14.121 Problem Set 3

Due: 10/4 in class

1. 16.D.2 (a pictorial example like in Lecture 6 slide 21 suffices)

2. Consider the neoclassical growth model economy described on slide 30 of lecture 6, with the additional assumption that f is weakly increasing, $f(0) = 0$, and $f'(0) > 1$, so there exists $\bar{k} > 0$ such that $f(k) > k$ for $k \in (0, \bar{k})$. Verify that assumptions 1-5 in Debreu (1954) are satisfied (lecture 6, slides 28-29). Given that this economy has exactly one Pareto optimal allocation, what do the results from Debreu (1954) imply about the existence and uniqueness of a valuation equilibrium?

3. Recall the environment from problem set 2 problem 2, but without the borrowing constraint. Consider a Pareto optimal allocation where L has a greater Pareto weight, or $\lambda_L/\lambda_B > 1$. Express prices $(1, p)$ and wealth levels (w_B, w_L) in terms of the relative Pareto weight $\lambda \equiv \lambda_L/\lambda_B$ that implement this allocation as a Walrasian equilibrium with transfers.

4. Suppose there is a continuum of agents indexed on $[0, 1]$ that is divided among n types. All agents have preferences defined on a finite set of bundles $X = \{c_1, .., c_n\} \subset \mathbb{R}^L_+$. Each type j comprises a fraction $\alpha_j > 0$ of the total population. Every agent of type j is endowed with a bundle c_j and has preferences on bundles given by $u_j : X \to \mathbb{R}$. Agents are able to buy lotteries on the consumption bundles. Formally, each agent has a consumption set $\tilde{X} \subset \mathbb{R}^n_+$ consisting of distributions $\tilde{X} = \Delta(X) = \{\pi : X \to \mathbb{R} : \pi(c_i) \geq 0, \sum_i \pi(c_i) = 1\}$ and expected utility preferences over lotteries, $U_j(\pi) = \sum_{i=1}^n \pi(c_i)u_j(c_i)$.

a. Write down the agent's optimization problem and the market clearing conditions for a competitive equilibrium in this economy.

b. (Optional) Suppose $n = 2$, $u_j(c_1) \neq u_j(c_2)$ for $j = 1, 2$ and $\alpha_1 \neq \alpha_2$. Show that in a competitive equilibrium, the agents must consume their endowments.

5. (Based on Prescott and Townsend, IER 1984) Consider a one-period economy populated by a continuum of agents indexed by $[0, 1]$. Each agent has an endowment e of an input and a household production function that turns this input into utility, $U(c, \theta)$. Here c is the amount of input used by the household and θ is a "type" or preference shock.

Importantly, types are assigned randomly: ex ante it is known that there are two types, $\Theta = \{\theta_1, \theta_2\}$, and that a fraction λ of agents will get the first type while $1 - \lambda$ will get the second. Note that since λ is fixed and there is a continuum of agents, there is no aggregate uncertainty.

Concretely $e = 1$ for all agents, $U(c, \theta_1) = \sqrt{c}$ and $U(c, \theta_2) = c$. Note that type θ_1 is risk averse but type θ_2 is risk neutral.

Assume first that types are publicly observed once assigned, and that contracts are enforceable. Agent i can agree ex ante to any insurance contract $c_i(\theta)$ that assigns them an amount $c(\theta_1)$ of the input if they are type 1 and $c(\theta_2)$ if they are type 2. Households have expected utility preferences over states, $W[c(\theta)|\theta \in \Theta] = \lambda U(c(\theta_1), \theta_1) + (1 - \lambda)U(c(\theta_2), \theta_2)$.

a. Solve for the symmetric Pareto optimal allocation (symmetric means that each i gets the same contract, not that types θ_1 and θ_2 are treated equally).

b. Imagine that there is an intermediary firm in the economy that trades insurance contracts. Concretely, at a price $p(\theta)$ the firm commits to give out $y(\theta)$ to type θ agents if $y(\theta) > 0$ or take in $-y(\theta)$ if $y(\theta) < 0$. Show how the allocation in part a can be supported as a Walrasian equilibrium. What are the equilibrium prices $(p(\theta_1), p(\theta_2))$, up to a positive scalar multiple?

Now suppose instead that types are private information. Agents can still agree to an insurance contract $(c(\theta_1), c(\theta_2))$ ex ante. However, ex post they can claim that their type is $\bar{\theta}$ and consequently receive $c(\bar{\theta})$, even if their true type is $\theta \neq \bar{\theta}$. Recall that an insurance contract is *incentive compatible* (IC) if ex post the agent never has an incentive to lie about their type.

c. Is the allocation you found above incentive compatible?

d. Solve for a symmetric Pareto optimal allocation. Hint: despite the IC constraint, can you still attain the same utilities as above? (Second hint: what happens if you make $c(\theta_2)$ a lottery instead of a certain allocation?)

e. (Optional) Imagine that there is an intermediary firm in the economy that trades insurance contracts. Concretely, for each c in a finite set $C \subset \mathbb{R}_+$, at price $p(c, \theta)$ the firm commits to give out $y(c, \theta)$ units of a bundle with c units of the consumption good (collectively) to agents who announce that they are of type θ if $y(c, \theta) > 0$ or take in $-y(c, \theta)$ if $y(c, \theta) < 0$. Consumers can buy incentive compatible insurance contracts $\{x(c, \theta) | c \in C, \theta \in \Theta\} \subset \mathbb{R}^{2n}$, where $x(c, \theta)$ is the probability of receiving c upon announcing type θ. Show how the allocation in part d can be supported as a Walrasian equilibrium. What are the equilibrium prices?

f. (Optional) Is the full insurance utility level still attainable with private information if neither agent is risk neutral?