# Lecture 9. Arbitrage Pricing Theory (APT), part 2

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# Outline

# Today: APT, Factor models, and empirical asset pricing tests

- Last time, we derived an APT result which held when returns followed an exact factor structure

$$\bar{r}_n - r_F = \sum_k \beta_{nk}(\bar{r}_k - r_F) \equiv \sum_k \beta_{nk}\lambda_k, \quad n = 1, \ldots, N.$$

  where $\lambda_k = \bar{r}_k - r_F$ is risk premium of the $k^{th}$ factor-mimicking portfolio

- Today, we will derive a similar pricing result when a large number of returns follow an approximate factor structure
- We'll discuss two common methods for testing these predictions in the data

# Factor Model for Returns

We will work with a generalized model for security returns.

## Definition (Factor Model for Returns)

In a factor model, security returns are given as follows:

$$r_n = \bar{r}_n + \sum_{k=1}^{K} \beta_{nk} F_k + \varepsilon_n, \quad n = 1, \ldots, N, \tag{1}$$

where

(1) $\mathbb{E}[F_k] = \mathbb{E}[\varepsilon_n] = \mathbb{E}[\varepsilon_n | F_k] = 0 \;\; \forall \; k, n$

(2) $\mathbb{E}[\varepsilon_n^2] = \sigma_n^2 < v < \infty, \;\; \mathbb{E}[\varepsilon_n \varepsilon_{n'}] = 0 \;\; \forall \; n \neq n'$.

- $F_1, \ldots, F_K$ define the risk factors common to all securities.
- $\varepsilon_1, \ldots, \varepsilon_N$ capture the risks specific/idiosyncratic to individual securities.
- Setting $\varepsilon_n = 0 \; \forall \; n$, we return to the exact factor model.
- Here, we assume $N$ very large (approaching infinity) while $K$ small.

# Factor Model for Returns

We can also express the factor model in matrix form. Let:

$$r \equiv [r_1, \ldots, r_N], \quad \bar{r} \equiv [\bar{r}_1, \ldots, \bar{r}_N],$$
$$F \equiv [F_1, \ldots, F_K], \quad \beta_n \equiv [\beta_{n1}; \ldots; \beta_{nK}], \quad \beta \equiv [\beta_1, \ldots, \beta_N],$$
$$\varepsilon \equiv [\varepsilon_1, \ldots, \varepsilon_N].$$

We can then express the factor model for returns as follows:

$$r = \bar{r} + F\,\beta + \varepsilon,$$

where

(1) $\mathbb{E}[F] = 0,\ \mathbb{E}[\varepsilon] = 0,\ \mathbb{E}[\varepsilon|F] = 0,$

(2) $\mathbb{E}[\varepsilon^\top \varepsilon] = \Sigma,$

and $\Sigma$ is a diagonal matrix with diagonal elements bounded by $v$.

# Generalizing our variance decomposition

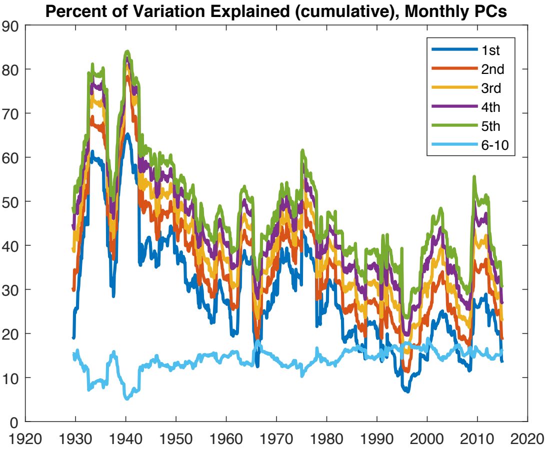
From last time,

- $Var[r_n] = \beta_n^2 Var[r_{-\eta}] + Var[\varepsilon_n]$
  where $r_{-\eta} - r_F$ was the SDF-mimicking portfolio and $\beta_n \in \mathbb{R}$ and $\varepsilon_n$ was defined via a regression equation.

Under our current assumptions,

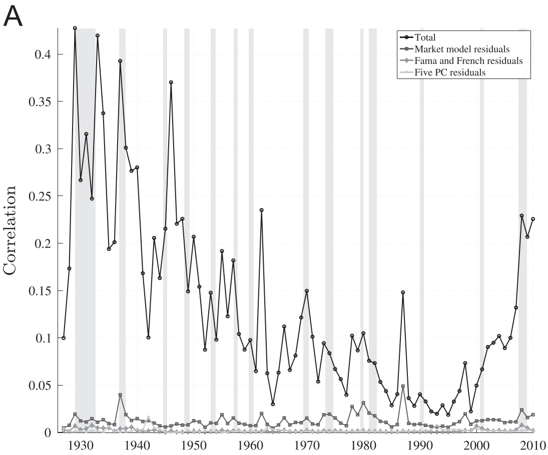
- $$Var(r_n) = \beta_n' \underbrace{E\left[\begin{pmatrix} F_1 \\ \vdots \\ F_K \end{pmatrix} \begin{pmatrix} F_1 & \dots & F_K \end{pmatrix}\right]}_{\text{VCV matrix of factors}} \beta_n + Var[\varepsilon_n]$$
- $$Cov(r_i, r_j) = \beta_i' E\left[\begin{pmatrix} F_1 \\ \vdots \\ F_K \end{pmatrix} \begin{pmatrix} F_1 & \dots & F_K \end{pmatrix}\right] \beta_j + \underbrace{Cov[\varepsilon_i, \varepsilon_j]}_{=0 \text{ by assumption}}$$

# Data: Factor Structure in Returns




- Principal components analysis decomposes the realized variance of a (potentially high-dimensional) set of random vectors into a set of $K$ independent factors and residuals uncorrelated w/ the factors.
- Chart shows the cumulative fraction of variation across all monthly common US stock returns explained by factors $1, 2, \ldots, 5$ using 36 month rolling estimation windows
- Number of stocks varies between 600 and 7000.
- Factors 6-10 explain another 10% of variance.

# Data: Factor Structure in Returns




- Graph uses daily stock returns and plots the average pairwise correlation between residuals of different stock returns after various factors have been removed
- Source: Herskovic, Lustig, Kelly, and Van Nieuwerburgh (2016), "The common factor in idiosyncratic volatility: Quantitative asset pricing implications", *Journal of Financial Economics*

## Diversification

- We need to know the potential influence of idiosyncratic risks on pricing.
- Let $\theta$ be a portfolio:

  $$\theta = [\theta_1; \ldots; \theta_N] \quad \text{where} \quad 1_N^\top \theta = 1.$$

- The return on the portfolio is:

  $$r_\theta = \bar{r}_\theta + F\,\beta_\theta + \varepsilon_\theta, \quad \text{where } r_\theta = r\theta,\ \bar{r}_\theta = \bar{r}\theta,\ \beta_\theta = \beta\,\theta,\ \varepsilon_\theta = \varepsilon\theta.$$

- Portfolio $\theta$ is called well diversified if

  $$\theta_n = O\left(\frac{1}{n}\right),$$

  where $O(\frac{1}{n})$ means of order $1/n$: i.e., as $n \to \infty$, $n \cdot ||\theta||^\infty$ is bounded

# Diversification

## Definition

A sequence of portfolios $\theta_1, \ldots, \theta_n, \ldots$ with $1_n^\top \theta = 1$ is well diversified if there exists $0 < \kappa < \infty$ such that for all $n$ and $i = 1, \ldots, n$:

$$\theta_{ni}^2 < \frac{\kappa}{n^2}.$$

- The definition applies to a sequence of portfolios, not a single portfolio.
- Portfolio $\theta = \left[\frac{1}{n}; \ldots; \frac{1}{n}\right]$ is well diversified while $\theta = [0; \ldots; 1; \ldots 0]$ is concentrated.
- Diversification is a limiting statement BUT modern financial markets include thousands of assets in the investment universe.

# Example: Variance of an equally weighted portfolio

- Suppose that we form an equal-weighted portfolio with weight $1/N$ on each of $N$ assets
- What is its variance?

$$
\begin{aligned}
Var\left[\sum_{j=1}^{N} \tfrac{1}{N} r_j\right] &= \sum_{i=1}^{N}\sum_{j=1}^{N} \tfrac{1}{N^2} Cov(r_i, r_j) \\
&= \tfrac{1}{N} \underbrace{\left[\tfrac{1}{N}\sum_{i=1}^{N} Var[r_i]\right]}_{\text{average variance}} + (1 - \tfrac{1}{N}) \underbrace{\left[\tfrac{1}{N(N-1)}\sum_{i=1}\sum_{j\neq i} Cov(r_i, r_j)\right]}_{\text{average covariance}}
\end{aligned}
$$

- Suppose $r_i = \bar{r} + F + \epsilon_i$, where $Var(\epsilon_i) = \sigma_\epsilon^2$

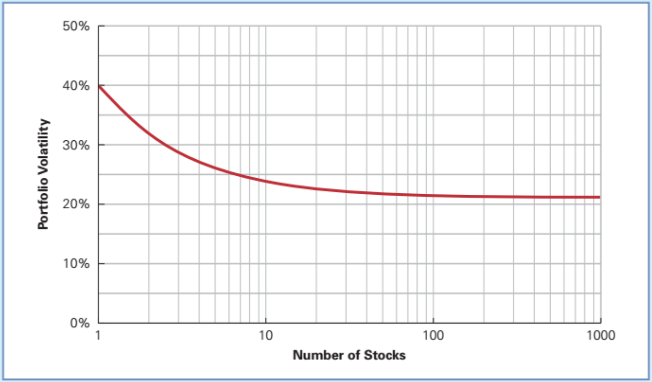
$$
Var\left[\sum_{j=1}^{N} \tfrac{1}{N} r_j\right] = \tfrac{1}{N}(\sigma_\epsilon^2 + Var(F)) + (1 - \tfrac{1}{N}) Var(F) \overset{N\to\infty}{\to} Var(F)
$$

# Variance of an equally weighted portfolio

FIGURE 11.2

**Volatility of an Equally Weighted Portfolio Versus the Number of Stocks**

The volatility declines as the number of stocks in the portfolio increases. Even in a very large portfolio, however, market risk remains.



# Systematic vs idiosyncratic risk

- An asset pricing model splits an asset's return variance into 2 components:
  - Diversifiable (unsystematic risk): Firm-specific or idiosyncratic risk, Can be eliminated by holding a well-diversified portfolio.
    Example: success or failure of a new product line
  - Non-Diversifiable (systematic risk): Common sources of risk that (e.g., factors $F$) cannot be eliminated through portfolio selection.
    Example: in a macroeconomic crisis, there is nowhere to hide

- Asset pricing models seek to identify sources of **systematic, priced risk**
  - Systematic: non-diversifiable, explains variation in returns across *all* assets. Present in well-diversified portfolios.
  - Priced: covariance with the risk affects an asset's risk premium. There can be systematic risks (common factors in realized returns) that are not priced.
    - Under our assumptions, this can happen if linear combinations of $\beta$s are not associated with changes in expected returns (e.g., $\lambda_k = 0$ for some $k$ in APT).

# Diversification

## Theorem (Diversification Theorem)

*For a sequence of well diversified portfolios, $\theta_n$, $n = 1, 2, \ldots$,*

$$\mathbb{V}\left[\varepsilon_{\theta_n}\right] = \mathbb{V}\left[\sum_i \theta_{ni}\varepsilon_i\right] \;\rightarrow\; 0$$

*of order $1/n$: i.e., as $n \rightarrow \infty$, $n \cdot \sup_{i=1,\ldots,n} \theta_i$ is bounded above by a constant.*

Proof. We have:

$$\mathbb{V}\left[\varepsilon_{\theta_n}\right] = \mathbb{V}\left[\sum_{i=1}^{n} \theta_{ni}\varepsilon_i\right] = \sum_i \theta_{ni}^2\sigma_i^2 < \sum_i v\theta_{ni}^2 < v\sum_i \kappa\left(\frac{1}{n}\right)^2 = \kappa v\left(\frac{1}{n}\right).$$

Thus, as $n \rightarrow \infty$, $\mathbb{V}[\varepsilon_{\theta_n}] \rightarrow 0$ of order $1/n$.

- Diversification eliminates idiosyncratic risks.
- Well diversified portfolios have only systematic/factor risks.

# Asymptotic Arbitrage

## Definition (Asymptotic Arbitrage)

An asymptotic arbitrage is a sequence of portfolios, $\theta_n$, $n = 1, \ldots$, that are self-financed ($1_n^\top \theta_n = 0$) such that as $n \to \infty$, we have:

(1) $\mathbb{E}\left[r_{\theta_n}\right] \to \alpha > 0$

(2) $\mathbb{V}\left[r_{\theta_n}\right] \to 0.$

- Asymptotic arbitrage is an arbitrage in the limit
- As $n \to \infty$, the self-financed portfolio yields a positive payoff with no risk as measured by volatility.
  - For any finite $n$, the portfolio does carry risk, however small.
  - Volatility may not be a sufficient measure of risk.

# Arbitrage Pricing Theory (APT): General Version

## Theorem (APT)

*Given a $K$-factor model for security returns, if there is no asymptotic arbitrage, then there exists a scalar, $r_F$, a $K$-vector, $\lambda = [\lambda_1; \ldots; \lambda_K]$, and a positive constant $A$, such that:*

$$\sum_{i=1}^{n} \left[\bar{r}_i - \left(r_F + \lambda^\top \beta_i\right)\right]^2 = \sum_{i=1}^{n} \left[\bar{r}_i - \left(r_F + \sum_k \lambda_k \beta_{ik}\right)\right]^2 < A < \infty. \tag{2}$$

- The APT states that in a factor model of returns, no asymptotic arbitrage (NAA) yields approximate factor pricing:

$$\bar{r}_i - r_F \approx \sum_{k=1}^{K} \lambda_k \beta_{ik}, \quad \forall\ i. \tag{3}$$

- The pricing error, denoted by $\delta_i$ for security $i$, is "non-trivial" only for a small number of securities:

$$\sum_i |\delta_i|^2 < A < \infty.$$

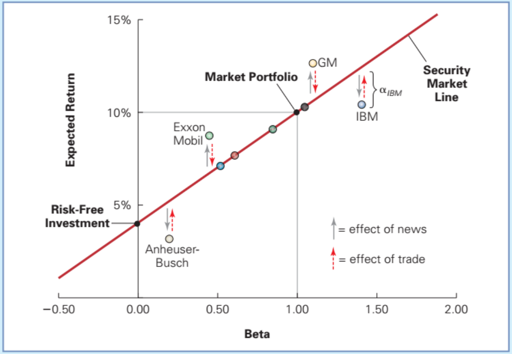
# Logic of the APT: price adjustment to news (e.g., change in $E[D_n]$)

**Investors will trade to eliminate asymptotic arbitrage opportunities**
**$\Rightarrow$ prices adjust until all expected returns line up w/ $\beta$**

FIGURE 13.2

**Deviations from the Security Market Line**

If the market portfolio is not efficient, then stocks will not all lie on the security market line. The distance of a stock above or below the security market line is the stock's alpha. We can improve upon the market portfolio by buying stocks with positive alphas and selling stocks with negative alphas, but as we do so, prices will change and their alphas will shrink toward zero.



# Arbitrage Pricing Theory (APT)

## Proof of APT

Proof. We only prove the case with one factor. Let $\bar{r} \equiv [\bar{r}_1, \ldots, \bar{r}_n]$ denote the (row) vector of expected returns of $n$ securities and $\beta \equiv [\beta_1, \ldots, \beta_n]$ the vector of their betas. Consider the projection of $\bar{r}$ on $1_n^\top$ ($1_n$ is column vector) and $\beta$:

$$\bar{r} = a_0 1_n^\top + a_1 \beta + \delta^\top, \quad \delta \equiv [\delta_1; \ldots; \delta_n].$$

We want to show that $\delta^\top \delta$ is bounded if $n \to \infty$. Since $\delta^\top$ is the projection residual, it is orthogonal to $1_n^\top$ and $\beta$:

$$1_n^\top \delta = 0 \quad \text{and} \quad \beta\,\delta = 0.$$

Thus, $\delta$ is a self-financing portfolio (first equation) with no factor risk (second equation). Then, $\delta' = b\,\delta$ $(b > 0)$ is also an arbitrage portfolio, and

$$\mathbb{E}[r_{\delta'}] = \mathbb{E}\left[b\,r\,\delta\right] = b\,\bar{r}\,\delta = b\,\left(a_0 1_n^\top + a_1\beta + \delta^\top\right)\,\delta = b\,\left(\delta^\top\delta\right),$$

where the last equality follows because $\delta^\top$ is orthogonal to $1_n^\top$ and $\beta$, and

$$\mathbb{V}\left[r_{\delta'}\right] = \mathbb{V}\left[b\,r\,\delta\right] = \mathbb{V}\left[b\left(\bar{r} + \beta F + \varepsilon\right)\delta\right] = b^2 \delta^\top \Sigma \delta < b^2 v \left(\delta^\top\delta\right).$$

# Arbitrage Pricing Theory (APT)

## Proof of APT

Proof. (Continued) Let

$$b = \frac{1}{\delta^\top \delta},$$

we have

$$\bar{r}_{\delta'} = b\left(\delta^\top \delta\right) = 1$$

and

$$\mathbb{V}\left[r_{\delta'}\right] < b^2 v\left(\delta^\top \delta\right) = \frac{v}{\delta^\top \delta} \to 0$$

as $n \to \infty$ if $\delta^\top \delta$ is not bounded. This is an asymptotic arbitrage. Thus, NAA requires $\delta^\top \delta$ to be bounded.

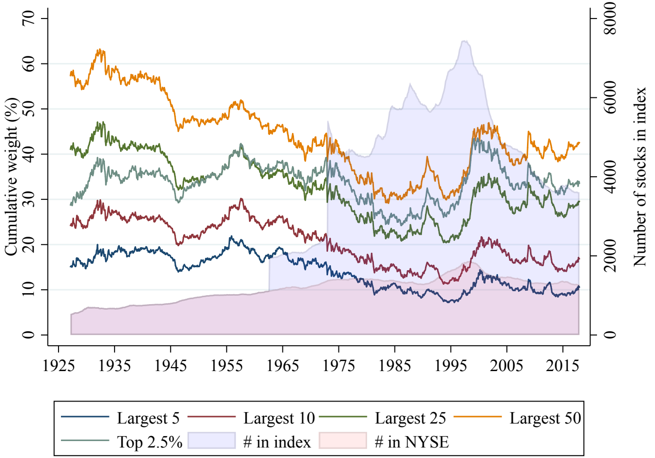
# Implications of APT

- With a large number of securities and a small number of risk factors, APT applies (to most – but not all – securities):

$$\bar{r}_i - r_F = \sum_{k=1}^{K} \beta_{ik} \left( \bar{r}_k - r_F \right) = \sum_{k=1}^{K} \beta_{ik} \lambda_k, \ \forall \ i.$$

- $\beta_{ik}$ is the factor loading of security $i$ on factor $k$.
- $\bar{r}_k - r_F = \lambda_k$ is risk premium of traded portfolio that mimics factor $k$.
- The implementation of APT requires the determination of factors and the corresponding factor premiums.
- Note that APT places no restrictions on signs/magnitudes of $\lambda_k$, and there could in principle be a large number of factors that affect risk premia
- Main prediction of APT is equivalent to a model of the SDF which is affine in the factors: $\eta = a + Fb$. See Cochrane Chapter 6 for details

# In Practice: Why might APT not apply to all securities?




- One reason: market value of public firms is extremely concentrated among a small number of stocks (source: my work with Sung Je Byun)
- Relative supply of these mega-cap stocks is so large that, in the aggregate, *someone* must be exposed to their idiosyncratic risks.

# Testing the APT (+ any linear model of the SDF)

- Suppose that I have an empirical proxy for the factor(s) and want to test whether the prediction of the theory holds
- Want to test if expected returns lie on the Securities Market Line (hyperplane w/ $>1$ factor) implied by the expected return of the factors
- Two (closely related) common approaches are taken in the literature:
  1. Time-series: Is alpha zero in a regression of excess asset returns on the factors? Only applicable w/ traded factors (or factor-mimicking portfolios)
  2. Cross-sectional: Do estimates of beta explain average returns? Make an empirical counterpart to the SML.
- Note: this material follows Campbell Chapter 3.3

## Time-series approach

- Get data (usually monthly) T-bill rates, asset returns, and factor returns
- Run a time-series regression:

$$r_{i,t} - r_{f,t} = \alpha_i + \sum_k \beta_{i,k} ( \underbrace{r_{k,t} - r_{F,t}}_{\substack{\text{factor } k \text{ mimicking} \\ \text{portfolio excess return}}} ) + \epsilon_{i,t}$$

  for $t = 1, \ldots, T$ for an individual tradable asset or a portfolio

- APT predicts that $\alpha_i = 0$
- Test whether $\hat{\alpha}_i = 0$. Can also do this for multiple assets and consider a joint statistical test for all $\{\hat{\alpha}_i\}_{i=1}^N = 0$
- Implicit assumption: $\beta_{i,k}$ is constant over time. Individual stock betas often time-varying and estimated imprecisely $\Rightarrow$ common to use portfolios

# Cross-sectional approach

**First step:**

- Get data (usually monthly) T-bill rates, asset returns, and factor returns
- Estimate betas for each asset on factors by running a time-series regression:

$$r_{i,t} - r_{F,t} = \alpha_i + \sum_k \beta_{i,k} F_{k,t} + \epsilon_{i,t}$$

and save $\{\alpha_i, \beta_i\}_{i=1}^N$

**Second step:**

- Use your data from the first step to run a cross-sectional regression:

$$\frac{1}{T}\sum_{t=1}^{T}(r_{i,t} - rR_{f,t}) \xrightarrow{p} E[r_i - r_F] = \lambda_0 + \sum_{k=1}^{K} \lambda_k \hat{\beta}_{i,k} + u_i$$

for $i = 1, \ldots, N$

- APT predicts $\lambda_0 = 0$ and $\lambda_k = E[f_{k,t}]$ if factors are tradable excess returns.
- Geometrically, you are estimating the SML with a linear regression
- Works w/ non-traded factors too! Difference: intercept $\lambda_0$ isn't restricted.
- Perfect model + enough data $\Rightarrow u_i = 0$ for all i, $R^2 = 100\%$

# Testing the APT: Practical considerations

- Goal: characterize all major sources of systematic risk and individual stocks/portfolios' exposures to them. Data $\Rightarrow$ risk prices $\lambda_k$
- Step 1: find data on the factors. Sources used in practice:
  - Returns (endogenous): Factor/principal component analysis (PCA).
  - Fundamentals: Macroeconomic shocks (consumption, GDP, etc.)
- Step 2: estimate $\beta$ and $\lambda$. Harder than it sounds! Potential challenges:
  - Validity of the factor model: Are residuals actually uncorrelated or are there missing factors?
  - Stability of factor structure and factor premiums: $\beta$ and $\lambda$ may vary over time and be hard to estimate
    - Exposures to macro factors often particularly hard to estimate: often only available at low frequencies and there may be lead/lag relationships
    - Realized returns may not accurately reflect ex-ante expected returns. Beware of data mining
  - Interpretation: factors may work empirically but not be interpretable

# The Cross-Sectional Asset Pricing Literature

- Eugene Fama and Kenneth French started a massive literature which tries to identify sources of priced systematic risk in the data
- Their approach (still the standard):
    1. Use observable characteristics of individual stocks (e.g., accounting variables) to form portfolios
    2. Test the ability of a given factor model to explain variation in expected returns across portfolios in the data
- If model can't explain the data, call it an **asset pricing anomaly**
- Because 1 factor explains majority of comovements across stocks + reasons that you'll learn about later in the course, the literature began by testing the Capital Asset Pricing Model (CAPM)
- CAPM is equivalent to the APT using the value-weighted portfolio of all traded assets as a single pricing factor.

# Asset pricing patterns unexplained by the CAPM

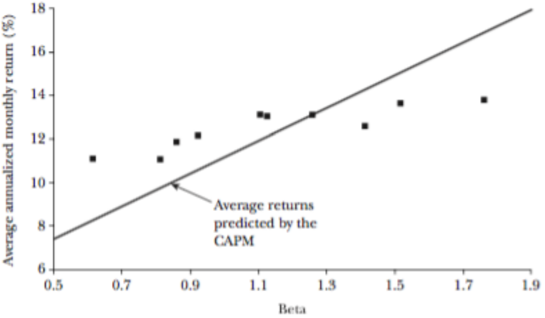
- Fama and French (1992) showed that the CAPM could not explain expected returns of portfolios sorted on several very simple firm characteristics:

  1. **Beta**: Using data from prior to time $t$ estimated $\beta$, then formed portfolios sorting on these estimates
  2. **Size**: A small cap stock is a stock with a low market capitalization. F-F compared returns of small cap vs large cap stocks
  3. **Book to Market Ratio**: is the ratio of its book value to its market value. Equivalently, it is the ratio of the book value per share to the stock price per share.
     - Growth stocks have low book-to-market ratios, tend to have high expected growth rates in earnings, sales, cash flow, or book value.
     - Value stocks have high book-to-market ratios, thought to have lower growth prospects

- Hundreds of other anomalies have been discovered (see Campbell book), one of the most notable and robust being **momentum**: sorting on returns over the prior year

# Historical average returns: Sorting on Beta

*Figure 2*
**Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003**




- Qualititatively, things look good.
- Quantitatively, line is too flat
- Source: Fama and French (2004)

# Historical average returns: Sorting on Size (market cap)

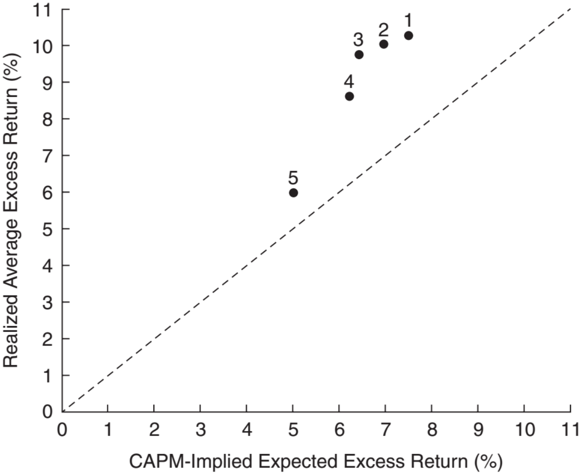



***Figure 3.5.*** *The CAPM and the Size Effect*

- Slope is too steep relative to what CAPM would predict

# Size effect was concentrated in January

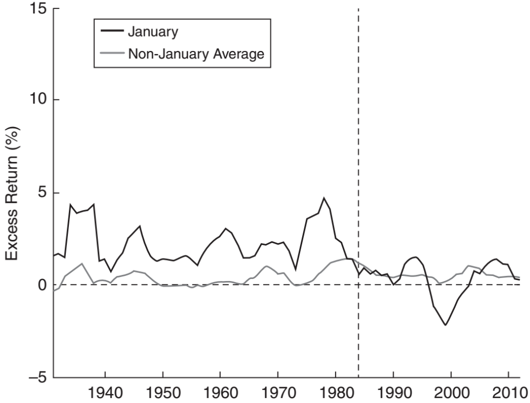



***Figure 3.9.*** *Five-Year Moving Average Excess Returns to Small-Cap Stocks, January vs. Other Months*

- Rationale: capital losses on stocks are tax-deductible, investors tend to sell at end of year to take deduction. Hits smaller, less-liquid stocks harder
- Vertical line is when original paper documenting the phenomenon was published. Magnitudes smaller now!

# Size effect was concentrated in January

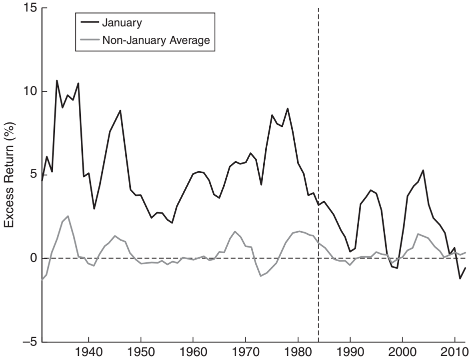



***Figure 3.10.*** *Five-Year Moving Average Excess Returns to Micro-Cap Stocks, January vs. Other Months*

- Magnitudes even bigger for really small (micro-cap) stocks, but also decay
- One reaction to smaller effects later in sample: investors are trading to eliminate them. McLean & Pontiff (2016) show that volumes & volatility increase following publication: this is anomaly elimination

# Historical average returns: Sorting on Book-to-market ratio

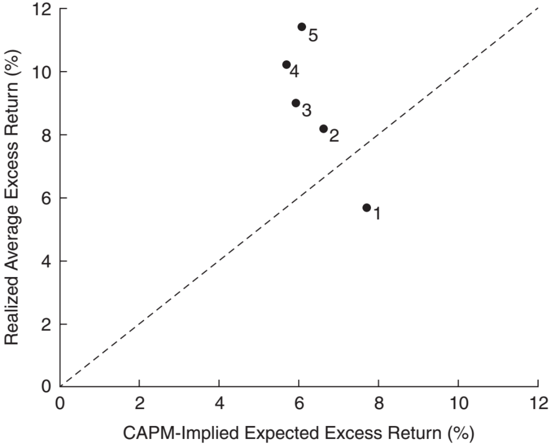
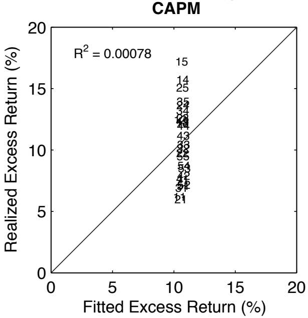



***Figure 3.6.*** *The CAPM and the Value Effect*

- Betas and expected returns not even *qualitatively* related in direction predicted by the CAPM (though this feature can change depending on sample)

# Things look even worse if we sort on Size AND Value




| CAPM$_\alpha$ | | | | | $t$-value | | | | |
|---|---|---|---|---|---|---|---|---|---|
| −5.14 | 1.84 | 3.84 | 7.65 | 8.08 | −1.38 | 0.61 | 1.45 | 2.90 | 2.82 |
| −3.99 | 1.51 | 4.58 | 5.87 | 6.79 | −1.62 | 0.79 | 2.12 | 2.64 | 2.90 |
| −2.23 | 2.45 | 3.36 | 5.33 | 6.31 | −1.34 | 1.52 | 1.97 | 2.56 | 2.55 |
| −0.55 | 0.90 | 3.68 | 4.65 | 3.98 | −0.36 | 0.66 | 2.41 | 2.48 | 1.96 |
| −0.46 | 0.40 | 2.22 | 1.85 | 1.88 | −0.38 | 0.44 | 1.99 | 1.26 | 0.98 |

Rows: sort on size from small to big

Columns sort on book-to-market from low (growth) to high (value)

Sample: 1954-2003. Source: Jagannathan and Wang (2007)

# Historical average returns: Prior returns (Momentum)

**Table I**

**Momentum Portfolio Returns**

This table reports the monthly returns for momentum portfolios formed based on past six-month returns and held for six months. P1 is the equal-weighted portfolio of 10 percent of the stocks with the highest returns over the previous six months, P2 is the equal-weighted portfolio of the 10 percent of the stocks with the next highest returns, and so on. The “All stocks” sample includes all stocks traded on the NYSE, AMEX, or Nasdaq excluding stocks priced less than $5 at the beginning of the holding period and stocks in the smallest market cap decile (NYSE size decile cutoff). The “Small Cap” and “Large Cap” subsamples comprise stocks in the “All Stocks” sample that are smaller and larger than the median market cap NYSE stock respectively. “EWI” is the returns on the equal-weighted index of stocks in each sample.

| | All Stocks | | | Small Cap | | | Large Cap | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1965–1998 | 1965–1989 | 1990–1998 | 1965–1998 | 1965–1989 | 1990–1998 | 1965–1998 | 1965–1989 | 1990–1998 |
| P1 (Past winners) | 1.65 | 1.63 | 1.69 | 1.70 | 1.69 | 1.73 | 1.56 | 1.52 | 1.66 |
| P2 | 1.39 | 1.41 | 1.32 | 1.45 | 1.50 | 1.33 | 1.25 | 1.24 | 1.27 |
| P3 | 1.28 | 1.30 | 1.21 | 1.37 | 1.42 | 1.23 | 1.12 | 1.10 | 1.19 |
| P4 | 1.19 | 1.21 | 1.13 | 1.26 | 1.34 | 1.05 | 1.10 | 1.07 | 1.20 |
| P5 | 1.17 | 1.18 | 1.12 | 1.26 | 1.33 | 1.06 | 1.05 | 1.00 | 1.19 |
| P6 | 1.13 | 1.15 | 1.09 | 1.19 | 1.26 | 1.01 | 1.09 | 1.05 | 1.20 |
| P7 | 1.11 | 1.12 | 1.09 | 1.14 | 1.20 | 0.99 | 1.09 | 1.04 | 1.23 |
| P8 | 1.05 | 1.05 | 1.03 | 1.09 | 1.17 | 0.89 | 1.04 | 1.00 | 1.17 |
| P9 | 0.90 | 0.94 | 0.77 | 0.84 | 0.95 | 0.54 | 1.00 | 0.96 | 1.09 |
| P10 (Past losers) | 0.42 | 0.46 | 0.30 | 0.28 | 0.35 | 0.08 | 0.70 | 0.68 | 0.78 |
| P1–P10 | 1.23 | 1.17 | 1.39 | 1.42 | 1.34 | 1.65 | 0.86 | 0.85 | 0.88 |
| $t$ statistic | 6.46 | 4.96 | 4.71 | 7.41 | 5.60 | 5.74 | 4.34 | 3.55 | 2.59 |
| EWI | 1.09 | 1.10 | 1.04 | 1.13 | 1.19 | 0.98 | 1.03 | 1.00 | 1.12 |

Source: Jegadeesh and Titman (2001)

# Historical CAPM alphas: Prior returns (Momentum)

| | CAPM Alpha |
|---|---|
| P1 | 0.46 |
| | (3.03) |
| P2 | 0.29 |
| | (2.86) |
| P3 | 0.21 |
| | (2.53) |
| P4 | 0.15 |
| | (1.92) |
| P5 | 0.13 |
| | (1.70) |
| P6 | 0.10 |
| | (1.22) |
| P7 | 0.07 |
| | (0.75) |
| P8 | −0.02 |
| | (−0.19) |
| P9 | −0.21 |
| | (−1.69) |
| P10 | −0.79 |
| | (−4.59) |
| P1–P10 | 1.24 |
| | (6.50) |

T-stats are in parentheses. Source: Jegadeesh and Titman (2001)

# Orders of magnitude are large

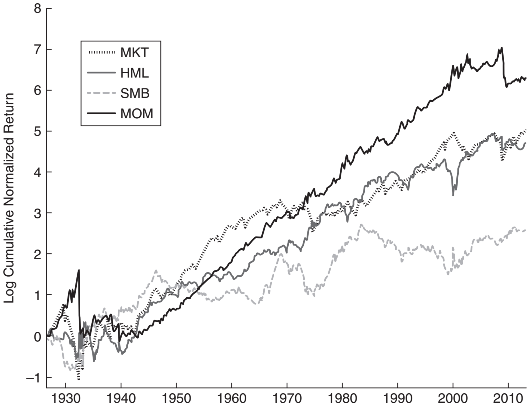



***Figure 3.7.*** *Fama-French-Carhart Log Cumulative Normalized Factor Returns, 1926–2013*

Some anomaly portfolios earn expected returns of the same order of magnitude, or even slightly larger, than the equity premium (MKT $\equiv E[r_m - r_f]$) (!!)
Source: Campbell Textbook

# Interpretations of CAPM failures

- These anomaly portfolios also feature stocks that strongly co-move ⇒ there are additional sources of systematic risk in the data
- APT ⇒ any systematic factor may be priced. Work with multi-factor representations. Most popular empirical specifications:
  - Fama-French 3 factor model: add Value and Size portfolios as additional pricing factors
  - Carhart 4 factor model: Fama French + momentum
  - Fama-French now have a 5 factor model
  - Lu Zhang + coauthors: factor models sorting on investment
  - Rise of the machines! Examples: Giglio and Xiu (2017); Kelly, Pruitt, & Siu (2018), among many others. This is a rapidly growing area
- Another reaction: much of this is a statistical fluke. Data snooping has led to a "zoo" of factors, many of which are spurious.
- Limits to arbitrage: many "overpriced" stocks are a small fraction of the market and hard to short-sell.

# One reaction to the "factor zoo"

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

## Predicting anomaly performance with politics, the weather, global warming, sunspots, and the stars

Robert Novy-Marx

*Simon Graduate School of Business, University of Rochester, 500 Joseph C. Wilson Blvd., Box 270100, Rochester, NY 14627, United States*

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ABSTRACT

Predictive regressions find that the party of the US president, the weather in Manhattan, global warming, the El Niño phenomenon, sunspots, and the conjunctions of the planets all have significant power predicting the performance of popular anomalies. The interpretation of these results has important implications for the asset pricing literature.



- Along these lines, if you want a laugh, check out `http://www.tylervigen.com/spurious-correlations` An Example!

# A similar reaction

## … and the Cross-Section of Expected Returns

**Campbell R. Harvey**
Duke University, National Bureau of Economic Research

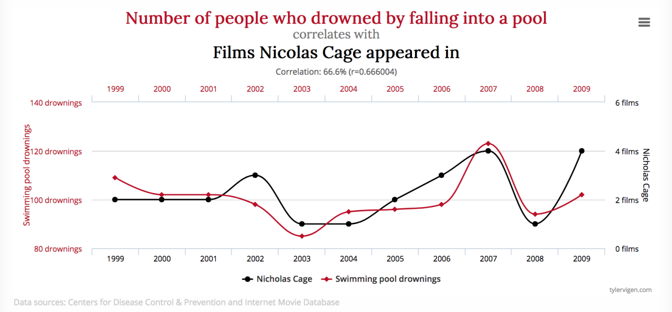
**Yan Liu**
Texas A&M University

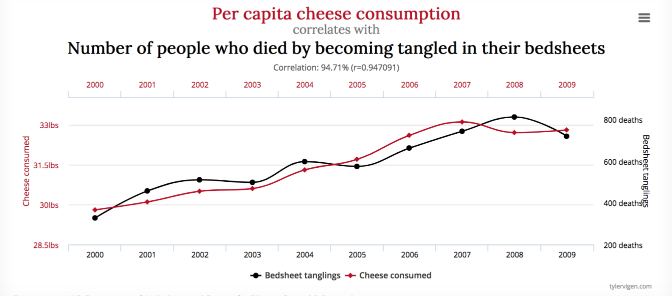
**Heqing Zhu**
The University of Oklahoma

Hundreds of papers and factors attempt to explain the cross-section of expected returns. Given this extensive data mining, it does not make sense to use the usual criteria for establishing significance. Which hurdle should be used for current research? Our paper introduces a new multiple testing framework and provides historical cutoffs from the first empirical tests in 1967 to today. A new factor needs to clear a much higher hurdle, with a $t$-statistic greater than 3.0. We argue that most claimed research findings in financial economics are likely false. (*JEL* C12, C52, G12)

## Where we're going next

- Roughly speaking, this concludes the "reduced-form" part of the course
- Starting next week, we will start working from the bottom up to build a micro-founded model
- One important lesson to keep in mind: the data are highly informative about models and investors' expectations. Ignore data at your own risk!
- Most successful researchers in asset pricing work with both theory and data because they complement one another.


Number of people who drowned by falling into a pool
correlates with
Films Nicolas Cage appeared in
Correlation: 66.6% (r=0.666004)
1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009
140 drownings
120 drownings
100 drownings
80 drownings
Swimming pool drownings
6 films
4 films
2 films
0 films
Nicolas Cage
Nicolas Cage
Swimming pool drownings
tylervigen.com
Data sources: Centers for Disease Control & Prevention and Internet Movie Database


Per capita cheese consumption
correlates with
Number of people who died by becoming tangled in their bedsheets
Correlation: 94.71% (r=0.947091)
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009
33lbs
31.5lbs
30lbs
28.5lbs
Cheese consumed
800 deaths
600 deaths
400 deaths
200 deaths
Bedsheet tanglings
Bedsheet tanglings
Cheese consumed
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