14.121, Fall 2019 | Problem Set 2
Pathak / Ridley | Due: Wednesday, September 18, 2019

1) MWG Exercise 3.G.5

2) MWG Exercise 3.G.6 parts (a), (b), and (c).

3) MWG Exercise 3.G.17

   Note: The minus sign at the beginning of the right-hand side of the indirect utility function should be deleted. That is, it should be $v(p, w) = (w/p_2 + b^{-1}(ap_1/p_2 + a/b + c)) \exp(-bp_1/p_2)$.

   Also, in (b), the minus sign in front of the first term of the right-hand side of the expenditure function should be deleted. That is, it should be $e(p, u) = p_2 u \exp(bp_1/p_2) - (1/b)(ap_1 + ap_2/b + p_2 c)$.

   Finally, in (c), the minus sign in front of the first term of the right-hand side of the Hicksian function should be deleted. That is, it should be $h(p, u) = ub \exp(bp_1/p_2) - a/b$.

4) There are two individuals in a household. You observe prices $p = p_1, ..., p_n$, total household income w, and total household demand $x_1^H, ..., x_n^H$ for each good. Only individual A benefits from good 1 and only individual B benefits from good 2, while goods 3 through n are private goods in the sense that each benefits only from the amount the individual consumes of the jointly observed amount of this good.

   For instance, they order pizza and A benefits only from the number of slices that A consumed and similarly for B. You get to see how much pizza was ordered, but now how much each ate.

   The preferences of A are constant in amount of goods $2, ..., n$ consumed by B, and the preferences of B are constant in the amount of goods $1, 3, ..., n$ consumed by A. We therefore can write their utilities as:

$$u^A(x_1, x_3^A, ..., x_n^A) \quad \text{and} \quad u^B(x_2, x_3^B, ..., x_n^B)$$

   Assume that preferences are locally non-satiated and continuous, and you can assume that we are always at an interior solution.

   1) Prove that the following two problems are *equivalent*, in the sense that for any function $\bar{u}^B(p, w)$ in part a), there is a function $f(p, w)$ in part b) that gives the same solution, and vice versa.

      a) The household maximizes the utility of person A subject to the constraint that person B obtains at least a certain utility level $\bar{u}^B(p, w)$ and subject to the household budget constraint, where the minimal utility level for B can depend on income and prices, but the function is homogenous degree zero.

      b) The household divides the household income among the two individuals according to some **sharing rule** that can depend on income and prices, but is homogenous degree 1, and individuals then each use their own income to maximize their utility subject to the induced budget constraint (That is, $w^A = f(p, w)$ where $0 \le f(p, w) \le w$ and $w^B = w - f(p, w)$).

   2) True or False:

      a) The function specifying the minimal utility for person B's utility in part 1a) is increasing in w if and only if the income given to B in part 1b) is increasing in w.

      b) The function specifying the minimal utility for person B's utility in part 1a) is linear in w if and only if the income given to B in part 1b) is linear in w.