

1.11.23

Module IIITRANSPORTATION PROBLEM

Transportation problem is a special class of allocation problem in which the objective is to transport various quantities of a single homogenous commodity that are kept at different origins to a number of destinations such as shops, markets etc.

Let c_{ij} be the cost of transporting 1 unit of commodity from m origins $O_1, O_2, O_3, \dots, O_m$ to the different shops / destinations d_1, d_2, \dots, d_n . Then the cost matrix of a TP can be expressed as a table.

	D_1	D_2	D_3	.	.	D_n	Available
O_1	c_{11}	c_{12}	c_{13}	.	.	c_{1n}	a_1
O_2	c_{21}	c_{22}	c_{23}	.	.	c_{2n}	a_2
O_m	c_{m1}	c_{m2}	c_{m3}	.	.	c_{mn}	a_m

to

where a_i 's a_i 's are the quantity of resource available in each of the origins $O_i = 1, 2, 3, \dots, m$.

and b_j 's are the requirement, demand of the shops.

Let x_{ij} be the no. of unit of commodity transported from the i^{th} origin to j^{th} destination. Then the mathematical form~~to~~ of TP may be expressed as

$$\text{min } Z = \sum_i \sum_j c_{ij} x_{ij}$$

st

$$\sum_{j=1}^n x_{ij} = a_i, \quad i = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^m a_{ij} = \sum_{j=1}^n b_j, a_{ij} \geq 0 \quad x_{ij} \geq 0$$

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- Balanced and Unbalanced Transportation Problem
A TP is said to be balanced if the demand of the shops = sum of the resources.
A feasible solution of a TP is a set of non-negative individual allocation which satisfies the row & column sum restrictions.

- Basic feasible solutions :-

A feasible solution of an $m \times n$ TP is said to be basic feasible soln if the total no. of non-zero allocations = $m+n-1$.

- Optimal solution of a TP :-

A feasible soln of an $m \times n$ TP is said to be optimal if it minimizes the total transportation cost.

- Non degenerate basic feasible solution :-

A feasible solution of an $m \times n$ TP is said to be non-degenerate basic feasible solution if the no. of non-zero allocations is exactly equal to $m+n-1$.

These allocations are at independent positions:

A set of allocations of a TP are said to be at independent positions if it is not possible to form a polygon joining occupied cells.

- Methods of finding IBFS :-

3 methods :-

- i) North-West corner rule
- ii) Matrix minimum / minimum cost method
- iii) VAM (Vogel's approximation method)

- Q. Find an initial feasible soln to the following TP using

- a) North-West corner rule
- b) Matrix - minimum
- c) Vogel's approximation method

A) a) North-West Corner Rule :-

	D ₁	D ₂	D ₃	Availability	no. of allocations
O ₁	5 2	2	4	5	0
O ₂	2 3	6 3	1	8	6
O ₃	5 3	4 4	7	X	0
O ₄	1 0	6 14	2	14	X
				18	
	X	X			
			X	3	14

North-West corner rule :-

Allocate the min. of 5 and 2 to the (1, 1) cell at the north-west corner of the table. Then, the resource of polygon origin O₁ is fully utilized. Then strike off the 1st row. Then allocate the min of (8, 7-5) to the cell (2, 1) which is at the n-w corner position of P₁ is fully satisfied. Strike off the first column. Continue this procedure until all demands are satisfied.

utilized.

b) Matrix - min method :-

	D ₁	D ₂	D ₃	availability
O ₁	5	2	4	5 0
O ₂	2	3	8	8 0
O ₃	3	4	1	4 0
O ₄	5	4	7	14 0
demand	7	8	18	
	0	1	10	
			0	

$$\text{cost} = 5 \times 7 + 8 \times 1 + 4 \times 4 + 3 \times 7 + 4 \times 1 + 7 \times 2 \\ = 76$$

c) VAM method :-

	D ₁	D ₂	D ₃	
O ₁	5	2	4	difference of min. of lowest & next lowest
O ₂	2	3	8	5(0) - 2(2)
O ₃	3	4	1	8(2) - 3(1)
O ₄	5	4	7	7(1) - 5(0)
	7(1)	8(1)	18(1)	
	7(1)	8(2)	10(2)	
	7(4)	8(2)	10(5)	
	7(4)	8(2)	10(5)	

- Q. Find an initial feasible solution for the following TP using north-west corner rule & then use modified distribution method to find the optimal solution, which will minimize the total cost of transportation.

	w ₁	w ₂	w ₃	w ₄	w ₅	Available res.
factory	20	2	4	6	8	9
	20	6	4			
	2	10	1	5	8	
	1	11	20	40	3	
F ₁	20	2	4	6	8	9
F ₂	20	6	4	5	8	
F ₃	2	10	1	5	8	
F ₄	1	11	20	40	3	
-	2	1	9	14	6	16
demand	40	6	8	18	6	
	20	0	4	10	0	
1st	0	0	0	0	0	

check total demand = total availability
 $40+6+8+18+6 = 20+30+15+13$
 $78 = 78$

(if shortage, add row or column)

check
 \rightarrow no. of non-zero allocation is $m+n-1 = 8 \Rightarrow 4+5-1 = 8$

\therefore no. of non-zero allocation is $= m+n-1$

MODI method :-

Find no. of

$u_1 = 0$	20	-6	5	13	14
	2	4	6	8	9
$u_2 = 0$	20	6	4	16	18
	2	10	1	5	8
$u_3 = 19$	4	18	4	11	3
	1	11	20	40	3
$u_4 = -7$	1	2	9	15	14
	2	1	9	14	16

$\rightarrow m+n-1$ variables are introduced
 $\rightarrow u_i + v_j = c_{ij}$

$\rightarrow m+n-1$ no. of equations

$$\begin{aligned} \rightarrow u_i + v_j &= c_{ij} \\ u_1 + v_1 &= 2 \\ u_2 + v_2 &= 2 \\ u_2 + v_3 &= 10 \\ u_2 + v_4 &= 1 \\ u_3 + v_3 &= 20 \\ u_3 + v_4 &= 40 \\ u_4 + v_4 &= 14 \\ u_4 + v_5 &= 16 \end{aligned}$$

assign values to u_i 's & v_j 's.
 (assume putting $u_1 = 0$)

cell evaluation / net evaluation

corresponding to each unoccupied cell

$$\text{calculate } -d_{ij} = c_{ij} - (u_i + v_j)$$

$$= 4 - (0 + 10) = -6$$

highest ~~neg~~ ve in d_{ij} , give max. allocation

- construct polygon starting from highest $-d_{ij}$ with whose corners are on the basic/unoccupied cell.

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- alternate cell's allocation along the polygon &
 - find min. $\min\{6, 11\} = 6$
 skip other allocations

- allocate that min. of to the cell that have the highest negative d_{ij} value.

$$V_1 = 2 \quad V_2 = 10 \quad V_3 = 1 \quad V_4 = 21 \quad V_5 = -16$$

$u_1 = 0$	20	6	5	13	14
	2	10	1	5	8
$u_2 = 0$	7	11	20	40	3
	1	1	20	40	3
$u_3 = 19$	4	2	5	13	14
	2	1	9	14	16
$u_4 = -7$	2	1	9	14	16

$$\min\{6, 11\} = 6$$

$$V_1 = 2 \quad V_2 = 10 \quad V_3 = 1 \quad V_4 = 21 \quad V_5 = -16$$

$u_1 = 0$	20	6	5	13	7
	2	10	1	5	8
$u_2 = 0$	7	11	20	40	3
	1	1	20	40	3
$u_3 = 1$	4	2	5	13	14
	2	1	9	14	16
$u_4 = -7$	2	1	9	14	16

$$\min\{5, 2\} = 2$$

corners only

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Q Solve the following transportation problem with the following post-matrix

	w ₁	w ₂	w ₃	w ₄	availability
P ₁	190	300	500	100	70
P ₂	700	300	400	600	90
P ₃	400	100	600	200	180
demand	50	80	70	140	

Using Vogel's method to find initial feasible soln.

-A

	w ₁	w ₂	w ₃	w ₄	
P ₁	190	300	500	100	70(90) 20(200) 30(100)
P ₂	700	300	400	600	90(100) 90(100) 90(200)
P ₃	400	100	600	200	180(100) 120(100) 100(400)
	50(210)	80(209)	70(209)	140(209)	

- = highest diff. - 80(200) = 20(100) 140(100)
- = lowest cost - 80(200) = 20(100) 140(100)
- = availability > demand min - 70(100) 120(400)
- = 110 - 70(200) 120(400)
- = 70(100) 20(600)
- = 70(100) -

$$\text{cost} = 90 \times 190 + 2000 + 28000 + 12000 + 8000 + 20000$$

9500

$$= 43,700$$

=====

$$= 49,500$$

Now, we use ~~modified~~ modified distribution method to find the optimum solution.

$$V_1 = 190 V_2 = 0 V_3 = 100 V_4 = 100$$

50	300	600 20		70
U ₁ = 0	190	300	500 100	90
U ₂ = 100	700 20	300 100	400 600	
U ₃ = 100	400 100	600 100	200	180

50 80 70 140

⇒ Unoccupied cells

$$d_{12} = 300 - 0 = 300$$

Some $d_{ij} < 0$, so the current soln is not optimum.

$$\min(80, 20) = 20$$

Allocate a ∞ quantity which is equal to the min. of those allocations in the alternate occupied cell connected by the polygon.

$$V_1 = 190 V_2 = 0 V_3 = 100 V_4 = 100$$

50	300	600 20		70
U ₁ = 0	190	300	500 100	90
U ₂ = 100	700 20	300 100	400 600	
U ₃ = 100	400 100	600 100	200	180

50 80 70 140

This is optimum.

$d_{ij} = c_{ij} - (u_i + v_j) \geq 0$
since d_{ij} is greater than 0, it gives the optimum solution.

$$\begin{aligned}\text{The min. cost of transportation} &= \\ &= 50 \times 190 + 40 \times 100 + 20 \times 300 + 40 \times 400 + \\ &\quad 60 \times 100 + 120 \times 200 \\ &= 775500\end{aligned}$$

Q) Solve the following TP to max. profit availability

15	51	42	33	25	
80	42	26	81	45	
90	40	66	60	35	
23	31	16	30	100	

The following table giving profit when 1 unit of commodity is transported from origin to destination.

A) Here there is excess resource of 5 units, so we introduce a dummy shop label $\begin{cases} \sum \text{availability} = 105 \\ \sum \text{demand} = 100 \end{cases}$



while adding profit column or row. $\begin{cases} \text{cost} \leq \text{min cost} \rightarrow 0 \\ \text{profit} \geq \text{max profit} \end{cases}$

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Since all d_{ij} are non-negative, this given table

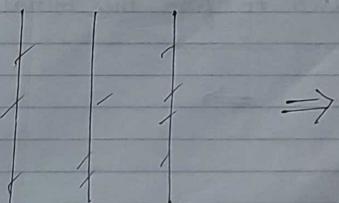
$\min \text{ cost} \rightarrow \min \text{ col/row}$

To min to w_3 , from

$$\begin{aligned}\text{The min. cost of transportation} &= 50 \times 190 + 20 \times 100 + \\ &\quad 20 \times 300 + 40 \times 400 + \\ &\quad 60 \times 100 + 120 \times 200 \\ &= 75500\end{aligned}$$

16	51	42	33	90	25
80	42	26	81	90	45
90	40	66	60	90	35
23	31	16	30	5	

Remove Replace each cell by highest profit minus each cell value.



79	39	48	57	0
10	48	64	91	0
0	50	24	30	0

when we are given an unbalanced TP, to make it balanced, we will be giving a dummy row when demand is higher than availability. (Demand is lesser than availability) with 0 cost in each cell if the given problem is minimising the cost.

If the given TP is of maxing profit, we will give the max. profit to adding dummy row or col.

MODI method is used to minimize the total cost, so when we are given a TP in which our aim is to maximize the profit, we will convert the given TP into 1 in which our aim is to min. the post, for this we will replace the each cell entry by the highest profit minus the corresponding cell entry.

To solve the following problem, Vogon's method is used to find the initial solution.



20(9)	16(39)	15(16)	-	-	25(31)	20(9)	20(9)	20(9)
10	48	64	91	0	23(10)	31(9)	16(24)	30(21)
0	50	24	30	0	23(10)	31(9)	16(24)	30(21)
					-	31(9)	16(24)	30(21)
					-	31(9)	16(24)	-
					-	31(9)	4(24)	-
					-	31(9)	-	-

16	4	5	25
15	51	42	33
80	42	26	90
23	12	81	90
90	40	66	60
23	31	16	30
			5

Assignment problem :-

It is a special case of TP in which the objective is to assign a no. of origins (person, machines etc) to an equal no. of jobs/tasks in such a way that the total cost of assignment is min (total profit max).
 Eg: suppose there are 4 persons in a dept available for doing 4 different jobs,

Now our objective is to assign 1 job to 1 person in such a way that total cost/time is min. Let c_{ij} be the cost if the i th person is assigned to j th job. Then the cost matrix can be represented as

	J_1	J_2	\dots	J_n	
P_1	c_{11}	c_{12}	\dots	\dots	\vdots
P_2	c_{21}	c_{22}	\dots	\dots	\vdots
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
P_n	\vdots	\vdots	\vdots	\vdots	\vdots
	1	1	1	...	1

Mathematical formulation of assignment problem :-

Let x_{ij} represents the no. of persons assigned from i th origin to j th destination. Then the mathematical formulation of given problem is

$$\min z = \sum_{j=1}^n \sum_{i=1}^n c_{ij} x_{ij}$$

$$\text{st } \sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, 3, \dots, n$$

$$\sum_{i=1}^n x_{ij} = 1, \quad j = 1, 2, 3, \dots, n$$

with $x_{ij} = 0 \text{ or } 1$

→ Method for solving AP :- Hungarian method

Q. Solve the following minimal AP

	J_1	J_2	J_3	J_4
P_1	12	30	21	15
P_2	18	33	9	31
P_3	44	25	21	21
P_4	14	30	28	14

-A-

- find the min. of each row & subtract it with from the cell value in each row.

0	18	9	3
9	24	0	22
23	4	0	0
0	16	14	0

Find the min. of each col & subtract this quantity min. from the cell of the respective cells cols.

0	14	9	3
9	20	0	22
23	0	X	X
0	12	14	0

→ Identify the row ~~having~~ having exactly one 0 (start with top row)
Then mark in square (~~with~~) we give an assignment
Next, each col & strike the 0's of that column

- Subtract row min with each cell value.

* The optimum assignment is ~~assign~~

$P_1 \rightarrow J_1, P_2 \rightarrow J_2, P_3 \rightarrow J_3, P_4 \rightarrow J_4$
Minimum cost of assignment = $12 + 9 + 25 + 14 = 60$

Q. Solve the following AP involving 5 jobs & 5 persons

	J_1	J_2	J_3	J_4	J_5
P_1	1	3	2	3	6
P_2	2	4	3	1	5
P_3	5	6	3	4	6
P_4	3	1	4	2	2
P_5	1	6	6	5	4

Form a table, by replacing each

Find the min. of each row & subtract with this min. with from the cells of respective rows.

0	2	1	2	5
1	3	2	0	4
2	3	0	1	3
2	0	3	1	1
0	4	5	4	3

↓ ↓

10	2	1	2	4
1	3	2	0	3
2	3	0	1	2
2	0	3	1	*
*	4	5	4	2

Using minimal no. of horizontal & vertical lines, cover all 0's

no. of lines = no. of either rows \Rightarrow optimum soln.

Find the min. of those entries over which no lines passes. Subtract this min from these entries & add this min. to those cells where 2 lines cross & leave the remaining entries as such

Find the min. of those entries over which no lines passes. Subtract this min from these entries & add it to the entries which have cross intersection of lines & leave the remaining entries as such

⊗	1	⊗	1	3
2	3	2	0	3
3	3	0	1	2
3	0	3	1	⊗
0	3	5	4	2

Using min. no. of horizontal & vertical lines, cover all zeroes. If the no. of lines = no. of tot rows, then soln is optimum. Hence, the above table is not optimum.

- * find the min. of those entries over which no lines passes.
- * subtract this min. from those entries.
- * add this min. to those cells where 2 lines cross and leave the remaining entries as such.

no line \rightarrow sub(min)
 line passing cells \rightarrow no change
 intersection \rightarrow add (min)

\times	1	\times	1	3	✓
2	3	2	\square	3	✓
3	3	\square	1	2	✓
2	\square	3	1	\times	✓
0	3	4	3	1	✓
✓	✓	✓	✓	✓	

using minimal
no. of lines,
cover all zeros

14/11/23 Rule to find the min. no. of lines

- 1) mark all rows for which there is no assignment.
- 2) mark cols which has ~~zero's~~ assignments in marked rows.
- 3) mark the rows which has assignment in marked ~~zero's~~ cols.

4) Draw lines through unmarked rows and marked cols.

0	\times	\times	\times	2
3	3	3	\square	3
3	2	\square	\times	1
4	\square	4	1	\times
0	2	4	2	\square

5 zeros = 5 rows \Rightarrow optimum

min. cost of assignment = $1 + 1 + 3 + 1 + 4$
 $= 9$

Q: The following table gives profits for different jobs done by diff. machines. Find the max. profit of assignment.

	M ₁	M ₂	M ₃	M ₄	M ₅
J ₁	30	37	40	48	40
J ₂	40	24	27	21	36
J ₃	40	32	33	30	35
J ₄	25	38	40	36	36
J ₅	29	62	41	34	39

The given problem is to max. the profit. We replace each cell ~~entire entry~~ by $60 - \text{the respective cell entry}$ (60 is the max. of all entries).

32	25	22	34	22
22	38	35	41	26
22	30	29	32	27
37	24	22	26	26
33	0	21	28	23

for each row
Find the row matrix minima. - subtract this min. from the elements of respective rows.
Form the new table.

10	3	0	12	0
0	16	13	19	4
0	8	7	10	5
15	2	0	4	4
33	0	21	28	23

Find col minima.

Subtract this minimum. from the elements of respective col. Form the new table.

10	3	0	8	*
0	16	13	15	4
*	8	7	6	5
5	2	*	0	4
33	0	21	24	23

✓② Cover all zeros by min. no. of horizontal & vertical lines.

Mark rows without assignment.
Draw lines through unmarked rows & marked cols.

- subtract min. of uncovered elements to this elements

③

14	3	0	8	*
*	12	9	11	0
0	4	3	2	1
19	2	*	0	4
34	0	21	24	23

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HW Q: A company has 4 machines to do 3 jobs. Each job can be assigned to one & only one machine. The following table gives the cost of each job on each machine. Determine an optimal assignment which will minimise the cost.

	1	2	3	4
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22

→ This AP is unbalanced. we introduce 1 dummy job machine D with 0 cost in the respective cells.

	1	2	3	4
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22
D	0	0	0	0

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MODULE 4

Network Analysis

A project is composed of a no. of jobs and activities or tasks that are related to each other. And all of these should be completed in order to finish the project. There are 2 prominent methods to do this

- 1) Critical path method (CPM)
- 2) Pert PERT (Project Evaluation Review Technique)

Q: Assume that a statue is to be erected in a village square on a stone base which is to be built in an cement concrete foundation. The statue is to be made on another place, then move to the base and install. The various operations of the entire project are below in a random order.

- A - make statue,
- B - lift the statue into place
- C - construct concrete foundation,
- D - compact & level the site
- E - move statue to village square,
- F - construct stone base.

Construct network diagram of the project

