14.121 Lecture 1: Utility and Revealed Preferences

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September 2018

Utility Theory

In most economic models, agents are assumed to have a utility function that they maximize. The utility function approach usually serves two purposes:

1. Model for predicting behavior (positive); i.e. agent will maximize utility
2. Framework for welfare analysis (normative); i.e. utility is "good"

While most of the time we do this blindly, it is useful to start by thinking more about this and what it means. Are there hidden assumptions? Undesirable assumptions?

This provides us with an opportunity to introduce one useful economic methodology: the **axiomatic approach**.

The idea is to find simple axioms which together are equivalent to less obvious assumptions of utility theory. One advantage is to understand what is behind the utility approach. Also, these axioms may be directly testable.

Let $X \subset \mathbb{R}^N$ be a set of states of the world.
(Often $x = (x_1, ..., x_N) \in X$ is taken to represent the state of the world where the agent has x_1 units of good, x_2 units of good 2, etc., but this need not be true. x_i could contain weather, represent past consumption, other people's consumption, etc.)

Definition

A **utility function** u defined on X is a real-valued function $u : X \to \mathbb{R}$.

A more foundational place to start to think about assumptions inherent in utility theory is treating preferences as a primitive.

Definition

A **preference** P is a subset of $X \times X$, where we write $x \succeq_p y$ if $(x, y) \in P$.

Interpretation of $x \succeq_P y$: agent thinks "x is at least as good as y".

Definition

The utility function $u : X \to \mathbb{R}$ **represents** the preference P if

$$u(x) \geq u(y) \quad \Leftrightarrow \quad x \succeq_P y.$$

Note: many different utility functions can represent the same set of preferences. e.g., if $X = \{x, y, z\}$, and $P = \{(x, y), (x, z), (y, z)\}$ (i.e $x \succeq_P y, x \succeq_p z, y \succeq_p z$).

One utility representation: $u(x) = 5, u(y) = 3$, and $u(z) = 1$
Another: $u(x) = 2.3$, $u(y) = 0$, and $u(z) = -17$.

In general, if u represents P, then so does $f \circ u$, for any <u>strictly</u> increasing function f.

Properties of Preferences:

Definition

The preference relation $\succeq_P$ is **complete** on X if $\forall x, y \in X$, either $x \succeq_P y$ or $y \succeq_P x$.

Definition

The preference relation $\succeq_P$ is **reflexive** on X if $\forall x \in X$, $x \succeq_P x$.

Definition

The preference relation $\succeq_P$ is **transitive** on X if $x \succeq_P y$ and $y \succeq_P z \Rightarrow x \succeq_P z$.

Our first result is:

Proposition

If $\succeq_P$ is represented by u on X, then $\succeq_P$ is complete, reflexive, and transitive.

Proof:

Complete: Given any $x, y \in X$, either $u(x) \geq u(y)$ or $u(y) \geq u(x)$ (property of real numbers). This implies $x \succeq_P y$ or $y \succeq_P x$ (by representation assumption).

Reflexive: Given any $x \in X$, $u(x) \geq u(x) \Rightarrow x \succeq_P x$.

Transitive: If $x \succeq_P y$ and $y \succeq_P z$, then $u(x) \geq u(y)$ and $u(y) \geq u(z)$.

Transitivity of $\geq$ on $\mathbb{R}$ implies that $u(x) \geq u(z) \Rightarrow x \succeq_P z$.

Utility theory apparently rules out preferences that do not satisfy these 3 assumptions. Should we focus on preferences which do not satisfy these relations?

- Reflexive: probably not

- Transitive: Hard to discuss **welfare** of policies without it.

  E.g., if $x \succeq_P y$ and $y \succeq_P z$ and $x \not\succeq_P z$, then we could let someone switch from z to y, then y to x, and then x to z (charging a small amount) and make the individual better off.

  This is sometimes called a Dutch book argument and is used to argue that such a preference is unreasonable.

  KT example

- Complete: Moral issues and bounded rationality can create problems

When seeing results like this in economic theory, it is often very useful to consider the converse: If $\succeq_P$ is complete, reflexive, and transitive, then can it be described by some utility function?

Turns out the answer depends on whether X is finite.

Proposition

Let X be a finite set. Let P be a preference on X. Assume $\succeq_P$ is complete on X, reflexive on X, and transitive on X. Then there exists a utility function u which represents P.

Some comments:

1. With finite sets, apparently assuming utility maximization is exactly the same as assuming preferences are complete, reflexive and transitive.

2. Any critique of utility theory is a critique of one of these assumptions (or our notion of preferences). If we are happy with these, then we are happy with utility functions.

   (Caveat: to turn preferences or utility into **predictive** or **welfare** tool need extra assumptions, e.g., people will do what they prefer. This is where additional objections may be raised.)

When you write your own papers with models, a suggestion:

You should always try to use the most intuitive notation and simplify your proofs into steps (as in claims).

This makes your arguments easier to follow and remember.

Economists, while often very good mathematicians, may have to communicate to audiences with various degrees of mathematical sophistication.

The exposition of the best economic theorists often follows this mode.

For tips, see Thomson, William. "A Guide for the Young Economist." MIT Press, 2011.

Common Criticisms of Utility Theory

- Realism: "no one chooses objects after consulting some numerical index of goodness"

  Interpretation: individuals act *as-if* they maximize utility (doesn't mean they actually do); Key question for us: what can we falsify (rationalizability/identification)

- Inconsistent or probabilistic choice: standard model assumes choices are innate and unchanging

  Consumer may choose an object with lower utility for some random reason

  In empirical work, often see people with same observable attributes choosing differently

  Luce/McFadden introduced idea of *random utility* or *probabilistic choice* $\Rightarrow$ behavioral foundation for discrete choice econometrics

- Where do prefs come from? Do they change?

  Becker-Stigler (1977): *De Gustibus Non Est Disputandum* – economists should assume tastes don't change, look for differences in prices, incomes, constraints to explain phenomenon

- Procedural vs. consequentialist aspects of choice (e.g., Sen)

  If x is chosen from set A, individual is just as well off if given a choice from A vs. simply given x without opportunity to choose

  Maybe individuals value being able to choose (though there is some evidence that too much choice is bad - *choice overload*)

Standard model still remains the workhorse model and remains useful, but important to remember it is based on assumptions on human behavior and not laws of nature

Definition

The preference relation $\succeq$ on X is **monotone** if $x \in X$ and $y \geq x$ implies $y \succeq x$. If $x \neq y$ and $y \geq x \Rightarrow y \succ x$, then **strictly monotone**. ("goods are goods")

Definition

The preference relation $\succeq$ on X is **locally non-satiated** if $\forall x$ and all $\delta > 0$, $\exists y$ such that

$$||y - x|| < \delta \text{ and } y \succ x.$$

("never a bliss point, even a local one")

Definition

The preference relation $\succeq$ on X is **convex** if $\forall x$, $y \succeq x, z \succeq x$ and $y \neq z$ implies that for all $\alpha \in (0, 1)$,

$$\alpha y + (1 - \alpha) z \succeq x.$$

("moderation in all things")

To extend our earlier representation theorem to commodity space X which is not countable, the main assumption is continuity:

Definition

The preference relation $\succeq$ on X is **continuous** if it is preserved under limits: for any sequence $\{(x^n, y^n)\}_{n=1}^{\infty}$ with $x^n \succeq y^n$ for all n, $x \succeq y$, where x and y are limit points of the sequence, respectively.
("no jumps in preferences")

Main representation theorem:

Proposition (Debreu's theorem)

Suppose the rational preference relation is $\succeq$ on X is continuous. Then there is a continuous utility function $u(x)$ that represents $\succeq$.

Proof will be covered in recitation (see also MWG p. 47)

Choice as primitive

So far we have taken preferences as a primitive.

Another approach is to take treat choice behavior as the primitive.

Let $\mathcal{B}$ be a set of nonempty subsets of X and $Ch(\cdot)$ be a choice rule that assigns a nonempty set of chosen elements $Ch(B) \subset B$ for each $B \in \mathcal{B}$.

A lot of modern literature begins this way. A classic and entertaining reference is Kreps, "Lecture Notes on the Theory of Choice."

Recap

So far there are two main lessons:

1. Axiomatic method is a useful way to think about complex assumptions we make
2. With finite choice sets, the assumption of a utility representation is equivalent requiring preferences to be complete, reflexive, and transitive. Continuity yields utility representation when choice set not finite.

Revealed Preference

Think in terms of choice experiment.

Consider a consumer who is repeatedly asked to choose a consumption bundle $x \in \mathbb{R}^{L+}$.

Suppose the consumer is confronted with different prices and we observe resulting choices $C = \{(p^1, x^1), (p^2, x^2), ...\}$.

What can we say about preferences and what axioms they satisfy?

Need some structure on preferences, otherwise indifference could rationalize any behavior.

Definition

The utility function $u : X \to \mathbb{R}$ satisfies **local nonsatiation** if $\forall x \in X$, and all $\epsilon > 0$, $\exists y \in X$ with $||y - x|| < \epsilon$ such that $u(y) > u(x)$.

Observe: x, y at price p and $p \cdot x \geq p \cdot y$.

If x chosen when y could have been chosen, x is **directly revealed preferred** to y.

Notation: $xR^D y$.

Observe: <u>sequence</u> of comparisons such that

$$xR^D z_1, z_1 R^D z_2, ...z_{n-1} R^D z_n, z_n R^D y.$$

Here, x is (indirectly) **revealed preferred** to y and write xRy.

Denote strict counterparts with P^D and P, respectively:

$$xP^D y \quad \Leftrightarrow \quad p \cdot x > p \cdot y$$

Definition

A set of consumption choices $\{(p^1, x^1), (p^2, x^2), ..., (p^n, x^n)\}$ satisfies the general axiom of revealed preferences (**GARP**) if and only if

$$x^i R x^j \Rightarrow p^j \cdot x^j \leq p^j \cdot x^i, \qquad \forall i, j$$

If x^i is revealed preferred to x^j, then x^i is only weakly affordable at x^j's prices.

Another way to say this is that if x^i is indirectly revealed preferred to x^j, then x^j cannot be strictly directly revealed preferred to x^i.

That is,

$$x^i R x^j \Rightarrow \text{ not } x^j P^D x^i$$

Since $\neg(x^j P^D x^i)$ means, $\neg(p^j \cdot x^j > p^j \cdot x^i)$ or $p^j \cdot x^j \leq p^j \cdot x^i$.

Some textbooks have other axioms of revealed preferences

- **Weak Axiom of Revealed Preference (WARP)**: If $x^i R^D x^j$ and $x^i \neq x^j$, then it is not the case that $x^j R^D x^i$.

- **Strong Axiom of Revealed Preference (SARP)**: If $x^i R x^j$ and $x^i \neq x^j$, then it is not the case that $x^j R x^i$.

These other axioms require slightly different conditions, but main difference is implication: it is not the case that $x^j R^{(D)} x^i$ vs. not the case that $x^j P^D x^i$.

GARP is seen as a generalization of these other tests (which allows the consumer to potentially choose multiple demand bundles at each budget), and because of the next theorem it is our main focus.

Drawback is that GARP is computationally demanding: requires computing all chains from one to every other observation (since WARP is based on R^D it may be quicker to check)

Proposition (Afriat 1967)

Let $\{(p^1, x^1) ..., (p^n, x^n)\}$ *be a finite set of consumption choices. If a finite set of demand data violates GARP, then these data are inconsistent with choice according to locally nonsatiated, complete, and transitive preferences.*

Conversely, if a finite set of demand data satisfies GARP, then these data are consistent with choice according to strictly increasing, continuous, convex, complete and transitive preferences.

- Central result of consumer theory because it provides succinct, testable conditions that a finite dataset must satisfy to be consistent with utility maximization

- Kreps (2012): modern proof of this theorem uses techniques that do not illustrate any general lesson.

  The idea is to show the equivalence of GARP and the existence of a solution to a set of linear inequalities.

  From this solution, we can construct a utility function rationalizing the choices of the consumer.

  (See Ch. 4 of Kreps for complete proof, if interested)

- Kreps and Varian emphasize that with finite data on (p, x), strictly increasing or continuous or convex adds no testable restrictions (if satisfy GARP, can construct a utility function satisfying these extra conditions)

Besides being simply well-educated, one sees GARP discussed in fields where people are interested in whether some utility function can explain things.

Two examples of literatures are:

1. Behavioral: can we explain altruism etc. with utility?
2. Family/household: are interfamily dynamics causing inefficiency or do families act like they have some common goal?

Andreoni and Miller, “Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism.” *Econometrica*, March 2002.

A well-known experiment is a the “dictator game”. Two players (anonymous, e.g. in separate in rooms). Player 1 given 10 dollars and told he can give some to player 2. Typical average about 2.

Question: Is this altruism like $u(x_{me}, x_{other})$ or do we need some other model?

Andreoni and Miller examine utility theory in a simple GARP experiment. Ask upper level economics undergraduates to make 8-14 decisions. Most are: here are M tokens. Divide between self and anonymous other. Each you keep gives you x cents. Each you give gives other y cents.

Various combinations: $M \in \{40, 60, 75, 80, 100\}$, $x \in \{1, 2, 3, 4\}$, $y \in \{1, 2, 3, 4\}$.

e.g. $M = 40, x = 3, y = 1$; if I keep 30 tokens, I get 90 cents, other gets 10 cents.

Look at each persons's choice and ask is it consistent with GARP.

Find 158 of 176 are consistent, 18 are not.

Most violations are small. A couple people violate GARP a lot.

Idea is to test altruism in a way that does not make strong functional form assumptions

Duflo, "Grandmothers and Granddaughters: Old-Age Pensions and Intrahousehold Allocation in South Africa." *World Bank Economic Review*, 2003.

People in family/household economics look at how spending depends on husband's income versus wife's income and use this to examine the *unitary model* of the family

Esther found nice source of variation: **unexpected** grants of large pensions to black South African retirees.

She shows whether money given to a grandfather or grandmother affects how it is spent. For grandmothers, more goes to daughters influenced evidenced by increased weight. Effect more pronounced for mother's mother than father's mother.

The relationship of this to GARP is that it is as if prices do not change, so choice should not change if unitary model is correct (other than if indifferent.)

Framing effects / “mental accounts”

Experiment due to Danny Kahneman and Amos Tversky (1984)
Imagine you are about to purchase a stereo for 125 and calculator for 15 dollars. You go to the MIT-Coop.

Salesman tells you that the calculator is on sale for 5 dollars less at Harvard Coop. Do you make the trip?

Suppose instead salesman tells you that stereo is 5 dollars less at Harvard Coop.

Kahneman and Tversky: fraction of respondents who would travel for cheaper calculator is much higher than for cheaper stereo

If instead you say: there was a stockout so you have to go to Harvard Coop, and will get 5 dollars off either item as compensation. Which item do you care to get money off?

People say they are indifferent.

If x = go to Harvard and get 5 dollars off calculator, y= go to Harvard and get 5 dollars off stereo, z = get both items at MIT.

For most, we have $x \succ z$ and $z \succ y$, but last question implies $x \sim y$.

Transitivity would imply that $x \succ y$, which is the contradiction.

Many other examples like this (appears to be even easier to demonstrate framing effects with uncertain prospects).

Common argument is that this is a mistake people won't make if they have experience.

The applicability of this type of argument is debated.

Nonetheless, how violations of transitivity should be analyzed and what can be said about a decision maker in this case is an active research question. We don't have compelling frameworks for how to make welfare statements in these situations.

Some current approaches considered explaining intransitive behavior as the result of several more primitive transitive preferences, e.g., voting, changing tastes, strategic intrapersonal conflicts, choice conditional on "frames."

Return