14.121 Fall 2017 Final Exam

You have 120 minutes to complete the exam. There are 100 points total. Please try to keep your answers concise.

Question 1 (10 points) Suppose you want to measure the welfare effect for a consumer if prices change from p to p'.

a. What is a problem with trying to measure the effect by the formula $v(p', w) - v(p, w)$, where v is the indirect utility function?

b. Define the equivalent and compensating variation.

Question 2 (10 points) a. Briefly describe an economy with non-convex consumption sets and strictly increasing preferences where the First Welfare Theorem fails. What condition for the First Welfare Theorem is violated in this economy?

b. Briefly describe an economy with non-convex consumption sets where the First Welfare Theorem holds but an extension of the commodity space to lotteries can be Pareto improving.

Question 3 (10 points) State and prove the Slutsky equation. You may invoke without proof any mathematical theorems or other results from consumer theory, but clearly indicate where any of these results are used.

Question 4 (10 points) Consider a pure trade economy. Answer true, false, or uncertain. Support your answer with a proof, explanation, or counterexample.

a. If preferences are identical, then aggregate demand only depends on the distribution of wealth through the aggregate wealth.

b. If preferences Gorman aggregate, then equilibrium prices only depend on the distribution of endowments through the aggregate endowments.

Question 5 (15 points) Suppose that you are given a finite consumer choice dataset $\{(x^t, p^t, w^t)\}_{t=1}^T$.

a. State a restriction that this dataset must satisfy to be consistent with optimization by an agent with locally non-satiated preferences.

b. State an additional restriction that this dataset must satisfy to be consistent with optimization by an agent with locally non-satiated and rational preferences.

d. Now suppose that you are given a set of observed prices $\{p^t\}_{t=1}^T$ normalized so that $\sum_{l=1}^L p_l^t = 1$ for $t = 1, 2, ..., T$. State the Sonnenschein-Mantel-Debreu ("Anything Goes") result.

e. As shown in Brown-Matzkin, what additional data could be collected in order for GE theory to be falsifiable?

Question 6 (15 points) Consider an economy with one physical good, available at infinitely many dates $t = 0, 1, ...$ One agent ("individual t") is born at each date $t = 0, 1, ...$ and lives and consumes at only two dates: dates t (when they are "young") and $t + 1$ (when they are "old"). Individual t is endowed with 1 unit of the good at date t and zero units of the good at date $t + 1$. Utility for agent t is given by $U^t(c_t^t, c_{t+1}^t) = c_t^t + c_{t+1}^t$.

a. Derive individual t's demand as a function of prices $p = (p_t)_{t=0}^{\infty}$.

b. Show that there is a unique Walrasian equilibrium allocation where every agent consumes his endowment when young. Are equilibrium relative prices uniquely determined?

c. Show that the allocation in part b is not Pareto optimal. In particular, describe explicitly a Pareto-dominating allocation.

d. What additional assumption, compared to the finite-agent case, is necessary for the First Welfare Theorem to hold in an economy with infinitely many agents?

Question 7 (15 points) Consider an economy $\mathcal{E} = \left\{\{X_i, \succsim_i, \omega_i\}_{i=1}^{I}, \{Y_j\}_{j=1}^{J}, \{\theta_{ij}\}_{i=1,j=1}^{I,J}\right\}$. Suppose this economy satisfies the following assumptions: consumption sets are given by the non-negative orthant $X_i = \mathbb{R}_+^L$, preferences $\succsim_i$ are represented by concave, locally non-satiated, and continuously differentiable utility functions $u_i : X_i \to \mathbb{R}$, the aggregate endowment is strictly positive $\bar{\omega} \gg 0$, there is one firm ($J = 1$) with production set $Y = \left\{y \in \mathbb{R}^L | F(y) \geq 0\right\}$ given by a concave and continuously differentiable transformation function F, and the set of feasible allocations is compact. You can assume that utility satisfies conditions so that all optimal allocations must be interior.

a. Show that if $\lambda \gg 0$ then any solution to the Pareto problem with weights λ (i.e. any allocation which maximizes a linear welfare function with weights λ over the set of feasible allocations) is Pareto optimal.

b. Conversely, given the stated assumptions about this economy, why can we be assured that all Pareto optimal allocations can be expressed as solutions to the Pareto problem for some set of Pareto weights $\lambda \geq 0$? What mathematical theorem is used to prove this result?

c. Write down the definition of a price equilibrium with transfers for this economy.

d. Describe the mapping between Pareto optimal allocations and price equilibria with transfers. In particular, given a set of Pareto weights λ and a corresponding Pareto optimal allocation $(x(\lambda), y(\lambda))$, discuss how you could obtain a price system $p(\lambda)$ and wealth levels $w(\lambda)$ that decentralize this allocation as a price equilibrium with transfers.

e. Describe Negishi's mapping, for which a fixed point corresponds to a Walrasian equilibrium. Explain why a fixed point of this mapping yields a Walrasian equilibrium (without transfers).

Question 8 (15 points) Consider a finite-horizon economy populated by households $i = 1, .., I$ with identical preferences that are separable in consumption and labor

$$U_i = E\left[\sum_{t=1}^{T} \beta^t \left(u(c_i(s^t)) - v(l_i(s^t))\right)\right]$$

$$= \sum_{t=1}^{T} \sum_{s^t} \beta^t \left(u(c_i(s^t)) - v(l_i(s^t))\right) Pr(s^t)$$

Suppose that u is continuously differentiable, strictly increasing, and strictly concave, and suppose v is continuously differentiable, strictly increasing, strictly convex. You can assume utility satisfies conditions so that all Pareto optimal allocations must be interior. Household i is able to produce output $y_i(s^t) = \phi_i(s^t) l_i(s^t)$ in terms of the consumption good, which depends on the household's labor supply $l_i(s^t)$ and history-dependent productivity $\phi_i(s^t)$. Note that labor is not tradable.

a. Setup the Pareto problem for this economy and derive the first order conditions.

b. Suppose that you are using the model to make predictions. In particular, consider the restrictions on data that are implied by supposing that observed allocations belong to the set of Pareto optimal allocations. Does individual i's consumption depend on idiosyncratic productivity after controlling for the productivities of all the agents at a given history?

c. For any two agents i and j, if individual i consumes more at history s^t compared to $\tilde{s}^t$, then are there any restrictions on how the consumption of individual j at history s^t compares to his consumption at $\tilde{s}^t$?

d. Answer the analogous questions (from parts b and c) for labor supply instead of consumption.

e. Now suppose preferences in consumption and labor are nonseparable. For concreteness suppose

$$U_i = \sum_{t=1}^{T} \sum_{s^t} \beta^t \frac{\left(c_i(s^t) e^{-l_i(s^t)}\right)^{1-\gamma}}{1-\gamma} Pr(s^t)$$

where $\gamma > 1$, so working more increases the marginal value of consumption. Show that in this case consumption is *not* fully insulated/insured against idiosyncratic risk in productivity $\phi_i(s^t)$. Can you provide any intuition for this result?