14.121 Microeconomic Theory I: Final Exam

Prof. Parag Pathak

Fall 2018

Instructions.

- This is a closed-book examination. Put away your books, handouts, notes, calculators, laptops, cellular phones, and other gadgets.
- **Print your name and email clearly on the front cover of each answer booklet you use**. Write your answers clearly. We won't grade unreadable answers.
- The examination is for 90 minutes.
- There are five questions and a total of 100 points.
- In the exam, you are asked to "prove" certain statements. In doing so, you may rely on any results from mathematics you wish, but you should clearly state the steps of your argument and the theorems you reference, if appropriate. Results from economics should be proven unless noted. When in doubt, state all assumptions and proceed. Partial credit will be given for clear logic and careful reasoning.

1) Consumer theory (25 points)

An individual has two budget constraints. That is, any bundle has to be purchased using points and dollars, and he has w_1 points and w_2 dollars. Price vectors in points are $p_1 \in \mathbb{R}^N_{++}$ and in dollars are $p_2 \in \mathbb{R}^N_{++}$. (Note that p_1 is not the price fo good 1, but a vector of prices.

We will say that a bundle is feasible if and only if it is feasible under both both constraints.

Suppose that preferences are represented by continuous and weakly monotone u, where weakly monotone means increasing means an increase in all goods is beneficial.

Let $x(p_1, p_2, w_1, w_2) \in \mathbb{R}^N_+$ denote optimal demand at prices p_1, p_2, and wealths w_1 and w_2.

(a) What is the FOC for x? **[6 points]**

Answer: The FOC are

$$\nabla u(x^*) \leq \lambda p_1 + \mu p_2$$

with equality if $x \gg 0$ where $\lambda \geq 0$ is the multiplier for the budget constraint on points, and $\mu \geq 0$ is that for the budget constraint on dollars. $\lambda > 0$ if the constraint on points is binding, and $\mu > 0$ if the constraint on dollars is binding. We could say

$$\lambda[xp_1 - w_1] = 0 \quad \text{and} \quad \mu[xp_2 - w_2] = 0.$$

Note: complete answer is careful about boundary conditions.

(b) If x homogenous of degree k in something? If so, what is k and what is the something? **[4 points]**

Answer: It is HD0 in (p_1, p_2, w_1, w_2), (p_1, w_1) or (p_2, w_2) because the constraints are not changed when we multiply any of these vectors by α.

Note: complete answer has all three arguments. 3 out of 4 points for subset.

(c) Suppose that preferences are homothetic. Suppose we had the simple case with only one budget constraint in terms of dollars. What does that imply the homogeneity of demand in wealth? Explain. **[5 points]**

Answer: In the simple case, $x(p, w) = \alpha x(p)$, when we have homothetic preferences. Income elasticity of demand is 1 (see proof from class).

(d) Does this property still hold in our situation with two budget constraints? Explain. **[5 points]**

Answer. Demand is not HD1 in either w_1 or w_2 alone. If you start with some (p_1, p_2, w_1, w_2) where $p_1 = p_2$ but $w_2 > w_1$, then if you multiply w_2 by 2 you do not change the intersection of these budget sets, but the solution is the same.

Demand is HD1 in (w_1, w_2); that is, for any $\alpha > 0$, $x(p_1, p_2, \alpha w_1, \alpha w_2) = \alpha x(p_1, p_2, w_1, w_2)$. The proof is the same one given in class.

(e) Let $v = u(x(p_1, p_2, w_1, w_2))$ be the indirect utility function. Is v quasi-convex? **[5 points]**

Answer. It may not be quasi-convex. The budget set $\alpha(p_1, p_2, w_1, w_2) + (1 - \alpha)(p_1, p_2, w_1', w_2')$ may contain points that are not feasible under either (p_1, p_2, w_1, w_2) or (p_1, p_2, w_1', w_2'). Suppose that we only have two goods and $p = p' = q = q' = (1, 1)$, $w_1 = 2$, $w_2 = 4$, $w_2' = 8$, $w_1' = 8$, and $\alpha = 1/2$. Then, there are points affordable under the combination of budgets, but not under either individual budget (e.g., those that cost larger than 4, but less than 5).

2) Consumer theory (25 points)

Suppose you estimate an expenditure function which takes the following form:

$$e(p_1, p_2, u) = f(\min\{p_1, p_2\}) \cdot g(u).$$

(a) What conditions on f and g are needed for this to be an expenditure function? [**15 points**]

Answer:

i) Expenditure functions must be HD_1 in prices [**2 points**]
This implies that $f = ax$ for $a \neq 0$. If $a > 0$ then $g(u) > 0$. If $a < 0$ then $g(u) < 0$. Why?

f must be HD_1. Since f is a function of only one variable, this means that

$$f(\alpha x) = \alpha f(x).$$

Set $\alpha = \frac{1}{x}$, so that $f(1) = \frac{1}{x} f(x)$, or

$$f(x) = f(1)x$$

Here $a = f(1)$, and we have $f(x) = a \min\{p_1, p_2\}$ [**3 points**]

ii) Expenditure functions must be increasing in u [**3 points**]
Therefore, g must be increasing

iii) Expenditure functions must be non-decreasing in price [**3 points**]
Since $f(x) = ax$ we already have this

iv) Expenditure functions must be concave in prices. [**2 points**]
This follows by definition here:

$$\begin{aligned}
e(\alpha p + (1-\alpha)p', u) &= a \min\{\alpha p_1 + (1-\alpha)p_1', \alpha p_2 + (1-\alpha)p_2'\} g(u) \\
&= a(\alpha p_i + (1-\alpha)p_i')g(u) \\
&\geq a(\alpha \min\{p_1, p_2\} + (1-\alpha)\min\{p_1', p_2'\} g(u) \\
&= \alpha e(p, u) + (1-\alpha)e(p', u)
\end{aligned}$$

v) Expenditure functions must be continuous [**2 points**]
f is already continuous; so we further need g to be continuous

A full credit answer must state that $f(\min\{p_1, p_2\}) = a \min\{p_1, p_2\}$ and $g(u)$ is such that $g(u) > 0$, $g(u)$ increasing, $g(u)$ is continuous.

(b) What utility function would generate this e? [**10 points**]

Answer:

Since there is a min in the price, we must have perfect substitutes, or some kind of linear utility. But we must be careful to adjust for the $g(u)$ term. Consider

$$u(x_1, x_2) = g^{-1}\left(\frac{1}{a}(x_1 + x_2)\right).$$

Why does this work?

To obtain u utils, it must be that $u = g^{-1}(\frac{1}{a}(x_1 + x_2))$ or

$$g(u) = (\frac{1}{a}(x_1 + x_2))$$

So if $p_1 < p_2$, we'd buy $ag(u)$ units of the cheaper good at price p_1. That will cost us

$$a \min\{p_1, p_2\} g(u)$$

Note: $x_1 + x_2$ doesn't work here. Why? Because the expenditure function is

$$e(p, u) = \min\{p_1, p_2\} u,$$

and we did not assume that $g(u) = u$. Partial credit given for linear utility.

Note: other monotonic transformations of $ag(u(x_1, x_2))$ do work.

3) Producer theory (10 points)

(a) Suppose a firm's production function is

$$f(x_1, x_2) = \min\{ax_1, bx_2\}.$$

What is the cost function? [**5 points**]

Answer: The firm will not waste any input with a positive price, so it will operate at the point where $y = ax_1 = bx_2$. Hence, to produce y units of output, it must use y/a units of good 1 and y/b units of good 2 for any input prices. Therefore, the cost function is:

$$c(w_1, w_2, y) = \frac{w_1 y}{a} + \frac{w_2 y}{b} = y(\frac{w_1}{a} + \frac{w_2}{b})$$

(b) Suppose a firm's production function is

$$f(x_1, x_2) = ax_1 + bx_2.$$

What is the cost function? [**5 points**]

Answer: Since the two factors are perfect substitutes, the firm will use which ever factor is cheaper. The cost function will therefore be:

$$c(w_1, w_2, y) = \min\{w_1/a, w_2/b\} y$$

4) Short answer questions (20 points)

(a) What is Samuelson's LeChatelier principle? Describe some conditions under which it is true. [**5 points**]

Answer: A firm reacts more to input price changes in the long-run than in the short-run because it has more inputs it can adjust. We saw in class that if we have two factors of production that are complements or substitutes then the result holds. We can't sometimes have complements, sometimes substitutes.

(b) You have data from a monopoly and estimate that the elasticity of demand is -0.5. Is this estimate consistent with theory? Explain. [**5 points**]

Answer: Recall the Lerner elasticity rule:

$$\frac{P(q^m) - c'(q_m)}{P(q^m)} = -\frac{1}{\epsilon}$$

If $\epsilon = -0.5$, then the percentage markup is 2. But the denominator is always greater than the numerator in the expression above, so this cannot be true. Said another way, the monopolist only prices at a point where $\epsilon < -1$, where demand is elastic.

(c) Does the Coase Theorem need to assume that there are no wealth effects? Elaborate [**5 points**]

Answer. The outcome can be a function of the initial assignment when there are wealth effects. That is, the initial assignment determines which of the efficient allocations is selected. So if the Coase theorem means the outcome + efficiency are unaffected as long as there are property rights and no transaction costs, this is not true with wealth effects. (The key idea is when we have quasi-linear preferences, there is one efficient point)

Note: received 4 points if interpret Coase theorem as stating outcome is efficient (since notes are about efficiency)

(d) Your friend doing a physics PhD criticizes economics by saying that it's not a *real* science, since neither microeconomics or macroeconomics have testable predictions. Is she right? [**5 points**]

Answer. Looking for discussion of Slutsky/compensated demand is downward sloping; Sonnenschein-Mantel-Debreu theorem on anything goes.

5) General equilibrium (20 points)

Consider an economy with $2I + 1$ consumers. Suppose I each own a right shoe and $I + 1$ each own a left shoe. Shoes are indivisible. Everyone has the same utility function which is $\min\{R, L\}$ where R and L are, respectively, the quantities of right and left shoes consumed.

(a) Show that any allocation of shoes where every individual consumes the same number of shoes of each kind (except possibly one individual, who consumes one more left shoe than right shoes) is a Pareto optimum. Show the converse statement. [**6 points**]

Answer: If some pair of shoes is not matched in a feasible allocation, than we can find a Pareto dominating allocation by combining the unmatched pair. If all pairs are matched, then it is Pareto optimal. (can formally show by contradiction)

(b) Which Pareto optima are in the core of this economy? [**7 points**]

Answer: Core allocations are those only where each owner of right shoes receives a pair of shoes.

Suppose x is feasible where each owner of right shoes receives a pair of shoes. Suppose there is a blocking coalition S which contains J owners of right shoes. The whole coalition S owns J right shoes and at x receives J pairs of shoes. Every member of S must receive at least a many pairs as before, and some must receive more. The whole S must thus receive more than J pairs at the blocking allocation, but this is not feasible since the whole S owns only J right shoes. Thus, x cannot be blocked.

Let x be Pareto in which some owner of right shoes receives no pair. Since there are $I + 1$ owners of the left shoe and only 1 right shoes are available in the economy, some owner of the left shoe receives none of the right shoes. Thus these two consumers are at zero. But if they form a coalition, then at least one of them can get a pair. Hence, that coalition will block x, implying that x does not have the core property.

(c) Let p_R and p_L be prices of the two kinds of shoe. Find the Walrasian equilbria of this economy. [**5 points**]

Answer: p must be a positive multiple of $(1, 0)$, and at x, each owner of right shoes receives a pair of shoes. If (x, p) is an equilibrium, it cannot be that $p \gg 0$. If $p \gg 0$, then each consumer can only afford one shoe. Thus the aggregate demand is at most $2I$ shoes, and hence a shoe would be left unsold in the market, contradicting market clearing. So one of the prices must be zero. If $p_R = 0$ then, every owner of the left shoe demands a pair, which contradicts the feasibility because there are $I + 1$ owners of left shoes. Hence, it must be the case that $p_L = 0$ and $p_R > 0$.

(d) Comment on the relationship between the core and Walrasian equilibria in this economy. [**2 points**]

Answer: Regardless of I, the core = walrasian equilibrium.