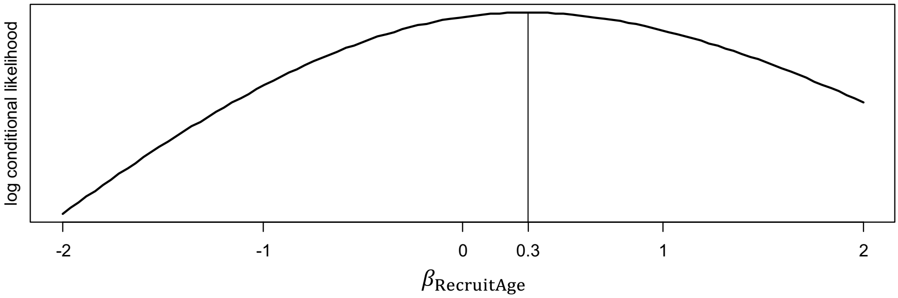

Fig. 1 Contribution of the example shooting to the log conditional likelihood function

$$\frac{1}{\sum_{\rho_1+\rho_2=7,\rho_1>0,\rho_2>0} \frac{3!4!}{\rho_1!\rho_2!} \frac{\exp(h(\mathbf{z}))^{\rho_1+\rho_2}}{\exp(h(\mathbf{z}))^7} \frac{\exp(\beta'\mathbf{x}_1)^{\rho_1}}{\exp(\beta'\mathbf{x}_1)^3} \frac{\exp(\beta'\mathbf{x}_2)^{\rho_2}}{\exp(\beta'\mathbf{x}_2)^4}} \tag{8}$$

$$= \frac{1}{\sum_{\rho_1+\rho_2=7,\rho_1>0,\rho_2>0} \frac{3!4!}{\rho_1!\rho_2!} \exp\left(\left(\rho_1 - 3\right)\beta'\mathbf{x}_1\right) \exp\left(\left(\rho_2 - 4\right)\beta'\mathbf{x}_2\right)} \tag{9}$$

Note that in (8), $\rho_1 + \rho_2$ always equals 7, so that $h(\mathbf{z})$ cancels out of the conditional likelihood entirely. This shooting's contribution to the conditional likelihood shown in (9) only depends on the features of the officers involved. *This* is what makes the conditional likelihood approach so valuable for analyzing the role of officer features in police use of force. There is no need to measure the environmental features at all since $\mathbf{z}$ has no information through the conditional likelihood function that is useful for estimating β.

We selected the shooting shown in Table 2 as an example because it involved only two officers that differed on one feature, age at recruitment, by only one year. In this special case, (9) further simplifies to

$$\frac{1}{\sum_{\rho_2=1}^{6} \frac{144}{(7-\rho_2)!\rho_2!} \exp\left(\left(\rho_2 - 4\right)\beta_{\text{RecruitAge}}\right)} \tag{10}$$

For any feature that all officers on the scene share, those coefficients drop out of the conditional likelihood. No variation in x gives no information for the associated β. Since these officers only differed on age when recruited, the associated coefficient is the only one that remains in the conditional likelihood. This shooting provides no information on any of the coefficients except $\beta_{\text{RecruitAge}}$.

Figure 1 plots the log of (10) for a range of values for $\beta_{\text{RecruitAge}}$. Consistent with the fact that the shooter who discharged one additional round was also one year older at the time of recruitment, this shooting's likelihood component is largest for positive values of $\beta_{\text{RecruitAge}}$. The value of $\beta_{\text{RecruitAge}}$ that maximizes (10) is 0.3, suggesting that one additional year of age at recruitment increases the shooting rate $\exp(0.3) = 1.3$ times. Indeed, Officer 2 shot four rounds, 1.3 times the number of rounds Officer 1 discharged.

Figure 1 and (10) only relate to one parameter and one shooting. The complete conditional likelihood incorporating all of the shootings, which provides information on all of the parameters, equals the product of terms like (9) where the number of officers involved and the number of rounds fired vary across shootings. The complete log conditional likelihood involves a sum over each of the S shootings

$$\log L(\beta) = -\sum_{s=1}^{S} \log\left(\sum_{\sum \rho_i = \sum r_{si}, \rho_i > 0} \prod_{i=1}^{n_s} \frac{r_{si}!}{\rho_i!} \exp\left(\left(\rho_i - r_{si}\right)\beta' \mathbf{x}_{si}\right)\right) \tag{11}$$

where S is the number of shooting incidents, r_{si} is the number of rounds that officer i involved in shooting s discharged, $\mathbf{x}_{si}$ are the features of officer i in shooting s, and n_s is the number of officers who fired at least one round in shooting s.

As mentioned in the Data section, shootings involving a single officer provide no information for β. To be clear, it is not the case that we "drop" single officer shootings. In (11), if shooting s has a single officer, then the inner sum will have a single term where $\rho_1 = r_{s1}$, which makes $\exp\left(\left(\rho_i - r_{si}\right)\beta' \mathbf{x}_{si}\right) = 1$. Single officer shootings contribute a constant to the likelihood and, therefore, provide no information for β. One can "include" single officer shootings in the analysis, but estimates of β will be unaffected.

While the inner sum in (11) is not formidable with two officers, more generally the number of terms in this sum is $\binom{\left(\sum r_i\right) - 1}{n_s - 1}$, where $\sum r_i$ is the total number of rounds. The number of terms grows quickly in the number of rounds, on the order of $\left(\sum r_i\right)^{n_s}$. The most complex shooting in the data involved 11 officers discharging 88 rounds producing a sum involving 4,000,751,045,226 terms. If we did not need to condition on $R > 0$ (that is, if we also had data on officers at the scene who shot 0 rounds) then the inner sum would be computable trivially as the expected value of a multinomial distribution. We developed a computationally efficient recursion algorithm to compute this sum that we derive in "Appendix A". However, to optimize the conditional likelihood numerically, we need to be able to evaluate (11) 500 to 700 times. Computing the conditional likelihood contribution for the *second* most complex shooting (8 officers shooting 58 rounds) takes 20 s and the most complex shooting, with 15,000 times more terms, would take nearly four days to evaluate once. Since the inner sum in (11) can be rewritten as the expected value of the inverse of the product of one plus a draw from a multinomial distribution, for these two complex shootings with the greatest number of terms in the sum we can compute a Monte Carlo estimate of their contributions to the likelihood (details in "Appendix A.2"). Even though (11) still involves over 4 trillion terms, with the code included in "Appendix A.3", we can evaluate it for our dataset on a standard modern computer in under 5 s.

Permutation Test for the Conditional Likelihood

In making the conditional independence assumption in (5), we risk that the standard errors will not be correct. To avoid leaning on asymptotic assumptions about the reference distribution of the model parameters, we use a permutation test (Good 1994). A permutation test randomly permutes the observed data in a manner that is consistent with the null hypothesis, computes new estimates based on the permuted data, and repeats this process numerous times. The collection of parameter estimates based on the permutated data provide a non-parametric, permutation-based reference distribution from which we can compute p-values. We adopt the same permutation test used in (Ridgeway 2016). The null hypothesis is that the officer features are unrelated to the number of rounds fired within shooting. Therefore, to generate each permuted dataset, we randomly shuffle the number of rounds among the officers within each shooting. Each shooting still will have the same values of

$\left(r_{s,1}, r_{s,2}, \ldots, r_{s,n_s}\right)$, but they will be shuffled among the n_s officers involved in shooting s. In these permuted datasets, the rounds fired are independent of the officer features, but will retain any dependence from contagion or anti-contagion. The collection of estimates of β based on these permuted datasets yield a valid reference distribution, the distribution of $\hat{\beta}$ under the hypothesis of no relationship between officer features and number of rounds fired. After generating 1000 permuted datasets and the associated parameter estimates $\hat{\beta}_{(1)}, \hat{\beta}_{(2)}, \ldots, \hat{\beta}_{(1000)}$, we compute the permutation p value for the kth coefficient as

$$p_k = \frac{1}{1000} \sum_{i=1}^{1000} I\left(\hat{\beta}_{k,(i)} \leq \hat{\beta}_k\right) \tag{12}$$

the fraction of estimates from the permuted data that were less than or equal to the estimate computed on the original data. In our findings we report the two-tailed p-value as $\min\left(2p_k, 2\left(1 - p_k\right)\right)$. We can back out a corrected standard error for $\hat{\beta}_k$ as $\left|\hat{\beta}_k / \Phi^{-1}\left(p_k/2\right)\right|$, where $\Phi^{-1}()$ is the inverse of the normal distribution function. This is the standard error that would have produced the permutation test p-value using a normal approximation. We compute 95% confidence intervals based on a normal distribution with this permutation test corrected standard error.

Results

Table 3 describes the officer features available in the dataset. The second column shows either the average (for age and experience measures) or the percentage of officers in the sample with that feature.

As discussed with the example shooting in the previous section, not all shootings have information on all features. Because age at recruitment was the only officer feature with variation in the example shooting, that is the only officer feature for which that shooting had information. The third column in Table 3 shows the number of shootings in the dataset that provide information for the model coefficient associated with that officer feature, equal to the number of shootings within which the feature's variance is non-zero. For features not included in the model, Table 3 does not report the number of shootings with information (age is a linear combination of other features and white, zero prior OIS, and patrol/police officer are reference categories). The number of shootings with information varies greatly. Rare features, such as officers in less common roles, naturally occur less frequently among the shootings. Other features, like officers with special assignments, have relatively few shootings with information because officers with those features tend to group together. Officers with special assignments were involved in 115 shooting incidents, but for 75 of those incidents *all* of the officers had a special assignment meaning that those officers' round count could not be contrasted with officers not on special assignment.

Table 4 reports the rate ratios, $\exp\left(\beta_k\right)$, indicating the association between a unit increase in the officer feature and the relative increase in the shooting rate. The permutation confidence intervals and p-values correctly account for dependence and do not find differences.

The permutation test p-values suggest that none of the officer features appear to influence the number of rounds fired. Age at recruitment and years of experience had rate ratios nearly equal to 1, indicating no relationship with shooting risk. Neither race nor sex appeared to be strongly associated with shooting risk.

Table 3 Summary of officer features and incident counts

Officer features	Average or percent	Number of shootings with information
Age at recruitment	26.1	272
Years of experience	9.6	277
Age at time of shooting	35.7	–
Female	4.9%	36
Race		
White	69.0%	–
Black	10.0%	49
Hispanic	14.7%	73
Other	5.8%	35
Prior OIS		
0	84.5%	–
1 or more	15.6%	86
2 or more	4.5%	30
Prior force complaint	13.2%	40
Role		
Patrol/police officer	79.2%	–
Detective	8.0%	21
Sergeant or more senior	10.5%	67
Other	2.4%	9
Special assignment	31.8%	40
Long gun	22.1%	54
Incident counts	**Average**	**Range**
Number of rounds per officer	5.9	1–31
Number of rounds per incident	15.9	2–88
Number of officers per incident	2.7	2–11
Number of shootings	317	
Number of officers	849	
Number of rounds	5026	

The estimated rate ratios for prior OISs and prior excessive force complaints were in the expected direction, but these too did not differ significantly from 1. Having a prior force complaint tends to increase the number of rounds fired by 25%, but the permutation test suggests it could plausibly be between 8% fewer rounds or 69% more rounds.

These results provide no evidence that any officer feature strongly influences the number of rounds fired. Few of the officer features come close to statistical significance and most features of primary interest have risk ratios that are close to 1.0.

When presented with null findings, the logical conclusions are that either there is no effect or that the magnitude of the effect is smaller than the study is powered to detect.

Table 4 Estimates of risk ratios associated with each officer feature

Officer features	Rate ratio	Permutation 95% CI	Permutation p-value
Age at recruitment	1.01	(0.99, 1.02)	0.31
Years of experience	1.00	(0.98, 1.01)	0.58
Female	0.86	(0.64, 1.14)	0.27
Race (relative to white)			
Black	1.05	(0.86, 1.28)	0.64
Hispanic	1.09	(0.89, 1.32)	0.39
Other	0.76	(0.57, 1.01)	0.05
Prior OIS (relative to 0)			
1 or more	1.02	(0.84, 1.24)	0.85
2 or more	1.23	(0.88, 1.72)	0.21
Prior force complaint	1.25	(0.92, 1.69)	0.14
Role			
Detective	1.09	(0.78, 1.52)	0.61
Sergeant or more senior	1.03	(0.87, 1.22)	0.75
Other	0.66	(0.32, 1.37)	0.26
Special assignment	1.28	(0.97, 1.68)	0.07
Long gun (relative to pistol)	1.01	(0.78, 1.30)	0.97

Power of the Conditional Likelihood Test

Since we found no significant relationship between officer features and the risk of shooting, we studied whether this might be due to a lack of power for some officer features. For some features, like age at recruitment and years of experience, the estimated relative risk is almost identical to 1.0 *and* we have many shootings supplying information to those estimates, at least 272. For other important, policy-relevant features, like having a prior use-of-force complaint, there is an elevated relative risk, but with confidence intervals too wide to conclude the existence of a relationship.

We use prior force complaints to demonstrate the power demands of the conditional likelihood model. We made no changes to the existing set of officer features and their participation in each of the shooting incidents. We set all coefficients equal to 0 except for the coefficient for prior force complaints, which we set to one of six values ranging from no effect to prior force complaints increasing the shooting rate by 50%. We then simulated the number of rounds fired consistent with the truncated Poisson model shown in (1) and (2). We generated 3000 datasets for each of the six selected values of the prior force complaints coefficient. For each dataset we conducted a permutation test and recorded whether we would reject the hypothesis of no effect.

Table 5 shows the power of the conditional likelihood test, the probability of rejecting the hypothesis of no effect, for each of the six values of $\beta_{\text{ForceComplaints}}$. As expected, when we simulate a dataset with no relationship between force complaints and shooting rate, we

Table 5 Conditional likelihood model power, the probability of finding a significant effect of prior force complaints for various choices of true values for $\beta_{\text{ForceComplaints}}$

Simulated value of $\exp(\beta_{\text{ForceComplaints}})$	Power	
	Study sample size	Double study sample size
1.0	0.05	0.05
1.1	0.19	0.33
1.2	0.53	0.84
1.3	0.82	0.99
1.4	0.96	1.00
1.5	0.99	1.00

reject the no effect hypothesis 5% of the time demonstrating that the size of the test is correct. As the true value of $\beta_{\text{ForceComplaints}}$ increases, the power of the test increases. Having a prior force complaint needs to elevate the shooting rate by at least 30% for the conditional likelihood to be sufficiently powered to detect an effect with this study's sample size. The third column recomputes the power on a dataset twice the size of our dataset, showing that doubling the sample size would allow us to detect a prior force complaint effect that increases shooting rate by 20% or more.

Table 5 shows detecting the effect of officer features with the conditional likelihood approach requires large effects and large datasets. Of the 849 officers in the analysis, 112 officers had a prior excessive force complaint. In typical analyses, we would have sufficient power to compare the 112 officers with prior complaints to the 737 officers without prior complaints. Matching officers eliminates numerous sources of confounding that would introduce bias, but pays the price in terms of increased variance. Only shootings that have variation on an officer feature provide information for its coefficient. As shown in Table 3, of the 317 shootings, only 40 of them have some officers with force complaints and some without force complaints. No other shooting incidents provide information.

The estimated $\exp(\hat{\beta}_{\text{ForceComplaints}}) = 1.25$ appears to be just below the boundary of the size of effects that our study is powered to detect.

Discussion

Policymakers and scholars have long pointed to the importance of both officer-level and system-level factors in explaining why police-involved shootings happen. To date, these two analytical frameworks have co-existed with little clarity on the general explanatory power of each. The present investigation demonstrated a novel estimator that allows for the estimation of the contribution of individual officer features to the number of rounds fired. Because a greater number of rounds fired correlates with the lethality of a shooting (Braga and Cook 2018), in essence this analysis goes beyond the question of which officers decide to shoot and tries to assess which officer features are linked with the risk of shooting lethality. As Zimring (2017) argues that, in addition to the decision to *start* shooting, the decision to *stop* shooting is also a critical determinant of whether a police shooting is fatal. Our approach enables the identification of specific officer features that predict the number of rounds fired and it simultaneously allows for an assessment of whether officer-level factors or system-level factors dominate.

In practice, we find that no individual officer characteristic is significantly related to the number of rounds fired in an OIS incident. Our analysis suggests that the expected number of rounds officers discharged did not differ significantly according to officers' demographics and law enforcement experience. These findings correspond with police scholars' supposition that duty assignment or other factors commonly associated with system-level explanations may be more responsible for explaining differences in police use of force than individual officer characteristics. These findings on the number of rounds fired contrast with the conditional likelihood analysis of NYPD officers that found black officers, officers recruited at a young age, and officers who quickly accumulate negative marks are more likely to decide to shoot (Ridgeway 2016). Our analysis indicates that individual officer features do not seem to be driving the decision to *stop* shooting even if prior research indicates that they influence the decision to *start* shooting.

Fortunately, OIS incidents are rare events, but this poses serious challenges for analyzing, understanding, and minimizing death and injury that result from police use of deadly force. We believe that the conditional likelihood approach is a valuable tool for learning about officer features, but carries with it demand for a substantial amount of data.

Data collection needs to improve in three ways if we are to improve our understanding of officer features and their relationship to shooting risk. First, as Klinger et al. (2015) note, more and better sources of national OIS data must be developed. The rarity of OIS incidents means that even when agencies agree to participate in data collection, the process of amassing sufficient cases to allow for a robust analysis takes time. A small fraction of the shootings collected, 4% (40/979), have information for the relationship between prior force complaints and shooting risk. Conditional likelihood requires multiple officers on the same scene with variation in their features, meaning larger data collections are essential. The MCCA OIS database represents one of the largest detailed data collection efforts surrounding officer-involved shootings to date, an excellent start on addressing this need. However, we cannot necessarily generalize the results we found outside of large police departments. It is possible that the lack of relationships observed in these large departments would also be observed in smaller agencies, but there is no sufficiently large data collection involving smaller departments containing information on multi-officer shootings.

Second, we need uniform reporting standards. Until such standards are available, research in this area is likely to be based on limited samples of incidents reported by a relatively small number of agencies. The MCCA and Police Foundation collaboration represents an important advance in data collection on OIS incidents, including a standardized data collection for many agencies. Further advances will depend on expanding the number of agencies and increasing the participation of smaller agencies, typically overlooked in use-of-force analyses.

Third, police need to document the presence of non-shooting officers on the scenes of shooting incidents. These officers and their features provide a substantial amount of information on officer risks. Ridgeway (2016) was able to identify the effects of officer features using data from a single police department containing 106 shootings involving 150 shooting officers and 141 non-shooting officers. While it lacked the diversity of departments that this study included, by including the non-shooting officers many more shootings provide information for the coefficients of interest. The consent decree for the Chicago Police Department represents an excellent model in this regard, requiring the documentation of all "CPD units identified in the incident report as being on the scene of the use of force incident" (State of Illinois vs. City of Chicago, Consent Decree 2018, ¶571f).

Ridgeway (2016) showed that the conditional likelihood approach can address the first of Zimring's (2017) four primary decisions, whether police will shoot. In this paper, we have demonstrated that the conditional likelihood framework can also address Zimring's (2017) second decision point, when police stop shooting and how many rounds they fire. Conditional likelihood offers a rigorous approach to avoid environmental, organizational, and situational confounding that previously has been a challenge.

The methodological advancements described here aid in untangling the relationship between officer features, system-level factors, and their effect on shooting risk. This helps us scientifically test whether individual-level or system-level features drive police shootings. Since our results indicate that individual-level officer features explain little of the variation in the number of rounds fired, it appears that system-level and organization-level frameworks for understanding police shootings merit renewed attention.

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Appendix A: Efficient Computation of the Conditional Log-Likelihood

A.1 Derivation of Recursive Computation

Estimating the maximum likelihood estimator requires repeated evaluation of the log conditional likelihood function shown in (11). Let

$$Q_{si}(\rho) = \frac{1}{\rho!}\exp\left(\left(\rho - r_{si}\right)\beta'\mathbf{x}_{si}\right) \tag{13}$$

In (13) we drop the $r_{si}!$ that had appeared in (11) since these are constants that can factor out. Equation (10) demonstrates this most clearly as the 144 can factor out of the denominator.

Let

$$A_s(k, n_r) = \sum_{\sum_{i=k}^{n_s} \rho_i = n_r, \rho_i > 0} \prod_{i=1}^{n_s} Q_{si}(\rho_i) \tag{14}$$

where n_s is the number of officers involved in shooting s and $n_r = \sum_{i=1}^{n_s} r_{si}$ is the total number of rounds fired in shooting s.

So that the log conditional likelihood is

$$\log L(\beta) = -\sum_{s=1}^{S} \log(A_s(1, n_r)) \tag{15}$$

We gain efficiency by factoring out $Q_{s,1}(\rho_1)$ in (14) so that we can define $A_s(k, n_r)$ recursively.

$$A_s(1, n_r) = \sum_{\rho_1=1}^{n_r-n_s+1} \left[Q_{s,1}(\rho_1) \sum_{\sum_{i=2}^{n_s} \rho_i = n_r - \rho_1, \rho_i > 0} \prod_{i=k+1}^{n_s} Q_{si}(\rho_i) \right] \tag{16}$$

$$= \sum_{\rho_1=1}^{n_r-n_s+1} Q_{s,1}(\rho_1) A_s(2, n_r - \rho_1) \tag{17}$$

Essentially, this recursion works by setting ρ_1 to a specific value in the summation, then summing over all the other combinations of ρ_is that sum to $n_r - \rho_1$ and multiplying the result by $Q_{s,1}(\rho_1)$. Then the sum is over fewer officers. We can repeat the recursion for $A_s(2, n_r - \rho_1)$, setting specific values for ρ_2 and summing over the remaining ρ_is that sum to $n_r - \rho_1 - \rho_2$. More generally

$$A_s(k, n) = \sum_{\rho_k=1}^{n_r-n_s+k} Q_{s,k}(\rho_k) A_s(k+1, n - \rho_k) \tag{18}$$

$$A_s(n_s, n) = Q_{s,n_s}(\rho_{n_s}) \tag{19}$$

A.2 Monte Carlo Estimation of $A_s(1, n_r)$

For shootings that are complex, such as when $\binom{n_r - 1}{n_s - 1} > 10^8$, the recursive computation in "Appendix A.1" is slow. We can approximate a shooting's contribution to the log-likelihood using a Monte Carlo estimate of the expected value of a function of a draw from a multinomial distribution. From (13) and (14) we have

$$A_s(1,n_r) = \sum_{\sum_{i=1}^{n_s} \rho_i = n_r, \rho_i > 0} \prod_{i=1}^{n_s} \frac{1}{\rho_i!} \exp\left((\rho_i - r_{si})\beta' \mathbf{x}_{si}\right) \tag{20}$$

Rewrite the indices on the summation so that the ρ_i start at 0. In this way the ρ_i count how many rounds in excess of 1 the ith officer discharges.

$$A_s(1,n_r) = \sum_{\sum_{i=1}^{n_s} \rho_i = n_r - n_s, \rho_i \geq 0} \prod_{i=1}^{n_s} \frac{1}{(\rho_i + 1)!} \exp\left((\rho_i + 1 - r_{si})\beta' \mathbf{x}_{si}\right) \tag{21}$$

$$= \sum_{\sum_{i=1}^{n_s} \rho_i = n_r - n_s} \prod_{i=1}^{n_s} \frac{1}{\rho_i + 1} \frac{1}{\rho_i!} \left(\exp(\beta' \mathbf{x}_{si})\right)^{\rho_i} \left(\exp(\beta' \mathbf{x}_{si})\right)^{1 - r_{si}} \tag{22}$$

$$= \frac{1}{(n_r - n_s)!} \prod_{i=1}^{n_s} \left(\exp(\beta' \mathbf{x}_{si})\right)^{1 - r_{si}} \sum_{\sum_{i=1}^{n_s} \rho_i = n_r - n_s} (n_r - n_s)! \prod_{i=1}^{n_s} \frac{1}{\rho_i + 1} \frac{1}{\rho_i!} \left(\exp(\beta' \mathbf{x}_{si})\right)^{\rho_i} \tag{23}$$

The term inside the sum resembles the components of a multinomial distribution with extra $1/(\rho_i + 1)$ terms. We need an additional term to normalize the $\exp(\beta' \mathbf{x}_{si})$ so that they sum to 1.

$$\frac{1}{(n_r - n_s)!} \frac{\left(e^{\beta' \mathbf{x}_{s1}} + \cdots + e^{\beta' \mathbf{x}_{sn_s}}\right)^{n_r - n_s}}{\left(e^{\beta' \mathbf{x}_{s1}}\right)^{r_{s1} - 1} \cdots \left(e^{\beta' \mathbf{x}_{sn_s}}\right)^{r_{sn_s} - 1}} \sum_{\sum_{i=1}^{n_s} \rho_i = n_r - n_s} \frac{1}{\rho_1 + 1} \cdots \frac{1}{\rho_{n_s} + 1} \binom{n_r - n_s}{\rho_1 \cdots \rho_{n_s}} \left(\frac{e^{\beta' \mathbf{x}_{s1}}}{e^{\beta' \mathbf{x}_{s1}} + \cdots + e^{\beta' \mathbf{x}_{sn_s}}}\right)^{\rho_1} \cdots \left(\frac{e^{\beta' \mathbf{x}_{sn_s}}}{e^{\beta' \mathbf{x}_{s1}} + \cdots + e^{\beta' \mathbf{x}_{sn_s}}}\right)^{\rho_{n_s}} \tag{24}$$

Let $p_i = e^{\beta' \mathbf{x}_{si}} / \left(e^{\beta' \mathbf{x}_{s1}} + \cdots + e^{\beta' \mathbf{x}_{sn_s}}\right)$.

$$= \frac{1}{(n_r - n_s)!} \frac{1}{p_1^{r_{s1} - 1} \cdots p_{n_s}^{r_{sn_s} - 1}} \sum_{\sum_{i=1}^{n_s} \rho_i = n_r - n_s} \frac{1}{\rho_1 + 1} \cdots \frac{1}{\rho_{n_s} + 1} \binom{n_r - n_s}{\rho_1 \cdots \rho_{n_s}} p_1^{\rho_1} \cdots p_{n_s}^{\rho_{n_s}} \tag{25}$$

If $\boldsymbol{\rho} \sim \text{Multinomial}(n_r - n_s, \boldsymbol{p})$ then the sum in (25) is $E\left(\frac{1}{(\rho_1+1)\cdots(\rho_{n_s}+1)}\right)$, which can be estimated via Monte Carlo draws from the multinomial distribution.

A.3 Computation

Here we provide R/C++ code for computing the negative log conditional likelihood. The analyst can insert `negCLL()` into an optimizer (like `nlm()`) to obtain maximum likelihood estimates of β.

```
# optimized C++ recursive function for computing A_s(k,n)
#   returned value is on the likelihood scale (not log scale)
cppFunction("
double Asum(int iStart,                  // k, 0-based
            int nRounds,                 // n, number of rounds
            IntegerVector ivR,           // vector of observed rounds
            NumericVector nvLogRate,     // log shooting rate
            NumericVector nvFactInverse) // precomputed 1/n! table
{
  double dResult = 0.0;
  int x          = 0;

  if(iStart == ivR.size()-1) // last officer, terminal case
  {
    dResult = std::exp((nRounds-ivR [iStart]) *
                       nvLogRate[iStart]) *
              nvFactInverse[nRounds];
  }
  else
  {
    dResult = 0.0;
    for(x=1; x<=nRounds-(ivR.size()-iStart-1); x++)
    {
      dResult += std::exp((x-ivR[iStart]) *
                          nvLogRate[iStart]) *
                 nvFactInverse[x] *
                 Asum(iStart+1, nRounds-x, ivR, nvLogRate, nvFactInverse);
    }
  }
  return dResult;
}
")

# Monte Carlo estimate of Asum
#   uses E(prod(1/x_i)) where x ~ multinomial()
#   return value is on the log likelihood scale
mcAsum <- function(r, logRate)
{
   p <- exp(logRate)
   p <- p/sum(p) # normalize to probabilities
   if(all(!is.na(p)))
   {
      a <- rmultinom(100000, sum(r)-length(r), p)
      A <- -sum((r-1)*log(p)) - lfactorial(sum(r)-length(r))
      a <- apply(a, 2, function(x) 1/prod(x+1))
      result <- A + log(mean(a))
   } else
   {  # exclude any implausible logRate by returning Inf
      result <- Inf
   }
   return(result)
}

# negative conditional log likelihood
negCLL <- function(beta0, X, y, id)
{
   logRate <- as.vector(X %*% beta0)

   likShoot <- sapply(unique(id),
                      function(id0)
                      {
                         # collect number of rounds, estimated rates
                         r <- y[id==id0]
                         logRate0 <- logRate[id==id0]
                         nterms <- choose(sum(r)-1, length(r)-1)
                         if(nterms < 10^8)
                         {  # compute exact likelihood
                            Aresult <- Asum(0, sum(r), as.integer(r),
                                            logRate0,
                                            1/factorial(0:sum(r)))
                            Aresult <- log(Aresult) # put on log scale
                         } else
                         {  # use Monte Carlo estimate
                            Aresult <- mcAsum(r, logRate0)
                         }
                         return(Aresult)
                      })
   return(sum(likShoot))
}
```

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Affiliations

Greg Ridgeway[1,2] · **Breanne Cave**[3] · **Julie Grieco**[4] · **Charles Loeffler**[1]

[1] Department of Criminology, University of Pennsylvania, Philadelphia, PA, USA

[2] Department of Statistics, University of Pennsylvania, Philadelphia, PA, USA

[3] Government Accountability Office, Washington, DC, USA

[4] National Police Foundation, Arlington, VA, USA