14.121 Recitation 1 – Debreu's Theorem

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Introduction

- Hello, and welcome to 14.121!
- I am Matthew, a fourth year econ Phd student whose interests lie in behavioral/development/labor economics
- I look forward to getting to know you all!

Some words on grad school

- The first quarter of the PhD course is sometimes a stressful and tiring experience (though hopefully the new structure will improve things)
- You will survive it. It gets better
- I am happy to offer advice on navigating the start of grad school to anyone who wants

Course logistics

- OH: 4-6pm Tuesdays, E52-532 but I am always accessible by email
- Problem Set submission: three options
  - In class
  - In my mailbox (3rd floor)
  - On Stellar (under the 'Assignments' tab)

Introduction to this course

- For many non-theorists 14.121 is not the most interesting material
- But whatever your interests, it gives you essential background that other economists may assume you know
- Please stop me with questions
- If you're confused, it's my fault for not explaining well
  - It's not because you're dumb
- Tell me if I'm going too slow or too fast

Today's Recitation

1. What is Debreu's theorem? Why should we care?
2. Overview of proof, and key intuitions
3. Details of proof
4. Concluding thoughts

Debreu's Theorem

- Any rational and continuous preference relation $\succeq$ on a (separable, connected) space X can be represented by a continuous utility function $u : X \mapsto \mathbb{R}$.

Motivation

- Why should we care? Why do we start your economics teaching with this?
- Choice and preference are the real-world concepts at the bottom of most things we do in micro
- This theorem provides (one) justification for using utility functions when thinking about choice
  - Links utility function to appealing and simple axioms
  - Much easier, mathematically, to work with utility functions than preference relations
- Also, a lot of econ theory work involves proofs of this kind– linking axioms to things we can work with practically
  - Particularly decision theory which you should see in 14.123

Overview of proof

- Today we will actually prove a simpler version of the theorem (the one in MWG)
- This considers only the special case in which:
  1. $X = \mathbb{R}^n_+$ for some $n \in \mathbb{N}$
  2. $\succeq$ is strictly monotone
- Note this still covers many (most?) cases of economic interest
- Proof by construction: we will come up with a utility function

Reminder of definitions

- **Rational:** Complete, transitive and reflexive
- **Continuous:** The sets $U(x) := \{y \in X : y \succeq x\}$ and $L(x) := \{y \in X : x \succeq y\}$ are closed (contain all their limit points).
- **Monotone:** $y \gg x \Rightarrow y \succ x$)

Overview of proof

See the board!

Detailed proof

See the board!

Concluding thoughts/questions

- Where does the proof break down with lexicographic preferences?
- What should we make of the continuity assumption?