1) Suppose $X = \mathbb{R}^k_+$, for some $k \geq 2$, and define $x = (x_1, ..., x_k) \succeq y = (y_1, ..., y_k)$ if $x \geq y$; i.e., if for each $i = 1, ..., k$, $x_i \geq y_i$. (This is known as the *Pareto ordering* on $\mathbb{R}^k_+$.)

   a) Show that $\succeq$ is transitive but not complete.

   b) Characterize $\succ$ defined from $\succeq$ in the usual fashion; i.e. $x \succ y$ if $x \succeq y$ and not $y \succeq x$. Is $\succ$ reflexive? transitive? symmetric? Prove your assertions.

   c) Characterize $\sim$ from $\succeq$ in the usual fashion; i.e. $x \sim y$ if $x \succeq y$ and $y \succeq x$. Is $\sim$ reflexive? transitive? symmetric? Prove your assertions.

2) MWG 1.D.5

3) Let $\succeq$ be some complete, transitive preference on a non-empty convex set $X \subseteq \mathbb{R}^L$. We say that preferences are strictly convex when $\succeq$ is convex and for all x, y, z such that $y \succeq x$ and $z \succeq x$ and $y \neq z$, we have that for $\alpha \in (0, 1)$:

$$\alpha y + (1 - \alpha) z \succ x$$

   a) Let $X^* \subseteq X$ be the set of maximal bundles of X:

$$X^* = \{x \in X : x \succeq y \text{ for all } y \in X\}.$$

   Show if that $\succeq$ is complete, transitive, and convex, then X^* is convex.

   b) Suppose that preferences are also strictly convex. Show that X^* has at most one element.

   c) Suppose that $\succeq$ is strictly monotone and that $X \subseteq \mathbb{R}^L$ is an open set. Show that $\succeq$ is also locally non-satiated.

   d) Suppose that X is a non-empty compact set and that $\succeq$ is complete, transitive, and continuous (but not necessarily convex). Prove that preferences cannot be locally non-satiated (Hint: show $X^* \neq \emptyset$). What does this imply about the relationship between monotonicity and local non-satiation?