How is the audit market affected by characteristics of the non-audit services market?*

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Abstract

How can features of the market for non-audit services (NAS) affect an audit firm's incentives to invest in audit quality, the price at which it offers audits, total welfare, and the effects of prohibitions on providing NAS to audit clients and generally? We address this and related questions in a model focusing on an audit firm that can provide both audit and NAS to a market of heterogeneous clients. The audit firm faces competition both in the audit market and in the NAS market. We show that bans on the provision of NAS to audit clients and non-audit clients can increase or decrease audit quality and total welfare. These contrasting effects suggest a more nuanced view of how regulating an auditor's provision of NAS might affect audit quality and total welfare, and operate through effects related to competition, pricing, and segmentation on ex ante quality investment incentives rather than previously identified auditor independence or knowledge spillover channels.

Keywords: Audit quality; Auditing services; Non-audit services; Competition.

JEL codes: L10; M41; M42.

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1 Introduction

Researchers, regulators, and practitioners have long-standing concerns about the negative effects of non-audit services (NAS) on audit quality (see, e.g., Che et al., 2018; DeAngelo, 1981*a*; Kowaleski et al., 2018). Payments for NAS by client firms can impair auditor independence and motivate auditors to ignore client deficiencies. Simunic (1984) and Bouwens (2018), in contrast, suggest positive spillovers from NAS provision to audit quality, due to, for instance, knowledge transfer across audit and NAS employees at the same firm. These arguments are generally constructed in a single-client setting, in which one auditor provides services to one client. Such a setting precludes a number of interesting economic forces from operating, including the potential for auditors to target audit and NAS client segments via quality and pricing decisions.[1]

In this paper, we focus on interactions between audit and NAS using an industrial organization perspective. We use a model to examine how competition in the market for NAS can affect the audit firm's choices related to firm-wide quality investments, audit pricing, and the implications of regulations prohibiting auditors from offering NAS. Our analysis provides a novel perspective on how economic forces related to NAS can affect audit outcomes through channels involving the industrial organization of the audit and NAS markets.

Our model features three types of economic agents and two services markets. A set of heterogeneous client firms demand professional services from auditors and consultants. Auditors provide assurance about financial reports, and consultants provide NAS to help firms gain value. Our primary interest lies in the choices made by a high-quality audit firm that can make audit-improving investments and offers both audit and NAS to clients. Our model also features: a low-quality audit firm who provides a base-level audit and competes with the high-quality auditor in the audit market; and a NAS-only supplier who competes with the high-quality auditor in the NAS market. Our setup follows from our primary focus on how features of the NAS market can influence the high-quality auditor's offerings.[2]

[1] A single-client model also cannot distinguish between auditor-wide and engagement-specific investments. The Public Company Accounting Oversight Board has highlighted firm-wide investments in human capital and infrastructure as important aspects contributing to audit quality (PCAOB, 2015).

[2] In our main analysis, we abstract away from features such as audit independence, knowledge spillovers, audit standards, and costs of audit failure studied in previous research, discussed further in Section 1.1. We introduce additional competition in the NAS market in Section 4.1 and show that our results still hold as long as there is some degree of differentiation. We present results related to changes in audit standards in Section 4.2.

Audit services for each client firm are modeled as in the standard single-client models of Dye (1995) and Laux and Newman (2010). Each client firm has a project that can be either good or bad. Good projects provide higher cash flows in expectation, via a greater likelihood of success. We refer to the probability of having a bad project as the business risk of a client firm, and allow this risk to vary across client firms. Each client firm's investors must choose whether to make a fixed investment in the firms' project. In order to value each client firm's project and subsequently make an informed investment, the investors require each client firm's manager to issue an accounting report. Absent an audit, each client firm's manager would report that the firm's project is good. The audit probabilistically detects misreporting in this accounting report, and therefore exposes bad projects with a probability that depends on the quality of the audit purchased by the firm.

Before offering its services to clients, the high-quality auditor can invest in firm-wide audit quality, with higher audit quality increasing the probability that its audits successfully detect misreporting.[3] The low-quality auditor's audit quality is fixed (e.g., at the quality required by auditing standards). The high-quality audit firm's investment in quality is observable to client firms, reflecting, for instance, the auditor's reputation. This reputation affects client firms' willingness to pay for audit services, and it is this audit revenue channel that motivates investment in audit quality. The revenue-based motive does not conflict with previously-studied motives related to auditor liability or audit standards, which are discussed further in Section 1.1.

Client firms also can purchase NAS, which increase client firm value, either from the high-quality auditor or from the NAS competitor, i.e., a pure-play consulting firm. Both the high-quality auditor and the NAS competitor offer equivalent-quality NAS and cannot use NAS quality or client business risk to price discriminate and segment the market.[4] However, the high-quality auditor and NAS competitor choose how to price the services they offer, and clients choose whom to purchase services from. Differentiation in the NAS market comes only from the high-quality auditor's ability to offer audit services, which helps maintain our focus on audit outcomes.

We begin our analysis with a setting in which there is no restriction on the provision of NAS to client firms. In the audit market, the high-quality auditor sells auditing services to high-risk clients

[3] Audit quality in our model is firm-wide, applying to all of the auditor's engagements. This captures constructs such as audit firm culture, internal controls, or reputation, rather than engagement-level auditor effort as captured by many prior single-client models (e.g., Dye, 1995). Such firm-wide audit quality can also be interpreted as capturing the probability that any given engagement partner or staff member acts in good faith and avoids shirking.

[4] In a prior version of this paper, we allowed price discrimination via bundling of audit services and NAS.

whereas the low-quality auditor sells auditing services to low-risk clients. The high-quality auditor engages in full Bertrand competition with the NAS competitor in the NAS market. This pushes the high-quality auditor's rents from NAS to zero, and causes the high-quality auditor's choice of audit quality to be independent from the features of the NAS market.

Next, we study a setting in which the high-quality auditor cannot provide NAS to audit clients. We show that banning the provision of NAS to audit clients may increase or decrease the high-quality auditor's audit quality depending on the correlation between the demand for auditing services and the demand for NAS. Perhaps surprisingly, regulatory bans on the provision of NAS to audit clients can make the high-quality auditor and the NAS competitor better off. Essentially, such restrictions create niches in which each supplier earns monopoly rents where before the rents were dissipated by Bertrand competition.

If the correlation between NAS demand and audit demand is positive or weakly negative, a restriction on NAS to audit clients leads to *higher* audit quality. The high-quality auditor has incentives to increase audit quality in order to reduce the number of audit clients and increase the number of NAS clients. This increases the NAS rents earned by the high-quality auditor.

An interesting effect emerges with a sufficiently negative correlation between NAS demand and audit demand, and the restriction on auditor provision of NAS to audit clients: here, a restriction on NAS to audit clients leads to *lower* audit quality. This occurs solely through the interaction between NAS restrictions and competitive effects. Specifically, NAS restrictions carve out a niche in which the NAS competitor is a monopolist, i.e., in offering NAS to high-quality audit clients. Due to decreased competition, the NAS competitor can choose a positive price and earn rents from this segment.[5] If the NAS competitor cannot price discriminate between high-quality and low-quality audit clients, a larger equilibrium set of high-quality audit clients increases the niche in which the NAS competitor is a monopolist, and causes the NAS competitor to raise prices. This allows the high-quality auditor to raise prices for the NAS it offers to non-audit clients. By lowering audit quality and expanding audit coverage (which are linked in our model via demand heterogeneity), the high-quality auditor indirectly increases the rents it earns in the NAS market.

We then demonstrate that total welfare may be higher or lower with NAS restrictions to audit

[5] In Section 4, we show that this result is robust to additional competition in the NAS market if NAS services are imperfectly competitive, e.g., through quality differentiation.

clients. Indeed, a direct implication from our findings is that NAS restrictions change auditors' incentives to deliver high-quality audits, which may increase or decrease total welfare depending on the client firms' business risks.

Some regulators and practitioners have proposed breaking up integrated audit-consulting firms, i.e., prohibiting audit firms from offering NAS to all clients (see, e.g., Kowaleski et al., 2018; Rapoport and Trentmann, 2018; *The Economist*, 2018). The main concern is that providing both services may create conflicts and inherent biases across services, even if not provided to common clients. Our analysis shows that a regulatory ban on the provision of NAS to non-audit clients may increase or decrease audit quality and total welfare relative to a prohibition on providing NAS solely to audit clients, because this removes the potential for the high-quality auditor to benefit from competition-related price increases in the NAS market.

In an extension of our main model, we study the impact of additional competition in the NAS market. First, we show that in the presence of two NAS competitors, if both the high-quality auditor and one NAS provider have a competitive advantage in the NAS market, NAS restrictions to audit clients decrease audit quality. Then, we analyze a setting in which only the high-quality auditor has a competitive advantage in the NAS market and competes against two quality-disadvantaged NAS providers. In that latter case, NAS restrictions to audit clients increase audit quality. This extension of our main model shows that, even with additional competition in the NAS market, NAS restrictions affect audit quality and total welfare.

Finally, we study the effect of increasing the low-quality auditor's audit quality. We interpret this as a regulator imposing higher auditing standards, as auditing standards set a floor on audit quality. An increase in auditing standards may increase audit fees charged by low-quality audit firms and, for a given audit coverage, decrease audit fees charges by high-quality audit firms. However, an increase in auditing standards can cause changes in audit coverage, which can increase high-quality audit firms' audit quality and audit fees. As result, there are optimal interior auditing standards that maximize total welfare.

1.1 Contribution and related literature

Several theoretical studies have examined the incentives of auditors to deliver high quality audits, primarily in models of a single audit engagement. Dye (1993), Gao and Zhang (2019) and Ye and

Simunic (2013) study the effects of auditing standards on audit quality. Laux and Newman (2010) analyze the incentive effects of legal penalties. Absent contingent contracts, which are generally restricted, incentives for auditors to deliver high audit quality in models where audit quality results from unobservable effort typically either come from reputation benefits or legal liability after an audit failure. In our study, audit quality is an initial investment (e.g., hiring high-quality auditors) observable to clients, and the incentive for higher audit quality comes from the auditor's ability to charge higher prices. Our paper adds to the literature by studying how these incentives interact with market features such as client heterogeneity and the potential to provide NAS to both audit and non-audit clients.

This paper contributes to auditing literatures that have independently studied the effects of NAS provision by auditors and the effects of competition between auditors. Simunic (1984) shows that auditors should provide NAS to audit clients because of potential knowledge spillovers. On the contrary, DeAngelo (1981*a*) argues that NAS may threaten auditors' independence by creating an economic bond between auditors and client firms.[6] In a recent study, Mahieux (2019) highlights a positive externality of NAS in that the potential to hire an auditor for NAS can give the auditor stronger motives to exert audit effort. Bar-Yosef and Sarath (2005) model competing auditors, providing a rationale for well-capitalized auditors to capture a greater share of the audit market when audit failure can result in litigation recoveries from auditors. Gerakos and Syverson (2015) estimate public firms' demand for differentiated audit services to provide numerical estimates of the potential effects of mandatory auditor rotation or the exit of one of the Big 4 auditors. Bleibtreu and Stefani (2018) argue that regulators' goals of simultaneously decreasing client importance and audit market concentration are in direct conflict. They conclude that mandating audit firm rotation might have unintended consequences. We study the interactions between the audit and the NAS markets in a setting in which an audit firm faces competition in both markets.

Empirical evidence on the effects of NAS and NAS restrictions to audit clients on audit quality is mixed.[7] For a focused review of such studies, see Bouwens (2018). While anecdotes and regulatory interventions related to NAS fees eroding audit quality provide support for restrictions on the provision of NAS to audit clients, large sample studies provide mixed findings on the association

[6]Chan and Liu (2018) show that while restricting NAS can restore faithful reporting, it also adversely influences audit effort.

[7]For a recent survey of the related archival auditing literature, see DeFond and Zhang (2014).

between NAS fees and audit quality. DeFond et al. (2002) find no association, while some studies find a negative impact of NAS on audit quality, (e.g., Bell et al., 2015; Causholli et al., 2014) and others find positive effects, at least for some clients (Kinney et al., 2004; Nam and Ronen, 2012). We show that the impact of NAS restrictions to audit clients on audit quality depends on the correlation between audit demand and NAS demand.

Our paper also contributes to the literature studying the impact of NAS restrictions to non-audit clients. There is a concern that audit firms have focused on consulting services that are more lucrative than auditing and that this may have impaired audit quality. Che et al. (2018) show that partners' compensation is positively associated with revenues from consulting services to non-audit clients, but not those to audit clients. Kowaleski et al. (2018) provide experimental evidence that suggests that providing NAS increases auditor cooperation with managers, increasing audit quality when managers prefer high audit quality and decreasing audit quality when managers prefer low audit quality. In the cross-section, heterogeneity in client preferences causes insignificant global average effects of providing NAS on audit quality. Lisic et al. (2014) argue that a higher proportion of NAS revenue to total revenue at the accounting firm level is either not associated with impaired audit quality or is sometimes associated with improved audit quality. However, they show that investors perceive a deterioration in audit quality when a higher proportion of revenue is generated from NAS. We demonstrate how NAS restrictions to all clients may increase or decrease audit quality relative to the case of NAS restrictions to audit clients. However, NAS restrictions to all clients do not impact audit quality relative to the case absent NAS restrictions.

Lastly, Bleibtreu and Stefani (2012) and Wu (2006) also analyze the interactions between the audit market and the NAS market. Bleibtreu and Stefani (2012) study the interactions between NAS restrictions as a measure intended to improve auditor independence and audit market concentration. In their model, NAS fees subsidize market entry, which increases audit market concentration and independence in the sense of reducing the fraction of an auditor's audit revenue coming from a given client. Independence in their model, however, does not affect audit quality. Wu (2006) analyzes the tradeoff between "knowledge spillovers" from auditing to NAS or vice versa, and "competition crossovers," i.e, how oligopolistic competition in one market influences the counterpart in the other market. Wu's main results speak to how knowledge spillovers between audit and NAS can change pricing strategies, similar to how retailers offer "loss leaders" to enhance demand for more profitable offerings. We study the interactions between the audit market and the NAS market in a setting without cross-subsidization or knowledge spillovers. Our results derive from market-based incentives for auditors to provide high-quality services.

The rest of the paper proceeds as follows. Section 2 describes the baseline model. In Section 3, we analyze the equilibrium without NAS restrictions, with NAS restrictions to audit clients, and with NAS restrictions to all clients respectively. We analyze two extensions of our main model in Section 4: the impact of additional competition in the NAS market and the effects of auditing standards. Finally, Section 5 concludes.

2 Model setup

This section describes our model setup and timing. Key assumptions are discussed in Section 2.1. We start from a standard one-auditor, one-client audit model following Dye (1995) and Laux and Newman (2010) expanded to three client firms indexed by $j \in \{A, B, C\}$. Each client firm has a project, w_j, that can be either good (g) or bad (b). A good project yields cash flow $R > 0$ whereas a bad project yields cash flow 0. The prior probability that project w_j is bad is $1 - p_j$. This probability is also client firm j's type. We assume that $1 > 1 - p_A > 1 - p_B > 1 - p_C > 0$. As in Lu and Sapra (2009), we refer to $1 - p_j$ as j's "client business risk" or "risk." There are two auditors indexed by $i \in \{1, 2\}$, and a consultant, who is a non-audit services (NAS) competitor.

The investors in each client firm can invest $I > 0$ in the firm's project.[8] In order to value the client firm and subsequently make an informed investment, the investors of a client firm require the client firm's manager to issue an accounting report. Each client firm's type is its private information at the time of audit/NAS provision, and is therefore non-contractible information for service provision. The client firms' managers do not have private information about project quality and always send favorable accounting reports. An auditor's job is to verify the accuracy of the accounting report.[9] Note that buying auditing services is not mandatory for client firms, but we assume the parameters are such that all client firms find it beneficial to procure audits. In the

[8] As in Gao and Zhang (2019), we assume that the client firms' investors make the investment decision. Alternatively, we could distinguish between current and new investors. The current investors sell the firm in a competitive market to new investors who in turn make the investment decision. Such a setting introduces additional notation without affecting the main results.

[9] This is a standard assumption in the audit literature that helps to focus the analysis on the role of the auditor. For a similar assumption, see, e.g., Lu and Sapra (2009).

rest of the paper, we use the terms "client firm" and "investors" interchangeably, as the role of managers in the model is primarily to send favorable reports subject to correction by audits.

We assume that the upper bound on client risk is $1 - I/R$, which implies that $p_j R > I$ is satisfied for all three client firms. Hence, as is common in theoretical auditing studies, the investors prefer to invest in a project in the absence of audit but would not invest in the project if the audit revealed a bad project.

We model audit services as follows. In return for an audit fee, $F_{a,i}$, auditor i issues a public audit report $r_{ij} \in \{b, g\}$ for client firm j's investors. Audit quality, a_i, determines the probability that an audit by auditor i detects a bad project in client firm j as follows:

$$\Pr(r_{ij} = g \mid w_j = g) = 1, \tag{1}$$

$$\Pr(r_{ij} = g \mid w_j = b) = 1 - \Pr(r_{ij} = b \mid w_j = b) = 1 - a_i. \tag{2}$$

The firm-wide audit quality, a_i, is publicly known, reflecting, for instance, auditor i's reputation for delivering high-quality audits. Audit reports are conditionally independent across client firms although a_i is the same for all client firms audited by auditor i.

Auditor 2 can invest *ex ante* in audit quality, $a_2 \in (0, 1)$. The low-quality auditor, for simplicity, offers a fixed audit quality $a_1 \in (0, 1)$. We assume that exogenous parameters are such that, in equilibrium, auditor 2's audit quality is higher than auditor 1's audit quality, i.e., $a_2 > a_1$ (see assumption in (5) below). The quality differentiation between high- and low-quality auditors could emerge from an unmodeled earlier stage in which ex ante symmetric audit firms both choose audit quality. Indeed, it is well-known from the industrial organization (IO) literature that two competing firms prefer to choose two different product qualities to soften the effect of competition (Shaked and Sutton, 1982).

The cost of providing audits of quality a_i to a number $Q_{a,i}$ of clients is $kQ_{a,i}a_i^2/2$, with $k > (1 - p_A)I$ a cost parameter sufficiently large so that the equilibrium audit quality is interior. As described below, audit quality captures the probability with which a bad project is detected and an adverse audit report (e.g., qualified or going-concern) is reported. This is consistent with the definition provided by DeAngelo (1981*b*, 186) in that it captures the "joint probability that a given auditor will both discover a breach in a client's accounting system, and report the breach." The notion of firm-wide audit quality is borne out in three of the five key drivers of audit quality provided by the Financial Reporting Council's (2008) Audit Quality Framework: audit firm culture; audit partners' and staffs' skills and personal qualities; and audit process effectiveness. We discuss the modeling of audit quality further in Section 2.1.

In the NAS market, auditor 2 is engaged in non-differentiated Bertrand competition with a consultant, who is a pure NAS provider.[10] The consultant chooses NAS fees $F_{c,c}$ whereas auditor 2 chooses NAS fees $F_{c,2}$. The value of NAS to client firm j is $(1 + p_j\theta)v$ regardless of whether auditor 2 or the NAS competitor supplies it, where $v \geq 0$. The parameter $\theta \in [-1, \infty)$ captures the correlation between client firm j's business risk and NAS value, and, therefore the correlation between audit demand and NAS demand. The restriction $\theta \geq -1$ ensures that the value of NAS to client firms is positive for any p. In practice, examples of NAS that add value to low-risk firms are: outsourcing, systems implementation, and tax advisory services. On the contrary, examples of NAS that benefit high-risk firms are bankruptcy and M&A advisory services.

The timing of the model is summarized in Figure 1. At $t = 1$, auditor 2 chooses audit quality. Then, at $t = 2$, all service providers choose their service fees. At $t = 3$, client firms procure services. We assume that each client firm must choose whether to purchase audit services and NAS at the same time. That is, audit and NAS markets clear simultaneously. This eliminates the possibility of purchasing NAS conditional on a positive audit outcome. With such a sequence, there would be a clear complementarity between audit and NAS. While such a spillover may be interesting, we do not explore it here.[11] At $t = 4$, service providers then conduct audits and provide NAS. Investors make investments conditional on the audit reports. Finally, at $t = 5$, investments that are undertaken pay off (as do the firm's other projects that may be affected by NAS) and firm cash flows are realized.

At $t = 3$, the expected utility of the investors of client firm j hiring auditor i with audit quality

[10] We introduce additional and differentiated competition in the NAS market in Section 4.1.

[11] Mahieux (2019) shows that having auditors providing NAS contingent on the audit outcome can provide an incentive for high-quality audits.

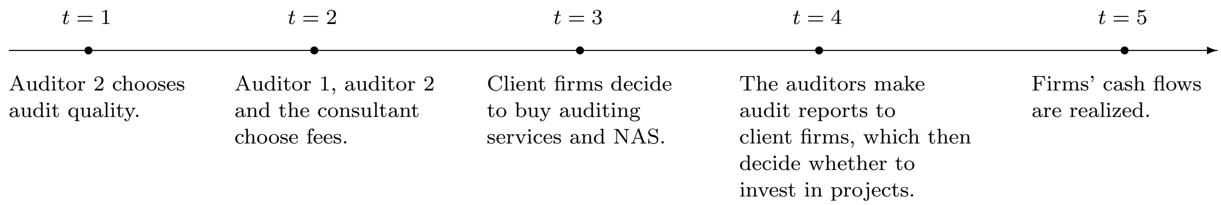

Figure 1: Timeline of the baseline model

a_i for a fee $F_{a,i}$ and buying NAS for a fee $F_{c,i}$ is

$$U_{client,j} = \underbrace{p_j R - I}_{\text{investment return}} + \underbrace{a_i(1-p_j)I}_{\text{audit benefit}} + \underbrace{(1+p_j\theta)v}_{\text{NAS value}} - \underbrace{F_{a,i}}_{\text{audit fee}} - \underbrace{F_{c,i}}_{\text{NAS fee}}. \tag{3}$$

Client firms with lower business risk (i.e. higher p) are less willing to pay for auditing services. Further, their willingness to pay for NAS depends on the sign and magnitude of the parameter θ. Recall that auditors and the consultant cannot contract on client business risk. Combined with auditor 2 setting firm-wide quality rather than client-specific quality, this implies that auditor 1 and auditor 2 have limited ability to price discriminate or screen clients. Similarly, the fixed firm-wide NAS quality precludes price discrimination and screening for NAS.

Finally, we make the following assumptions concerning parameter values:

$$a_1 < \frac{2I}{k}\left(2(1-p_C) - (1-p_B)\right), \tag{4}$$

$$a_1 < \frac{(1-p_B)I}{k}, \tag{5}$$

$$I^2\left(3(1-p_C)^2 - 2(1-p_B)^2\right) < ka_1\left(10(1-p_C)I - 4(1-p_B)I - 3ka_1\right), \tag{6}$$

and

$$\max\left(\sqrt{2}(1-p_A), (1-p_A)+(1-p_C)\right) < 2(1-p_B) < 4(1-p_C). \tag{7}$$

The assumption in (4) ensures that client firm C's business risk is sufficiently large so that auditor 1 always finds it optimal to sell auditing services to client firm C. The assumption in (5) ensures that auditor 2 has a higher audit quality than auditor 1 in equilibrium. The assumption in (6) ensures that auditor 2 does not sell auditing services to all three client firms. Lastly, the assumption in (7) implies that client firms are not too heterogeneous with respect to their business risks. This is consistent with our assumption of no price-discrimination, as limited client heterogeneity would make screening more difficult for the service providers.

2.1 Discussion of the main assumptions

Audit quality: Audit quality is an ex ante investment by the auditors and, therefore, audit quality for a given auditor is the same for all audit clients. Although audit quality in practice varies across engagements, firm-wide audit quality is a construct of both academic and practical importance. Several studies have examined empirically whether certain types of audit firms (i.e., larger, wealthier, or Big-N) provide higher quality audits, on average (e.g., Lawrence et al., 2011), consistent with the relevance of firm-wide quality. In a model of heterogeneous clients, Bar-Yosef and Sarath (2005) also assume a level of quality across an audit firm's engagements. From a practitioner/regulatory perspective, the notion of firm-wide audit quality is consistent with the aforementioned "key drivers of audit quality" outlined in the Financial Reporting Council's (2008) Audit Quality Framework. The PCAOB, in their 2015 concept release on audit quality indicators, highlights ex ante investments firms make related to human capital (e.g., expertise and training hours of audit personnel) and audit quality supporting infrastructure (PCAOB, 2015). As a stark example, note that a series of audit failures led to the demise of not only a few engagement teams, but the entire Arthur Andersen firm, suggestive of firm-wide reputations for audit quality.[12]

The ex ante choice of audit quality can also be interpreted as an investment in internal controls. Aobdia (2019) finds a negative association between audit firms' quality control system deficiencies, mainly performance-related, and audit quality. Further, several studies find that PCAOB inspections improve perceived or actual firm-wide audit quality (see, e.g., DeFond and Lennox, 2017; Fung et al., 2017; Gipper et al., 2019). The model of Gao and Zhang (2019) also features ex ante investment by auditors, termed "expertise," which is one interpretation of our audit quality variable, a_i.

Audit quality is observable in our setup, while in practice audit quality for a given engagement

[12] Knechel (2016) observes that "audit quality is generally defined as consisting of two important attributes: competence (expertise) and independence (objectivity)". A straightforward interpretation of our ex ante investment in quality a_i is that of competence/expertise. However, our ex ante quality choice also captures investments in culture, which would relate to independence/objectivity.

Moreover, a recent article in the *Financial Times*, (Masters, 2019), provides anecdotal evidence that audit quality investments get passed on to client firms (broadly, audit-firm-wide) as higher fees. It also mentions various ways that audit firms are investing in quality, including technology and human capital. KPMG UK's 2017 Annual Report that states, "We invest heavily in delivering and developing our audit offering and this includes significant investment in training and research and development on audit and assurance" (p. 62).

may not be observable even after the audit is done. In our setting, firm-wide audit quality, a_i, can represent reputation and skill based on prior engagements, the capacity to deliver high quality audits, and/or the average quality of auditors assigned to audits, who then make optimizing decisions to adjust audit quality. Although we abstract away from ex post adjustments in audit effort/quality, our model captures the ex ante skill and capacity of auditors, who will adjust optimally to the information they get in each client engagement. The idea that an auditor can adjust audit quality ex post implies both that the auditor is of sufficient quality to provide valuable effort, and that she possesses the ability, based on prior experience or training, to make judgments regarding audit scope, client risk factors, and her team's capacity and skill. In any model involving ex post costly audit effort, a certain measure of ex ante auditor quality is therefore implicit. Our focus is on this ex ante quality.

Cost of audit: One interpretation of the cost of audit quality and capacity, $kQ_{a,i}a_i^2/2$, is that an audit firm needs to hire and train enough high-quality auditors to service the market and it needs to do this before offering to audit client firms. Conditional on the size of the audit firm, quality improvements are increasingly costly as audit quality increases. While we do not model a labor market explicitly, our cost function captures in reduced form the scarcity of human capital necessary to carry out high quality audits. Duguay et al. (2019) provide evidence consistent with labor input scarcity being an important factor in audit markets.[13]

Price discrimination: Client business risk is non-contractible and auditors make the same offers to all clients. The inability to price discriminate is consistent with Liu and Simunic (2005), who argue that, "client type is unlikely to be contractible because a client's operating characteristics that determine its type are generally difficult to verify." In practice, when performing an audit, an auditor uses risk assessment procedures to assess the risk that material misstatement exists. However, this risk assessment is not perfect. Our model plausibly applies to a set of client firms that have received similar misstatement risk *assessments*. In other words, our model may be of one segment of audit clients, among whom business risk is difficult to gauge and price ex ante (see, e.g., the assumption in (7)). It may be that relevant client risks are only revealed after audits have been priced or that auditors face other constraints that inhibit price discrimination (e.g., standard

[13]Our linear-in-$Q_{a,i}$ cost of audit quality implies constant returns to scale, rather than economies of scale. Audit firms may benefit from economies of scale due to the ability to withstand or spread litigation risk (e.g., Bar-Yosef and Sarath, 2005).

hourly rates). We view price discrimination as an interesting extension for future work.

3 Main analysis

3.1 Analysis without NAS restrictions

We first study the equilibrium in which auditor 2 can provide NAS to all client firms. Note that the auditors rely on client firms to self-select into auditing services. The auditors cannot screen client firms, except through pricing and, for auditor 2, quality choices. Further, the consultant does not observe which client firms are audited by auditor 2. As a result, the consultant cannot price discriminate between the client firms audited by auditor 2 and the other client firms.

We solve the model by backward induction. Investors' decisions at $t = 4$ are reflected in the utilities of the client firms' investors in equation (3). So we start from client firms' decisions to buy services at $t = 3$.

First, we analyze the equilibrium in the NAS market. Auditor 2 is engaged in Bertrand competition with the consultant, who is a pure NAS provider. A client firm with business risk $1 - p_j$ buys NAS at a price F_c if and only if $(1 + p_j\theta)v \geq F_c$.

Lemma 1 *Without NAS restrictions, the equilibrium in the NAS market is as follows. Both auditor 2 and the consultant offer NAS at a price $F^*_{c,2} = F^*_{c,c} = 0$ as they engage in undifferentiated Bertrand competition.*

Lemma 1 implies that, absent NAS restrictions, the NAS market does not interact with the audit market.[14]

We now analyze the equilibrium in the audit market. Client firm A is high-risk and buys auditing services from the high-quality auditor 2. Similarly, client firm C is low-risk and buys auditing services from the low-quality auditor 1.

Client firm B has an intermediate business risk and buys audit services from auditor 1 only if

$$a_1(1 - p_B)I - F_{a,1} \geq 0.$$

[14] Note that introducing features modeled elsewhere, such as independence threats or knowledge spillovers, would break this independence.

Further, client firm B buys audit services from auditor 1 and not from auditor 2 only if

$$a_2(1-p_B)I - F_{a,2} < a_1(1-p_B)I - F_{a,1}.$$

Now, suppose auditor 1 sells auditing services to client firm C, and auditor 2 sells auditing services to client firms A and B. Auditor 1 sets audit fees $F_{a,1}$ such that client firm C is indifferent between buying auditing services from auditor 1 or not buying auditing services, i.e., $F^*_{a,1} = a_1(1-p_C)I$. Auditor 2 sets audit fees $F_{a,2}$ such that client firm B is indifferent between buying auditing services from auditor 2 or buying auditing services from auditor 1, i.e.,

$$F^*_{a,2} = (1-p_B)(a_2-a_1)I + F^*_{a,1} = (1-p_B)(a_2-a_1)I + a_1(1-p_C)I.$$

Given those fees, auditor 2's maximization problem is

$$\max_{a_2} 2(1-p_B)(a_2-a_1)I + 2a_1(1-p_C)I - ka_2^2.$$

Therefore, auditor 2 chooses audit quality $a_2^* = \frac{(1-p_B)I}{k}$, and auditor 2's expected utility is

$$U_{auditor,2} = 2F^*_{a,2} - ka_2^{*2} = \frac{(1-p_B)^2}{k}I^2 - 2a_1\left((1-p_B)-(1-p_C)\right)I.$$

Alternatively, suppose auditor 2 sells auditing services to client firm A, and auditor 1 sells auditing services to client firms B and C. As before, auditor 1 sets audit fees $F_{a,1}$ such that client firm C is indifferent between buying auditing services from auditor 1 or not buying auditing services, i.e., $F^*_{a,1} = a_1(1-p_C)I$. Auditor 2 sets audit fees $F_{a,2}$ such that client firm A is indifferent between buying auditing services from auditor 2 or buying auditing services from auditor 1, i.e.,

$$F^*_{a,2} = (1-p_A)(a_2-a_1)I + F^*_{a,1} = (1-p_A)(a_2-a_1)I + a_1(1-p_C)I.$$

Given those fees, auditor 2's maximization problem is

$$\max_{a_2}(1-p_A)(a_2-a_1)I + a_1(1-p_C)I - ka_2^2/2,$$

which implies that auditor 2's audit quality is $a_2^* = \frac{(1-p_A)I}{k}$. Auditor 2's expected utility is then

$$U_{auditor,2} = F_{a,2}^* - k a_2^{*2}/2 = \frac{(1-p_A)^2}{2k} I^2 - a_1 \left((1-p_A) - (1-p_C)\right) I.$$

We now compare the two subgames. Auditor 2 prefers to sell to client firms A and B rather than only to client firm A if and only if

$$\frac{(1-p_B)^2}{k} I^2 - 2a_1 \left((1-p_B) - (1-p_C)\right) I > \frac{(1-p_A)^2}{2k} I^2 - a_1 \left((1-p_A) - (1-p_C)\right) I, \quad (8)$$

which is equivalent to

$$k < \frac{2(1-p_B)^2 - (1-p_A)^2}{2a_1(2(1-p_B) - (1-p_A) - (1-p_C))} I. \quad (9)$$

In order to simplify the notation in the rest of the paper, we define the function

$$\bar{k}(x) \equiv \frac{2(1-p_B)^2 - (1-p_A)^2}{2a_1(2(1-p_B) - (1-p_A) - (1-p_C)) + x} I.$$

The assumption in (7) ensures that both the numerator and the denominator of $\bar{k}(x)$ are positive for any non-negative value of x. Condition (9) is then equivalent to $k < \bar{k}(0)$. At $t = 1$, via its choice of audit quality, auditor 2 chooses the subgame that will be played from $t = 2$ onwards.

The assumption in (4) ensures that auditor 1 prefers to sell to client firms B and C rather than to sell only to client firm B. This condition also implies that auditor 1 would rather sell to client firms B and C than just to client firm C.[15]

Lastly, we also need to check that auditor 2 is not better off selling auditing services to client firms A, B, and C rather than selling to client firms A and B. If auditor 2 sells auditing services to the three client firms, auditor 2 sets audit fees $F_{a,2}$ such that client firm C is indifferent between buying auditing services from auditor 2 or buying auditing services from auditor 1, i.e.,

$$F_{a,2}^* = (1-p_C)(a_2 - a_1)I + F_{a,1}^* = (1-p_C)(a_2 - a_1)I + k a_1^2/2.$$

[15]Formally, auditor 1 is better off selling audit services to both client firms B and C rather than selling auditing services only to client B, i.e.,

$$2a_1(1-p_C)I - k a_1^2 > a_1(1-p_B)I - k a_1^2/2,$$

which is equivalent to $k < \frac{2I}{a_1}\left(2(1-p_C) - (1-p_B)\right)$. This is always satisfied by the assumption in (4).

Given those fees, auditor 2's maximization problem is

$$\max_{a_2} 3(1-p_C)(a_2-a_1)I + 3ka_1^2/2 - 3ka_2^2/2.$$

Therefore, auditor 2 chooses audit quality $a_2^* = \frac{(1-p_C)I}{k}$, and auditor 2's expected utility is

$$U_{auditor,2} = F_{a,2}^* - ka_2^{*2}/2 = \frac{3(1-p_C)^2}{2k}I^2 - 3a_1\left((1-p_C)I - ka_1/2\right).$$

Auditor 2 is better off selling to client firms A and B only rather than selling to all three client firms if

$$\frac{(1-p_B)^2}{k}I^2 - 2a_1\left((1-p_B)-(1-p_C)\right)I > \frac{3(1-p_C)^2}{2k}I^2 - 3a_1\left((1-p_C)I - ka_1/2\right),$$

which is satisfied by the assumption in (6). The following proposition summarizes our findings thus far regarding the audit market.

Proposition 1 *Without NAS restrictions, the equilibrium in the audit market is as follows:*

- *If $k < \bar{k}(0)$, then auditor 2 sells auditing services to client firms A and B, and auditor 1 sells auditing services to client firm C. Auditor 1 sets audit fees $F_{a,1}^* = a_1(1-p_C)I$. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_B)I}{k}$ and sets audit fees $F_{a,2}^* = \frac{(1-p_B)^2I^2}{k} - a_1((1-p_B)-(1-p_C))I$.*
- *Otherwise, if $k > \bar{k}(0)$, then auditor 2 sells auditing services to client firm A and auditor 1 sells auditing services to client firms B and C. Auditor 1 sets audit fees $F_{a,1}^* = a_1(1-p_C)I$. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_A)I}{k}$ and sets audit fees $F_{a,2}^* = \frac{(1-p_A)^2I^2}{k} - a_1((1-p_A)-(1-p_C))I$.*

Auditor 2 faces a tradeoff between selling auditing services only to one client firm at a higher price, and selling auditing services to two client firms at a lower price. Recall that auditor 2's cost of providing auditing services increases in the number of audit clients, and that the client firms are not too heterogeneous (assumption in (7)). Hence, if the cost of audit is low, auditor 2 is better off selling auditing services to both client firms A and B. On the contrary, if the cost of audit is high, auditor 2 is better off selling auditing services only to client firm A with a higher audit quality at a higher fee. The following corollary is a direct consequence of Proposition 1.

Corollary 1 *Auditor 2's audit quality a_2^* and auditor 2's number of audit clients $Q_{a,2}^*$ are negatively correlated, in that variation in audit costs causes opposite-signed changes in a_2^* and $Q_{a,2}^*$.*

A low cost of audit quality implies auditor 2 covers more clients but at a lower audit quality. On the contrary, a high cost of audit quality implies the opposite. Note that the assumption in (5) guarantees that auditor 2 has always a higher audit quality than auditor 1.

Before analyzing the equilibrium with NAS restrictions, we compare total welfare absent NAS restrictions when auditor 2 provides auditing services only to client firm A, and when auditor 2 sells audits to client firms A and B. Total welfare in the economy is defined as the sum of the expected utilities of the three clients firms, the two auditors, and the NAS competitor:

$$W = U_{client,A} + U_{client,B} + U_{client,C} + U_{auditor,1} + U_{auditor,2} + U_{consultant}. \tag{10}$$

If the audit cost is low, i.e., $k < \bar{k}(0)$, then auditor 2 sells auditing services to client firms A and B, and auditor 1 sells auditing services to client firm C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_B)I}{k}$. Hence, auditors' expected utilities are

$$U_{auditor,1} = F_{a,1}^* - ka_1^2/2 = a_1(1-p_C)I - ka_1^2/2, \tag{11}$$

$$U_{auditor,2} = 2F_{a,2}^* - ka_2^{*2} = \frac{(1-p_B)^2}{k}I^2 - 2a_1\left((1-p_B)-(1-p_C)\right)I, \tag{12}$$

and the expected utility of the consultant engaged in Bertrand competition is $U_{consultant} = 0$. The expected utility of client firm j is given in equation (3). As result, the total welfare is

$$\begin{aligned} W \quad = \quad & \overbrace{3(pR-I)}^{\text{investment return}} + \overbrace{(1+p_A\theta)v + (1+p_B\theta)v + (1+p_C\theta)v}^{\text{NAS value for clients } A,\, B, \text{ and } C \text{ respectively}} + \overbrace{\frac{(1-p_B)(1-p_A)}{k}I^2}^{\text{audit value for client } A} \\ & + \underbrace{\frac{(1-p_B)^2}{k}I^2 - k\left(\frac{(1-p_B)I}{k}\right)^2}_{=0,\text{ audit value for client } B \text{ minus auditor 2's audit cost}} + \underbrace{a_1(1-p_C)I}_{\text{audit value for client } C} - \underbrace{ka_1^2/2}_{\text{auditor 1's audit cost}} . \end{aligned}$$

Otherwise, if the audit cost is high, i.e., if $k > \bar{k}(0)$, then auditor 2 sells auditing services to client firm A and auditor 1 sells auditing services to client firms B and C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_A)I}{k}$. Hence, the auditors' expected utilities are

$$\begin{aligned} U_{auditor,1} &= 2F_{a,1}^* - ka_1^2 = 2a_1(1-p_C) - ka_1^2, && (13) \\ U_{auditor,2} &= F_{a,2}^* - ka_2^{*2}/2 = \frac{(1-p_A)^2}{2k} I^2 - a_1\left((1-p_A) - (1-p_C)\right) I, && (14) \end{aligned}$$

and the expected utility of the consultant engaged in Bertrand competition is $U_{consultant} = 0$. The expected utility of client firm j is given in equation (3). As result, the total welfare is

$$\begin{aligned} W \quad = \quad & \overbrace{3(pR-I)}^{\text{investment return}} + \overbrace{(1+p_A\theta)v + (1+p_B\theta)v + (1+p_C\theta)v}^{\text{NAS value for clients } A,\, B, \text{ and } C \text{ respectively}} + \overbrace{a_1(1-p_B)I}^{\text{audit value for client } B} \\ & + \underbrace{a_1(1-p_C)I}_{\text{audit value for client } C} + \underbrace{\frac{(1-p_A)^2 I^2}{2k}}_{\text{audit value for client } A \text{ minus auditor 2's audit cost}} - \underbrace{ka_1^2}_{\text{auditor 1's audit cost}} . \end{aligned}$$

Corollary 2 *The total welfare is higher when auditor* 2 *sells auditing services to client firms A and B compared to when auditor* 2 *sells only auditing services to client firm A if and only if*

$$1 - p_B > \frac{1}{2I}\left((1-p_A)I + ka_1\right). \tag{15}$$

Further, if $k > \max\left(\bar{k}(0), (2(1-p_B) - (1-p_A))I/a_1\right)$ *or* $k < \min\left(\bar{k}(0), (2(1-p_B) - (1-p_A))I/a_1\right)$, *then auditor* 2 *serves the set of clients that maximizes total welfare.*

If client B's business risk is sufficiently high, then total welfare is higher if auditor 2 sells auditing services to client firm B, as auditor 2 has a higher audit quality than auditor 1. If the audit cost is sufficiently low, both conditions (9) and (15) are satisfied, which means that auditor 2 sells auditing services to clients A and B, and this choice maximizes total welfare. On the contrary, if the audit cost is sufficiently large, both conditions (9) and (15) are violated, which means that auditor 2 sells auditing services only to client A and this choice maximizes total welfare. In the event of mismatch between the welfare-maximizing allocation of clients and auditor 2's preferences (i.e., if client B's business risk and audit costs are both low), then Corollary 2 provides a potential rationale for mandating changes to audit coverage.

3.2 Analysis with NAS restrictions to audit clients

We now study the equilibrium in which auditor 2 cannot provide NAS to its audit clients. First, assume that there is a positive correlation between NAS demand and audit demand, i.e., $\theta \in [-1, 0]$. If auditor 2 provides auditing services to client firms A and B, the consultant sets NAS fees $F^*_{c,c} = (1 + p_B\theta)v$ and sells NAS to client firms A and B.[16] Similarly, auditor 2 sets NAS fees $F^*_{c,2} = (1 + p_C\theta)v$ and sells NAS to client firm C. Auditor 2's expected utility is then

$$U_{auditor,2} = 2F^*_{a,2} + F^*_{c,2} - ka_2^{*2} = \frac{(1 - p_B)^2}{k} I^2 - 2a_1 \left((1 - p_B) - (1 - p_C)\right) I + (1 + p_C\theta)v$$

and the expected utility of the consultant is $U_{consultant} = 2F^*_{c,c} = 2(1 + p_B\theta)v$.

Otherwise, if auditor 2 sells auditing services only to client firm A, the consultant sets NAS fees $F^*_{c,c} = (1+p_A\theta)v$ and sells NAS to client firm A. Similarly, auditor 2 sets NAS fees $F^*_{c,2} = (1+p_C\theta)v$ and sells NAS to client firms B and C.[17] Auditor 2's expected utility is

$$U_{auditor,2} = F^*_{a,2} + 2F^*_{c,2} - ka_2^{*2}/2 = \frac{(1 - p_A)^2}{2k} I^2 - a_1 \left((1 - p_A) - (1 - p_C)\right) I + 2(1 + p_C\theta)v,$$

and the expected utility of the consultant is $U_{consultant} = F^*_{c,c} = (1 + p_A\theta)v$.

We now compare the two subgames. Auditor 2 prefers to sell to client firms A and B rather than only to client firm A if and only if

$$\begin{aligned} &\frac{(1 - p_B)^2}{k} I^2 - 2a_1 \left((1 - p_B) - (1 - p_C)\right) I + (1 + p_C\theta)v \\ &\qquad > \frac{(1 - p_A)^2}{2k} I^2 - a_1 \left((1 - p_A) - (1 - p_C)\right) I + 2(1 + p_C\theta)v, \end{aligned} \tag{16}$$

which is equivalent to

$$k < \frac{2(1 - p_B)^2 - (1 - p_A)^2}{2a_1(2(1 - p_B) - (1 - p_A) - (1 - p_C)) + 2(1 + p_C\theta)v} I = \bar{k}(2(1 + p_C\theta)v). \tag{17}$$

[16] Note that the consultant is better off selling NAS to both client firms A and B rather than selling NAS only to client firm A. This is true if $(1 + p_A\theta)v < 2(1 + p_B\theta)v$, which is true if $2p_B - p_A < 1$. This last inequality is satisfied by the assumption in (7): $2(1 - p_B)^2 > (1 - p_A)^2$ implies $2(1 - p_B) > (1 - p_A)$.

[17] Note that auditor 2 is better off selling NAS to both client firms B and C rather than selling NAS to client firm B only. This is true if $(1 + p_B\theta)v < 2(1 + p_C\theta)v$, which is true if $2p_C - p_B < 1$. This last inequality is satisfied by the assumption in (7).

Next, assume that there is a negative correlation between NAS demand and audit demand, i.e., $\theta \in (0, \infty)$. If auditor 2 provides auditing services to client firms A and B, the consultant sets NAS fees $F^*_{c,c} = (1 + p_B\theta)v$ and sells NAS only to client firm B. Similarly, auditor 2 sets NAS fees $F^*_{c,2} = (1 + p_B\theta)v$ and sells NAS to client firm C. Auditor 2's expected utility is then

$$U_{auditor,2} = 2F^*_{a,2} + F^*_{c,2} - ka^{*2}_2 = \frac{(1-p_B)^2}{k} I^2 - 2a_1\left((1-p_B) - (1-p_C)\right) I + (1 + p_B\theta)v$$

and the expected utility of the consultant is $U_{consultant} = F^*_{c,c} = (1 + p_B\theta)v$.

This is true as long as the consultant is better off selling NAS only to client firm B rather than selling NAS to both client firms A and B, i.e., as long as $(p_B - 2p_A)\theta > 1$. If $(p_B - 2p_A)\theta < 1$, then the consultant sets NAS fees $F^*_{c,c} = (1 + p_A\theta)v$ and sells NAS to client firms A and B. Similarly, auditor 2 sets NAS fees $F^*_{c,2} = (1 + p_A\theta)v$ and sells NAS to client firm C. Auditor 2's expected utility is then

$$U_{auditor,2} = 2F^*_{a,2} + F^*_{c,2} - ka^{*2}_2 = \frac{(1-p_B)^2}{k} I^2 - 2a_1\left((1-p_B) - (1-p_C)\right) I + (1 + p_A\theta)v$$

and the expected utility of the consultant is $U_{consultant} = 2F^*_{c,c} = 2(1 + p_A\theta)v$.

Otherwise, if auditor 2 sells auditing services to client firms A only, the consultant sets NAS fees $F^*_{c,c} = (1+p_A\theta)v$ and sells NAS to client firm A. Similarly, auditor 2 sets NAS fees $F^*_{c,2} = (1+p_A\theta)v$ and sells NAS to client firms B and C. Auditor 2's expected utility is

$$U_{auditor,2} = F^*_{a,2} + 2F^*_{c,2} - ka^{*2}_2/2 = \frac{(1-p_A)^2}{2k} I^2 - a_1\left((1-p_A) - (1-p_C)\right) I + 2(1 + p_A\theta)v,$$

and the expected utility of the consultant is $U_{consultant} = F^*_{c,c} = (1 + p_A\theta)v$.

We now compare the two subgames. If $(p_B - 2p_A)\theta > 1$, auditor 2 prefers to sell to client firms A and B rather than only to client firm A if and only if

$$\begin{aligned}&\frac{(1-p_B)^2}{k} I^2 - 2a_1\left((1-p_B) - (1-p_C)\right) I + (1 + p_B\theta)v \\ &\qquad > \frac{(1-p_A)^2}{2k} I^2 - a_1\left((1-p_A) - (1-p_C)\right) I + 2(1 + p_A\theta)v, \qquad (18)\end{aligned}$$

which is equivalent to

$$k < \frac{2(1-p_B)^2 - (1-p_A)^2}{2a_1(2(1-p_B)-(1-p_A)-(1-p_C)) + 2(1+(2p_A-p_B)\theta)v} I = \bar{k}(2(1+(2p_A-p_B)\theta)v). \quad (19)$$

For simplicity, we assume that θ is not too large so that the denominator in (19) stays positive.[18]

Otherwise, if $(p_B - 2p_A)\theta < 1$, auditor 2 prefers to sell to client firms A and B rather than only to client firm A if and only if

$$\begin{aligned}\frac{(1-p_B)^2}{k} I^2 - 2a_1\left((1-p_B)-(1-p_C)\right) I + (1+p_A\theta)v \\ > \frac{(1-p_A)^2}{2k} I^2 - a_1\left((1-p_A)-(1-p_C)\right) I + 2(1+p_A\theta)v, \quad (20)\end{aligned}$$

which is equivalent to

$$k < \frac{2(1-p_B)^2 - (1-p_A)^2}{2a_1(2(1-p_B)-(1-p_A)-(1-p_C)) + 2(1+p_A\theta)v} I = \bar{k}(2(1+p_A\theta)v). \quad (21)$$

Our results derived above are summarized in the following proposition.

Proposition 2 *With NAS restrictions to audit clients, the equilibrium is as follows.*

- *If $\theta \in [-1, 0]$, then audit and NAS demand are positively correlated and*
  - *if $k < \bar{k}(2(1+p_C\theta)v)$, then auditor 2 sells auditing services to clients A and B, and NAS to client C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_B)I}{k}$ and sets $F_{c,2}^* = (1+p_C\theta)v$. The consultant sets $F_{c,c}^* = (1+p_B\theta)v$ and sells NAS to clients A and B;*
  - *otherwise, if $k > \bar{k}(2(1+p_C\theta)v)$, then auditor 2 sells auditing services to client A and NAS to clients B and C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_A)I}{k}$ and sets $F_{c,2}^* = (1+p_C\theta)v$. The consultant sets $F_{c,c}^* = (1+p_A\theta)v$ and sells NAS to client A.*
- *If $\theta > 0$ and if $(p_B - 2p_A)\theta < 1$, then audit and NAS demand are weakly negatively correlated and*

[18] Formally, $2a_1(2(1-p_B)-(1-p_A)-(1-p_C)) + 2(1+(2p_A-p_B)\theta)v > 0$ is equivalent to $\theta < \frac{a_1(2(1-p_B)-(1-p_A)-(1-p_C))+v}{(p_B-2p_A)v}$.

- *if $k < \bar{k}(2(1+p_A\theta)v)$, then auditor 2 sells auditing services to clients A and B, and NAS to client C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_B)I}{k}$ and sets $F_{c,2}^* = (1+p_A\theta)v$. The consultant sets $F_{c,c}^* = (1+p_A\theta)v$ and sells NAS to clients A and B;*
- *otherwise, if $k > \bar{k}(2(1+p_A\theta)v)$, then auditor 2 sells auditing services to client A and NAS to clients B and C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_A)I}{k}$ and sets $F_{c,2}^* = (1+p_A\theta)v$. The consultant sets $F_{c,c}^* = (1+p_A\theta)v$ and sells NAS to client A.*

- *If $\theta > 0$ and if $(p_B - 2p_A)\theta \geq 1$, then audit and NAS demand are strongly negatively correlated and*
  - *if $k < \bar{k}(2(1+(2p_A-p_B)\theta)v)$, then auditor 2 sells auditing services to clients A and B, and NAS to client C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_B)I}{k}$ and sets $F_{c,2}^* = (1+p_B\theta)v$. The consultant sets $F_{c,c}^* = (1+p_B\theta)v$ and sells NAS to client B;*
  - *otherwise, if $k > \bar{k}(2(1+(2p_A-p_B)\theta)v)$, then auditor 2 sells auditing services to client A and NAS to clients B and C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_A)I}{k}$ and sets $F_{c,2}^* = (1+p_A\theta)v$. The consultant sets $F_{c,c}^* = (1+p_A\theta)v$ and sells NAS to client A.*

Recall from Proposition 1 that auditor 2 faces a tradeoff between selling auditing services only to one client firm at a higher price, and selling auditing services to two client firms at a lower price. Without NAS restrictions, auditor 2 earns no rent in the NAS market as auditor 2 engages in Bertrand competition with the consultant. However, with NAS restrictions, auditor 2 earns NAS rents and auditor 2's choice between selling audit services only to client A and selling audit services to clients A and B depends on auditor 2's NAS rent.

Proposition 2 implies that regulatory bans on the provision of NAS to audit clients can make auditor 2 and the NAS competitor better off. Essentially, such restrictions create niches in which each supplier earns monopoly rents where before the rents were dissipated by Bertrand competition.

We now compare audit quality and total welfare between regimes without NAS restrictions and with NAS restrictions to audit clients. If $\theta \in [-1, 0]$, the threshold on k is smaller with NAS restrictions to audit clients, i.e., $\bar{k}(2(1+p_C\theta)v) < \bar{k}(0)$. Therefore, if $\bar{k}(2(1+p_C\theta)v) < k < \bar{k}(0)$, then NAS restrictions to audit clients increase audit quality.

Similarly, if $\theta > 0$ and if $(p_B - 2p_A)\theta < 1$, the threshold on k is smaller with NAS restrictions to audit clients, i.e.,

$$\bar{k}(2(1+p_A\theta)v) < \bar{k}(0).$$

Therefore, if $\bar{k}(2(1+p_A\theta)v) < k < \bar{k}(0)$, then NAS restrictions to audit clients increase audit quality.

Otherwise, if $\theta > 0$ and if $(p_B - 2p_A)\theta \geq 1$, the threshold on k is larger with NAS restrictions to audit clients, i.e., $\bar{k}(2(1+(2p_A-p_B)\theta)v) > \bar{k}(0)$. Therefore, if $\bar{k}(2(1+(2p_A-p_B)\theta)v) > k > \bar{k}(0)$, then NAS restrictions to audit clients decrease audit quality.

Corollary 3 *The impact of NAS restrictions to audit clients on auditor 2's audit quality depends on θ.*

- *If $\theta \in [-1, 0]$ or $(p_B - 2p_A)\theta < 1$, then NAS restrictions to audit clients increase auditor 2's audit quality a_2^*;*
- *otherwise, if $\theta > 0$ and if $(p_B - 2p_A)\theta \geq 1$, then NAS restrictions to audit clients decrease auditor 2's audit quality a_2^*.*

The effects of NAS restrictions to audit clients on auditor 2's audit quality depend on the correlation between the NAS and audit demand. With a positive or a weak negative correlation between NAS demand and audit demand, auditor 2 has incentives to increase audit quality and to sell audit services only to client firm A. This increases auditor 2's number of NAS clients and increases auditor 2's NAS rents. Therefore, NAS restrictions to audit clients increase audit quality.

Otherwise, if there is a strong negative correlation between NAS demand and audit demand, NAS restrictions to audit clients change auditor 2's incentives and can encourage it to supply lower-quality audits more broadly. This may increase the market niche in which the NAS competitor is a monopolist supplier of NAS, which may increase the price of the NAS competitor's offering. This price in turn sets a ceiling on the price that auditor 2 can charge for NAS to non-audit clients, such that auditor 2 benefits from the NAS competitor's price increase. Therefore, NAS restrictions to audit clients decrease audit quality.

Next, let us compare total welfare absent NAS restrictions and with NAS restrictions to audit clients. If there is a positive or a weak negative correlation between NAS demand and audit demand, i.e., $\theta > 0$ and if $(p_B - 2p_A)\theta < 1$, then all client firms buy NAS, whether audits of NAS clients are restricted or not. Hence, as in Corollary 2, the welfare difference between regimes is driven by auditor 2's audit coverage. Otherwise, if there is a strong negative correlation, i.e., $\theta > 0$ and if $(p_B - 2p_A)\theta \geq 1$, the welfare difference between regimes is driven by auditor 2's audit coverage and the fact that client firm A may not buy NAS with NAS restrictions to audit clients.

Corollary 4 *The welfare comparisons are as follows.*

- *If $\theta \in [-1, 0]$, then*
  - *If $k < \bar{k}(2(1+p_C\theta)v)$ or $k > \bar{k}(0)$, then total welfare is the same absent NAS restrictions and with NAS restrictions to audit clients.*
  - *Otherwise, if $\bar{k}(2(1+p_C\theta)v) < k < \bar{k}(0)$, then total welfare with NAS restrictions to audit clients is higher than total welfare without NAS restrictions if and only if*

$$1 - p_B < \frac{1}{2I}\left((1 - p_A)I + ka_1\right). \tag{22}$$

- *If $\theta > 0$ and if $(p_B - 2p_A)\theta < 1$, then*
  - *If $k < \bar{k}(2(1+p_A\theta)v)$ or $k > \bar{k}(0)$, then total welfare is the same absent NAS restrictions and with NAS restrictions to audit clients.*
  - *Otherwise, if $\bar{k}(2(1+p_A\theta)v) < k < \bar{k}(0)$, then total welfare with NAS restrictions to audit clients is higher than total welfare without NAS restrictions if and only if*

$$1 - p_B < \frac{1}{2I}\left((1 - p_A)I + ka_1\right). \tag{23}$$

- *If $\theta > 0$ and if $(p_B - 2p_A)\theta \geq 1$, then*
  - *If $k < \bar{k}(0)$, then total welfare is lower with NAS restrictions to audit clients than absent NAS restrictions.*
  - *Otherwise, if $\bar{k}(0) < k < \bar{k}(2(1+(2p_A-p_B)\theta)v)$, then total welfare with NAS restrictions to audit clients is higher than total welfare without NAS restrictions if and only if*

$$1 - p_B > \frac{2k(1+p_A\theta)v + (1-p_A)^2I^2 - k^2a_1^2}{2(1-p_A)I^2 - 2ka_1I}. \tag{24}$$

– *Otherwise, if* $\bar{k}(2(1+(2p_A-p_B)\theta)v) < k$*, then total welfare is the same absent NAS restrictions and with NAS restrictions to audit clients.*

First, for large values of the audit cost, total welfare is the same absent NAS restrictions and with NAS restrictions to audit clients. A large value of the audit cost implies that NAS restrictions to audit clients do not change audit coverage. In that case, auditor 2 only sells auditing services to client firm A and all three client firms buy NAS.

For small values of the audit cost, total welfare is weakly lower with NAS restrictions to audit clients. A low value of the audit cost implies that NAS restrictions to audit clients do not change audit coverage, and auditor 2 sells auditing services to client firms A and B. If there is a strong negative correlation between NAS demand and audit demand, welfare is lower with NAS restrictions to audit clients because auditor 2 sells NAS only to client firm B.

Otherwise, for intermediate values of the audit cost, NAS restrictions to audit clients may change audit coverage. On the one hand, if there is a positive or a weak negative correlation between NAS and audit demand, NAS restrictions to audit clients imply that client firm B becomes auditor 1's audit client. As a result, as already shown in Corollary 2, total welfare is higher with NAS restrictions to audit clients if and only if client B's business risk is sufficiently low. On the other hand, if there is a strong negative correlation between NAS and audit demand, NAS restrictions to audit clients imply that client firm B becomes auditor 2's audit client. As a result, total welfare is higher with NAS restrictions to audit clients if and only if client B's business risk is sufficiently high.

3.3 Analysis with NAS restrictions to all clients

Assume now that the regulator bans the provision of NAS by auditors to any client. The equilibrium in the audit market is similar to the equilibrium in the audit market when there is no NAS restrictions at all. Therefore, compared to the case in which the regulator bans the provision of NAS to audit clients only, a regulatory ban on the provision of NAS to all clients may increase or decrease audit quality.

In the NAS market, the consultant is now a monopolist. If there is a positive correlation between NAS demand and audit demand, i.e., if $\theta \in [-1, 0]$, then there are two possible cases. First, the consultant may sell NAS to all three client firms and the NAS fees are $F^*_{c,c} = (1+p_C\theta)v$. Otherwise, the consultant may sell NAS to client firms A and B and the NAS fees are $F^*_{c,c} = (1+p_B\theta)v$.[19]

Otherwise, if there is a negative correlation between NAS demand and audit demand, i.e., if $\theta > 0$, then there are three possible cases. First, the consultant may sell NAS to all three client firms and the NAS fees are $F^*_{c,c} = (1+p_A\theta)v$. Second, the consultant may sell NAS to client firms B and C and the NAS fees are $F^*_{c,c} = (1+p_B\theta)v$. Third, the consultant may sell NAS to client firm C and the NAS fees are $F^*_{c,c} = (1+p_C\theta)v$

Proposition 3 *With NAS restrictions to all clients, the equilibrium in the audit market is the same as in the case without NAS restrictions. In the NAS market, the consultant is a monopolist and may sell NAS to one, two or all three clients firms.*

Lastly, we analyze the impact of NAS restrictions to all clients relative to the cases absent NAS restrictions and with NAS restrictions to audit clients.

Corollary 5 *Auditor 2's audit quality is the same absent NAS restrictions and with NAS restrictions to all clients. Relative to the case with NAS restrictions to audit clients, the impact of NAS restrictions to all clients on auditor 2's audit quality depends on θ.*

- *If $\theta \in [-1, 0]$ or $(p_B - 2p_A)\theta < 1$, then NAS restrictions to all clients decrease auditor 2's audit quality a^*_2;*
- *otherwise, if $\theta > 0$ and if $(p_B - 2p_A)\theta \geq 1$, then NAS restrictions to audit clients increase auditor 2's audit quality a^*_2.*

The equilibrium audit quality, a^*_2, is the same whether auditor-provided NAS are unrestricted or completely banned. The latter part of Corollary 5 (comparing NAS restrictions to all clients versus just to audit clients) and related intuition thus follow directly from Corollary 3. The next Corollary compares total welfare absent NAS restrictions to welfare with NAS restrictions to all clients.

Corollary 6 *Total welfare is higher without NAS restrictions than with NAS restrictions to all clients.*

[19] The consultant is always better off selling NAS to client firms A and B rather than only selling NAS to client A. See footnote 16.

Recall that the equilibrium in the audit market is the same without NAS restrictions and with NAS restrictions to all clients. In the NAS market, all three client firms buy NAS without NAS restrictions. With NAS restrictions to all clients, the consultant is a monopolist and may sell NAS to one, two or all three client firms. As result, total welfare is always weakly larger without NAS restrictions. Lastly, relative to NAS restrictions to audit clients, NAS restrictions to all clients may increase or decrease total welfare. The difference in total welfare is driven by the change in audit coverage and the number of NAS clients.

4 Extensions

4.1 Additional competition in the NAS market

In the main text, auditor 2 has only one competitor in the NAS market. In this extension, we study the impact of additional competition in the NAS market on auditor 2's audit quality. For brevity, we do not make welfare comparisons as we have already shown how a change in auditor 2's audit quality affects total welfare in Corollary 2.

Assume now that there are two NAS competitors in the NAS market: consultant 1 and consultant 2. If there is undifferentiated Bertrand competition, NAS fees are $F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$ whatever the NAS restrictions. As a result, without differentiation in the NAS market, there is no interaction between the audit market and the NAS market. We therefore introduce NAS market differentiation in two different ways. First, we assume that both auditor 2 and consultant 2 have a competitive advantage in the NAS market (alternatively, that consultant 1 has a competitive disadvantage). Second, we assume that only auditor 2 has a competitive advantage in the NAS market.

4.1.1 Competitive advantage for auditor 2 and a NAS competitor

We assume that the value of NAS for client firm B is $(1 + p_B\theta)v$ if NAS are provided by consultant 1, and $\phi(1 + p_B\theta)v$ if NAS are provided by consultant 2 or auditor 2, with $\phi > 1$ capturing quality differentiation in the NAS market. Otherwise, the value of NAS for client firm A is $(1 + p_A\theta)v$, and the value of NAS for client firm C is $(1 + p_C\theta)v$ regardless of NAS provider. Without NAS restrictions to audit clients, there is Bertrand competition in the NAS market, which implies that

$F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$.

Suppose now that there are NAS restrictions to audit clients. First, assume that there is a positive correlation between audit demand and NAS demand, i.e., $\theta \in [-1, 0]$. If auditor 2 sells auditing services only to client firm A, then Bertrand competition in the NAS market implies $F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$. Indeed, the consultants are engaged in Bertrand competition in selling NAS to client firms A and C. Similarly, auditor 2 and consultant 2 are engaged in Bertrand competition in selling NAS to client firm B. This drives NAS fees down to zero.

Next, assume that auditor 2 sells auditing services to client firms A and B. Client firm B prefers to buy NAS from consultant 2 rather than consultant 1 if and only if

$$\phi(1 + p_B\theta)v - F_{c,c2} > (1 + p_B\theta)v - F_{c,c1}.$$

If the value of NAS provided by consultant 2 is lower for client firm A than for client firm B, i.e, $(1 + p_A\theta)v < \phi(1 + p_B\theta)v$, then $F^*_{c,c1} = (1 + p_A\theta)v$, $F^*_{c,c2} = \phi(1 + p_B\theta)v$ and $F^*_{c,2} = (1 + p_C\theta)v$. Otherwise, if the value of NAS provided by consultant 2 is greater for client firm A than for client firm B, i.e. $(1 + p_A\theta)v > \phi(1 + p_B\theta)v$, then Bertrand competition in the NAS market implies $F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$.

Now, suppose that there is a negative correlation between audit demand and NAS demand, i.e., $\theta \in (0, \infty)$. If auditor 2 sells auditing services only to client firm A, then Bertrand competition in the NAS market implies $F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$. Otherwise, if auditor 2 sells auditing services to client firms A and B, then consultant 1 sells NAS to client firm A at a price $F^*_{c,c1} = (1 + p_A\theta)v$, consultant 2 sells NAS to client firm B at a price $F^*_{c,c2} = (\phi - 1)(1 + p_B\theta)v + (1 + p_A\theta)v$, and auditor 2 sells NAS to client firm C at a price $F^*_{c,2} = (1 + p_A\theta)v$. Therefore, compared to the equilibrium without NAS restrictions, auditor 2 has incentives to choose broader coverage and lower audit quality in order to capture NAS rents whatever the correlation between NAS and audit demand. The following proposition summarizes our findings.

Proposition 4 *If there are two NAS competitors, and both auditor* 2 *and consultant* 2 *have a competitive advantage in the NAS market, NAS restrictions to audit clients decrease auditor* 2*'s audit quality,* a^*_2.

4.1.2 Competitive advantage for auditor 2

We now assume that the value of NAS provided by auditor 2 for client firm j is $\phi(1+p_j\theta)v$, whereas the value of NAS provided by consultant 1 or consultant 2 for client firm j is $(1+p_j\theta)v$, where $\phi > 1$. Hence, auditor 2 has a competitive advantage in the NAS market.

First, note that Bertrand competition in the NAS market between consultant 1 and consultant 2 implies that $F^*_{c,c1} = F^*_{c,c2} = 0$. As a result, client firm j buys NAS from auditor 2 if and only if

$$\phi(1 + p_j\theta)v - F_{c,2} > (1 + p_j\theta)v.$$

Without NAS restrictions to audit clients and if there is a positive correlation between NAS demand and audit demand, auditor 2 may sell NAS to all three client firms at a fee $F^*_{c,2} = (\phi - 1)(1 + p_C\theta)v$, or sell NAS to client firms A and B at a fee $F^*_{c,2} = (\phi - 1)(1 + p_B\theta)v$.[20] Otherwise, if there is a negative correlation between NAS demand and audit demand, auditor 2 may sell NAS to the three clients firms at a fee $F^*_{c,2} = (\phi - 1)(1 + p_A\theta)v$, sell NAS to client firms B and C at a fee $F^*_{c,2} = (\phi - 1)(1 + p_B\theta)v$, or sell NAS only to client firm C at a fee $F^*_{c,2} = (\phi - 1)(1 + p_C\theta)v$.

With NAS restrictions to audit clients and a positive correlation between NAS demand and audit demand, if auditor 2 only sells audit services to client A, then auditor 2 sells NAS to client firms B and C at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$.[21] Otherwise, if auditor 2 sells audit services to both clients A and B, then auditor 2 sells NAS to client firm C at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$. With a negative correlation between NAS demand and audit demand, if auditor 2 only sells audit services to client A, auditor 2 may sell NAS to client firms B and C at a fee $F^*_{c,2} = (\phi-1)(1+p_B\theta)v$, or sell NAS only to client firm C at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$. Otherwise, if auditor 2 sells audit services to both clients A and B, then auditor 2 sells NAS to client firm C at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$. As a result, NAS restrictions to audit clients may increase audit quality. Our results derived above are summarized in the following proposition.

Proposition 5 *If there are two NAS competitors and auditor 2 has a competitive advantage in the NAS market, NAS restrictions to audit clients increase auditor 2's audit quality, a^*_2.*

[20] Auditor 2 is always better off selling NAS to client firms A and B rather than only selling NAS to client A. See footnote 16.

[21] Note that auditor 2 is better off selling NAS to both client firms B and C rather than selling NAS to client firm B only. See footnote 17.

4.2 Auditing standards

In practice, regulators may also intervene in the audit market by setting auditing standards, which set a floor on audit quality. In this extension, we study the impact of setting auditing standards in an environment in which auditors face competition both in the audit market and the NAS market. To that end, we derive comparative statics with respect to auditor 1's audit quality a_1, with the idea that stricter audit standards would force auditor 1's quality to increase.

We first analyze the impact of an increase in a_1 on audit fees and auditor 2's audit quality a_2^*.

Corollary 7 *Regardless of NAS restrictions, the comparative statics on audit fees and auditor 2's audit quality with respect to a_1 are as follows:*

- *Auditor 1's audit fees $F_{a,1}^*$ increase in a_1;*
- *for a given auditor 2's audit coverage, auditor 2's audit fees $F_{a,2}^*$ decrease in a_1. However, $\bar{k}(x)$ decreases in a_1, such that an increase in a_1 may change auditor 2's audit coverage from clients A and B to client A only. In that case, audit quality a_2^* and audit fees $F_{a,2}^*$ both increase.*

Corollary 7 shows that an increase in auditing standards leads to an increase in audit fees charged by low-quality audit firms and, for a given audit coverage, a decrease in audit fees charged by high-quality audit firms. This effect occurs through a decrease in the differentiation between the low- and high-quality auditors' services. However, an increase in auditing standards may change audit coverage, as intermediate-risk firms find lower-quality auditors' services more attractive. This would then increase high-quality audit firms' audit quality and audit fees, as their services would then be directed more precisely at higher-risk firms who place a higher value on audit quality. We now analyze the impact of a_1 on total welfare.

Corollary 8 *Regardless of the NAS restrictions, the comparative statics on total welfare with respect to a_1 are as follows.*

- *If auditor 2 sells auditing services to client firms A and B, then*
  - *if $a_1 < \frac{(1-p_C)I}{k}$, then total welfare increases in a_1;*

- *otherwise, if* $a_1 > \frac{(1-p_C)I}{k}$, *then total welfare decreases in* a_1.

- *If auditor* 2 *only sells auditing services to client firm* A, *then*

  - *if* $a_1 < \frac{(1-p_B)+(1-p_C)}{2k}I$, *then total welfare increases in* a_1;
  - *otherwise, if* $a_1 > \frac{(1-p_B)+(1-p_C)}{2k}I$, *then total welfare decreases in* a_1.

Corollary 8 implies that, for a given audit coverage, there are optimal auditing standards that maximize total welfare: if auditor 2 sells auditing services to client firms A and B, then $a_1^* = \frac{(1-p_C)I}{k}$, whereas if auditor 2 only sells auditing services to client firm A, then $a_1^* = \frac{(1-p_B)+(1-p_C)}{2k}I$. In particular, setting higher auditing standards than the optimal may reduce total welfare. Note that increasing auditing standards may also change audit coverage and auditor 2's audit quality (as $\bar{k}(x)$ decreases in a_1), which may in turn increase or decrease total welfare depending on client B's risk as shown in Corollary 2.

5 Conclusion

We study the impact of non-audit services (NAS) on audit quality, audit coverage, audit fees, NAS pricing, and total welfare. We consider three potential features of the NAS setting relevant to auditors: 1) competition for NAS clients; 2) competition for audit clients; and 3) restrictions on the provision of NAS to audit clients and non-audit clients. Our study yields several interesting results, which contribute to our understanding of the interplay between audit and NAS beyond the potential for NAS to impair auditor independence and the countervailing possibility that NAS provide knowledge spillovers that improve audit quality. Instead, our results follow from economic forces related to competition, pricing, and ex ante investments in quality and capacity.

First, we show how regulatory bans on the provision of NAS to audit clients can make a high-quality auditor and a NAS competitor better off. Essentially, such restrictions create niches in which each supplier earns rents where before the rents were dissipated by Bertrand competition.

Second, and perhaps most interesting, we find that the effects of NAS restrictions to audit clients on audit quality depend on the cost of audit quality, the correlation between the NAS demand and the audit demand, whether NAS for non-audit clients are also restricted, and the distribution of competitive advantage in the NAS market. With a positive or weakly negative correlation between NAS demand and audit demand, NAS restrictions to audit clients lead to higher audit quality. Otherwise, with a strong negative correlation between NAS demand and audit demand, NAS restrictions to audit clients change the auditor's incentives and can encourage it to supply lower-quality audits more broadly. This can increase the market niche in which the NAS competitor is a monopolist supplier of NAS, which tends to increase the price of the NAS competitor's offering. This price in turn sets a ceiling on the price that the auditor can charge for NAS to non-audit clients, such that the auditor benefits from the NAS competitor's price increase.

Third, we demonstrate that total welfare may be higher or lower with NAS restrictions to audit clients. Indeed, NAS restrictions change auditors' incentives to deliver high-quality audits, which may increase or decrease total welfare depending on the client firms' business risks.

Lastly, our analysis shows that a regulatory ban on the provision of NAS to non-audit clients may increase or decrease audit quality and total welfare relative to a prohibition on providing NAS solely to audit clients, because this removes the potential for the high-quality auditor to benefit from competition-related price increases in the NAS market.

There are many interesting routes for future research. The model could be extended via the addition of price discrimination or an explicit labor market in which both auditors and NAS firms compete explicitly to hire talented workers. Dynamic features related to auditor independence, knowledge spillovers, and legal liability could be added to examine how these interact with competition in the audit and NAS markets. Additionally, empirical studies could examine how the effects of NAS competition on audit markets manifest in U.S. and non-U.S. settings and vary across different types of NAS whose demand may be positively or negatively correlated with demand for audit services (i.e., management consulting versus tax advisory).

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Proofs

Proof of Lemma 1:

See the main text.□

Proof of Proposition 1:

See the main text. Note that auditor 2 is better off selling audit services to client firms A and B rather than not selling audit services:

$$U_{auditor,2} = \frac{(1-p_B)^2}{k} I^2 - 2a_1\left((1-p_B)-(1-p_C)\right) I > 0. \tag{25}$$

Indeed, the assumption in (5) implies that

$$2\frac{(1-p_B)}{k}\left((1-p_B)-(1-p_C)\right) I^2 > 2a_1\left((1-p_B)-(1-p_C)\right) I. \tag{26}$$

Further,

$$\frac{(1-p_B)^2}{k} I^2 > 2\frac{(1-p_B)}{k}\left((1-p_B)-(1-p_C)\right) I^2 \tag{27}$$

is equivalent to

$$2(1-p_C) > 1-p_B, \tag{28}$$

which is always satisfied by the assumption in (7). As a result, condition (25) is always satisfied.

We also show that auditor 1 is better off selling audit services to client firm C rather than not selling audit services:

$$U_{auditor,1} = a_1(1-p_C)I - ka_1^2/2 > 0, \tag{29}$$

which is equivalent to

$$\frac{2(1-p_C)I}{k} > a_1. \tag{30}$$

The assumptions in (5) and in (7) imply that

$$a_1 < \frac{(1-p_B)}{k} I < \frac{2(1-p_C)}{k} I. \tag{31}$$

As a result, condition (29) is always satisfied. □

Proof of Corollary 1:

Direct consequence of Proposition 1.□

Proof of Corollary 2:

If $k < \bar{k}(0)$, then auditor 2 sells auditing services to client firms A and B, and auditor 1 sells auditing services to client firm C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_B)I}{k}$. Hence, we have

$$U_{auditor,1} = F_{a,1} - ka_1^2/2 = a_1(1-p_C)I - ka_1^2/2, \tag{32}$$

$$U_{auditor,2} = 2F_{a,2} - ka_2^2 = \frac{(1-p_B)^2}{k} I^2 - 2a_1\left((1-p_B)-(1-p_C)\right) I, \tag{33}$$

and the expected utility of the consultant is

$$U_{consultant} = 0. \tag{34}$$

As result, the total welfare is

$$\begin{aligned}
W &= U_{auditor,1} + U_{auditor,2} + U_{consultant} + U_{client,A} + U_{client,B} + U_{client,C} \\
&= F_{a,1} - ka_1^2/2 + 2F_{a,2} - ka_2^2 + 3(pR-I) + (1+p_A\theta)v + (1+p_B\theta)v \\
&\quad +(1+p_C\theta)v + a_1(1-p_C)I - F_{a,1} + a_2(1-p_B)I - F_{a,2} + a_2(1-p_A)I - F_{a,2} \\
&= 3(pR-I) + (1+p_A\theta)v + (1+p_B\theta)v \\
&\quad +(1+p_C\theta)v + a_1(1-p_C)I + a_2(1-p_B)I + a_2(1-p_A)I - ka_1^2/2 - ka_2^2 \\
&= 3(pR-I) + (1+p_A\theta)v + (1+p_B\theta)v \\
&\quad +(1+p_C\theta)v + a_1(1-p_C)I + \frac{(1-p_A)(1-p_B)}{k} I^2 - ka_1^2/2.
\end{aligned}$$

Otherwise, if $k > \bar{k}(0)$, then auditor 2 sells auditing services to client firm A and auditor 1 sells auditing services to client firms B and C. Auditor 2 chooses audit quality $a_2^* = \frac{(1-p_A)I}{k}$. Hence, we have

$$U_{auditor,1} = 2F_{a,1} - ka_1^2 = 2a_1(1-p_C) - ka_1^2, \tag{35}$$

$$U_{auditor,2} = F_{a,2} - ka_2^2/2 = \frac{(1-p_A)^2}{2k} I^2 - a_1\left((1-p_A) - (1-p_C)\right) I, \tag{36}$$

and

$$U_{consultant} = 0. \tag{37}$$

As result, the total welfare is

$$\begin{aligned} W &= U_{auditor,1} + U_{auditor,2} + U_{consultant} + U_{client,A} + U_{client,B} + U_{client,C} \\ &= 2F_{a,1} - ka_1^2 + F_{a,2} - ka_2^2/2 + 3(pR - I) + (1 + p_A\theta)v + (1 + p_B\theta)v \\ &\quad +(1 + p_C\theta)v + a_1(1-p_C)I - F_{a,1} + a_1(1-p_B)I - F_{a,1} + a_2(1-p_A)I - F_{a,2} \\ &= 3(pR - I) + (1 + p_A\theta)v + (1 + p_B\theta)v \\ &\quad +(1 + p_C\theta)v + a_1(1-p_C)I + a_1(1-p_B)I + a_2(1-p_A)I - ka_1^2 - ka_2^2/2 \\ &= 3(pR - I) + (1 + p_A\theta)v + (1 + p_B\theta)v \\ &\quad +(1 + p_C\theta)v + a_1(1-p_C+1-p_B)I + \frac{(1-p_A)^2I^2}{2k} - ka_1^2. \end{aligned}$$

The total welfare is higher when auditor 2 sells auditing services to client firms A and B compared to when auditor 2 sells only auditing services to client firm A if and only if

$$1 - p_B > \frac{(1-p_A)^2I^2 - k^2a_1^2}{2(1-p_A)I^2 - 2ka_1I}, \tag{38}$$

which is equivalent to

$$1 - p_B > \frac{1}{2I}\left((1-p_A)I + ka_1\right).\square \tag{39}$$

Proof of Proposition 2:
See the main text.□

Proof of Corollary 3:
Direct consequence of Proposition 2.□

Proof of Corollary 4:

- If $\theta \in [-1, 0]$, then
  - If $k < \bar{k}(2(1+p_C\theta)v)$, then total welfare is the same absent NAS restrictions and with NAS restrictions to audit clients:

$$\begin{aligned} W &= 3(pR - I) + (1 + p_A\theta)v + (1 + p_B\theta)v \\ &\quad +(1 + p_C\theta)v + a_1(1-p_C)I + \frac{(1-p_A)(1-p_B)}{k}I^2 - ka_1^2/2. \end{aligned}$$

  - Otherwise, if $k > \bar{k}(0)$, then total welfare is the same absent NAS restrictions and with NAS restrictions to audit clients:

$$\begin{aligned} W &= 3(pR - I) + (1 + p_A\theta)v + (1 + p_B\theta)v \\ &\quad +(1 + p_C\theta)v + a_1(1-p_C+1-p_B)I + \frac{(1-p_A)^2I^2}{2k} - ka_1^2. \end{aligned}$$

  - Otherwise, if $\bar{k}(2(1+p_C\theta)v) < k < \bar{k}(0)$, then total welfare with NAS restrictions to audit clients is higher than total welfare without NAS restrictions if and only if

$$3(pR-I)+(1+p_A\theta)v+(1+p_B\theta)v+(1+p_C\theta)v+a_1(1-p_C+1-p_B)I+\frac{(1-p_A)^2I^2}{2k}-ka_1^2$$
$$>3(pR-I)+(1+p_A\theta)v+(1+p_B\theta)v+(1+p_C\theta)v+a_1(1-p_C)I+\frac{(1-p_A)(1-p_B)}{k}I^2-ka_1^2/2,$$

which is equivalent to

$$1-p_B<\frac{1}{2I}\left((1-p_A)I+ka_1\right). \tag{40}$$

- If $\theta>0$ and if $(p_B-2p_A)\theta<1$, then
  - If $k<\bar{k}(2(1+p_A\theta)v)$ then total welfare is the same absent NAS restrictions and with NAS restrictions to audit clients:

$$\begin{aligned} W &= 3(pR-I)+(1+p_A\theta)v+(1+p_B\theta)v \\ &\quad +(1+p_C\theta)v+a_1(1-p_C)I+\frac{(1-p_A)(1-p_B)}{k}I^2-ka_1^2/2. \end{aligned}$$

  - Otherwise, if $k>\bar{k}(0)$, then total welfare is the same absent NAS restrictions and with NAS restrictions to audit clients:

$$\begin{aligned} W &= 3(pR-I)+(1+p_A\theta)v+(1+p_B\theta)v \\ &\quad +(1+p_C\theta)v+a_1(1-p_C+1-p_B)I+\frac{(1-p_A)^2I^2}{2k}-ka_1^2. \end{aligned}$$

  - Otherwise, if $\bar{k}(2(1+p_A\theta)v)<k<\bar{k}(0)$, then total welfare with NAS restrictions to audit clients is higher than total welfare without NAS restrictions if and only if

$$3(pR-I)+(1+p_A\theta)v+(1+p_B\theta)v+(1+p_C\theta)v+a_1(1-p_C+1-p_B)I+\frac{(1-p_A)^2I^2}{2k}-ka_1^2$$
$$>3(pR-I)+(1+p_A\theta)v+(1+p_B\theta)v+(1+p_C\theta)v+a_1(1-p_C)I+\frac{(1-p_A)(1-p_B)}{k}I^2-ka_1^2/2,$$

    which is equivalent to

$$1-p_B<\frac{1}{2I}\left((1-p_A)I+ka_1\right). \tag{41}$$

- If $\theta>0$ and if $(p_B-2p_A)\theta\geq 1$, then
  - If $k<\bar{k}(0)$, then total welfare is lower with NAS restrictions to audit clients than absent NAS restrictions:

$$3(pR-I)+(1+p_B\theta)v+(1+p_C\theta)v+a_1(1-p_C)I+\frac{(1-p_A)(1-p_B)}{k}I^2-ka_1^2/2$$
$$<3(pR-I)+(1+p_A\theta)v+(1+p_B\theta)v+(1+p_C\theta)v+a_1(1-p_C)I+\frac{(1-p_A)(1-p_B)}{k}I^2-ka_1^2/2,$$

  - Otherwise, if $\bar{k}(0)<k<\bar{k}(2(1+(2p_A-p_B)\theta)v)$, then total welfare with NAS restrictions to audit clients is higher than total welfare without NAS restrictions if and only if

$$3(pR-I)+(1+p_A\theta)v+(1+p_B\theta)v+(1+p_C\theta)v+a_1(1-p_C+1-p_B)I+\frac{(1-p_A)^2I^2}{2k}-ka_1^2$$
$$<3(pR-I)+(1+p_B\theta)v+(1+p_C\theta)v+a_1(1-p_C)I+\frac{(1-p_A)(1-p_B)}{k}I^2-ka_1^2/2,$$

    which is equivalent to

$$1-p_B>\frac{2k(1+p_A\theta)v+(1-p_A)^2I^2-k^2a_1^2}{2(1-p_A)I^2-2ka_1I}. \tag{42}$$

  - Otherwise, if $\bar{k}(2(1+(2p_A-p_B)\theta)v)<k$, then total welfare is the same absent NAS restrictions and

with NAS restrictions to audit clients:

$$\begin{aligned} W &= 3(pR - I) + (1 + p_A\theta)v + (1 + p_B\theta)v \\ &\quad +(1 + p_C\theta)v + a_1(1 - p_C + 1 - p_B)I + \frac{(1 - p_A)^2 I^2}{2k} - ka_1^2.\square \end{aligned}$$

Proof of Proposition 3:

We provide the conditions under which the consultant is better off selling NAS to one, two, or all three client firms.

- If $\theta \in [-1, 0]$, the consultant is better off selling NAS to all three client firms rather than selling NAS to client firms A and B if and only if
$$3(1 + p_C\theta)v > 2(1 + p_B\theta)v, \tag{43}$$
which is equivalent to
$$1 > (2p_B - 3p_C)\theta. \tag{44}$$
- Otherwise, if $\theta > 0$, the consultant is better off selling NAS to all three client firms rather than selling NAS to client firms B and C if and only if
$$3(1 + p_A\theta)v > 2(1 + p_B\theta)v, \tag{45}$$
which is equivalent to
$$1 > (2p_B - 3p_A)\theta. \tag{46}$$
Further, the consultant is better off selling NAS to client firms B and C rather than selling NAS to client firm C if and only if
$$2(1 + p_B\theta)v > (1 + p_C\theta)v, \tag{47}$$
which is equivalent to
$$1 > (p_C - 2p_B)\theta. \tag{48}$$

See the main text for the remainder of the proof. $\square$

Proof of Corollary 5:

Direct consequence of Corollary 3.$\square$

Proof of Corollary 6:

See the main text.$\square$

Proof of Proposition 4:

Without NAS restrictions to audit clients, there is Bertrand competition in the NAS market, which implies that $F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$.

Suppose now that there are NAS restrictions to audit clients.

- First, assume that there is a positive correlation between audit demand and NAS demand, i.e., $\theta \in [-1, 0]$.
  - If auditor 2 sells auditing services only to client firm A, then Bertrand competition in the NAS market implies $F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$. Indeed, the consultants are engaged in Bertrand competition upon selling NAS to client firms A and C. Similarly, auditor 2 and consultant 2 are engaged in Bertrand competition upon selling NAS to client firm B. This drives NAS fees down to zero. Auditor 2's expected utility is
  $$U_{auditor,2} = \frac{(1 - p_A)^2}{2k} I^2 - a_1\left((1 - p_A) - (1 - p_C)\right) I.$$
  - Next, assume that auditor 2 sells auditing services to client firms A and B. Client firm B prefers to buy NAS from consultant 2 rather than consultant 1 if and only if
  $$\phi(1 + p_B\theta)v - F_{c,c2} > (1 + p_B\theta)v - F_{c,c1}.$$
  If the value of NAS provided by consultant 2 is lower for client firm A than for client firm B, i.e, $(1 + p_A\theta)v < \phi(1 + p_B\theta)v$, then $F^*_{c,c1} = (1 + p_A\theta)v$, $F^*_{c,c2} = \phi(1 + p_B\theta)v$ and $F^*_{c,2} = (1 + p_C\theta)v$. Auditor 2's expected utility is
  $$U_{auditor,2} = \frac{(1 - p_B)^2}{k} I^2 - 2a_1\left((1 - p_B) - (1 - p_C)\right) I + (1 + p_C\theta)v.$$

Otherwise, if the value of NAS provided by consultant 2 is greater for client firm A than for client firm B, i.e. $(1+p_A\theta)v > \phi(1+p_B\theta)v$, then Bertrand competition in the NAS market implies $F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$. Auditor 2's expected utility is

$$U_{auditor,2} = \frac{(1-p_B)^2}{k} I^2 - 2a_1 \left((1-p_B) - (1-p_C)\right) I.$$

- Now, suppose that there is a negative correlation between audit demand and NAS demand, i.e., $\theta \in (0, \infty)$.
  - If auditor 2 sells auditing services only to client firm A, then Bertrand competition in the NAS market implies $F^*_{c,2} = F^*_{c,c1} = F^*_{c,c2} = 0$. Auditor 2's expected utility is

$$U_{auditor,2} = \frac{(1-p_A)^2}{2k} I^2 - a_1 \left((1-p_A) - (1-p_C)\right) I.$$

  - Otherwise, if auditor 2 sells auditing services to client firms A and B, then consultant 1 sells NAS to client firm A at a price $F^*_{c,c1} = (1+p_A\theta)v$, consultant 2 sells NAS to client firm B at a price $F^*_{c,c2} = (\phi-1)(1+p_B\theta)v + (1+p_A\theta)v$, and auditor 2 sells NAS to client firm C at a price $F^*_{c,2} = (1+p_A\theta)v$. Auditor 2's expected utility is

$$U_{auditor,2} = \frac{(1-p_B)^2}{k} I^2 - 2a_1 \left((1-p_B) - (1-p_C)\right) I + (1+p_A\theta)v.$$

Therefore, compared to the equilibrium without NAS restrictions, auditor 2 has incentives to choose broader coverage and lower audit quality in order to capture NAS rents whatever the correlation between NAS and audit demand.

Proof of Proposition 5:

First, note that Bertrand competition in the NAS market between consultant 1 and consultant 2 implies that $F^*_{c,c1} = F^*_{c,c2} = 0$. As a result, client firm j buys NAS from auditor 2 if and only if

$$\phi(1+p_j\theta)v - F_{c,2} > (1+p_j\theta)v.$$

- Without NAS restrictions to audit clients:
  - If there is a positive correlation between NAS demand and audit demand, auditor 2 may sell NAS to all three client firms at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$, or sell NAS to client firms A and B at a fee $F^*_{c,2} = (\phi-1)(1+p_B\theta)v$. Note that auditor 2 is always better off selling NAS to client firms A and B rather than only selling NAS to client A (see footnote 16).
  - Otherwise, if there is a negative correlation between NAS demand and audit demand, auditor 2 may sell NAS to the three clients firms at a fee $F^*_{c,2} = (\phi-1)(1+p_A\theta)v$, sell NAS to client firms B and C at a fee $F^*_{c,2} = (\phi-1)(1+p_B\theta)v$, or sell NAS only to client firm C at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$.
- With NAS restrictions to audit clients:
  - If there is a positive correlation between NAS demand and audit demand,
    * if auditor 2 only sells audit services to client A, then auditor 2 sells NAS to client firms B and C at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$. Note that auditor 2 is better off selling NAS to both client firms B and C rather than selling NAS to client firm B only (see footnote 17). Auditor 2's expected utility is

$$U_{auditor,2} = \frac{(1-p_A)^2}{2k} I^2 - a_1 \left((1-p_A) - (1-p_C)\right) I + 2(\phi-1)(1+p_C\theta)v.$$

    * Otherwise, if auditor 2 sells audit services to both clients A and B, then auditor 2 sells NAS to client firm C at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$. Auditor 2's expected utility is

$$U_{auditor,2} = \frac{(1-p_B)^2}{k} I^2 - 2a_1 \left((1-p_B) - (1-p_C)\right) I + (\phi-1)(1+p_C\theta)v.$$

  - With a negative correlation between NAS demand and audit demand,
    * if auditor 2 only sells audit services to client A, auditor 2 may sell NAS to client firms B and C at a fee $F^*_{c,2} = (\phi-1)(1+p_B\theta)v$, or sell NAS only to client firm C at a fee $F^*_{c,2} = (\phi-1)(1+p_C\theta)v$.

Auditor 2's expected utility is

$$U_{auditor,2} = \frac{(1-p_A)^2}{2k} I^2 - a_1 \left((1-p_A) - (1-p_C)\right) I + 2(\phi - 1)(1 + p_B\theta)v$$

or

$$U_{auditor,2} = \frac{(1-p_A)^2}{2k} I^2 - a_1 \left((1-p_A) - (1-p_C)\right) I + (\phi - 1)(1 + p_C\theta)v.$$

* Otherwise, if auditor 2 sells audit services to both clients A and B, then auditor 2 sells NAS to client firm C at a fee $F^*_{c,2} = (\phi - 1)(1 + p_C\theta)v$. Auditor 2's expected utility is

$$U_{auditor,2} = \frac{(1-p_B)^2}{k} I^2 - 2a_1 \left((1-p_B) - (1-p_C)\right) I + (\phi - 1)(1 + p_C\theta)v.$$

As a result, if there are two NAS competitors and auditor 2 has a competitive advantage in the NAS market, NAS restrictions to audit clients increase audit quality.□

Proof of Corollary 7:

- If auditor 2 only sells audit services to client A, auditor 2 sets $F^*_{a,2} = \frac{(1-p_A)^2}{k} I^2 - a_1 \left((1-p_A) - (1-p_C)\right) I$ and auditor 1 sets $F^*_{a,1} = a_1(1-p_C)I$. We have

$$\frac{\partial F^*_{a,1}}{\partial a_1} = (1-p_C)I > 0$$

and

$$\frac{\partial F^*_{a,2}}{\partial a_1} = -\left((1-p_A) - (1-p_C)\right) I < 0.$$

- Otherwise, if auditor 2 sells audit services to both clients A and B, then auditor 2 sets $F^*_{a,2} = \frac{(1-p_B)^2}{2k} I^2 - a_1 \left((1-p_B) - (1-p_C)\right) I$ and auditor 1 sets $F^*_{a,1} = a_1(1-p_C)I$. We have

$$\frac{\partial F^*_{a,1}}{\partial a_1} = (1-p_C)I > 0$$

and

$$\frac{\partial F^*_{a,2}}{\partial a_1} = -\left((1-p_B) - (1-p_C)\right) I < 0.$$

Furthermore, we know that

$$\bar{k}(x) = \frac{2(1-p_B)^2 - (1-p_A)^2}{2a_1(2(1-p_B) - (1-p_A) - (1-p_C)) + x} I.$$

Hence, we have

$$\frac{\partial \bar{k}(x)}{\partial a_1} = \frac{-2\left(2(1-p_B)^2 - (1-p_A)^2\right)\left(2(1-p_B) - (1-p_A) - (1-p_C)\right)}{\left(2a_1(2(1-p_B) - (1-p_A) - (1-p_C)) + x\right)^2} I < 0.\square$$

Finally we prove our claim that auditor 2's audit fees are larger when auditor 2 sells audit services only to client A:

$$\frac{(1-p_A)^2 I^2}{k} - a_1 \left((1-p_A) - (1-p_C)\right) I > \frac{(1-p_B)^2 I^2}{k} - a_1 \left((1-p_B) - (1-p_C)\right) I$$

is equivalent to

$$\frac{(1-p_A)^2 I}{k} - \frac{(1-p_B)^2 I}{k} > a_1 \left((1-p_A) - (1-p_B)\right),$$

which is equivalent to

$$\frac{(1-p_A) + (1-p_B)}{k} I > a_1,$$

which is always satisfied by the assumption in (5).

Proof of Corollary 8:

If auditor 2 sells auditing services to client firms A and B, the total welfare is

$$W = 3(pR - I) + X_{NAS} + a_1(1-p_C)I + \frac{(1-p_A)(1-p_B)}{k} I^2 - k a_1^2/2, \tag{49}$$

where X_{NAS} is the NAS value to the client firms. For instance, if all three client firms buy NAS, $X_{NAS} = (1 + p_A\theta)v + (1 + p_B\theta)v + (1 + p_C\theta)v$.

Hence, we have

$$\frac{\partial W}{\partial a_1} = (1 - p_C)I - ka_1.$$

Otherwise, if auditor 2 sells auditing services to client firm A, the total welfare is

$$W = 3(pR - I) + (1 + p_A\theta)v + (1 + p_B\theta)v + (1 + p_C\theta)v + a_1(1 - p_C + 1 - p_B)I + \frac{(1 - p_A)^2 I^2}{2k} - ka_1^2. \tag{50}$$

Hence, we have

$$\frac{\partial W}{\partial a_1} = (1 - p_B + 1 - p_C)I - 2ka_1.\square$$