# 14.121: Microfoundations for GE

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## Informational Efficiency of Walrasian Equilibrium

Earlier, I criticized the value of the SWT since we can only construct the supporting prices if we have full knowledge of the economy – but this begs the question, if we have full knowledge of the economy why do we need prices?

Hayek (1945) conjectured prices may play an informational role

If individuals have private information about their preferences, endowments, technologies, etc, this information is too enormous to be communicated to a central planner.

Market mechanism works by using prices to give people concise sufficient statistics allowing them to make coordinated choices and arrive at a socially optimal allocation.

Part of broader debate on the comparative merits of alternative economic systems

Hurwicz (1960) proposed formalization of informational role of prices in terms of choosing a "mechanism" from a set of alternative mechanisms

We will not develop his ideas in detail, but they involved trying to formalize the statement that it is impossible to have a mechanism verifying a Pareto optimal allocation without revealing a supporting Walrasian equilibrium (or common marginal rates of substitution, which gives supporting prices)

Provides a formalization of the necessity of price revelation for informational reasons

See Hurwicz (1977) and Mount and Reiter (1974) who bounded the number of real variables that need to be communicated to verify Pareto optimality as at least the number of variables needed to describe a Walrasian equilibrium

## Microfoundations and the Core

Many formulations: core, Ostroy's no surplus condition, bargaining set, Shapley-Shubik market games, other non-cooperative games (i.e., Gale-Rubinstein-Wolinsky)

Core is most common. The **core** is the set of all allocations such that no coalition (set of agents) can improve on or block the allocation (make all of its members better off) by seceding from the economy and only trading among its members.

Edgeworth (1881) thought core, not set of Walrasian equilibria, was the best positive description of outcomes from market mechanism

Core is institution-free concept: no mention of prices

**Central result**: "Core convergence" $\Rightarrow$ for economies with a large number of agents, core allocations are "approximately Walrasian"

- approximately Walrasian means different things in different contexts

## Definition

In an exchange economy, a **coalition** is a set $S \subset \{1, ..., I\}$. A coalition $S$ **blocks** or **improves on** allocation $x$ by $\hat{x}$ if

$$\sum_{i \in S} \hat{x}^i = \sum_{i \in S} \omega^i,$$

and for all agents $i$,

$$\hat{x}^i \succeq_i x^i, \text{ with at least one preference strict.}$$

## Definition

The **core** (by weak domination) is the set of all allocations which cannot be improved on by any coalition.

Note: MWG requires $\hat{x}^i \succ_i x^i$ for all $i \in S$. Some find this definition of the core natural if you think of the status quo as a focal point, one needs a strict improvement to join a coalition to upset the status quo.

Two basic results:

- **Result 1**: In an exchange economy, every core allocation is Pareto optimal.

- **Result 2**: In an exchange economy, every Walrasian Equilibrium lies in the core.

  Can be thought of strengthening FWT
  - Since Walrasian equilibrium are Pareto optimal, no way to upset equilibrium and make all individuals weakly better off
  - Walrasian equilibrium in the core means that no group of individuals would choose to upset equilibrium by recontracting among group and make group better off

Some remarks:

- Our existence of Walrasian equilibrium, together with the last theorem, shows the nonemptiness of the core. The fact that the core exists is not a general property of **games**, however.

- In a Walrasian equilibrium, each consumer acts independently of all other consumers. In contrast, in the core, consumers can get together, take stock of total resources available to them, and exploit all potential gains from trade. Such a scenario might seem to require a great deal of coordination.

  Might suggest it would be hard to coordinate especially without a centralized authority.

- Last result shows that it is possible to achieve core outcomes without the aid of a central planner $\Rightarrow$ **decentralization**

Is it true that every core allocation is Walrasian?

In the two consumer Edgeworth box that every allocation in the contract curve is in the core, but only one is a Walrasian allocation.

The remarkable result is that when there are numerous consumers, the converse does approximately hold.

As we increase the size of the economy, the non-Walrasian allocations gradually drop from the core until in the limit only the Walrasian allocations are left.

Hence, while the fictitious Walrasian auctioneer is of course unrealistic, it might be seen as a “short-cut”

# Core Convergence

The set of **types** of consumers is $\mathcal{I} = \{1, ..., I\}$.

The $R$-**replica economy** $\mathcal{E}_R$ is one with $R$ consumers of each type. The total number of consumers is $RI$.

This provides a simple way of constructing a sequence of economies, with more and more consumers.

A typical allocation in an $R$-replica economy is

$$x = (x^{i,r}) = (\underbrace{x^{1,1}, ..., x^{I,1}}_{\text{First Economy}}, ..., \underbrace{x^{1,R}, ...., x^{I,R}}_{\text{Rth Economy}}) \in \mathbb{R}_+^{LRI}.$$

where $x^{i,r}$ is bundle of the $r$th consumer of type $i$.

Note that

$$\sum_{i \in \mathcal{I}} \sum_{r=1}^{R} x^{i,r} = R \sum_{i \in \mathcal{I}} \omega^i$$

An important simplification comes from the equal treatment property.

### Proposition (Equal Treatment)

*Suppose consumers have strictly convex preferences. If $x$ is a core allocation in $\mathcal{E}_R$, then $x^{i,r} = x^{i,r'}$ for all $r, r'$.*

i.e., consumers of the same type consume the same bundle.

This result allows us to regard the core allocations as vectors of fixed size $LI$, irrespective of the replica that we are concerned with.

As a matter of terminology, we call a vector $(x_1, ..., x_I) \in \mathbb{R}_+^{LI}$ a **type allocation** and, for any replica $R$, interpret it as the equal-treatment allocations where each consumer of type $i$ gets $x^i$.

Note that the core allocations of an $R$ replica economy are in $\mathbb{R}^{LIR}$. Strictly speaking, this means that we cannot compare the core allocations of an $R$ replica with those of an $R'$ replica.

But the equal treatment result provides a way to doing so. Given the equal treatment property, we can represent in $\mathbb{R}^{LI}$ the core allocations of an $R$-replica economy, or an $R'$-replica economy - just by looking at type allocations.

Let $C_R \subset \mathbb{R}^{LI}_{+}$ denote the set of type allocations corresponding to the (equal treatment) core allocations of the $R$-replica economy.

Let $W_R \subset \mathbb{R}^{LI}_{+}$ denote the type allocations corresponding to the (equal treatment) Walrasian allocations of the $R$-replica economy.

**Observation 1**. Suppose coalition $S$ has an objection against a type allocation $x$ in the $R$-replica. Then the same is true in the $R+1$ replica. This implies:

$$C_{R+1} \subseteq C_R \text{ for all } R$$

**Observation 2**. If a type allocation $x$ is Walrasian in the $R$-replica, it is also Walrasian in the $R+1$-replica, and vice versa:

$$W_R = W_{R+1} = W \text{ for all } R.$$

**Observation 3**. By our earlier result

$$W \subset C_R \text{ for all } R.$$

Putting together these observations:

$$W \subseteq ... \subseteq C_{R+1} \subseteq C_R \subseteq ... \subseteq C_1.$$

## Theorem (Debreu-Scarf 1963)

$$\cap_R C_R = W,$$

*i.e. if $x^* \in C_R$ for all $R$, then $x^*$ is Walrasian.*

Is replication too strong?

- Aumann (EMA 1964): demonstrated an exact result by looking at a model where there is a continuum of consumers and all summations are replaced by integrals. In such a setting, he was able to establish that an allocation belongs to the core if and only if it is a Walrasian equilibrium allocation.
- Dierker (JME 1975)-Anderson (EMA 1978): if $x$ is a core allocation, then we can find a price vector such that $(p, x)$ nearly satisfies the definition of Walrasian equilibrium. When there are many more agents than goods and the endowments are not too large, the bound will be small.