Time-dependent magnetospheric flow

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ABSTRACT

Key words:

1 DIMENSIONLESS FORMULATION

Let us assume $GM_* = c = \varkappa = 1$, where M_* is the mass of the accretor, and $\varkappa$ is the opacity (cm^2 g^{-1}, assumed constant). We have the right. This implies the following units for time, length, mass, and energy:

$$\Delta t = \frac{GM}{c^3}, \tag{1}$$

$$\Delta l = \frac{GM}{c^2}, \tag{2}$$

$$\Delta m = \frac{G^2 M^2}{\varkappa c^4}, \tag{3}$$

and

$$\Delta E = \frac{G^2 M^2}{\varkappa c^2}. \tag{4}$$

Luminosity is then scaled to Eddington units, more precisely

$$\Delta L = \frac{GMc}{\varkappa} = \frac{L_{\rm Edd}}{4\pi}. \tag{5}$$

For a one-dimensional formulation, let us integrate in the direction perpendicular to the magnetic field lines. If the strip we are integrating over is narrow, the cross-section of the area we integrate over is

$$A = 4\pi a R_{\rm e} dR_{\rm e} \frac{\sin^3\theta}{\sqrt{1+3\cos^2\theta}}. \tag{6}$$

Here, $0 < a \leq 1$ is the part of the full 2π azimuthal extent subtended by the flow. As there is no perfect axisymmetry, we expect the magnetospheric flow exist only at certain longitudes. Computation involves a conservative scheme for the three conserved quantities, mass, momentum along the field line, and energy, expressed per unit length along the field line:

$$m = \int \rho dS = \frac{\partial M}{\partial l}, \tag{7}$$

$$s = \int \rho v dS = \frac{\partial p}{\partial l}, \tag{8}$$

$$e = \int \left(u + \rho \left(\frac{v^2}{2} - \frac{1}{r} - \frac{1}{2}\omega^2 r^2 \sin^2\theta \right) \right) dS = \frac{\partial E}{\partial l}. \tag{9}$$

For each of the three quantities, conservation laws have the general form

$$\frac{\partial q}{\partial t} + \frac{\partial F_q}{\partial l} = S_q, \tag{10}$$

where F_q and S_q are, respectively, the flux and source for the particula quantity. Fluxes

$$F_m = \int \rho v dS = s, \tag{11}$$

$$F_s = \int \left(\rho v^2 + p\right) dS, \tag{12}$$

and

$$F_e = \int \left(v\rho \left(\frac{u+p}{\rho} + \frac{v^2}{2} - \frac{1}{r} - \frac{1}{2}\omega^2 r^2 \sin^2\theta \right) - D\frac{du}{dl} \right) dS, \tag{13}$$

where the pressure $p = u/3$, as we consider only radiation-dominated case, and $D = \frac{1}{3\rho}$ is diffusion coefficient. We do not consider any sources or losses of mass ($S_m = 0$), while for momentum, gravitational and centrifugal forces were taken into account

$$S_s = -\frac{1}{r^2}\sin(\theta+\alpha)\left(1-\Gamma\right) + \omega^2 r \sin\theta\cos\alpha. \tag{14}$$

Here, $\Gamma = \eta_{\rm irr} L/\tau$ is the correction for radiation pressure (Eddington factor), $\eta_{\rm irr} \lesssim$ is assumed constant, L is the total power lost by the flow as radiation, and τ is the optical depth across the flow in poloidal direction, estimated as $\tau = \rho\Delta\theta = \frac{2R\sin\theta}{A} m$.

For energy, there are two contributions: work done by the forces and energy loss due to radiation,

$$S_e = vS_s - \xi_{\rm rad} a r \sin\theta \frac{u}{\rho+1}. \tag{15}$$

maybe a good idea is to introduce a bulk viscousity term?

Resulting system of three differential equations was then solved using HLLE Riemann solver (see for instance **?**) with the signal velocities fixed by $v \pm 1$. No relativistic effects were taken into account.

Boundary conditions were profoundly different from those in **?**: we assume no matter or energy flow through the lower boundary (NS surface), and a constant mass inflow at a fixed velocity at the right (disc) boundary. Energy input at the outer boundary assumes pressure balance condition at the edge of the magnetosphere $p_{\rm mag} = p_{\rm rad} = 3u_{\rm rad}$.