14.121 Problem Set 4

Due: 10/11 in class

1. Suppose a consumer has preferences given by

$$X = \left\{ x \in \mathbb{R}_+^L : x_l \geq \underline{x}_l \right\}$$

$$U(x) = \left[\sum_{l=1}^{L} \alpha_l (x_l - \underline{x}_l)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma \geq 0$, $\alpha_l \geq 0$, and $\sum_{l=1}^{L} \alpha_l = 1$, and the $\{\underline{x}_l\}_{l=1}^{L}$ are the minimum subsistence levels. Note that in the case where $\underline{x}_l = 0$ for $l = 1, .., L$ this is called a *constant elasticity of substitution* (CES) utility function.

a. Find the Marshallian demands $x(p, w)$. Denote the *indirect utility function* as the utility function evaluated at the demands, $v(p, w) = U(x_1(p, w), ..., x_L(p, w))$. Show that the indirect utility function can be written

$$v(p, w) = -\frac{\sum_{l=1}^{L} p_l \underline{x}_l}{\left[\sum_{l=1}^{L} \alpha_l^{\sigma} p_l^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} + \frac{1}{\left[\sum_{l=1}^{L} \alpha_l^{\sigma} p_l^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} w$$

b. Verify that in the limit as $\sigma \to \infty$ the utility function is linear

$$U(x) = \sum_{l=1}^{L} \alpha_l (x_l - \underline{x}_l)$$

as $\sigma \to 1$ the utility function is Cobb-Douglas in $x_l - \underline{x}_l$ (this is also called *Stone-Geary* preferences)

$$U(x) = \Pi_{l=1}^{L} (x_l - \underline{x}_l)^{\alpha_l}$$

and as $\sigma \to 0$ the utility function is Leontief in $x_l - \underline{x}_l$

$$U(x) = \min_l \{x_l - \underline{x}_l\}$$

c. Find the demands $x(p, w)$ and indirect utility function $v(p, w)$ in each of the limit cases.

2. Consider a pure trade economy with I agents. Preferences for each agent are given by Cobb-Douglas preferences:

$$u^i(x^i) = \sum_{l=1}^{L} \alpha_l^i \log x_l^i$$

with $\alpha_l^i \geq 0$ and $\sum_{l=1}^{L} \alpha_l^i = 1$ for $i = 1, .., I$. Each agent has an endowment $\omega^i = (\omega_l^i)_{l=1}^{L}$.

a. Following Negishi's method, derive a system of equations in the Pareto weights $\lambda = (\lambda_i)_{i=1}^n \in \Delta^I$ that must be satisfed at a Walrasian equilibrium.

b. Suppose that all consumers have the same endowment $\omega^i = \bar{\omega}/I$. Use the system from part a to find the Pareto weights corresponding to the Walrasian equilibrium.

c. Suppose that consumer $\omega_l^1 > \omega_l^2$ for $l = 1, .., L$. Use the system from part a to show that $\lambda_1 > \lambda_2$ in the Pareto optimal allocation corresponding to the Walrasian equilibrium.

3. Consider a pure trade economy, with 2 consumers and 2 commodities. Consumer 1 has utility function $u_1(x_1, x_2) = \alpha \log x_1 + (1 - \alpha) \log x_2$, and consumer 2 has preferences given by utility function $u_2(x_1, x_2) = x_1 + \beta x_2$, with $\beta > 0$. Endowments are given by $\omega^i = (\omega_1^i, \omega_2^i)$.

a. Find all Walrasian equilibria for this economy, for all parameters α, β and endowments. It suffices to present the relative price of good 2, p, and express the allocation as a function of p using the individual demands, but you should also note if there are any restrictions on the parameters which must hold in order for the implied allocation to be feasible (including satisfying nonnegativity constraints) or for the price to be consistent with the demands at that allocation.

b. Suppose now that we are given data on volumes traded for each agent in this economy in the table below, where "value" is the price times the quantity. Using this data, calibrate the parameters α, β. For it, use Harberger's convention and take prices to be 1.

c. Suppose now a value added tax of 10% is added to the price of commodity 1. The income generated from this tax is thrown to the sea. Given the data in b), what will be the effect on the equilibrium prices?

	Consumer 1	**Consumer 2**
Values of purchases commodity 1	10	5
Values of purchases commodity 2	20	15
Total purchases	30	20
Value of endowment of commodity 1	5	10
Value of endowment of commodity 2	25	10