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Optimal Contracts with a Utility-Maximizing Auditor

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1. Introduction

This paper presents a principal–agent model in which the agent becomes strictly better informed than the principal after the contract agreement. To mitigate the inefficiency caused by this information asymmetry, the principal and agent agree that the agent will communicate the private information to the principal.[1] They can further reduce this inefficiency by hiring a utility-maximizing auditor, whose effort is not observable, to attest to the validity of the agent's message. To improve efficiency with this attestation, the principal must motivate the auditor to audit effectively and to report honestly. Thus, hiring a utility-maximizing auditor to mitigate the inefficiency in the principal–agent contractual relationship may introduce a moral hazard problem and a new source of inefficiency.

Our objectives are (*a*) to characterize an optimal pair of contracts (principal–agent and principal–auditor) so as (*b*) to analyze both how

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[1] See Christensen [1981], Baiman and Evans [1983], Dye [1983], Penno [1985], and Melumad and Reichelstein [1985] for analyses of the value of communication.

hiring the auditor improves the principal–agent contractual relationship and how the principal overcomes the moral hazard problem with the auditor, and (c) to identify conditions which are sufficient to ensure that hiring a utility-maximizing auditor improves efficiency.

In section 2, we introduce our model and relate it to others in the literature. Section 3 presents the major results, and section 4 summarizes the paper.

2. *The Model and Related Literature*

2.1 MODEL DESCRIPTION

Our model consists of three individuals: a risk-neutral principal, a strictly risk-averse agent, and a weakly risk-averse, strictly work-averse auditor. The principal owns a technology which generates an uncertain outcome which is liquid and transferable, e.g., cash. The outcome is:

$$x_i \in X \subseteq R^+, \qquad i = 1, 2, \ldots n,$$

where $x_k < x_m$ for $k < m$ and $x_1 \geq 0$. The ex ante probability distribution over the outcomes is *uniform*, exogenous, and common knowledge.

We assume that, because of geographic distance, lack of requisite skill, lack of time, and other reasons, the principal cannot operate the technology himself or directly observe or consume its outcome realization. Instead the principal must hire a risk-averse agent to (costlessly) observe, report on, and transfer the outcome to the principal. However, the agent takes no productive action and cannot affect the outcome realization.[2] The agent's utility function for wealth of w is $U(w)$ with $U'(w) > 0$ and $U''(w) < 0$ for $w \geq 0$. The agent has no initial endowment,[3] and his expected utility from employment elsewhere is $\bar{U} > 0$. The agent can consume any of the outcome which he does not transfer to the principal, so he has a potential incentive to misreport the outcome.

The principal has access to an exogenous public information system which, with probability π, reports the true outcome, and with probability $(1 - \pi)$ produces a signal which reveals no information, i.e., the null signal.[4] Because the agent's report has no value if it is issued after the

[2] This model is substantially equivalent to one in which the agent takes a productive action which affects the outcome, but either the principal can observe the agent's action or the agent is indifferent among actions.

The relationship between the principal and the agent is an agency relationship even though the agent takes no productive action. The agent influences the principal's consumption through his reporting behavior and his ability to misappropriate the outcome rather than, as is the more common assumption, through his action choice.

[3] All results would hold if the agent had finite initial wealth, as long as the amount was common knowledge.

[4] This exogenous public signal might result from investigations by a taxing or regulatory authority, a labor union, creditor, shareholder, stock analyst, or news reporter. For example, an SEC review of corporate financial statements may reveal misstatements. Such information might also be provided by, or inferred from, reports supplied by a trade association or competitor.

exogenous public signal occurs, the principal requires the agent to report on the realized outcome *before* the exogenous public signal occurs.

If π were zero, there would be no value to reporting by the agent. If π were one, the first-best solution would be achieved without the agent's report. Further, for $0 < \pi < 1$, if the agent's punishment for lying could be made arbitrarily large, a first-best solution could be achieved. We therefore assume that $0 < \pi < 1$, and that penalties are bounded so that the maximum punishment reduces the agent's wealth to zero, where $U(0) = 0$. These assumptions restrict consideration to a class of problems for which there is a potential demand for auditing. We next consider the auditor and the auditing technology.

The risk-averse, work-averse auditor has a utility function:

$$D(f, e) = H(f) - V(e),$$

where $H(f)$ is the auditor's utility for wealth f (his audit fee) with $H'(f) > 0$, $H''(f) \leq 0$, and $H(0) = 0$. $V(e)$ is the auditor's disutility from audit effort $e \in R^+$, with $V'(e) > 0$ and $V(0) = 0$. The auditor has no initial endowment[5] and has alternative opportunities for which his expected utility is $\bar{D} > 0$.

The minimum audit effort is $e = 0$, from which the auditor learns nothing. There also exists some positive level of audit effort (termed *effective auditing*), $e > 0$, from which the auditor learns the outcome with certainty. We initially assume that the greater the number of outcomes consistent with the agent's report, the greater the auditing effort ($\bar{e}$) necessary for the auditor to audit effectively. For certain results later in the paper, we simplify this assumption by making $\bar{e}$ a constant.[6] This audit technology is common knowledge, but only the auditor can observe the actual level of auditing effort.[7] As in the case of the agent, we assume bounded penalties by requiring the principal's minimum payment to the auditor to be zero.

The principal and the auditor agree on a contract before the agent observes the outcome. The ex ante probability distribution over the outcomes, as well as each individual's utility function, is common knowledge. The contract allows for conditional auditing by specifying the agent reports for which the principal precommits to direct the auditor to audit. The auditor will be hired only if he improves the risk sharing between the principal and agent beyond the best result achievable with just reporting by the agent. Because the principal is risk neutral, the auditor is not hired to share risk (as in Demski and Swieringa [1974]) but to influence the agent's reporting behavior.

[5] All results hold if the auditor had finite initial wealth, as long as the amount was common knowledge.

[6] This simplified assumption is consistent with a high fixed cost of auditing and a negligible variable cost.

[7] By not allowing the principal to observe any signal correlated with auditor's effort, we rule out the use of working papers as imperfect monitors of the auditor's level of effort (see Nagarajan [1984] for an example in which working papers are used as evidence).

Because the agent has sole control over the outcome, the auditor cannot unilaterally steal any part of it. In addition, we exogenously preclude any collusion between the agent and the auditor.[8] As a result, the auditor and the agent have fundamentally different opportunities in this model. While the agent can unilaterally misappropriate and consume the outcome, the auditor is limited to deciding whether to work for the firm, how much audit effort to exert, and what to report. The auditor's inability to misappropriate the outcome is one reason it may cost less to hire and motivate an auditor than to motivate honest reporting by the agent directly. Finally, we assume that both the auditor and the agent restrict themselves to pure strategy choices.[9]

We consider only *double-transfer* arrangements between the principal and the agent. The first stage of a double-transfer arrangement requires that when the agent reports an outcome of x_i, he must transfer x_i to the principal.[10] Because the agent has no initial wealth, the double transfer prevents him from reporting a larger outcome than he observed. In the second stage of the double transfer the principal transfers back to the agent an amount which depends on the first-stage transfer x_i and the jointly observed public signal. Using the double-transfer system in this way makes the amount transferred effectively serve as the agent's report.

The principal offers a contract pair, C_1 for the agent and C_2 for the auditor. The contract pair consists of a specification by the principal concerning (1) the set of reports which the agent can issue; (2) those agent reports for which the auditor will be hired; (3) the agent's reporting strategy; (4) a payment schedule for the agent based on all jointly observed variables; (5) the set of audit reports which the auditor can issue; (6) the auditor's auditing and reporting strategy; and (7) a payment schedule for the auditor based on all jointly observed variables.

We assume that the principal can precommit to implementing the agreed-upon contracts. Given a contract pair, the auditor and the agent each choose their strategies taking the other's strategy choice into consideration. A solution to this three-person problem consists of a pair of contracts and the agent's and auditor's induced Nash strategy pair. The time line for the game is represented in figure 1.

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[8] Any collusive agreement between the agent and the auditor would have to be self-enforcing because it would not be legally enforceable. Baiman, Evans, and Nagarajan [1987] show that our assumption of no collusion is without loss of generality, because a minor variation in the contracts which we derive prevents self-enforcing collusion between the agent and the auditor. Further, this variation in the contracts does not affect the resulting expected utilities.

[9] However, Lemma 8 later shows that given the contract pair derived assuming pure strategies by the agent and auditor, there are no mixed strategy pairs for the agent and auditor which are subgame perfect.

[10] Alternatively, if the agent's report states that the outcome is an element in a set of outcomes rather than a specific outcome, then the agent must transfer to the principal the smallest outcome in that set.

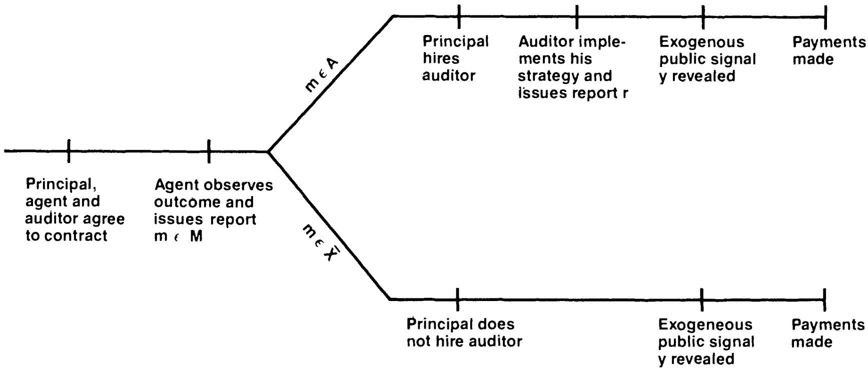

FIG. 1.—Time line for model.

2.2 COMPARISON OF OUR MODEL TO RELATED LITERATURE

Recent theoretical work in auditing has attempted to capture more fully the auditing environment by incorporating the auditor's legal liability (DeJong [1984], Nagarajan [1984], and Melumad and Thoman [1985]), the multiperiod nature of the auditor–client relationship (Datar [1985]), and a richer set of auditor activities (Balanchandran and Ramakrishnan [1980] and Antle [1982]). To make these models tractable, exogenous constraints were imposed on the behavior of the participants, typically by restricting the contracts which the principal could offer. Our primary focus is on characterizing the optimal contractual relationships among principal, agent, and auditor. This approach necessitates using a simpler model of auditing in which the contractual relationships are not restricted and then solving for the optimal relationships.[11]

Our model is similar to Townsend [1979] in concentrating on the acquisition of costly information for improving risk sharing between asymmetrically informed individuals. However, we replace Townsend's verification technology with a utility-maximizing individual who is subject to moral hazard.

The assumption of a self-interested auditor who is subject to moral hazard is similar to that made by Hamilton [1975], Magee [1976], Evans [1979], Noel [1981], Antle [1982], and Yoon [1987]. We compare our model with this previous work by discussing the Antle model, which is the most general of this set. Like our model, Antle's model includes a principal, an agent, and an auditor, but in his the agent's action choice is subject to moral hazard. By suppressing the agent's action choice our model focuses on the auditor's role in attesting to the accuracy of the agent's report, rather than in monitoring the agent's action choice.

The two models also differ in how the auditor's behavior depends on the agent's report. In Antle's model, the auditor issues the audit report

[11] This approach is consistent with that advocated by Harris [1982, p. 151].

at the same time that the agent makes his report (the auditor and the agent move simultaneously). As a result, the auditor cannot base the audit report on the agent's report and therefore cannot attest to its truthfulness, which is a basic function of the auditor. Antle's model is similar to Demski and Sappington [1984], in which two agents perform the same job independently and the results of each are used to assess the performance of the other. Because in our model the auditor does observe the agent's report *before* conducting his audit and issuing his opinion, he is able to attest to the agent's report.[12]

A final characteristic of our model that distinguishes it from some prior auditing models is that the principal's consumption cannot depend on the outcome because he does not observe it. The agent consumes everything that he does not transfer to the principal. This approach, used in Townsend [1979], Evans [1980], Noel [1981], Penno [1985], Reigenaum and Wilde [1985], and Mookherjee and P'ng [1986], gives rise to the possibility of the agent misappropriating the outcome. A quite different approach has been taken by Ng and Stoeckenius [1979], Gjesdal [1981], Antle [1982], and Yoon [1987]. In these models the agent cannot misappropriate the outcome and the principal can consume the outcome without knowing how large it is. This difference in assumptions can have important implications for the value of reporting by the agent.[13]

In summary, we focus on analyzing the optimal contractual relationship among the principal, agent, and auditor in a context which produces a demand for auditing in two steps. First, private observation of the firm's uncertain outcome by the risk-averse agent creates a potential demand for reporting to improve risk sharing. Second, the agent's ability to affect his own consumption by his report creates the demand for an auditor. Because the agent does not affect the outcome and the principal is risk neutral, the auditor is hired to attest to the agent's report rather than to monitor the agent's effort, to provide any productive effort of his own, or to share risk. This attestation directly affects the agent's reporting behavior, resulting in improved risk sharing between the principal and the agent.[14] Our assumption that the agent can consume the outcome

[12] In our model, both the audit work and the audit report are chosen after the agent's report is revealed. In Antle [1982] both are chosen simultaneously with the release of the agent's report. An intermediate model is one in which the audit work is performed before the release of the agent's report, but the audit report is issued after the agent's report is released. This intermediate model is closer to ours than to Antle's, because the essential element of our model is that the auditor knows the agent's report before issuing his own report. Many of the results in this paper also hold in a modified form for the intermediate model.

[13] See Penno [1985] and Noel [1981] on this point, and Baiman [1979] for a discussion of other implications of this assumption.

[14] Auditing could also be valuable in our model even if the agent were risk neutral, as long as he did not have the wealth to rent the firm from the principal. In this case auditing might be valuable because it would allow the principal to reduce the informational rents earned by the agent. We thank Stefan Reichelstein for pointing this out to us.

restricts the model to situations in which fraud by the agent is a significant concern of the principal.[15]

3. *Results*

In this section we characterize the form of the optimal contracts and the induced behaviors for the three-person auditing problem assuming that the first-best solution to the principal–agent problem cannot be achieved and hence that there is a demand for auditing. We also provide a set of sufficient conditions for auditing to have value.

The following notation is used in the statement of our results and proofs.

NOTATION

x	the outcome $x_i \in \{x_1, x_2, \ldots x_n\}$, where $\cup_i x_i = X$ and $x_i < x_j$ iff $i < j$.
m	the agent's report, $m \in M = \{\bar{x}_1, \ldots \bar{x}_j, A_1 \ldots A_p\}$.
A_k	a subset of outcomes, $A_k \subseteq X$. A_k also specifies a report by the agent that the outcome is in A_k, i.e., that $x \in A_k$ (see Observation 1 below). The principal precommits to hire the auditor for any such report, A_k. Therefore, we refer to A_k as audit *subregion* A_k and to $\cup_k A_k = A$ as the *audit region*. Further, we require that $A_k \cap A_j = 0$ for $k \neq j$.
$\bar{x}_i$	a subset of outcomes, $\bar{x}_i \subseteq X$. $\bar{x}_i$ also specifies a report by the agent that the outcome is in $\bar{x}_i$, i.e., that $x \in \bar{x}_i$ (see Observation 1 below). The principal precommits to not hiring the auditor for any such report, $\bar{x}_i$. Therefore, we refer to $\bar{x}_i$ as *nonaudit subregion* $\bar{x}_i$ and to $\cup_i \bar{x}_i = \bar{X}$ as the *nonaudit region*. Further, we require that $\bar{x}_i \cap \bar{x}_j = \emptyset$ for $i \neq j$, $\bar{X} \cap A = \emptyset$, and $\bar{X} \cup A \cdot = X$.
r	the auditor's report, $r \in \{x_1, x_2, \ldots x_n\}$ (see Observation 3 below).
y	the signal from the public information system $y \in \{y_0, x_1, \ldots, x_n\}$, where y_0 is the null report.
π	the exogenous probability that y reveals the true outcome (i.e., $\pi = 1 - \text{prob}(y = y_0)$).
$U(w)$	the agent's utility function, with $U(0) = 0$, $U'(\cdot) > 0$, and $U''(\cdot) < 0$.

[15] An alternative interpretation of this model is that the auditor acts as a utility-maximizing claims adjuster who audits the claim made by an insurance policyholder (the agent) to an insurance company (the principal).

$\bar{U}$	the agent's expected utility from working elsewhere, $\bar{U} > 0$.
$D(f, e) = H(f) - V(e)$	the auditor's utility function, with $H(0) = 0$, $H'(\cdot) > 0$, $H''(\cdot) \leq 0$, $V(0) = 0$, $V'(\cdot) > 0$.
$\bar{D}$	the auditor's minimum expected utility from working elsewhere, $\bar{D} > 0$.
e_k	the auditor's audit effort in audit subregion A_k, $e_k \in \{0, \bar{e}_k\}$; $\bar{e}_k$ is an effective audit in audit subregion A_k, i.e., the auditor learns the actual outcome for certain. $\bar{e}_k$ may depend on the number of outcomes in A_k. $V(\bar{e}_k) > 0\ \forall k$.
(C_1, C_2)	the principal's contracts with the agent (C_1) and the auditor (C_2).
$t(m = A_k, r, y)$	the agent's net transfer *to* the principal when the agent reports $m = A_k$, the auditor reports r, and the public signal is $y \in \{y_0, x_1, \ldots, x_n\}$.[16]
$t(m = \bar{x}_i, y)$	the agent's net transfer *to* the principal when the agent reports $m = \bar{x}_i$ and the public signal is $y \in \{y_0, x_1, \ldots, x_n\}$.[17]
$f(m = A_k, r, y)$	the principal's payment to the auditor when the agent reports $m = A_k$, the auditor reports r, and the public signal is $y \in \{y_0, x_1, \ldots, x_n\}$.
$n(A_k)$	the number of outcomes in audit subregion A_k; $n(A) = \sum_k n(A_k)$ is the number of outcomes in the entire audit region.
R	the agent's reporting strategy where $R = R^0$ denotes consistent reporting (see Observation 1 below) and $R = R^i$, $i \neq 0$, denotes any other feasible reporting strategy.
I	the auditor's auditing and reporting strategy where $I = I^0$ denotes effective auditing/honest reporting and $I = SG$ denotes shirking and guessing the outcome.
$ED(I \mid R)$	the auditor's expected utility, given that the agent has chosen reporting strategy $R \in \{R^0, R^i\}$ which results in the auditor being hired and choosing auditing strategy $I \in \{I^0, SG\}$.

Lemma 1 is a preliminary result which rules out one (apparently desirable) form that the agent's reporting might take. This result is

[16] Although we do assume the double-transfer system, the model is written in terms of the (potentially negative) *net* transfers to be made from the agent to the principal.

[17] Note that the agent's transfer schedule has three arguments when the auditor is hired (when the agent reports $m = A_k$) and two arguments when the auditor is not hired (when the agent reports $m = \bar{x}_i$).

followed by a series of observations (whose proofs are straightforward and are omitted) about the form of the optimal solution.

One potential reporting strategy for the agent would be to "report fully," i.e., to issue a different report for each outcome so that the mapping from outcomes to reports is one-to-one. However, Lemma 1 (the proof of which is straightforward and hence omitted) states that full reporting by the agent and effective auditing/honest reporting by the auditor cannot be an equilibrium solution.

LEMMA 1. There exists no equilibrium in which the agent reports fully, the auditor observes the agent's report prior to auditing, and the auditor audits effectively.

The proof of Lemma 1 follows immediately from the fact that with full reporting by the agent, the work-averse auditor will have no incentive to audit because he can infer the truth from the agent's report. The principal cannot detect this shirking by the auditor. As a result, all equilibria in which the agent reports fully result in the auditor expending no audit effort and hence in there being no value to hiring the auditor. Thus, Lemma 1 implies that an equilibrium solution to the three-person game in which there is value to hiring the auditor must be one in which the agent represents two or more outcomes with a single audit region report.

A second implication of Lemma 1 is that the Revelation Principle cannot be applied to the agent's report in the three-person setting.[18] The Revelation Principle requires that any individual receiving a message about the private information can precommit to how the information will be used. While the principal can precommit to the payment schedules for the agent and auditor, the auditor cannot credibly precommit to his auditing strategy because his audit effort is not publicly observed. In contrast, Antle [1982] could apply the Revelation Principle to the agent's report in his model because the auditor did not observe the agent's report prior to issuing his audit report.[19]

Full reporting by the agent would give the auditor perfect predecision information. As Christensen [1981] has noted, giving an agent predecision information can improve his decision and/or increase his incentive to shirk. Lemma 1 shows that in our model full reporting by the agent completely eliminates the auditor's incentive to work, so the second effect dominates.

While Lemma 1 is robust with respect to many of our assumptions,[20]

[18] However, the Revelation Principle can be applied to the agent's report in the two-person setting, resulting in full and honest reporting.

[19] An alternative approach to the problem noted in Lemma 1 would require the agent to communicate privately to the principal and have the principal reduce the information content when he communicates to the auditor. However, because in our model the principal can specify the level of detail which the agent is allowed to communicate in his report, the two representations are equivalent.

[20] In particular, Lemma 1 would hold if: (a) the auditor's audit technology produced imperfect information concerning the true outcome; (b) the probability distribution over

it does depend on (*a*) the agent's report being observed before the auditor issues his report (otherwise the auditor's strategy cannot depend on the agent's report and hence the Revelation Principle does hold) and (*b*) the agent observing as much information about the outcome as the auditor can observe with effective auditing (otherwise, the agent's report would leave the auditor with an incentive to collect additional information and thereby affect the risk which the auditor faces).

In our model, the agent's report m is both a report and a subset of outcomes. If the principal and auditor know that, given the contract pair, it is in the agent's best interest to issue report m_j only when outcome x_i or x_j occurs, then, regardless of the label attached to report m_j, the principal and auditor will interpret report m_j as implying that either x_i or x_j occurred. Therefore, we can consider only contracts which are incentive compatible, that is:

OBSERVATION 1. Without loss of generality, we need consider only contract pairs such that:

(*a*) The set of outcomes which result in the agent reporting A_k is identical to the set of outcomes contained in A_k, for all k; and
(*b*) The set of outcomes which result in the agent reporting $\bar{x}_i$ is identical to the set of outcomes contained in $\bar{x}_i$, for all i.

We shall refer to this as *consistent reporting* (Townsend [1979]), by the agent. Observation 1 is a convenient result concerning the labeling of reports. Consistent reporting does not imply full reporting because with consistent reporting each audit and nonaudit region need not be a singleton (as would be true if the Revelation Principle were applicable). However, consistent reporting does imply honest reporting.

If the auditor did not audit effectively, his reporting decision would be based on the common initial beliefs and the agent's report, information which the principal also has. It follows that:

OBSERVATION 2. The principal will hire the auditor only for those reports by the agent for which it is in the best interest of the auditor to audit effectively/report honestly.

Because the auditor moves last, the Revelation Principle implies that:

OBSERVATION 3. Without loss of generality, we need consider only contract pairs in which the auditor's message space is $\{x_1, \ldots, x_n\}$ and which induce the auditor to audit effectively/report fully and honestly whenever he is hired.

the outcome space were more general, as long as it was still common knowledge; (*c*) the set of outcome realizations were infinite rather than finite; (*d*) the auditor had to consider reputational effects of his action in a multi- (but finite) period setting because it would still be period-by-period rational for the auditor to shirk and confirm the agent's report; (*e*) related to (*d*), there were different types of auditors as in Datar [1985] and the types were privately known; (*f*) there were a legal liability structure in place; (*g*) the principal could consume the outcome realization, less the amount paid to the agent; (*h*) collusion were self-enforceable, because then there would be even less incentive to actually audit; and (*i*) the public signal were imperfect even when it did report an outcome realization.

If the agent reports that the outcome is in the *nonaudit* region, then the agent moves last. From the Revelation Principle it follows that:

OBSERVATION 4. Without loss of generality, we need consider only contract pairs in which each nonaudit subregion is a single outcome and in which the agent reports honestly.

Using Observation 4 we can change the notation for nonaudit region reports from $m = \bar{x}_i \subseteq \bar{X}$ to $m = x_i \in \bar{X}$.

Observations 1–4 allow us to consider only those contract pairs for which consistent reporting by the agent and effective auditing/honest reporting by the auditor is a Nash strategy pair.[21] Together with the zero initial wealth assumption for the agent and the auditor, this implies that:

OBSERVATION 5.

(*a*) Whenever the public signal contradicts
(1) the *agent's* report, the optimal transfer payment should reduce the agent's wealth to zero;
(2) the *auditor's* report, the optimal payment to the auditor is zero.

(*b*) Whenever the auditor's report contradicts the agent's report and the public signal is null (y_0), the optimal transfer payment should reduce the agent's wealth to zero.

These observations allow us to formulate the three-person auditing problem as a mathematical programming problem. The solution requires searching over the Pareto optimal payment schedules, audit subregions, nonaudit region, and their implied message spaces subject to (1) the agent reporting consistently, and (2) the auditor auditing effectively/reporting honestly. Appendix B contains the resulting mathematical programming representation of the three-person auditing problem.

The first major result characterizes the auditor's payment schedule. (All proofs are in Appendix A.)

THEOREM 1. For each audit subregion A_k, the optimal contract for the auditor can be written to pay him:

(1) $K_1(A_k)$ when the agent reports $m = A_k$, the auditor reports $r = x_j$, and the public information signal is $y = x_j$, for all x_j; or
(2) $K_2(A_k)$ when the agent reports $m = A_k$, the auditor reports $r = x_j$, and the public information signal is $y = y_0$, for all x_j; or
(3) 0 when either the agent reports $m \in \bar{X}$ (the auditor is not hired) or when the agent reports $m = A_k$, the auditor reports $r = x_j$, and the public information signal is $y = x_i$ where $x_i \neq x_j$; with
(4) $K_1(A_k) > 0\ \forall A_k$, $K_2(A_k) = K\ \forall A_k$, and $K_1(A_k) \geq K\ \forall A_k$.

[21] Like the Revelation Principle, Observations 1–4 refer to the achievability of an equilibrium rather than to the implementability of the equilibrium. Observations 1–4 indicate that every utility triple achievable by some contract pair can be achieved with a contract pair which induces consistent reporting by the agent and effective auditing/honest reporting by the auditor. We consider implementability, or the issue of alternative equilibria, in Theorem 4.

The auditor's risk is reduced by making his fee independent of the actual outcome and the auditor's report (unless the audit report is contradicted by the public information signal). This contract also eliminates any incentive for the auditor to lie if he audits effectively. An implication of Theorem 1 is that the choice of $K_2(A_k) = K \; \forall A_k$ has no effect on the auditor's incentives.

While $K_2(A_k)$ is constant for all A_k, $K_1(A_k)$ may vary in A_k because $\bar{e}_k$ may vary across audit subregions.[22] Hence different levels of compensation may be needed to induce the auditor to audit effectively in different audit subregions. This interpretation of why $K_1(A_k)$ may vary in A_k is made clearer later by Theorem 3. Theorem 3 establishes that if the effort to audit effectively is constant regardless of the size of the audit subregion, then a *single* audit subregion is optimal and the auditor's fee is independent of the agent's report, the audit report, and the outcome.

The optimal auditor contract form described in Theorem 1 is similar to the auditor's contract observed in practice. In both cases the auditor does not share risk with the firm by having his fee depend on his report. Further, in practice, the auditor's fee varies in audit effort but not in the firm's outcome. The corresponding result in Theorem 1 is that the audit fee depends on $\bar{e}_k$ indirectly through A_k but does not depend on x_i.

While Theorem 1 seems to describe auditor contracts similar to those observed in practice, the result depends critically on the assumption that the outcomes are generated from a uniform probability distribution. Without this distributional assumption, the result that the auditor's fee is independent of the realized outcome would not hold in general.

The next step in characterizing the optimal contracts is to determine the audit region. The number of outcomes in A will depend on the particular parameters of the problem. If the auditor's disutility for effort is minimal and his outside opportunity wage is small, then the audit region could be all of X. If the auditor's disutility for effort is great and his outside opportunity wage is high, then it may not be worthwhile hiring the auditor at all. Lemma 2 provides some additional characterization as to the size of the optimal audit region.

LEMMA 2. Let (C_1, C_2) be a Pareto optimal pair of contracts with the characteristics established in Observations 1–5 and Theorem 1. Then: (1) $n(A) \neq 1$; and (2) if $n(A) < n$, then (without loss of generality) $x_n \in \bar{X}$.

Lemma 2 provides three insights. First, neither the audit region nor any audit subregion can consist of a single outcome. If it did, then from Lemma 1 and Observation 2, there would be no incentive for the auditor to audit effectively and the principal would be strictly better off not hiring the auditor. Second, if the audit region contains fewer than all n outcomes, then without loss of generality, we need consider only contracts in which x_n is in the nonaudit region.

[22] $K_1(A_k) = K$ if and only if at optimality, none of the auditor's action self-selection constraints is binding. See Corollary 1 in Appendix A.

Finally, if the auditor were a technology not subject to moral hazard and $n(A) = n$, then the principal would be strictly better off by excluding x_n from the audit region. This would save the cost of auditing without affecting the agent's reporting incentives because, given the double-transfer system, the agent could never falsely report x_n. With a utility-maximizing auditor, however, the size of the audit region affects the *auditor's* incentives. Increasing the size of any audit subregion decreases the attractiveness of shirking to the auditor in that subregion by reducing the probability that a shirking auditor will correctly guess the outcome.

The optimal audit region can be further characterized using the concept of a lower audit region, in which every outcome in the audit region is smaller than every outcome in the nonaudit region. That is, with a lower audit region, for every $x_i \in A$ and $x_j \in \bar{X}$, $x_i < x_j$. If $n(A) = n$, then we trivially have a lower audit region. If $n(A) = n - 1$, then by Lemma 2, $x_n \in \bar{X}$ and we have a lower audit region. The optimality of lower audit region contracts in general is established by Theorem 2 using Lemmas 3 and 4 (see Appendix A).

THEOREM 2. (1) Any contract pair with (a) $0 < n(A) < n - 1$, (b) the agent's consumption not constant over all outcomes, and (c) a nonlower audit region can be strictly Pareto dominated by a contract with a lower audit region.

(2) Any contract pair with (a) $0 < n(A) < n - 1$, (b) the agent's consumption constant over all outcomes, and (c) a nonlower audit region is Pareto equivalent to another contract pair with a lower audit region.

Theorem 2, part (1) states that a contract pair with a nonlower audit region of fewer than $n - 1$ outcomes in which the agent bears risk cannot be optimal. Such a contract pair can be improved by replacing the largest outcome in the audit region with the smallest nonaudit outcome, while adjusting the agent's transfers to reduce his risk. Replacing an outcome in an audit subregion with an outcome from the nonaudit region does not change the auditor's incentives because Theorem 1 implies that the auditor's incentives depend only on the size of the audit subregions. The result in Theorem 2 is similar to that in Townsend [1979] and Evans [1980] for the case of audit technologies. Theorem 2 and Lemma 2 yield the positive implication that auditing will be observed over a range of lower outcomes. However, a direct comparison to external financial audits of public corporations is complicated by the presence of institutionally mandated audits, whereas we allow for conditional auditing.

The proof of Theorem 2, part (1) also establishes that, with a lower audit region and the double-transfer system, the agent's consumption can be made a constant over the entire audit region (unless the agent's report is contradicted by either the auditor's report or the public signal). This new contract can be constructed without affecting the agent's reporting incentives. The source of the strict Pareto improvement in Theorem 2, part (1) and the value of auditing in our model comes from improved risk sharing with the lower audit region.

In the analysis so far, the audit region A has consisted of smaller audit subregions, with the agent announcing the specific subregion in his report. If the auditor's effort function is convex in the number of outcomes in the audit subregion, multiple audit subregions may be efficient. However, motivating the auditor to audit effectively in a given subregion becomes easier as the number of outcomes in that audit subregion increases. Therefore, if the effort necessary for an effective audit does not depend on the size of the audit subregion, Theorem 3 establishes the optimality of a single audit subregion.

THEOREM 3. Assume that the effort necessary for an effective audit does not depend on the size of the audit subregion. Then any contract pair with multiple audit subregions can be at least weakly Pareto dominated by a contract pair with a single audit subregion.

Theorem 3 implies that the auditor's compensation is one constant when his audit report is confirmed by the public signal, and a different constant when the public signal is null. Theorem 3 is a refinement of Theorem 1, in that the auditor's compensation is now also independent of the agent's report.

Recall that without loss of generality we consider only those contracts in which consistent reporting by the agent and effective auditing/honest reporting by the auditor (jointly referred to as the obedient strategy pair) is a Nash strategy pair. We then characterized the optimal contract pair which maintained effective auditing/honest reporting by the auditor and consistent reporting by the agent as Nash strategies. An implementation problem may arise, however, if this optimal contract pair yields *other* Nash strategy pairs, particularly if they Pareto dominate the obedient strategy pair with respect to the agent–auditor subgame. This issue is addressed next, using the assumption that the effort necessary for effective auditing does not depend on the size of the audit subregion and therefore that there is a single audit subregion.

In order to determine whether other Nash strategy pairs exist under (C_1, C_2), we must consider the feasible strategies available to the agent and auditor. First, given the auditor's contract, regardless of the agent's strategy choice, the auditor strictly prefers effective auditing/honest reporting to effective auditing and dishonest reporting. Therefore, the only alternative auditor strategy that we need consider is shirking and guessing the outcome. Second, the lower audit region, the double-transfer system, and Observation 4 imply that the agent's only feasible alternative to consistent reporting is observing an outcome in the nonaudit region and reporting that the outcome is in the audit region. Lemmas, 5, 6, 7, and 8 (see Appendix A) examine all possible nonobedient pure or mixed agent–auditor strategy pairs given (C_1, C_2). Lemmas 5 and 6 eliminate certain nonobedient pure strategy pairs because they are not Nash. Lemma 7 proves that all remaining pure Nash strategy pairs except the obedient pair are not subgame perfect (see Selten [1975]). Lemma 8 generalizes Lemmas 5–7 to mixed strategy pairs. Lemmas 5–8 are used to establish the following theorem.

THEOREM 4. Assume that the effort required to audit effectively does not depend on the size of the audit region. Given the optimal contract pair, the only subgame perfect agent–auditor Nash strategy pair is consistent reporting and effective auditing/honest reporting.

Theorem 4 addresses the possibility of the agent and auditor preferring and implementing a strategy pair different from consistent reporting and effective auditing/honest reporting. It demonstrates that all such alternative strategy pairs are either not Nash or (if Nash) not reasonable (i.e., not subgame perfect). Theorem 4 does not consider the subgame perfectness of the principal's strategy because we have assumed throughout (as does essentially all of agency theory) that the principal has precommitted to implement (C_1, C_2). It should be noted that Lemmas 5–8 establish that effective auditing/honest reporting is a weakly dominant strategy for the auditor. Unlike Demski and Sappington [1984], we establish this result without explicitly requiring dominance and hence without incurring the cost of enforcing dominance.

The remaining issue is to establish a set of sufficient conditions for hiring an auditor to have value when compared to the optimal two-person principal–agent solution. Given our assumption that the solution to the two-person problem is not first-best, we can use the same logic as in Lemma 4 (see Appendix A) to establish that risk sharing will be inefficient over the lowest outcome in the optimal solution to the two-person problem. Consequently, there are potential gains to hiring the auditor to audit the region $\{x_1, x_2\}$ and thereby improve the principal–agent risk sharing. Define Δ as the agent's risk premium for the risk which is eliminated when the auditor is hired over $\{x_1, x_2\}$ and let the auditor's fee schedule be $K_1(A) = K_2(A) = \Delta - \epsilon$ where $A = \{x_1, x_2\}$ and $\epsilon > 0$. Theorem 5 establishes a set of sufficient conditions under which the auditor is worth hiring. (The proof is straightforward and hence omitted.)

THEOREM 5. Assume that the optimal principal–agent solution is second-best and that the effort required to audit effectively does not depend on the size of the audit region. If there exists $\epsilon > 0$ satisfying (5.1) and (5.2), then auditing can achieve a strict Pareto improvement with contracts which have as the unique Nash strategies of the agent and auditor consistent reporting, and effective auditing/honest reporting, respectively:

$$H(\Delta - \epsilon) \geq V(\bar{e}) + \bar{\bar{D}} \tag{5.1}$$

$$H(\Delta - \epsilon) > (2V(\bar{e}))/\pi. \tag{5.2}$$

Condition (5.1) ensures that the auditor would accept employment by the firm if he planned to audit effectively/report honestly. Condition (5.2) ensures that auditing effectively/reporting honestly will be the auditor's strictly dominant strategy. Although conditions (5.1) and (5.2) appear restrictive, it is useful to relate them to previous research on the gains to verification, such as Townsend [1979] and Baiman and Demski

[1980]. In both papers the results state that if monitoring is "not too costly," then the optimal contracts will be of a given form. Conditions (5.1) and (5.2) specify what "not too costly" means in this context. These conditions relate the benefits from auditing (Δ) to the cost of inducing effective auditing and honest reporting $(2V(\bar{e}))/(\pi)$. It should be noted that Theorem 5 does not depend on using a lower audit region with only two outcomes. Additional sufficient conditions can be generated with lower audit regions of size $n(A) > 2$.

These conditions depend only on the optimal principal–agent contract and the exogenously specified characteristics of the auditor. Thus, to the extent that auditors with sufficiently low costs exist, and the principal–agent problem does not have a first-best solution, the addition of an auditor results in a *strict* Pareto improvement.

5. *Summary and Conclusions*

This paper analyzes the role of auditing in a principal–agent context with a utility-maximizing auditor. We characterize the contractual relationships among the three individuals and thus analyze the effects of adding the auditor to the original principal–agent problem. In addition, we establish conditions which are sufficient to ensure the value of hiring a utility-maximizing auditor when the optimal principal–agent contract is second-best.

A number of extensions can be made to these results. First, our analysis assumes a uniform probability distribution over outcomes and direct extension to other distributions does not hold in general. A second generalization would allow the auditor's technology to be imperfect. A problem which arises with imperfect auditing in our model is the feasibility of the agent's transfer when the auditor's report is incorrect. Third, the model does not consider productive action by the agent. Allowing for such action potentially creates an additional demand for auditing to help overcome the agent's moral hazard problem.

APPENDIX A

Proofs or sketches of proofs of all lemmas and theorems except Lemma 1 and Theorem 5 are presented here. Where only a sketch is provided, more detailed proofs are available from the authors.

THEOREM 1. For each audit subregion A_k, the optimal contract for the auditor can be written to pay him:

(1) $K_1(A_k)$ when the agent reports $m = A_k$, the auditor reports $r = x_j$, and the public information signal is $y = x_j$, for all x_j; or
(2) $K_2(A_k)$ when the agent reports $m = A_k$, the auditor reports $r = x_j$, and the public information signal is $y = y_0$, for all x_j; or
(3) 0 when either the agent reports $m \in \bar{X}$ (the auditor is not hired) or when the agent reports $m = A_k$, the auditor reports $r = x_j$, and the public information signal is $y = x_i$ where $x_i \neq x_j$; with

(4) $K_1(A_k) > 0\ \forall A_k$, $K_2(A_k) = K\ \forall A_k$, and $K_1(A_k) \geq K\ \forall A_k$.

Proof. Assume that we have an optimal pair of contracts (C_1, C_2) satisfying Observations 1–5, but C_2 does not have the form described in Theorem 1. We will construct a new contract pair $(C_1, \tilde{C}_2)$ which is weakly Pareto superior to (C_1, C_2) (strictly if the auditor is strictly risk averse).

Based on C_2, define a new contract, $\tilde{C}_2$, as in the statement of Theorem 1. For each audit subregion A_k, the constants $K_1(A_k)$ and $K_2(A_k)$ are defined implicitly as:

$$H(K_1(A_k)) = \frac{1}{n(A_k)} \sum_{x \in A_k} (H(f(m = A_k,\, r = x,\, y = x))) \geq 0. \tag{A1}$$

$$H(K_2(A_k)) = \frac{1}{n(A_k)} \sum_{x \in A_k} (H(f(m = A_k,\, r = x,\, y = y_0))) \geq 0. \tag{A2}$$

The following constraints held for the original contract pair (C_1, C_2):

$$\begin{aligned}
&\frac{1}{n(A_k)} \sum_{x \in A_k} \{\pi H(f(m = A_k,\, r = x,\, y = x)) \\
&\qquad\qquad + (1 - \pi) H(f(m = A_k,\, r = x,\, y = y_0))\} - V(\bar{e}_k) \\
&\geq \max_{x_j \in A_k} \left\{ \frac{\pi}{n(A_k)} H(f(m = A_k,\, r = x_j,\, y = x_j)) \right. \\
&\qquad\qquad \left. + (1 - \pi) H(f(m = A_k,\, r = x_j,\, y = y_0)) \right\} - V(0) \\
&\geq \sum_{x \in A_k} \frac{1}{n(A_k)} \left\{ \frac{\pi}{n(A_k)} H(f(m = A_k,\, r = x,\, y = x)) \right. \\
&\qquad\qquad \left. + (1 - \pi) H(f(m = A_k,\, r = x,\, y = y_0)) \right\} - V(0).
\end{aligned} \tag{A3}$$

The first inequality in (A3) states that the auditor prefers effective auditing/honest reporting to shirking and selecting whichever audit report within A_k yields the largest audit fee (i.e., his best guess, see (B6k) in Appendix B). The second inequality in (A3) states that the auditor's best guess must be at least as good as randomizing over reports in A_k with equal probability.

Inserting (A1) and (A2) into the first and last expressions of (A3) establishes that the auditor's strategy of effective auditing/honest reporting which was optimal under (C_1, C_2) is also optimal under $(C_1, \tilde{C}_2)$, if the agent's reporting strategy remains unchanged. Because the auditor's best response to the agent's strategy of consistent reporting is the same under $(C_1, \tilde{C}_2)$ as under (C_1, C_2) and contract C_1 remains unchanged, the agent's best response to effective auditing/honest reporting remains consistent reporting. Therefore, the agent and auditor strategy pair which was Nash under (C_1, C_2) is also Nash under $(C_1, \tilde{C}_2)$. Finally, under $\tilde{C}_2$

the principal is able to hire the auditor at a cost which is no greater than the cost under C_2. Therefore, $(C_1, \tilde{C}_2)$ produces at least a weak Pareto improvement.

The results so far allow us to rewrite constraint (B6k) as (B6k') and constraint (B3) as (B3$'$) (see Appendix B). $K_1(A_k) > 0\ \forall A_k$ follows from (B6k') and the fact that $H(0) = 0$, $H'(0) > 0$, and $V(\bar{e}_k) > 0$. $K_2(A_k) = K_2(A_j) = K$ follows from the *FOC* with respect to $K_2(A_k)$. $K_1(A_k) \geq K$ $\forall A_k$ follows from the *FOC* with respect to $K_1(A_k)$ and K.

COROLLARY 1. $K_1(A_k) = K$ if and only if (C_1, C_2) is such that there is no moral hazard problem associated with the auditor auditing effectively/reporting honestly, i.e., the auditor's action self-selection constraint is not binding.

Proof. The proof follows from the *FOC* for $K_1(A_k)$ and K.

LEMMA 2. Let (C_1, C_2) be a Pareto optimal pair of contracts with the characteristics established in Observations 1–5 and Theorem 1. Then: (1) $n(A) \neq 1$; and (2) if $n(A) < n$, then (without loss of generality) $x_n \in \bar{X}$.

Proof. Assume that $n(A) = 1$. Then by Lemma 1 and Observation 2 the auditor would not be hired, so this cannot be the size of the audit region. By the same reasoning, no A_k can consist of a singleton.

Assume that $1 < n(A) < n$ and that $x_n \in A_k$ for some A_k. Consider a new contract pair $(\tilde{C}_1, \tilde{C}_2)$ with an audit region which is unchanged except that A_k is changed by deleting x_n and adding x_l, the smallest outcome in the nonaudit region. Call this new audit subregion $\tilde{A}_k$. x_n becomes part of the nonaudit region. All other audit subregions remain unchanged. Let the auditor's fee under $\tilde{C}_2$ be the same as under C_2. Let the transfers from the agent under C_1 be $t(\cdot)$ and under $\tilde{C}_1$ be $\tilde{t}(\cdot) = t(\cdot)$ except for the following:

$$\tilde{t}(m = x_n, y = x_j) = t(m = A_k, r = x_n, y = x_j) \quad j = 1, \ldots, n$$

$$\tilde{t}(m = x_n\ y = y_0) = t(m = A_k, r = x_n, y = y_0)$$

$$\tilde{t}(m = \tilde{A}_k, r = x_l, y = x_j) = t(m = x_l, y = x_j) \quad j = 1, \ldots, n$$

$$\tilde{t}(m = \tilde{A}_k, r = x_l, y = y_0) = t(m = x_l, y = y_0)$$

$$\tilde{t}(m = \tilde{A}_k, r = x_j, y = y_0)$$
$$= t(m = A_k, r = x_j, y = y_0) \quad x_j \neq x_n, \quad x_j \in A_k$$

$$\tilde{t}(m = \tilde{A}_k, r = x_j, y = x_j)$$
$$= t(m = A_k, r = x_j, y = x_j) \quad x_j \neq x_n, \quad x_j \in A_k$$

$$\tilde{t}(m = \tilde{A}_k, r = x_n, y = y_0) = t(m = \tilde{A}_k, r = x_n, y = x_n) = x_n.$$

Using the double-transfer system and the feasibility constraints (B8), it is straightforward to show that this new contract pair with $x_n \in \bar{X}$ is

Pareto equivalent to (C_1, C_2) and satisfies the agent's consistent reporting incentive compatibility constraints assuming that the auditor continues to audit effectively/report honestly. Further, because the size of each individual audit subregion remains unchanged and the auditor's payment schedule remains the same as under $(\tilde{C}_1, \tilde{C}_2)$, the auditor's best response to consistent reporting under (C_1, C_2) is the same as under $(\tilde{C}_1, \tilde{C}_2)$. Therefore, the strategy pair that was Nash under (C_1, C_2) is Nash under $(\tilde{C}_1, \tilde{C}_2)$. Finally, the cost of auditing is the same under $(\tilde{C}_1, \tilde{C}_2)$ as under (C_1, C_2). Therefore, if $x_n \in A$ under (C_1, C_2), we can construct a Pareto equivalent contract whereby $x_n \in \bar{X}$.

LEMMA 3. Let x_l be the lowest nonaudit outcome. Assume that the optimal solution does not give the agent constant consumption over all outcomes. Then the agent's consumption over the outcomes x_l and higher cannot be constant.

Proof. The optimal solution must either have $x_l = x_1$ or $x_l > x_1$. If $x_l = x_1$ and the agent's consumption over the outcomes x_l and higher were constant, the agent's consumption over all outcomes would be constant, contrary to the assumption of the lemma. Therefore, we need consider only the case in which $x_l > x_1$. Using the double-transfer system it is straightforward to establish that the agent's consumption over those outcomes smaller than x_l (all of which are, by definition of x_l, in the audit region) must be constant.

The rest of the proof proceeds by contradiction. Assume that the agent's consumption over the outcomes x_l and higher is constant, say G_2. We already know that the agent's consumption over the outcomes lower than x_l must be a constant, say G_1. If the agent's consumption over all the outcomes is not constant, then $G_1 \neq G_2$.

It cannot be that $G_1 > G_2$, for then the agent's welfare could be improved by increasing $t(m = A_k, r = x_1, y = x_1)$, $x_1 \in A_k$ by ϵ and decreasing $t(m = x_l, y = x_l)$ by ϵ. This variation does not violate any of the incentive compatibility contraints. Further, it is feasible because $G_1 > G_2 \geq 0$. But this variation results in a strict Pareto improvement which violates the assumption that the original contract was optimal. The same analysis holds for $G_1 < G_2$, but where the agent's consumption is shifted from x_l to x_1 without violating the *IC* constraints. Therefore, if the agent's consumption over all outcomes is not constant, the agent's consumption in outcomes x_l and higher cannot be constant.

LEMMA 4. Assume an optimal solution in which (a) $n(A) < n - 1$, and (b) the agent's consumption is not constant over all outcomes. Let x_l be the lowest outcome in the nonaudit region. Then $x_l - t(m = x_l, y = x_l) \neq x_l - t(m = x_l, y = y_0)$.

Proof (Sketch). The proof is by contradiction. Assume an optimal solution with $t(m = x_l, y = x_l) = t(m = x_l, y = y_0)$. The proof is established by showing that if $t(m = x_l, y = x_l) = t(m = x_l, y = y_0)$, the assumed optimality of the contract implies that the agent's consumption must be constant over all outcomes x_l and higher and hence that (by Lemma 3) the agent's consumption must be constant over all outcomes. But this contradicts assumption (b) in the lemma. Finally, note that this result holds even for the case of $n(A) = 0$, i.e., the principal–agent problem without the auditor.

THEOREM 2. (1) Any contract pair with (a) $0 < n(A) < n - 1$, (b) the agent's consumption not constant over all outcomes, and (c) a nonlower audit region can be strictly Pareto dominated by a contract with a lower audit region.

(2) Any contract pair with (a) $0 < n(A) < n - 1$, (b) the agent's consumption constant over all outcomes, and (c) a nonlower audit region is Pareto equivalent to another contract pair with a lower audit region.

Proof (Sketch). The proof of part (1) is by contradiction. Assume that a nonlower audit region contract pair is optimal. Let $x_k \in A_k$ be the largest outcome in A and x_l be the smallest outcome in $\bar{X}$. From Lemma 2 and the nonlower audit region assumption $x_n > x_k > x_l$. Consider *Condition A*:

$$\forall x_p > x_k \ \pi U(x_p - t(m = x_p, y = x_p)) + (1 - \pi)U(x_p - t(m = x_p, y = y_0)) \geq (1 - \pi)U(x_p - t(m = A_k, r = x_k, y = y_0)).$$

The proof will be established first, given that Condition A holds (Case 1) and then, given that Condition A does not hold (Case 2).

CASE 1. Assume that Condition A holds. Switching x_k from A_k to $\bar{X}$ and moving x_l to A_k will not change the agent's reporting incentives given Condition A. Therefore, adjust audit subregion A_k and the transfers in a manner similar to Lemma 2. The agent's reporting and the auditor's auditing incentives are unchanged. Using Lemma 4 and the fact that all outcomes x_l and below are now in the audit region, a strict Pareto improvement can then be achieved without affecting incentives by making the agent's consumption at outcomes x_l and lower a constant. Therefore, the original nonlower audit region contract pair could not have been optimal if Condition A held.

CASE 2. Assume the negation of Condition A. Define the following transfer under the original contract:

$$t(m = x_g, y = y_0) = \min_{i<k} \{t(m = x_i, y = y_0)\}.$$

Construct a new contract pair by moving x_k from A_k to $\bar{X}$ and x_l from $\bar{X}$ to A_k. Label the new kth audit subregion $\tilde{A}_k$. Let transfers to the auditor and the other audit subregions remain the same. Let the agent's transfers under the new contract be $\tilde{t}(\cdot) = t(\cdot)$ except for the following:

$$\tilde{t}(m = x_k, y = x_k) = t(m = A_k, r = x_k, y = x_k)$$

$$\tilde{t}(m = x_k, y = y_0) = t(m = x_g, y = y_0)$$

$$\tilde{t}(m = \tilde{A}_k, r = x_l, y = x_l) = t(m = x_l, y = x_l)$$

$$\tilde{t}(m = \tilde{A}_k, r = x_l, y = y_0) = t(m = x_l, y = y_0) \\ + t(m = A_k, r = x_k, y = y_0) - t(m = x_g, y = y_0).$$

Given the assumed optimality of the original contract pair, the agent's new transfers, and the fact that the size of each audit region is unchanged, it is straightforward to show that consistent reporting and effective auditing/honest reporting is a Nash strategy pair under the new contract pair. Further, it can be shown that because of the assumed negation of Condition A, the new contract pair results in a strict Pareto improvement in risk sharing over the original contract pair. This follows because the new transfers preserve the agent's expected wealth while producing a mean-preserving reduction in risk for the agent by second-order stochastic dominance. In addition, if:

$$\tilde{t}(m = \tilde{A}_k, r = x_l, y = x_l) \neq \tilde{t}(m = \tilde{A}_k, r = x_l, y = y_0)$$

then as in Lemma 3 a *further* strict Pareto improvement can be made, without affecting incentives, by revising the agent's transfers to make his consumption a constant over outcomes x_l and lower. Therefore, the original nonlower audit region contract pair could not have been optimal if Condition A did not hold.

The proof of part (2) is similar.

THEOREM 3. Assume that the effort necessary for an effective audit does not depend on the size of the audit subregion. Then any contract pair with multiple audit subregions can be weakly Pareto dominated by a contract pair with a single audit subregion.

Proof. Assume an optimal contract pair (C_1, C_2) with multiple audit subregions. Under (C_1, C_2), for audit subregions A_k and A_j the following incentive compatibility constraints (B6k') for the auditor are satisfied (dropping the index k from $\bar{e}_k$ because all effective audits require the same effort):

$$\pi\left(\frac{n(A_k) - 1}{n(A_k)}\right)H(K_1(A_k)) \geq V(\bar{e}). \tag{A4}$$

$$\pi\left(\frac{n(A_j) - 1}{n(A_j)}\right)H(K_1(A_j)) \geq V(\bar{e}). \tag{A5}$$

Replacing $\dfrac{n(A_k) - 1}{n(A_k)}$ in (A4) and $\dfrac{n(A_j) - 1}{n(A_j)}$ in (A5) with $\dfrac{n(A_k) + n(A_j) - 1}{n(A_k) + n(A_j)}$ strictly increases with LHS of both, giving:

$$\pi\left(\frac{n(A_k) + n(A_j) - 1}{n(A_k) + n(A_j)}\right)H(K_1(A_k)) > V(\bar{e}). \tag{A6}$$

$$\pi\left(\frac{n(A_k) + n(A_j) - 1}{n(A_k) + n(A_j)}\right)H(K_1(A_j)) > V(\bar{e}). \tag{A7}$$

Additional arithmetic manipulation and Jensen's inequality yield:

$$\pi\left(\frac{n(A_k + n(A_j) - 1}{n(A_k) + n(A_j)}\right)\left[H\left(\frac{n(A_k)K_1(A_k) + n(A_j)K_1(A_j)}{n(A_k) + n(A_j)}\right)\right] > V(\bar{e}).$$

This condition implies that a new contract $(\tilde{C}_1, \tilde{C}_2)$ which combines any two audit subregions and pays the auditor his weighted average compensation preserves his incentive compatibility constraints. The agent's incentives are unchanged as a result and hence the same strategy pair which was Nash under (C_1, C_2) is also Nash under $(\tilde{C}_1, \tilde{C}_2)$. The new contract makes the auditor no worse off, the principal no worse off, and the agent no worse off. Repeat this process for all the audit subregions. Therefore, we have demonstrated a weak Pareto improvement from using a single audit region.

LEMMA 5. Assume that the effort required for effective auditing does not depend on the size of the audit region. Given the optimal contract pair (C_1, C_2), the strategy pair involving the auditor shirking and guessing and the agent not reporting consistently $((SG, R^i), R^i \neq R^0)$ is not a Nash strategy pair.

Proof. Because (I^0, R^0) is a Nash pair given contracts (C_1, C_2), we know that:

$$ED(I^0 \mid R^0) \geq ED(SG \mid R^0). \tag{A8}$$

It follows from Theorem 1 that:

$$ED(I^0 \mid R^0) = ED(I^0 \mid R^i) \quad \forall R^i \neq R^0. \tag{A9}$$

Any feasible alternative to consistent reporting by the agent, i.e., any R^i, $i \neq 0$, would increase $n(A)$ and thereby reduce the auditor's odds of correctly guessing the outcome. Therefore:

$$ED(SG \mid R^i) < ED(SG \mid R^0) \quad \forall R^i \neq R^0. \tag{A10}$$

Combining (A8)–(A10) gives:

$$\begin{aligned} ED(I^0 \mid R^i) = ED(I^0 \mid R^0) &\geq ED(SG \mid R^0) \\ &> ED(SG \mid R^i) \quad \forall R^i \neq R^0 \end{aligned} \tag{A11}$$

which implies that strategy SG cannot be a best response to any $R^i \neq R^0$. Hence, (SG, R^i) with $R^i \neq R^0$ cannot be a Nash pair given contract pair (C_1, C_2).

LEMMA 6. Assume that the effort required for effective auditing does not depend on the size of the audit region. Given the optimal contract pair (C_1, C_2), the strategy pair involving the auditor auditing effectively and reporting honestly and the agent not reporting consistently $((I^0, R^i)$, for some $R^i \neq R^0)$ is not a Nash pair.

Proof. Assume that (I^0, R^i) for some $R^i \neq R^0$ is a Nash strategy pair under contract pair (C_1, C_2). We already know that (I^0, R^0) is Nash under (C_1, C_2). Therefore, the agent must be indifferent between R^0 and some R^i, $i \neq 0$, given I^0.

Under the contract R^i, where $R^i \neq R^0$, the agent must be calling for an audit for at least one outcome x_j in the nonaudit region. But given that the auditor uses strategy I^0, the agent's false report will be detected and the transfer will reduce the agent's net wealth to zero. Therefore, if the agent is indifferent between R^0 and R^i when the auditor uses strategy I^0, then under (C_1, C_2) the agent must also obtain zero wealth for this nonaudit outcome (i.e., $x_j - t(m = x_j, y = x_j) = x_j - t(m = x_j, y = y_0) = 0$) when he reports consistently, strategy R^0. This implies that x_j must be x_l, the smallest outcome in $\bar{X}$. Otherwise, there would exist an $x_k \in \bar{X}$ with $x_k < x_j$ and the following incentive compatibility constraint would hold:

$$\pi U(x_j - t(m = x_j, y = x_j)) + (1 - \pi) U(x_j - t(m = x_j, y = y_0)) \geq (1 - \pi) U(x_j - t(m = x_k, y = y_0)) > 0 \tag{A12}$$

where the strict inequality follows from the fact that:

$$x_k - t(m = x_k, y = y_0) \geq 0$$

and $x_j > x_k$. However, given that $U(0) = 0$, this contradicts the requirement that:

$$x_j - t(m = x_j, y = x_j) = x_j - t(m = x_j, y = y_0) = 0.$$

Therefore, x_j must be x_l, the smallest nonaudit outcome.

The preceding argument implies that under (C_1, C_2):

$$x_l - t(m = x_l, y = x_l) = x_l - t(m = x_l, y = y_0) = 0.$$

This together with $U(0) = 0$ and $\bar{U} > 0$ implies that under (C_1, C_2) the agent's consumption cannot be constant over all outcomes. But we know from Lemma 4 that if under (C_1, C_2) the agent's consumption is not constant over all outcomes, then it must be that:

$$x_l - t(m = x_l, y = x_l) \neq x_l - t(m = x_l, y = y_0).$$

Therefore, this establishes a contradiction and (I^0, R^i), where $R^i \neq R^0$, cannot be a Nash equilibrium pair given contract pair (C_1, C_2).

LEMMA 7. Assume that the effort required to audit effectively does not depend on the size of the audit region. If (SG, R^0) were a Nash strategy pair given (C_1, C_2), it would not be a subgame perfect equilibrium.

Proof. We will use Selten's [1975] concept of a trembling hand equilibrium by creating a sequence of perturbed games for the agent and auditor. In each game they *must choose* mixed strategies where the minimum probability assigned to each feasible strategy is $\epsilon_n > 0$ for the auditor and $\delta_n > 0$ for the agent with $\epsilon_n \to 0$ and $\delta_n \to 0$ as $n \to \infty$. We will then show that no such sequence of perturbed games converges to (SG, R^0), implying that (SG, R^0) is not a subgame perfect equilibrium.

Given that (I^0, R^0) is Nash, if (SG, R^0) were also a Nash pair under

(C_1, C_2), then:

$$ED(SG \mid R^0) = ED(I^0 \mid R^0) = ED(I^0 \mid R^i) > ED(SG \mid R^i) \quad \forall R^i \neq R^0. \tag{A13}$$

(A13) implies that *for any perturbed game n* and $\delta_n > 0$ the auditor's best response against the strategy by the agent which assigns a positive probability to R^i is to choose SG with minimum probability ϵ_n and I^0 with probability $1 - \epsilon_n$. As $n \to \infty$, it is clear that the sequence of best response pairs cannot converge to (SG, R^0) because the auditor's component is converging to I^0. Therefore, even if (SG, R^0) were a Nash strategy pair against (C_1, C_2), it would not be subgame perfect.

LEMMA 8. Assume that the effort required to audit effectively does not depend on the size of the audit region. Then (1) the following mixed strategy pairs cannot be Nash equilibria: (*a*) $((I^0, SG), (R^0, R^i))$, (*b*) $(I^0, (R^0, R^i))$, (*c*) $(SG, (R^0, R^i))$, (*d*) $((I^0, SG), R^i)$ $R^i \neq R^0$; (2) the mixed strategy pair $((I^0, SG), R^0)$ cannot be subgame perfect.

Proof. (1*a*) If $((I^0, SG), (R^0, R^i))$ were a Nash equilibria it would be true that:

$$\delta(ED(I^0 \mid R^0) - ED(SG \mid R^0)) = (1 - \delta)(ED(SG \mid R^i) - ED(I^0 \mid R^i)). \tag{A14}$$

where $0 < \delta < 1$ is the agent's probability of using strategy R^0. This would imply that:

$$\delta(ED(I^0 \mid R^0) - ED(SG \mid R^0)) = (1 - \delta)(ED(SG \mid R^i) - ED(I^0 \mid R^i)). \tag{A14}$$

Because (I^0, R^0) is a Nash equilibrium the *LHS* of (A14) must be nonnegative. However, from (A11) the *RHS* of (A14) is strictly negative. This leads to a contradiction.

In a similar manner: (1*b*) follows from Lemma 6; (1*c*) and (1*d*) follow from Lemma 5. Part (2) follows from Lemma 7.

THEOREM 4. Assume that the effort required to audit effectively does not depend on the size of the audit region. Given the optimal contract pair, the only subgame perfect agent–auditor Nash strategy pair is consistent reporting and effective auditing/honest reporting.

Proof. Given the optimal contract pair, we know that neither (SG, R^i) nor (I^0, R^i), where $R^i \neq R^0$, is a Nash strategy pair (Lemmas 5 and 6). Even if (SG, R^0) were a Nash strategy pair, it would not be subgame perfect (Lemma 7). Lemma 8 generalizes these results to mixed strategy pairs. The only remaining mixed or pure strategy pair is (I^0, R^0), which we know is a Nash pair. Therefore, because every game must have at least one subgame perfect Nash equilibrium, (I^0, R^0) must be a subgame perfect Nash equilibrium.

APPENDIX B

The Three-Person (Principal–Agent–Auditor) Problem

The principal's problem is to select the transfer schedule from the agent, the transfer schedule to the auditor, and to select the audit subregions. Based on Observations 1–5, the principal's problem can be represented as:

$$\max_{\substack{t(\cdot),f(\cdot)\\A_1,\ldots,A_p}} \frac{1}{n} \sum_{A_k\subseteq A} \sum_{x_j\in A_k} \{\pi(t(m = A_k,\, r = x_j,\, y = x_j) \tag{B1}$$

$$- f(m = A_k,\, r = x_j,\, y = x_j)) + (1 - \pi)$$

$$\cdot(t(m = A_k,\, r = x_j,\, y = y_0) - f(m = A_k,\, r = x_j,\, y = y_0))\}$$

$$+ \frac{1}{n} \sum_{x_j\in\bar{X}} \{\pi t(m = x_j,\, y = x_j) + (1 - \pi)(t(m = x_j,\, y = y_0)\}$$

subject to:

$$\frac{1}{n} \sum_{A_k\subseteq A} \sum_{x_j\in A_k} \{\pi U(x_j - t(m = A_k,\, r = x_j,\, y = x_j)) \tag{B2}$$

$$+ (1 - \pi)U(x_j - t(m = A_k,\, r = x_j,\, y = y_0))\}$$

$$+ \frac{1}{n} \sum_{x_j\in\bar{X}} \{\pi U(x_j - t(m = x_j,\, y = x_j))$$

$$+ (1 - \pi)U(x_j - t(m = x_j,\, y = y_0))\} \geq \bar{U}$$

$$\frac{1}{n(A)} \sum_{A_k\subseteq A} \sum_{x_j\in A_k} \{\pi H(f(m = A_k,\, r = x_j,\, y = x_j)) \tag{B3}$$

$$+ (1 - \pi)H(f(m = A_k,\, r = x_j,\, y = y_0)) - V(\bar{e}_k)\} \geq \bar{D}$$

$$\pi U(x_j - t(m = A_k,\, r = x_j,\, y = x_j)) \tag{B4ji}$$

$$+ (1 - \pi)U(x_j - t(m = A_k,\, r = x_j,\, y = y_0))$$

$$\geq (1 - \pi)U(x_j - t(m = x_i,\, y = y_0))$$

$$\forall x_j \in A_k, \quad \forall\{x_i \in \bar{X} \mid x_j > x_i\}, \quad \forall A_k \subseteq A$$

$$\pi U(x_j - t(m = x_j, y = x_j)) + (1 - \pi)U(x_j - t(m = x_j, y = y_0)) \tag{B5ji}$$

$$\geq (1 - \pi)U(x_j - t(m = x_i, y = y_0))$$

$$\forall x_j, \quad x_i \in \bar{X} \quad x_i < x_j$$

$$\frac{1}{n(A_k)} \sum_{x_j \in A_k} \{\pi H(f(m = A_k, r = x_j, y = x_j)) + (1 - \pi) H(f(m = A_k, r = x_j, y = y_0))\} - V(\bar{e}_k)$$
$$\geq \max_{x_i \in A_k} \left\{ \frac{1}{n(A_k)} H(f(m = A_k, r = x_i, y = x_i)) + (1 - \pi) H(f(m = A_k, r = x_i, y = y_0)) \right\} - V(0) \quad \forall A_k \subseteq A \tag{B6k}$$

$$\frac{1}{n(A_k)} \sum_{x_j \in A_k} \{\pi H(f(m = A_k, r = x_j, y = x_j)) + (1 - \pi) H(f(m = A_k, r = x_j, y = y_0))\} - V(\bar{e}_k)$$
$$\geq \max_{x_i \in (X - A_k)} (1 - \pi) H(f(m = A_k, r = x_i, y = y_0)) - V(0) \quad \forall A_k \subseteq A \tag{B7k}$$

$$\left.\begin{array}{l} x_j - t(m = A_k, r = x_j, y = x_j) \geq 0 \\ x_j - t(m = A_k, r = x_j, y = y_0) \geq 0 \end{array}\right\} \quad \forall x_j \in A_k, \quad \forall A_k \subset A \tag{B8}$$

$$\left.\begin{array}{l} x_j - t(m = x_j, y = x_j) \geq 0 \\ x_j - t(m = x_j, y = y_0) \geq 0 \end{array}\right\} \quad \forall x_j \in \bar{X}$$

$$\left.\begin{array}{l} f(m = A_k, r = x_j, y = x_j) \geq 0 \\ f(m = A_k, r = x_j, y = y_0) \geq 0 \end{array}\right\} \quad \forall x_j \in A_k, \quad \forall A_k \subseteq A.$$

(B1) is the principal's objective function. (B2) and (B3) are respectively the agent's and auditor's minimum expected utility constraints, assuming that the agent reports honestly and that the auditor audits effectively/ reports honestly.

(B4) and (B5) are the agent's incentive compatibility constraints. (B4ji) states that, assuming that the auditor audits effectively/reports honestly, given that the agent observed $x_j \in A_k$, he will prefer to report A_k than to report any smaller x_i which is not in an audit subregion. The double-transfer system prevents the agent from reporting an outcome higher than the actual outcome. If the agent lies by reporting a nonaudit outcome and $y = x_j$, the agent's net worth would be zero, by Observation 5. This is reflected in the *RHS* of (B4ji), because $\pi U(0) = 0$. If the agent reported an audit subregion different from the actual, the auditor would be hired and would contradict him, given that the auditor's best strategy is to audit effectively/report honestly (Observations 2 and 3). By Observation 5 the agent would then be left with zero wealth. Therefore, this alternative of claiming a false audit outcome is clearly no better than the agent reporting a nonaudit outcome and hence is not considered in the program. (B5ji) is similar to (B4ji) except that $x_j \in \bar{X}$.

(B6k) and (B7k) are the auditor's incentive compatibility constraints which state that, assuming that the agent reports consistently, the auditor prefers effective auditing/honest reporting to shirking and guessing an outcome x_i within the announced audit region A_k (B6k) or outside of the announced audit region (B7k). We have used Observation 5 which states that if the auditor is contradicted by the public information system he receives a payment of zero and $H(0) = 0$. Given that the agent is reporting consistently, this contradiction would occur with probability $\frac{n(A_k) - 1}{n(A_k)}$ if the auditor shirked and guessed an outcome within A_k (*RHS* of (B6k)) and with probability 1 if the auditor shirked and guessed an outcome outside of A_k (*RHS* of (B7k)). The auditor's alternative of auditing effectively and lying is clearly dominated by the alternative of shirking and guessing and hence is not considered. Constraints (B8) are the feasibility constraints on transfers from the agent to the principal and from the principal to the auditor.

Given parts (1)–(3) of Theorem 1, constraint (B7k) can be dropped and (B6k) rewritten as:

$$\pi H(K_1(A_k)) + (1 - \pi)(H(K_2 A_k)) - V(\bar{e}_k) \geq \frac{\pi}{n(A_k)} H(K_1(A_k))$$
$$+ (1 - \pi)H(K_2(A_k)) - V(0) \quad \forall A_k \subseteq A$$

or:

$$\pi\left(\frac{n(A_k) - 1}{n(A_k)}\right)H(K_1(A_k)) \geq V(\bar{e}_k) \quad \forall A_k \subseteq A. \qquad \text{(B6}k'\text{)}$$

Constraint (B3) can be rewritten as:

$$\sum_{A_k \subseteq A} \frac{n(A_k)}{n(A)} \{\pi H(K_1(A_k)) + (1 - \pi)H(K_2(A_k)) - V(\bar{e}_k)\} \geq \bar{D}. \qquad \text{(B3}'\text{)}$$

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