14.121 Problem Set 5

Due: 10/23 in class

1. A consumer has a continuously differentiable, concave, and strictly increasing utility function $u : \mathbb{R}^n_+ \to \mathbb{R}$. Let $v(p, w)$ be the indirect utility function.

a. Show that $u(x) \leq v(p, p \cdot x)$ for all x

b. Show that for all $x \gg 0$ we have $u(x) = \min_{p \gg 0} v(p, p \cdot x)$.

c. (Optional) Suppose that $v(p, w) = \left(p_1^{1-\sigma} + p_2^{1-\sigma}\right)^{-\frac{1}{1-\sigma}} w$ with $\rho > 0$. Use the result from part b to find a utility function u for which v is the indirect utility function.

2. (MWG 4.B.1) Show that if preferences admit Gorman-form indirect utility functions with the same $b(p)$, then preferences admit expenditure functions of the form $e_i(p, u_i) = c(p)u_i + d_i(p)$.

3. Suppose that preferences on the consumption set $X = \mathbb{R} \times \mathbb{R}^{L-1}_+$ are continuous, strictly convex and quasi-linear in the first good, represented by a utility function of the form $u(x_1, x_{-1}) = x_1 + f(x_{-1})$. Normalize $p_1 = 1$ and let $p = (1, p_2, .., p_L)$.

a. (MWG 3.D.4 (a-b)) Show that the Marshallian demand functions are independent of wealth for $l = 2, .., L$ and the indirect utility function is of the form $v(p, w) = \phi(p) + w$.

b. (MWG 3.E.7) The Hicksian demand functions are independent of wealth for $l = 2, ..., L$ and the expenditure function is of the form $e(p, u) = \psi(p) + u$. This follows from a similar argument as in problem 2. (Optional) Verify this.

c. (MWG 3.I.5) Show that if we fix $p_1 = 1$ then $CV(p, p', w) = EV(p, p', w)$ for any (p, p', w).

4. A consumer has a continuous and strictly increasing utility function defined on $X = \mathbb{R}^2_+$. You know that the consumer's expenditure function is $e(p_1, p_2, u) = \frac{p_1 p_2}{p_1 + p_2} u$.

a. Show that by considering what happens when one price goes to infinity you can find $u(x_1, 0)$ and $u(0, x_2)$. Hint: start by finding the Hicksian demands

b. For what values of x_1 does there exist a nonnegative x_2 such that $u(x_1, x_2) = u_0$?

c. For x_1 in the range identified in part b, trace out the indifference curve containing x_1. In particular, find the function $\tilde{x}_2(x_1, u_0)$ such that $u(x_1, \tilde{x}_2(x_1, u_0)) = u_0$. Hint: try to express x_1 and $\tilde{x}_2$ as the Hicksian demands at some (p, u_0)

d. Can you see how to use this information to very quickly find $u(x_1, x_2)$?

5. There are $L = 2$ commodities. A consumer's demand for the second commodity is given by

$$x_2(p_1, p_2, w) = a + \frac{p_2}{p_1} b$$

Apply the integrability theorem to determine restrictions on the parameters a and b that must be satisfied if this demand is generated by a continuous, locally non-satiated, and strictly quasiconcave utility function.