DATA 8

Spring 2020

Lecture 28

Sample Sizes and Designing Experiments

Weekly Goals

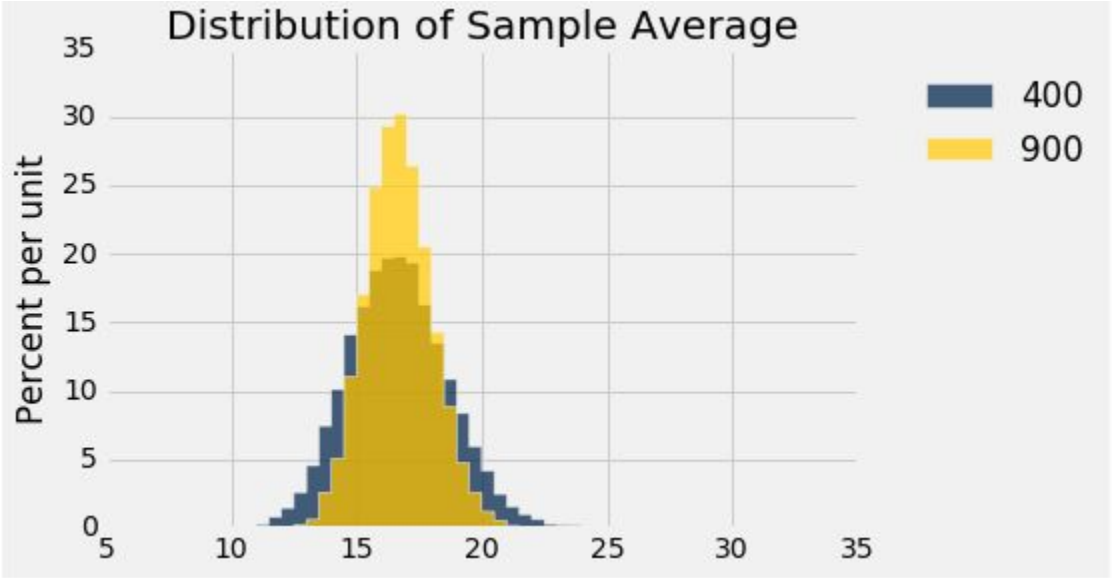
- Monday
  - Defining the “spread” of a distribution
  - How big are most of the values?
- Wednesday
  - The bell shaped curve and its relation to large random samples
- **Today**
  - The variability in a random sample average
  - Choosing the size of a random sample

Averages of Large Samples

The Effect of Sample Size

CLT: If the sample size is large, the distribution of a random sample average is roughly normal.

The bigger the sample, the smaller the spread of the distribution.



Variability of the Sample Average

- The distribution of all possible sample averages of a given size is called the *distribution of the sample average.*
- We approximate it by an empirical distribution.
- By the CLT, it's roughly normal:
  - Center = the population average
  - SD = (population SD) / $\sqrt{\text{sample size}}$

(Demo)

Central Limit Theorem

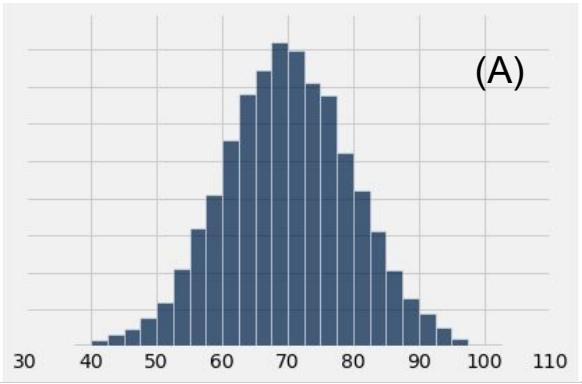
If the sample is large and drawn at random with replacement,

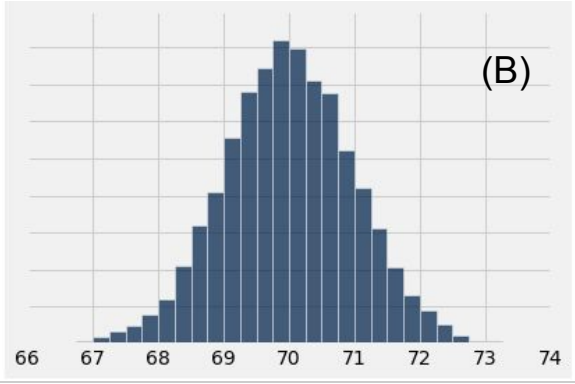
Then, *regardless of the distribution of the population,*

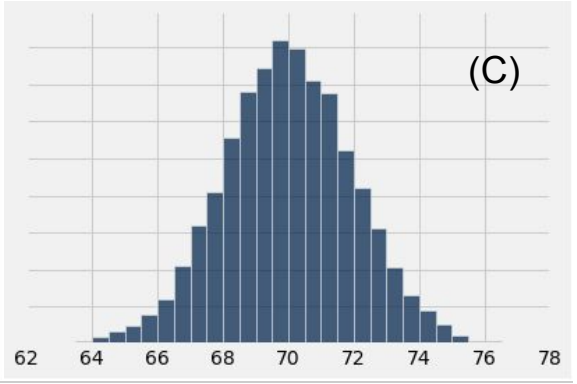
- **the probability distribution of the sample average:**
  - is roughly normal
  - mean = population mean
  - SD = (population SD) / $\sqrt{\text{sample size}}$

Discussion Question

A population has average 70 and SD 10. One of the histograms below is the empirical distribution of the averages of 10,000 random samples of size 100 drawn from the population. Which one?







Discussion Question

A city has 500,000 households. The annual incomes of these households have an average of $65,000 and an SD of $45,000. The distribution of the incomes [pick one and explain]:

(a) is roughly normal because the number of households is large.
(b) is not close to normal.
(c) may be close to normal, or not; we can't tell from the information given.

Discussion Question

A city has 500,000 households. The annual incomes of these households have an average of $65,000 and an SD of $45,000. A random sample of 900 households is taken.

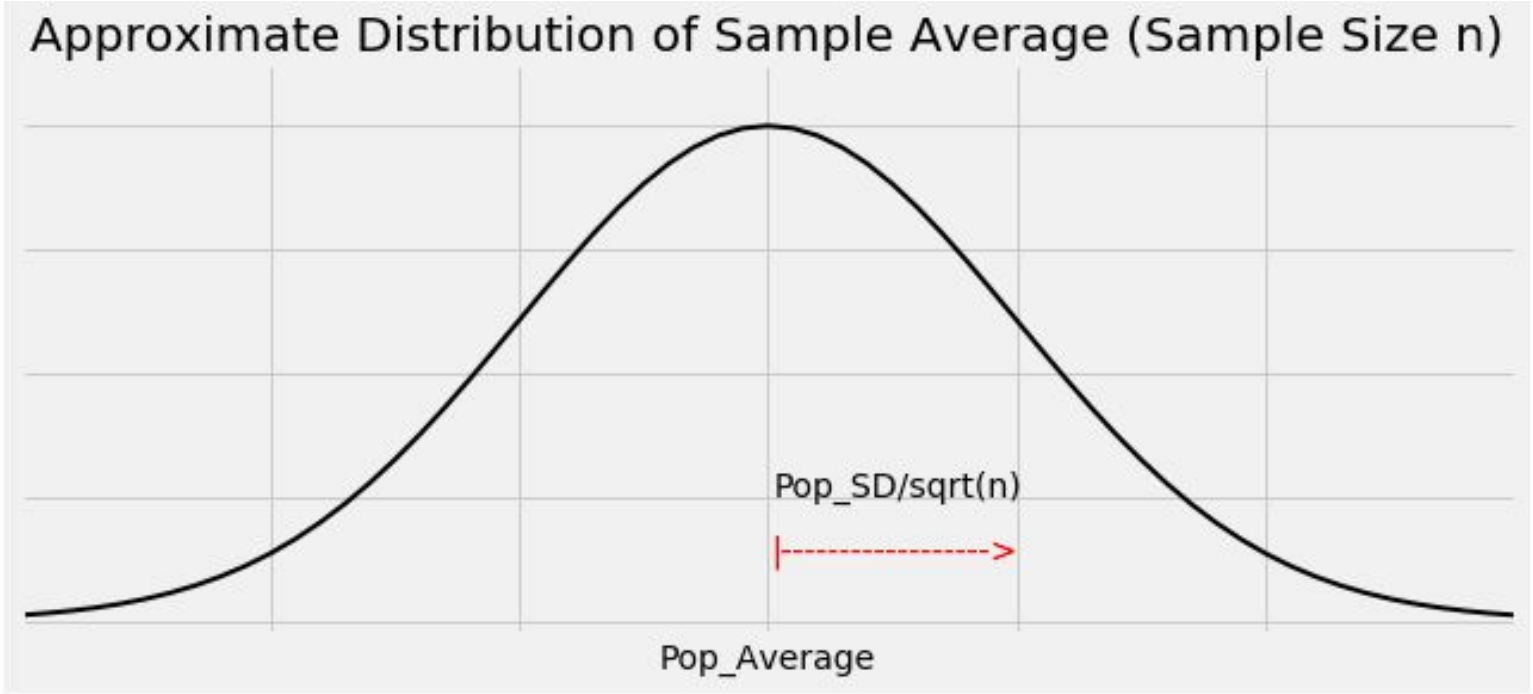
Fill in the blanks and explain:

There is about a 68% chance that the average annual income of the sampled households is in the range

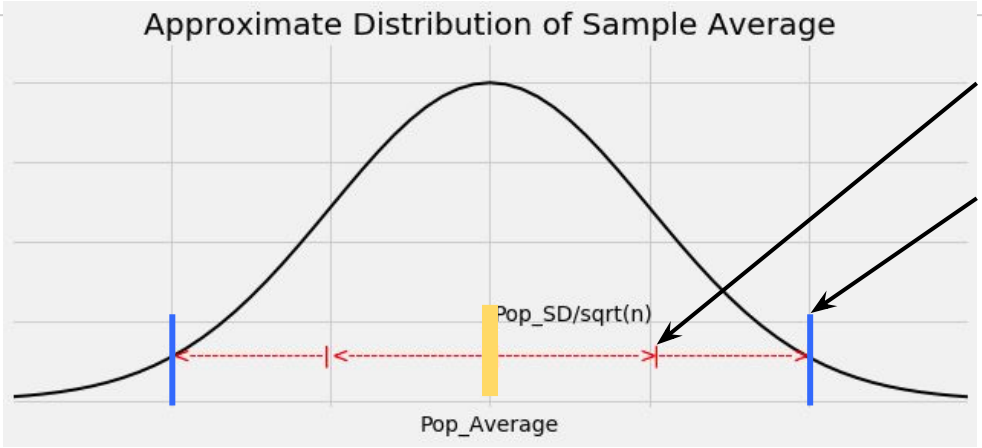
$_____ plus or minus $_____

Confidence Intervals

Graph of the Distribution



The Key to 95% Confidence



- For about 95% of all samples, the sample average and population average are within **2 SD**s of each other.
- **SD** = SD of sample average

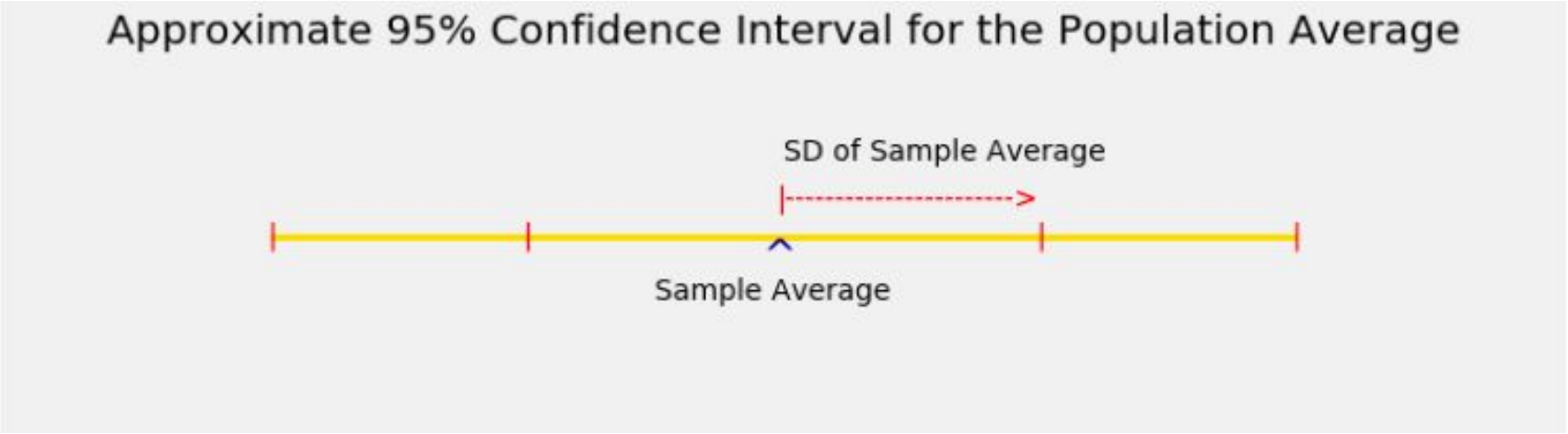
  = (population SD) / $\sqrt{\text{sample size}}$

Constructing the Interval

For 95% of all samples,

- If you stand at the population average and look two **SD**s on both sides, you will find the sample average.
- Distance is symmetric.
- So if you stand at the sample average and look two **SD**s on both sides, you will capture the population average.

The Interval



Width of the Interval

Total width of a 95% confidence interval for the population average

= 4 * SD of the sample average

= 4 * (population SD) / $\sqrt{\text{sample size}}$

Sample Proportions

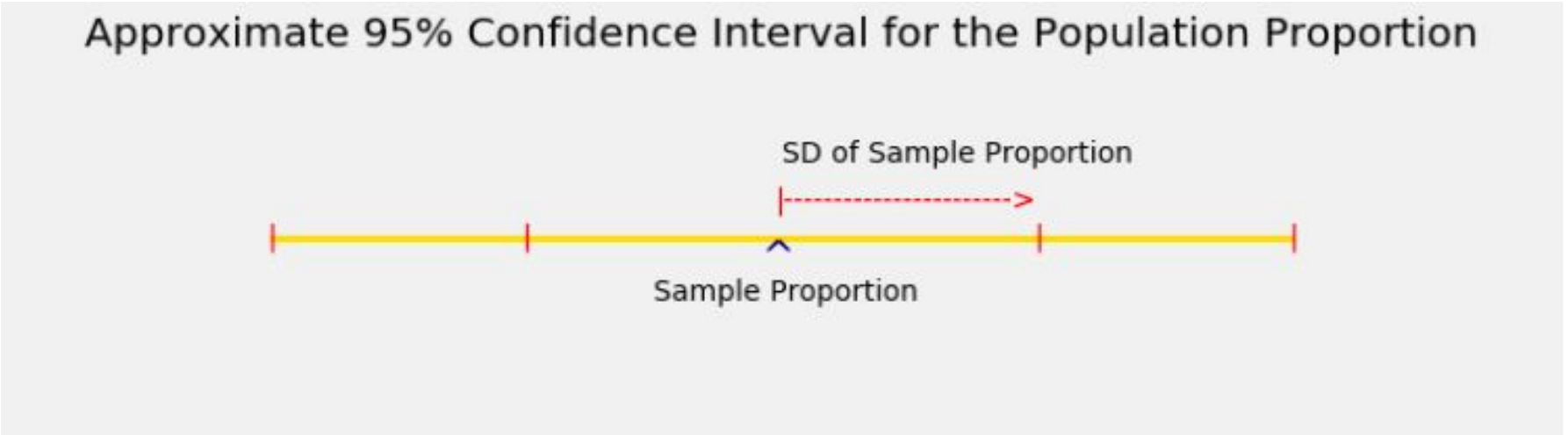
Proportions are Averages

- Data: 0 1 0 0 1 0 1 1 0 0 (10 entries)
- Sum = 4 = number of 1's
- Average = 4/10 = 0.4 = proportion of 1's

If the population consists of 1's and 0's (yes/no answers to a question), then:

- the population average is the proportion of 1's in the population
- the sample average is the proportion of 1's in the sample

Confidence Interval



Controlling the Width

- Total width of an approximate 95% confidence interval for a population proportion

  $= 4 * (\text{SD of 0/1 population}) / \sqrt{\text{sample size}}$

- The narrower the interval, the more precise your estimate.
- Suppose you want the total width of the interval to be no more than 1%. How should you choose the sample size?

The Sample Size for a Given Width

$$0.01 = 4 * (\text{SD of 0/1 population}) / \sqrt{\text{sample size}}$$

- Left side: 1%, the max total width that you'll accept
- Right side: formula for the total width

$$\sqrt{\text{sample size}} = 4 * (\text{SD of 0/1 population}) / 0.01$$

(Demo)

“Worst Case” Population SD

- $\sqrt{\text{sample size}} = 4 * (\text{SD of 0/1 population}) / 0.01$
- SD of 0/1 population is at most 0.5
- $\sqrt{\text{sample size}} \geq 4 * 0.5 / 0.01$
- $\text{sample size} \geq (4 * 0.5 / 0.01) ** 2 = 40000$
- The sample size should be 40,000 or more

Discussion Question

https://www.scientificamerican.com/article/howcan-a-poll-of-only-100/

Discussion Question

- A researcher is estimating a population proportion based on a random sample of size 10,000.

Fill in the blank with a decimal:

- With chance at least 95%, the estimate will be correct to within _____.

Discussion Question

- I am going to use a 68% confidence interval to estimate a population proportion.
- I want the total width of my interval to be no more than 2.5%.
- How large must my random sample be?

2 * (0.5) / sqrt(sample size) = 0.025