DATA 8

Spring 2020

Lecture 24

Interpreting Confidence Intervals

Announcements

- Online tutoring sections sign ups start on Thursday at noon
  - Officially commence after Spring Break
- Midterm scores out - regrade requests due Tuesday at 11:59pm
- Grade reports are available on Gradescope
- Lab due date has been extended to Saturday at 11:59pm
- GSIs are available via Zoom during usual lab hours!

Weekly Goals

- Wednesday
  - Estimation
  - Variability of the Estimate
- **Today**
  - Bootstrap
  - Confidence intervals
- Friday
  - Describing a distribution
  - Center and spread

Estimation

Quantifying Uncertainty

- The estimate is usually not exactly right:

Estimate = Parameter + Error

- How accurate is the estimate, usually?
- How big is a typical error?
- When we have a census, we can do this by simulation

(Demo)

Where to Get Another Sample?

- We want to understand errors of our estimate
- Given the **population**, we could simulate
  - ...but we only have the **sample**!
- To get many values of the estimate, we needed many random samples
- Can’t go back and sample again from the population:
  - No time, no money
- Stuck?

The Bootstrap

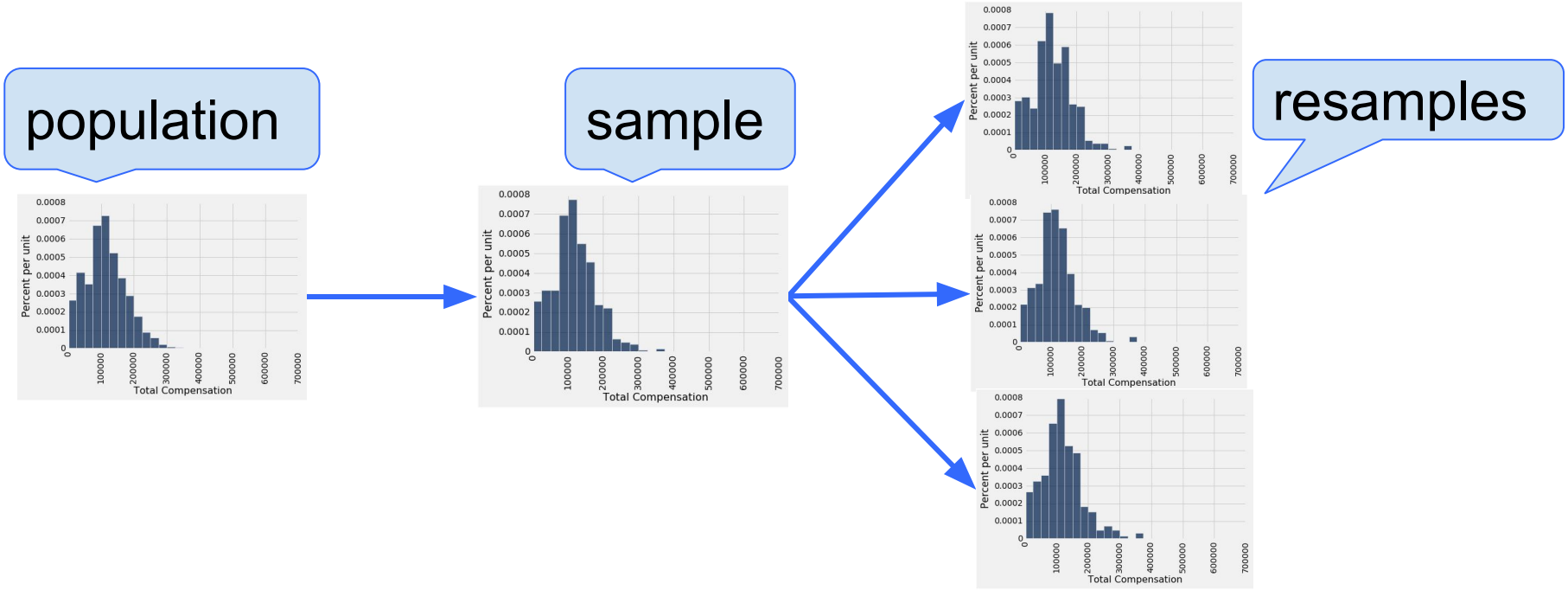
The Bootstrap

- A technique for simulating repeated random sampling

- All that we have is the original sample
  - … which is large and random
  - Therefore, it probably resembles the population

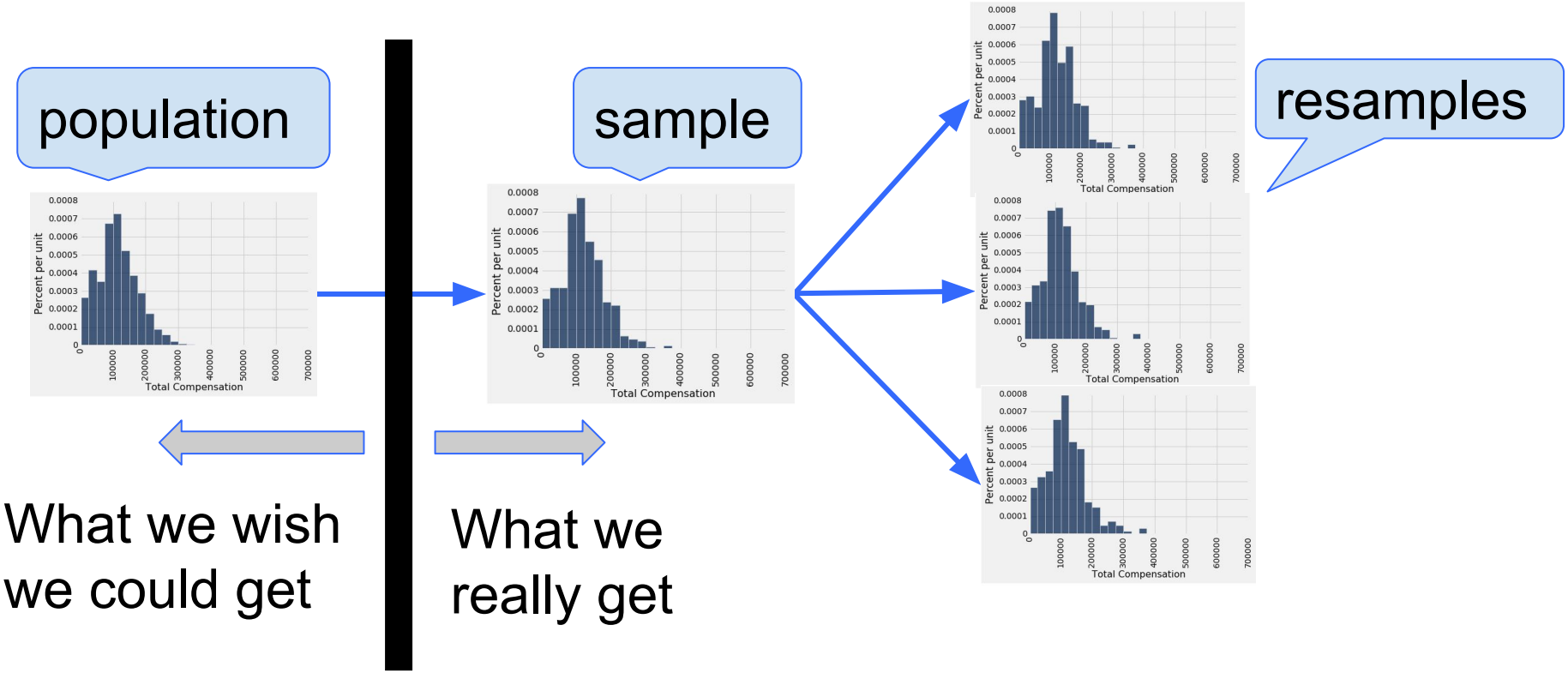
- So we sample at random from the original sample!

Why the Bootstrap Works



All of these look pretty similar, most likely.

Why We Need the Bootstrap



Real World vs. Bootstrap World

Real world:

- True probability distribution (**population**)
  - → Random sample 1
    - → Estimate 1
  - → Random sample 2
    - → Estimate 2
  - …
  - → Random sample 10000
    - → Estimate 10000

Bootstrap world:

- Empirical distribution of original sample (**“population”**)
  - → Bootstrap sample 1
    - → Estimate 1
  - → Bootstrap sample 2
    - → Estimate 2
  - ...
  - → Bootstrap sample 1000
    - → Estimate 1000

Hope: these two scenarios are analogous

The Bootstrap Principle

- The bootstrap principle:
  - **Bootstrap-world** sampling ≈ **Real-world** sampling
- Not always true!
  - … but reasonable if sample is large enough
- We hope that:
  a. Variability of bootstrap estimate
  b. Distribution of bootstrap errors

  ...are similar to what they are in the real world

Key to Resampling

- From the original sample,
  - draw at random
  - with replacement
  - as many values as the original sample contained

- The size of the new sample has to be the same as the original one, so that the two estimates are comparable

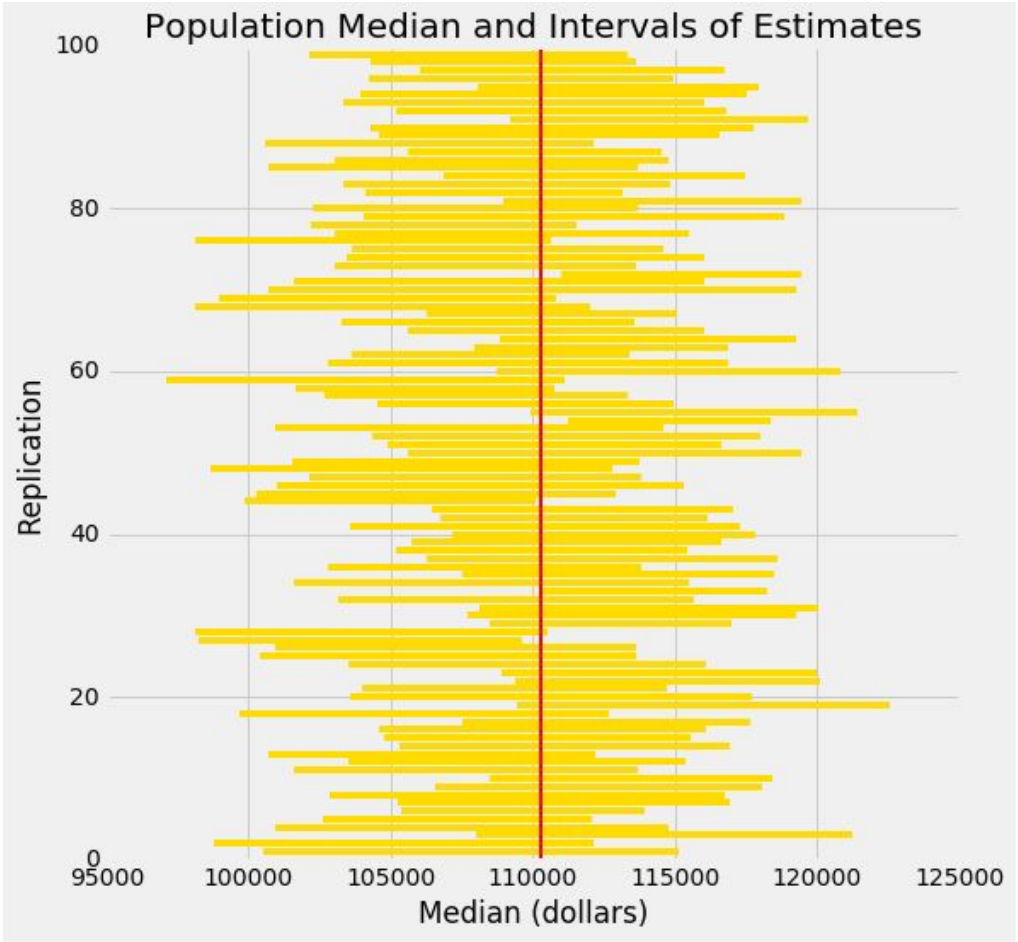
(Demo)

Confidence Intervals

95% Confidence Interval

- Interval of **estimates of a parameter**
- Based on random sampling
- 95% is called the confidence level
  - Could be any percent between 0 and 100
  - Higher level means wider intervals
- The **confidence is in the process** that gives the interval:
  - It generates a “good” interval about 95% of the time.

(Demo)



Each line here is a confidence interval from a fresh sample from the population

Use Methods Appropriately

Can You Use a CI Like This?

By our calculation, an approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

- About 95% of the mothers in the population were between 26.9 years and 27.6 years old.

Answer: False. We're estimating that their **average age** is in this interval.

Is This What a CI Means?

An approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

- There is a 0.95 probability that the average age of mothers in the population is in the range 26.9 to 27.6 years.

Answer: False. The average age of the mothers in the population is unknown but it's a constant. It's not random. No chances involved.

When *Not* to Use The Bootstrap

- If you're trying to estimate very high or very low percentiles, or min and max
- If you're trying to estimate any parameter that's greatly affected by rare elements of the population
- If the probability distribution of your statistic is not roughly bell shaped (the shape of the empirical distribution will be a clue)
- If the original sample is very small

Confidence Intervals For Testing

Using a CI for Testing

- Null hypothesis: **Population average = *x***
- Alternative hypothesis: **Population average ≠ *x***
- Cutoff for P-value: *p*%
- Method:
  - Construct a (100-*p*)% confidence interval for the population average
  - If *x* is not in the interval, reject the null
  - If *x* is in the interval, can't reject the null