# 14.121 Lecture 3: Putting Consumer Theory to Work

Parag Pathak
MIT

September 2019

## Putting Consumer Theory to Work

1. Price changes and welfare
2. Welfare from new goods
3. Price indexes
4. Choosing functional forms
5. Integrability
6. Estimating demand
7. Demand aggregation

# 1. Price Changes and Welfare

Central question in applied micro: how do price changes affect welfare?

Comes up in various ways:

1. effects of taxes or subsidies
2. effects of inflation, e.g. Social Security cost of living increase
3. loses due to monopoly pricing
4. distributional national accounting: do the poor pay more?
5. welfare benefits of innovation

Consider a consumer with wealth $w$ who originally faces prices $p^0$, who obtains utility $u_0 = v(p^0, w)$.

Write $u_1 = v(p^1, w)$ for utility after the price change if she gets no compensation.

If $u_1 < u_0$, then consumer is worse off.

Problem: we typically do not directly observe these quantities

Usually, we want a measure of welfare change in dollar units.

There are two famous measures of welfare change economists use.

### Definition

The **equivalent variation** EV is

$$EV(p_0, p_1, w) = e(p^0, u_1) - e(p^0, u_0) = e(p^0, u_1) - w.$$

Question: How much of a wealth shock would have been as bad as a price shock?

i.e. How much would you pay for research to prevent the spread of an agricultural pest (yesterday's dollars)

### Definition

The **compensating variation** CV is

$$CV(p_0, p_1, w) = e(p^1, u_1) - e(p^1, u_0) = w - e(p^1, u_0).$$

Question: How much would consumer need to be paid to make her as well off after the price change?

i.e. This is the question the social security committee on living adjustment is interested in - how much worse off is the person in today's dollars?

Neither measure is "better."

The standard way to think about evaluating these given observable objects is to use the Hicksian-Expenditure theorem.

$$
\begin{aligned}
EV &= e(p^0, u_1) - w \\
&= e(p^0, u_1) - e(p^1, u_1)
\end{aligned}
$$

If we assume only the price of good $i$ changes and all other prices say the same, this gives

$$
EV = -\int_{p_i^0}^{p_i^1} h_i(p_i, p_{-i}, u_1) dp_i.
$$

Similarly,

$$
\begin{aligned}
CV &= w - e(p^1, u_0) \\
&= e(p^0, u_0) - e(p^1, u_0) \\
&= -\int_{p_i^0}^{p_i^1} h_i(p_i, p_{-i}, u_0) dp_i.
\end{aligned}
$$

We could evaluate these in practice by using the Slutsky equation to go from observed Marshallian demand to Hicksian demands.

When price changes are small, you could also often see people cheating and assuming $h(p, u) = D(p)$ independent of $u$ (using observed wealth level and assuming it does not matter).

Then $CV = EV =$ area under the Marshallian demand curve.

This is cheating, but is often done.

# 2. Welfare from New Goods

Welfare changes from invention of new goods or loss of old ones can be calculated by taking integrals out to $\infty$, e.g. CV from invention of new good $i$ is

$$\int_{p_i}^{\infty} h_i(p_i, p_{-i}, u_0) dp_i.$$

A difficulty with estimating this empirically is that you rely on estimates with $p$ outside the range of observed prices.

It is also equal to

$$\int_{p_i'}^{\bar{p}_i} h_i(p_i, p_{-i}, u_0) dp_i$$

where $\bar{p}_i$ is the virtual price at which the demand for good $i$ would go to zero.

Three practical approaches one can take to estimate welfare changes.

1) Assume no income effects in demand: $x(p, w) = x(p)$. Then

$$EV = CV = -\int_{p_i^0}^{p_i^1} x(p_i, p_{-i}) dp_i.$$

May be reasonable (or all one can you do) for inexpensive items.

It can be justified, e.g. by assuming that

$$u(x_0, x_1, ..., x_L) = ax_0 + f(x_1, ..., x_L)$$

e.g., recent example is Einav, Finkelstein, Cullen (QJE 2010)

2) Non-parametrically go from Marshallian to Hicksian

We noted how one can use Slutsky to go from Marshallian to derivatives of Hicksian. One can solve the PDE to find the Hicksian demand.

Hausman & Newey (1995): specify demand as $x(p, w) + \eta$, for $\eta$ scalar heterogeneity, and solve PDE

Recent interest in making this more flexible with non-separable heterogeneity

Hausman & Newey (2011): $x(p, w, \eta)$ where $\eta$ is unrestricted vector; show how you can bound welfare measures

3) Convert from Marshallian to Hicksian via functional form assumption

Most econometric estimates need *some* functional form assumption, e.g.

$$x_i(p, w) = \alpha_0 - \sum_j \beta_j p_j - \gamma w + \epsilon$$

As long as you're making *some* assumption, might as well assume a form for $x(p, w)$ that has good properties, and corresponds to a known utility function

You can then convert to $h(p, u)$ by just plugging in parameters.

# 3. Price Indexes

An important application of welfare change measures is price indexes. The CPI is used to account for inflation. It affects social security, union contracts, estimates of real GDP growth, etc.

At $t = 0$ we see prices $p^0$ consumption $x^0$.
At $t = 1$, we see prices $p^1$ and consumption $x^1$.

Two most common price indices are

### Definition

The **Laspeyres price index** is

$$\frac{p^1 \cdot x^0}{p^0 \cdot x^0}.$$

The **Paasche price index** is

$$\frac{p^1 \cdot x^1}{p^0 \cdot x^1}.$$

A basic theorem on the biases in these indexes, which dates to Konüs (1924, in Russian)

## Proposition

a) *The Laspeyres index overstates inflation in the sense that*

$$\frac{p^1 \cdot x^0}{p^0 \cdot x^0} \geq \frac{e(p^1, u_0)}{e(p^0, u_0)}.$$

b) *The Paasche index understates inflation in the sense that*

$$\frac{p^1 \cdot x^1}{p^0 \cdot x^1} \leq \frac{e(p^1, u_1)}{e(p^0, u_1)}.$$

How else could one measure price changes?

The ideal would be to estimate the expenditure function changes

$$e(p^1, u^0)/e(p^0, u^0)$$

at some reference utility.

This is infeasible for a government that collects data on thousands of products and could not possible estimate price and wealth elasticities for each.

One recommendation sometimes see is the Fisher Ideal Index which is square root of Paasche times Laspeyres.

## Bias in the US CPI

In 1996, the Boskin commission issued a report that received a lot of attention and has led to big changes in the CPI. It concluded that the CPI was probably overstated by about 1.1 percent points per year.

The sources of bias they cited are:

- **Substitution bias** (0.4%)

  The BLS used a Laspeyres index. They changed $x^0$ very infrequently - it took 3 years to collect data, 3 years to start to use, and then another 10 years to the next changeover. So 1982-1984 weights were used for 1987-1997.

- **Outlet bias** (0.1%)

  The BLS treats different goods at different stores as different. If Walmart enters a market and offers much lower prices, they do not count as a price cut.

- **New goods and quality change** (0.6%)

  Cars used to break down a lot and now they rarely do. TVs have much clearer pictures. Cell phones didn't exist. Ulcers needed surgery, not drugs. The Internet exists.

  BLS would treat each new TV as a new good and only record changes in price. Cell phone would wait 10+ years to get in the index at all and then only change would be tracked, not big CV like Hausman estimates (100 billion per year in 1998)

Many economists believe that the 0.6% number is much too small.

# 4. Choosing Functional Forms

Usually consumers have demand over many goods, but we want to work with simpler functions. Why?

- Household survey, containing data on groups (food, housing); want to think of in terms of choice over composite groups
- Want to have auto demand could depend on prices of cheese via income effect, but we don't want it to depend on relative price of cheese

Two common approaches:

1) **Restrict preferences** – often made when do not observe consumption choices
2) **Restrict price movements** – restricts number of parameters we need to estimate

**Restricting preferences**: want demand within group of goods to be independent of prices outside group

Given group $g$, for each $i \in g$,

$$x_i = x_i(p, w) = x_i(p_g, w_g).$$

Interpret as two-step budgeting: 1) first allocate group expenditures

$$(w_{g_1}, ..., w_{g_N}) \quad \text{s.t. } \sum_{a=1}^{N} w_{g_a} = w$$

then 2) make choices within groups.

Preferences are weakly separable: $u(x) = U(u_1(x_{g_1}), ..., u_N(x_{g_N}))$ and $\frac{\partial U}{\partial u_g} > 0$ for all $g$

$$\text{Weak separability} \Leftrightarrow x_i = x_i(p_g, w_g)$$

See Deaton and Muelbauer (1980) for proof

**Restricting price movements**: assume prices within group move proportionately

Suppose 3 goods, goods 2 and 3 in group. We'd like to treat as if there were 2 goods, $x_1$ and $X$

Let $\bar{p}_2$ and $\bar{p}_3$ be "base" prices, so that

$$p_2 = q\bar{p}_2 \qquad p_3 = q\bar{p}_3$$

For any $(x_2, x_3)$ let

$$X = \bar{p}_2 x_2 + \bar{p}_3 x_3$$

with composite price $q$

Transform:

$$\max u(x_1, x_2, x_3) \text{ s.t. } p_1 x_1 + p_2 x_2 + p_3 x_3 \leq w$$

to

$$\max \tilde{u}(x_1, X) \text{ s.t. } p_1 x_1 + qX \leq w$$

Do we have the same solutions?

Start with indirect utilities, and note that

$$\tilde{V}(p_1, q, w) = V(p_1, q\bar{p}_2, q\bar{p}_3, w)$$

Use Roy:

$$\frac{\partial \tilde{V}}{\partial q} = \frac{\partial V}{\partial p_2} \times \frac{\partial p_2}{\partial q} + \frac{\partial V}{\partial p_3} \times \frac{\partial p_3}{\partial q} = -\lambda x_2 \times \bar{p}_2 - \lambda x_3 \times \bar{p}_3 = -\lambda X$$

May be ok when prices within group move together, or few within group substitution so that $q$ works as price index

# 5. Integrability

Following our earlier discussion of GARP and preferences, one might want to know if a set of demand functions corresponds to utility maximization.

Why?

1) foundations for utility theory
2) have we proved all we could?
3) Macro: often simpler to work with a single consumer economy (i.e. "representative agent"). Nice if observed demand functions as if chosen by one consumer.

We know that rationality implies

1) $x$ is homogenous of degree 0
2) $x$ satisfies Walras's law ($\sum p_i x_i(p, w) = w$)
3) Slutsky matrix symmetric and negative semi-definite

Will all such $x$'s correspond to some $u$ or are there other important implications of rationality that we've missed?

Answer turns out to be no.

### Proposition (Antonelli, Hurwicz-Uzawa)

*A set of continuously differentiable functions $x_i : \mathbb{R}^{L+} \times \mathbb{R} \to \mathbb{R}^+$ are the demand functions generated by some increasing, quasi-concave utility function u if they satisfy Walras's law and have a symmetric negative semidefinite Slutsky substitution matrix.*

What is integrability good for?

In applied micro, most start with specific parametric specification of consumer demand $x$.

One approach is to write a nice form for $U$ and solve UMP to compute demand $x$.

But for most $U$, the Marshallian demands that result are a mess.

Can be more convenient to write a tractable form for $x$.

Integrability allows us to make sure the $x$ corresponds to a particular $u$ (i.e. are we playing by the rules of standard microeconomic theory?)

# 6. Estimating Demand

So far we've studied the properties of a demand functions.

Now we are going to provide an overview on how modern applied microeconomists estimate demand

There are two main approaches:

- Treat **products as primitives**
- Treat product **characteristics as primitives**

## Demand Estimation: Product Space

A longstanding question of great interest is what patterns of substitution are there between different goods.

Doing this in full generality with $L$ goods requires $L^2$ cross price elasticities.

Imposing restrictions implied by utility theory cuts down on this somewhat. Slutsky symmetry cuts from $L^2$ to $L^2/2$.

Homogeneity of degree 0 and Walras's law each saves 1 degree of freedom.

Historically, datasets were very small so people chose functional forms with few parameters.

Sir Richard Stone (1954) had data on prices and quantities for 48 foods in Britain: butter, margarine, cheese, oranges, etc. He had 19 years of annual data, 1920-38.

There was no way to regress $x_{\text{butter}}$ on 47 parameters and wealth, so he used the Stone linear expenditure system.

## Stone-Geary preferences

$$u(x_1, ..., x_L) = \prod_{i=1}^{L} (x_i - \gamma_i)^{\beta_i} \text{ with } \sum \beta_i = 1$$

When $\gamma_i = 0$ get Cobb-Douglas, so can be thought of consumer having minimum consumptions

Demands are

$$x(p, w): \quad p_i x_i = p_i \gamma_i + \beta_i (w - \sum_j p_j \gamma_j)$$

$$h(p, u): \quad p_i h(p, u_0) = p_i \gamma_i + \beta_i \beta_o \prod_j p_j^{\beta_j} u_0$$

where $\beta_0$ is a function of the parameters. Amount spent is <u>linear</u> in prices and incomes (also known as linear expenditure system)

Expenditure function (can be used directly for welfare):

$$e(p, u) = \sum_j p_j \gamma_j + \beta_0 \prod_j p_j^{\beta_j} u_0$$

Stone-Geary demands are a special case of **quasi-homothetic** preferences, which have demands of:

$$x(p, w) = \alpha(p) + w \cdot \beta(p)$$

Preferences are **homothetic** if for all $x, y$, $\alpha > 0$,

$$x \succeq y \Rightarrow \alpha x \succeq \alpha y$$

Implies that each indifference curves magnified or reduced version of the other

MWG 3.C.5: Homothetic preferences admit representation

$$u(x) = f(x)$$

where $f(\alpha x) = \alpha f(x)$.

## Proposition

*If preferences are homothetic, then demand is homogenous of degree* 1 *in income:*

$$x(p, \alpha w) = \alpha x(p, w).$$

*Moreover, the income elasticity of demand is 1; that is, for all $j$,*

$$\frac{\partial \log(x_j)}{\partial \log(w)} = 1.$$

- Often see homogeneity of degree 1 expressed as:

$$x(p, w) = w\tilde{x}(p)$$

(where $\alpha = \frac{1}{p}$)

# Gorman (1961) polar form

Expenditure function is:

$$e(p,u) = a(p) + ub(p)$$

where $a(p)$ and $b(p)$ are homogenous of degree 1 in price and

$$\beta_j(p) = \frac{1}{b(p)} \frac{\partial b(p)}{\partial p_j}$$
$$\alpha_j(p) = \frac{\partial a(p)}{\partial p_j} - \beta_j(p) a(p)$$

Gorman showed that this form holds **if and only if** the demand is quasi-homothetic of form:

$$x(p,w) = \alpha(p) + w \cdot \beta(p)$$

This form implies an indirect utility function of:

$$v(p,w) = (w - a(p))/b(p)$$

Alternative approach pursued by Deaton and Muelbauer (1980) who model expenditures shares $s_i = p_i x_i / w$ as functions of logs of prices and wealth. Known as the **Almost-Ideal Demand System**

Start with expenditure functions of the form:

$$\log e(p, u) = a(p) + ub(p)$$

Implies share equations of the form:

$$s_i = \frac{p_i h_i(p_i, v(p, w))}{w} = \frac{p_i}{w} \frac{\partial e}{\partial p_i} = \frac{\partial \log e}{\partial \log p_i} = A_i(p) + B_i(p) \log w$$

D-M propose:

$$a(p) = \alpha_0 + \sum_j \alpha_j \log p_j + \frac{1}{2} \sum_j \sum_k \gamma^*_{jk} \log p_j \log p_k$$

$$b(p) = \beta_0 \prod_k p_k^{\beta_k}$$

where $\sum_j \alpha_j = 1$ and $\sum_j \gamma^*_{jk} = \sum_k \gamma^*_{jk} = \sum_j \beta_j = 0$

With substitution for $a(p)$ and $b(p)$ we get budget share equations of the form:

$$s_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(w/P)$$

where $P$ is a "price index":

$$\log P = \alpha_0 + \sum_j \alpha_j \log p_j + \frac{1}{2} \sum_j \sum_k \gamma_{jk} \log p_j \log p_k$$

and

$$\gamma_{ij} = (\gamma^*_{ij} + \gamma^*_{ji})/2 = \gamma_{ji}$$

Often see that the price index will be approximated well by a simpler index of the form:

$$\log P = \sum_j s_j \log p_j$$

- Almost ideal: cannot guarantee expenditure function is concave until you estimate

## Demand Estimation: Characteristics Space

The alternate way to economize on parameters in empirical work is via **characteristics** space: goods are seen as bundles of characteristics and consumers derive utility from these characteristics and some idiosyncratic taste.

**Example**: multinomial logit model

Posit utility $u_{ij}$ of individual $i$ for choice $j$:

$$u_{ij} = X_i \beta_j + Z_j \gamma_i + \epsilon_{ij} = v_{ij} + \epsilon_{ij}$$

- individual characteristics $X_i$ have choice specific effects
- choice characteristics $Z_j$ have individual-specific effects
- $\epsilon_{ij}$: unobserved (to econometrician) component of tastes, randomly distributed across population

# Foundations

- $X$: universe of objects of choice
- $S$: universe of attributes of individuals
- $B$: feasible choice set ($x \in B \subset X$)

Assume a behavior rule $h$ which maps attributes to choices:

$$h(B, S) = x$$

Why is this random?

- Population view: we've drawn a person from the population randomly with attributes $s$
- There are unobserved characteristics behind choices; Luce was in particular interested in unmeasured psychological factors. Found support in experiments where subjects did not make the same choice when same choice set presented twice

Let probably that a randomly drawn individual chooses $x$ be:

$$P(x|S, B) = Pr\{h|h(S, B) = x\}$$

# Luce Axioms

Goal: place some restrictions on $P(x|S, B)$ and derive implications for functional form of $P$

1) No zero probability choices

$$P(y|s, B) > 0 \qquad \forall B$$

2) **Independence of Irrelevant Alternatives** (IIA)

For $x, y \in B$, $s \in S$,

$$\frac{P(x|s, \{xy\})}{P(y|s, \{xy\})} = \frac{P(x|s, B)}{P(y|s, B)}$$

# IIA and the "duplicates problem"

- Debreu anticipated the best known shortcoming of the Luce model, known as the **blue bus/red bus problem**
- There are three options: 1) commute by car, 2) by red bus, 3) by blue bus
- Assume red and blue bus generate same utility, so choice between buses is random. Say, choice probabilities are $(0.50, 0.25, 0.25)$

$$\left.\frac{P(\text{car})}{P(\text{red bus})}\right|_{3\text{ choices}} = \left.\frac{P(\text{car})}{P(\text{red bus})}\right|_{2\text{ choices}}$$

- When we remove blue bus, would expect all to take red bus, leaving to $(0.50, 0.50)$ split, but IIA requires the same relative ratio $(0.66, 0.33)$

- Let
$$P_{xy} = P(x|s,\{xy\})$$
- IIA implies:
$$P(y|s,B) = \frac{P_{yx}}{P_{xy}} P(x|s,B)$$
- Summing over alternatives:
$$\sum_{y \in B} P(y|s,B) = \sum_{y \in B} \frac{P_{yx}}{P_{xy}} P(x|s,B) = 1$$
- Hence,
$$P(x|s,B) = \frac{1}{\sum_{y \in B} \frac{P_{yx}}{P_{xy}}}$$

- Compared to another alternative $z$, we have

$$P(y|s,B) = \frac{P_{yz}}{P_{zy}} P(z|s,B) \qquad P(x|s,B) = \frac{P_{xz}}{P_{zx}} P(z|s,B)$$

and therefore

$$\frac{P_{yx}}{P_{xy}} = \frac{P(y|s,B)}{P(x|s,B)} = \frac{P_{yz}/P_{zy}}{P_{xz}/P_{zx}}$$

- Denote the **Luce value** as

$$\tilde{v}(s,y,z) = \log(P_{yz}/P_{zy}) \qquad \tilde{v}(s,x,z) = \log(P_{xz}/P_{zx})$$

- We have:

$$\frac{P_{yx}}{P_{xy}} = \frac{e^{\tilde{v}(s,y,z)}}{e^{\tilde{v}(s,x,z)}}$$

- Third and last axiom is **separability**:

$$\tilde{v}(s,x,z) = v(s,x) - v(s,z)$$

  - Says that the Luce value can be decomposed relative to some "reference" choice

- Then

$$\begin{aligned} P(x|s,B) &= \frac{1}{\sum_{y\in B} \frac{P_{yx}}{P_{xy}}} \\ &= \frac{1}{\sum_{y\in B} \frac{e^{v(s,y)-v(s,z)}}{e^{v(s,x)-v(s,z)}}} \\ &= \frac{e^{v(s,x)}}{\sum_{y\in B} e^{v(s,y)}} \end{aligned}$$

# Random Utility Models

- Assume utility from alternative $j$ is:

$$u_j = v_{ij} + \epsilon_{ij}$$

  where $v_{ij}$ is non-stochastic

- Let $d_{ij} = 1$: individual $i$ chooses $j$ then for given distribution:

$$P(d_{ij} = 1 | X_i, Z_j) = P(v_{ij} + \epsilon_{ij} > v_{ik} + \epsilon_{ik} \quad \forall k \neq j)$$

  - Only relative utilities matter - can do a location-shift to everything
  - Scale is arbitrary: same choices if $v_{ij} \Rightarrow \lambda v_{ij}$ and $\epsilon_{ij} \Rightarrow \lambda \epsilon_{ij}$
  - $u_{ij}$ represents indirect utility; should depend on income and price

# Quasi-linearity

- Often assume quasi-linear direct utility in terms of numeraire:

$$U^i(n, d_{ij}) = \alpha n + \phi_i(Z_j) + \epsilon_{ij},$$

  where $n$ is the numeraire good
  - Assumes away income effects (EV=CV), can do standard welfare comparisons even though choices discrete (see Rosen and Small EMA 1981)
- With choice $j$, we have:

$$u_{ij} = \alpha(y_i - p_j) + \phi_i(Z_j) + \epsilon_{ij}$$

  equivalent to (location-shift)

$$u_{ij} = -\alpha p_j + \phi_i(Z_j) + \epsilon_{ij}$$

- Hard to justify for large budget share purchases (cars, houses, health)

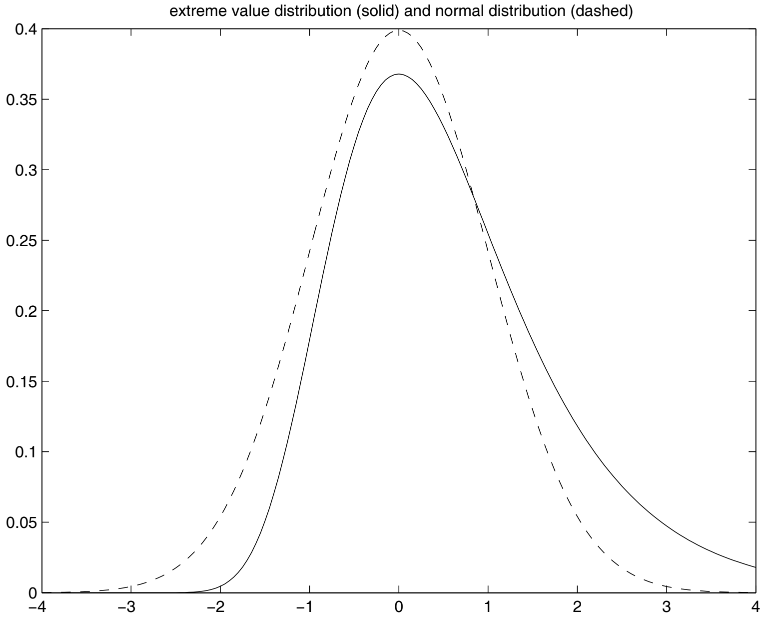
- To evaluate probability $d_{ij} = 1$ we need to solve $J - 1$ dimensional integral
- When $J$ is small, conventional assumption is $\epsilon_{ij}$ is normal, but beyond that need simulation methods to solve integral
- When $J > 3$, often assume error distribution:

  $$F(\epsilon) = e^{-e^{-\epsilon}}$$

  with mode 0, mean of 0.577 (Euler's constant) and variance of $\pi^2/6$

  Known as Extreme Value Type I (or Gumbel) distribution
- Has analytic solution:

  $$P(d_{ij} = 1) = \frac{\exp(v_{ij})}{\sum_{k=1}^{J} \exp(v_{ik})}$$
- McFadden (1973) showed how this distribution allows us to construct a random utility for the Luce model and therefore estimate Luce values as a function of observed characteristics; link to utility maximization




**From: Imbens-Woolridge (2007)**

IIA may help in some situations:

- Choice set is large – as in residential choice; McFadden (1978) proposed that we can randomly select a subset of other choices together with what has been chosen and compute "conditional" likelihood
- Forecasting demand for new product with known characteristics, e.g.,

$$u_{ij} = Z_j(\gamma_0 + \gamma_1 X_i) + \epsilon_{ij}$$

  Since we observe $X_i$ and $Z_j$, when we estimate $\hat{\gamma}$ we can product demand for new product $J+1$

- Can simplify rank data:

$$P(1-2-3) = P(\text{1st} \mid \text{3 total })P(\text{2nd} \mid \text{2 remaining })$$

Some ways to relax IIA

- Consider other distributions
  - Integrate over other distribution (e.g., multinomial probit) but computationally costly when $J$ is large
  - Generalizations of logit with closed-forms: nested logit and other generalized extreme value (GEV) distributions
- Mixed logit: each consumer type follows the MNL model, but the population is a mixture of various types (mixture model)
  - Often need to use simulation or Bayesian methods to estimate, see Train (2003)
  - Key references: Hausman and Wise (1978) and Berry, Levinsohn, and Pakes (1995)

# 7. Demand Aggregation

The formal theory that we have developed has been about a single consumer maximizing utility.

An important question is whether the theorems are useful for talking about aggregate demand and welfare.

Macroeconomists want to think of demand as derived from a "representative consumer."

Microeconomists want to estimate aggregate demands for goods and use Marshallian-Hicksian relations to talk about welfare.

We will consider three questions:

1) Does it make sense to estimate aggregate demand as a function of $p$ and aggregate $w$, or do we need to deal with wealth distributions?

2) Does aggregate demand (if defined) satisfy the same relations as individual demand? ("positive representative consumer")

3) Do welfare measures derived from aggregate demand have meaning? ("normative representative consumer")

## Aggregate demand - can we use $X(p, w)$?

Setup: $K$ consumers with Marshallian demand $x^k(p, w^k)$.

Aggregate demand: $X(p, w^1, ..., w^K) = \sum_{k=1}^{K} x^k(p, w^k)$.

Two classic cases:

1) Consider identical homothetic preferences:

$$x^k(p, w^k) = w^k \beta(p)$$

2) More generally, if individual preferences are homothetic, but not identical and incomes are proportional, then there exists a single preference ordering that generates aggregate demand (Chipman 1974).

Question: when can we write $X(p, w^1, ..., w^K) = X(p, \sum_k w^k)$?

Answer: Need for any $(w^k)$ and $(w'^k)$ such that $\sum_k w^k = \sum_k w'^k$, $X(p, \sum_k w^k) = X(p, \sum_k w'^k)$.

### Proposition

*Demand is a function of aggregate wealth only if*

$$\frac{\partial x_i^k}{\partial w^k}(p, w^k) = \frac{\partial x_i^j}{\partial w^j}(p, w^j), \quad \forall \textit{ goods } i, \textit{individuals } (j, k)$$

Strong requirement: requires that all wealth effects are the same across individuals and across wealth levels.

Since quasi-homothetic preferences give demands as:

$$x^k(p, w^k) = \alpha^k(p) + w^k \beta(p)$$

Here, we have person-specific $\alpha$, but identical $\beta$

Aggregate demand is:

$$X(p, w) = \sum_k x^k(p, w^k) = \alpha(p) + w \cdot \beta(p)$$

where $\alpha = \sum_k \alpha^k$ and $w = \sum_k w^k$

Aggregate demand is quasi-homothetic

### Proposition

*Demand is a function of aggregate wealth $\Leftrightarrow$ the preferences are represented by indirect utility functions of the form:*

$$v^k(p, w^k) = a^k(p) + b(p)w^k$$

Note: indirect utility has equal wealth coefficients $b(p)$

See Deaton Muelbauer (1980) for proof

Cases where satisfied:

- Preferences which are quasi-linear with respect to same good
- Identical (quasi)-homothetic preferences

## Does aggregate demand corresponds to some "representative" consumer?

No: not true even though these individuals have utility functions Consider two individuals and two goods, coffee and cookies, and both have $w = 4$. Agent 1 likes coffee and agent 2 likes cookies, but is fairly satiated at 2 units.

$$p = (1,2) : x^1 = (2,1), \quad x^2 = (0,2), \quad x^{agg} = (2,3).$$
$$\hat{p} = (2,1) : \hat{x}^1 = (2,0), \quad \hat{x}^2 = (1,2), \quad \hat{x}^{agg} = (3,2).$$

Bundle $(3,2)$ is strictly more affordable at prices $(1,2)$ and $(2,3)$ chosen

$$(2,3) R^D (3,2).$$

Bundle $(2,3)$ affordable at prices $(2,1)$

$$(3,2) R^D (2,3).$$

Contradicts GARP.

Hard to write conditions where GARP holds for aggregate demand

## Aggregate demand and welfare

Suppose we have some social welfare function $W(u^1, ..., u^K)$ which represents society's welfare for a particular collection of individual utilities.

Can we treat problems involving the maximization of $W(u^1, ..., u^K)$ as if we were maximizing $\sum_k u^k(x^k)$?

Not in general. Problem is that $W$ is may depend on the scale of the utility and would imply different selections of $x^{agg}$, while the $u^k$ is immune to monotonic transformations.

Consider two societies with utility profile $(u_1, u_2, ..., u_K)$ and $(\tilde{u}_1, u_2, ..., u_K)$, where $\tilde{u}_1 = f(u_1)$ for some monotonic $f$.

Taking monotonic transformations does not affect $x^k(p, w)$, so it should not affect $\sum_k x^k(p, w)$.

But preferences of the representative consumer depend on the form of $W$.

Gorman form is a special case where we can do aggregation:

$$v^k(p, w^k) = a^k(p) + b(p)w^k.$$

In this case, aggregate demand is independent of the wealth distribution and maximizes:

$$V(p, w) = \sum_k a^k(p) + b(p)w$$

Hence, the "representative consumer's" utility is the **utilitarian** social welfare function $W(u^1, ..., u^K) = \sum u^k$.

Intuition: Suppose there exists a good, "leisure," with constant marginal utility for everyone. Then regardless of wealth distribution each consumer consumes other goods until marginal utility declines to $b$ and puts the rest in leisure.

"Representative consumer" behaves like this: consumes each good until he consumes the sum of individual demands, and puts rest in leisure.