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Module - 2

Duality

Associated with every LPP, there is always a corresponding LPP called the dual problem of the given LPP.

The original (given) LPP is ~~also~~ called the primal problem. However, if we state the dual problem as the primal one, then the other can be considered to be the dual of this primal.

The 2 problems can thus be said to constitute a pair of dual problems and the 2 can be derived from eachother.

While solving a given LPP by simplex method, we shall simultaneously be solving its associated dual problem as well.

Formation of primal-dual pairs

There are 2 important forms of primal-dual pairs namely symmetric form and unsymmetric form.

Symmetric form

Primal ~~Problem~~ problem

An LPP of $x^T \in \mathbb{R}^n$, maximize $z = cx$, $c \in \mathbb{R}^n$ subject to the constraints $Ax \leq b$, $x \geq 0$

$b^T \in \mathbb{R}^m$

A is an mxn matrix (real)

is called primal problem

Note: It is not necessary that the above ~~general~~ LPP be always taken as the primal problem. We can also take the minimization LPP as our primal problem.

Dual Problem

An LPP of $w^T \in \mathbb{R}^m$, minimize $z^* = b^T w$, $b^T \in \mathbb{R}^m$ subject to the constraints $A^T w \geq c^T$, $w \geq 0$,

$c \in \mathbb{R}^n$

A^T is an $n \times m$ matrix (real)

is called the Dual problem.

The variables are called dual variables and the constraints of this dual problem are called dual constraints.

Unsymmetric form

Primal problem

An LPP of $x^T \in \mathbb{R}^n$, maximize $z = cx$, $c \in \mathbb{R}^n$
subject to the constraints $Ax = b$, $x \geq 0$, $b^T \in \mathbb{R}^m$
A is an $n \times m$ matrix (real)
is called primal problem

Dual Problem

An LPP of $w^T \in \mathbb{R}^m$, minimize $z^* = b^T w$, $b^T \in \mathbb{R}^m$
subject to the constraints $A^T w \geq c^T$, $c \in \mathbb{R}^n$
w is unrestricted in sign

Qn. Obtain the dual problem of the following LPP

Maximize $z = 2x_1 + 5x_2 + 6x_3$ subject to the constraints

$$5x_1 + 6x_2 - x_3 \leq 3$$
$$-2x_1 + x_2 + 4x_3 \leq 4$$
$$x_1 - 5x_2 + 3x_3 \leq 1$$
$$-3x_1 - 3x_2 + 7x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

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$$\text{Min } z^* = 3w_1 + 4w_2 + w_3 + 6w_4$$

subject to

$$5w_1 - 2w_2 + w_3 - 3w_4 \geq 2$$
$$6w_1 + w_2 - 5w_3 - 3w_4 \geq 5$$
$$-w_1 + 4w_2 + 3w_3 + 7w_4 \geq 6$$

$$w_1, w_2, w_3, w_4 \geq 0$$

Qn. Write the dual of Minimize $z = 4x_1 + 6x_2 + 18x_3$

subject to

$$x_1 + 3x_2 \geq 3$$
$$x_2 + 2x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0$$

$$\text{Max } z^* = 3w_1 + 5w_2$$

subject to,

$$w_1 \leq 4$$
$$3w_1 + w_2 \leq 6$$
$$2w_2 \leq 18$$

$$w_1, w_2 \geq 0$$

Qn. $\text{Max } z = x_1 - 2x_2 + 3x_3$

subject to,

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$x_1, x_2, x_3 \geq 0.$$

$$\text{Min } z^* = 2w_1 + w_2$$

subject to

$$-2w_1 + 2w_2 \geq 1$$
$$w_1 + 3w_2 \geq -2$$
$$3w_1 + 4w_2 \geq 3$$

w_1, w_2 unrestricted.

Qn. Min $z = x_1 - 3x_2 - 2x_3$

subject to, $3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 - 4x_2 \geq 12$

$-4x_1 + 3x_2 + 18x_3 = 10$

$x_1, x_2 \geq 0$, x_3 unrestricted.

ans: Convert to canonical form $-3x_1 + x_2 - 2x_3 \geq -7$

$2x_1 - 4x_2 \geq 12$

$-4x_1 + 3x_2 + 18x_3 = 10$

Max $z^* = -7w_1 + 12w_2 + 10w_3$

subject to, $-3w_1 + 2w_2 - 4w_3 \leq 1$

$w_1 - 4w_2 + 3w_3 \leq -3$

$-2w_1 + 18w_3 = -2$

$w_1, w_2 \geq 0$, w_3 unrestricted.

Qn. Max $z = 2x_1 + x_2$

subject to, $x_1 + 5x_2 \leq 10$

$x_1 + 3x_2 \geq 6$

$2x_1 + 2x_2 \leq 8$

$x_2 \geq 0$, x_1 unrestricted.

ans: Canonical form $x_1 + 5x_2 \leq 10$

$-x_1 - 3x_2 \leq -6$

$2x_1 + 2x_2 \leq 8$

Ans. Max $z = 6x_1 + 6x_2 + x_3 + 7x_4 + 5x_5$

subject to, $3x_1 + 7x_2 + 8x_3 + 5x_4 + x_5 = 2$

$2x_1 + x_2 + 3x_4 + 9x_5 = 6$

$x_1, x_2, x_3, x_4 \geq 0$, x_5 unrestricted.

Rule for Primal-dual conversion.

Primal	Dual
1) Objective function	
maximize	→ minimize
minimize	→ maximize
2) No. of Primal constraints	No. of primal variables.
3) No. of primal variables	No. of constraints
4) Primal variable unrestricted in sign	Corresponding constraint is an equation
5) Max $z = CX$, $AX \leq b$ & $X \geq 0$	Min $z^* = b^T w$, $A^T w \geq C^T$, $w \geq 0$

Also

Note: infeasible solution can be identified from an iteration, if all the net evaluations are non-negative and there is an artificial variable in the basis (not at the zero level).

Qn. Find the maximum of $z = 6x_1 + 8x_2$

subject to the constraints $5x_1 + 2x_2 \leq 20$

$$x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0$$

by solving its dual problem.

$$\min z^* = 20w_1 + 10w_2$$

subject to $5w_1 + w_2 \geq 6$

$$2w_1 + 2w_2 \geq 8$$

$$w_1, w_2 \geq 0$$

Standardization

$$\max z^* = (-z^*) = z' = -20w_1 - 10w_2 - 0S_1 - 0S_2 - MA_1 - MA_2$$

$$5w_1 + w_2 - S_1 + A_1 = 6$$

$$2w_1 + 2w_2 - S_2 + A_2 = 8$$

$$w_1, w_2, S_1, S_2, A_1, A_2 \geq 0$$

IBFS

$m = 2, \; n = 6$

$w_1 = w_2 = S_1 = S_2 = 0$

$A_1 = 6$

$A_2 = 8$

C_B	BV	X_B	y_1 (-20) w_1	y_2 (-10) w_2	y_3 (0) S_1	y_4 (0) S_2	y_5 ($-M$) A_1	y_6 ($-M$) A_2	
$-M$	A_1	6	5	1	-1	0	1	0	
$-M$	A_2	8	2	2	0	-1	0	1	
$z' = -14M$			$-7M + 20$	$-3M + 10$	M	M	0	0	
-20	w_1	$6/5$	1	$1/5$	$-1/5$	0	$1/5$	0	$R_1/5$
$-M$	A_2	$28/5$	0	$8/5$	$2/5$	-1	$-2/5$	1	$R_2 - 2R_1$
$z' = -\frac{28}{5}M - 24$			0	$-\frac{8}{5}M + 6$	$-\frac{2}{5}M + 4$	M			
-20	w_1	$1/2$	1	0	$-1/4$	$1/8$	$1/4$	$-1/8$	$R_1 - \frac{R_2}{5}$
-10	w_2	$7/2$	0	1	$1/4$	$-5/8$	$-1/4$	$5/8$	$\frac{5}{8}R_2$
$z' = -45$			0	0	$5/2$	$30/8$	X	X	

$w_1 = 1/2$

$w_2 = 7/2$

Side work: $\frac{28}{5} \times \frac{5}{8}$, $2 - \frac{2}{5}$, $-10 - \frac{35}{2}$, $8 - 2 \times \frac{6}{5}$, $-1 - \frac{1}{4}$, $-4 - \frac{8}{5}M + 10$, $\frac{40 - 12}{5}$, $\frac{2}{5} \times \frac{5}{8}$, 28, $6 - \frac{1}{2}$, $\frac{12 - 1}{2}$, $6, \frac{28}{8} = \frac{7}{2}$, $1 + \frac{1}{4}$, $-\frac{2}{5} \times \frac{5}{8}$, $20 \times \frac{6}{8}$, $24 -$, $4 - \frac{2M}{5}$

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Use duality to solve the following LPP.

Maximize $z = 2x_1 + x_2$

subject to the constraints
$$x_1 + 2x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$x_1 - 2x_2 \le 1$$
$$x_1, x_2 \ge 0$$

Min

$z^* = 10w_1 + 6w_2 + 2w_3 + w_4$

Subject to
$$w_1 + w_2 + w_3 + w_4 \ge 2$$
$$2w_1 + w_2 - w_3 - 2w_4 \ge 1$$
$$w_1, w_2, w_3, w_4 \ge 0$$

Standardization

$\max(-z^*) = -10w_1 - 6w_2 - 2w_3 - w_4 + 0s_1 + 0s_2 - MA_1 - MA_2$

subject to
$$w_1 + w_2 + w_3 + w_4 - s_1 + A_1 = 2$$
$$2w_1 + w_2 - w_3 - 2w_4 - s_2 + A_2 = 1$$
$$\cancel{w_1, w_2, w_3, w_4 \ge 0}$$
$$w_1, w_2, w_3, w_4, s_1, s_2, A_1, A_2 \ge 0$$

IBFS

$n = 8 \quad m = 2 . \quad n - m = 6$

$w_1 = w_2 = w_3 = w_4 = s_1 = s_2 = 0$

$A_1 = 2$

$A_2 = 1$

C_B	BV	X_B	-10 y_1 w_1	-6 y_2 w_2	-2 y_3 w_3	-1 y_4 w_4	0 y_5 s_1	0 y_6 s_2	$-M$ y_7 A_1	$-M$ y_8 A_2	
$-M$	A_1	2	1	1	1	1	-1	0	1	0	
$-M$	A_2	1	2	1	-1	-2	0	-1	0	1	
$z' = -3M$			$-3M+10$	$-2M+6$	2	$M+1$	M	M	0	0	
$-M$	A_1	$+3/2$	0	$+1/2$	$+3/2$	$+2$	-1	$+1/2$	$+1$	$-1/2$	$R_1 - R_2$
-10	w_1	$1/2$	1	$1/2$	$-1/2$	-1	0	$-1/2$	0	$1/2$	
$z' = \frac{-3M}{2} - 5$			0	$\frac{-M}{2}+1$	$\frac{-3M}{2}+3$	$-2M+11$	M	$\frac{-M}{2}+5$	0	X	
-1	w_4	$3/4$	0	$1/4$	$3/4$	1	$-1/2$	$1/4$	$1/2$	$-1/4$	
-10	w_1	$5/4$	1	$3/4$	$1/4$	0	$-1/2$	$-1/4$	X	X	$R_2 + R_1$
$z' = -53/4$			0	$-7/4$	$-5/4$	0	$11/2$	$9/4$	X	X	
-1	w_4	$1/3$	$-1/3$	0	$2/3$	1	$-1/3$	$1/3$	X	X	$R_1 - \frac{1}{4}R_2$
-6	w_2	$5/3$	$4/3$	1	$1/3$	0	$-2/3$	$-1/3$	X	X	$\frac{4}{3}R_2$
$z' = -31/3$			$7/3$	0	$-2/3$	0	$13/3$	$5/3$	X	X	
-2	w_3	$1/2$	$-1/2$	0	1	$3/2$	$-1/2$	$1/2$	X	X	$\frac{3}{2}R_1$
-6	w_2	$3/2$	$3/2$	1	0	$-1/2$	$-1/2$	$-1/2$			$R_2 \to R_2 - \frac{1}{3}R_1$
$z = -10$			2	0	0	1	4	2			

$w_1 = 0,\ w_3 = 1/2,\ w_2 = 3/2,\ w_4 = 0$

$x_1 = 4,\ x_2 = 2$

$\max(-z^*) = -10$

$\min z^* = 10$

Qn. Use principle of duality to solve

Minimize $z = 15x_1 + 10x_2$ subject to the constraints

$3x_1 + 5x_2 \geq 5$

$5x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0$

Dual

Max $z^* = 5w_1 + 3w_2$

$3w_1 + 5w_2 \leq 15$

$5w_1 + 2w_2 \leq 10$

Standardization

Max $z^* = 5w_1 + 3w_2 + 0s_1 + 0s_2$

subject to $3w_1 + 5w_2 + s_1 = 15$

$5w_1 + 2w_2 + s_2 = 10$

$w_1, w_2, s_1, s_2 \geq 0$

IBFS

$n = 4 \quad m = 2 \quad n - m = 2$

$w_1 = w_2 = 0$

$s_1 = 15, \; s_2 = 10$

C_B	BV	X_B	y_1 (w_1), $c=5$	y_2 (w_2), $c=3$	y_3 (s_1), $c=0$	y_4 (s_2), $c=0$	
0	s_1	15	3	5	1	0	
0	s_2	10	5	2	0	1	
$z^* = 0$			-5 ↑	-3	0	0	
0	s_1	9	0	$19/5$	1	$-3/5$	$R_1 - 3R_2$
5	w_1	2	1	$2/5$	0	$1/5$	$R_2/5$
$z^* = 10$			0	-1	0	1	
3	w_2	$45/19$	0	1	$5/19$	$-3/19$	$\frac{5}{19}R_1$
5	w_1	$20/19$	1	0	$-2/19$	$25/95 = 5/19$	$R_2 - \frac{2}{5}R_1$
$z^* = \frac{235}{19}$			0	0	$5/19$	$16/19$	

$z^* = \frac{235}{19}$

$w_1 = \frac{20}{19}$

$w_2 = \frac{45}{19}$

Primal solution

$x_1 = 5/19$

$x_2 = 16/19$

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Revised Simplex Method

Qn. Using Reverse Simplex Method Solve the following LPP

Maximize $Z = 2x_1 + x_2$

subject to, $3x_1 + 4x_2 \leq 6$

$6x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

Standard form,

$Z - 2x_1 - x_2 + 0s_1 + 0s_2 = 0$

$3x_1 + 4x_2 + s_1 = 6$

$6x_1 + x_2 + s_2 = 3$

$x_1, x_2, s_1, s_2 \geq 0$

Matrix form

$$\begin{bmatrix} 1 & -2 & -1 & 0 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 6 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

IBFS

$s_1 = 6$ NBV: $x_1 = 0$, $x_2 = 0$

$s_2 = 3$

BV	B^{-1}			X_B	X_k	Ratio X_B/y_i
	B_0	B_1	B_2			
z	1	0	0	0	-2	
S_1	0	1	0	6	3	6/3
$\leftarrow S_2$	0	0	1	3	$\boxed{6}$	3/6

$a_1 = x_1$	$a_2 = x_2$
-2	-1
3	4
6	1

Net Evaluation $= \{ [1\ 0\ 0]\, x_1,\ [1\ 0\ 0]\, x_2 \}$

$= \{ \underset{\uparrow}{-2}, -1 \}$

x_1 enters the basis $B^{-1} x_1 = X_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}$$

I iteration

Modify B^{-1}

B_1	B_2	X_B	$X_{k=1}$
0	0	0	-2
1	0	6	3
0	1	3	$\boxed{6}$

$R_1 \to R_1 + 2R_3$

$R_2 \to R_2 - 3R_3$

$R_3 \to R_3 / 6$

Improved table

B_1	B_2	X_B	$X_{k=1}$
0	1/3	1	0
1	-1/2	9/2	0
0	1/6	1/2	1

BV	B^{-1}			X_B	X_k	Ratio X_B/y_i
	B_0	B_1	B_2			
Z	1	0	$1/3$	1	$-2/3$	
S_1	0	1	$-1/2$	$9/2$	$7/2$	$\boxed{9/7}$
x_1	0	0	$1/6$	$1/2$	$1/6$	3

S_2	x_2
0	-1
0	4
1	1

Net evaluation, $z_j - c_j = \{ [1\ 0\ 1/3]\, s_2 \, ; [1\ 0\ 1/3]\, x_2 \}$

$= \{ 1/3 \, ; -2/3 \}$

$4 - \frac{1}{2}$

$\frac{8-1}{2}$

x_2 enters the basis

$B^{-1} x_2 = X_2$

$$B^{-1} \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1/6 \end{bmatrix} \overset{x_2}{\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}} = \begin{bmatrix} -2/3 \\ 7/2 \\ 1/6 \end{bmatrix}$$

$\frac{9}{2} \times \frac{2}{7}$

$\frac{1}{2} \times \frac{2}{7}$

$-\frac{1}{2} \times \frac{2}{7}$

III iteration

Modify B^{-1}

B_1	B_2	X_B	$X_{k=2}$
0	$1/3$	1	$-2/3$
1	$-1/2$	$9/2$	$\boxed{7/2}$
0	$1/6$	$1/2$	$1/6$

$R_1 \to R_1 + \frac{2}{3} R_2$

$R_2 \to \frac{2}{7} R_2$

$R_3 \to R_3 - \frac{1}{6} R_2$

Improved table

B_1	B_2	X_B	$X_{k=2}$
		0	0
$-1/7$	$9/7$	1	1
	$4/21$	0	0

B_1	B_2	X_B	X_k
4/21	5/21	13/7	0
2/7	-1/7	9/7	1
-1/21	4/21	2/7	0

Improved table

BV	B^{-1}			X_B	X_k
	B_0	B_1	B_2		
z	1	4/21	5/21	13/7	
x_2	0	2/7	-1/7	9/7	
x_1	0	-1/21	4/21	2/7	

S_2	S_1
0	0
0	1
1	0

Net evaluation $= \{ [1 \ 4/21 \ 5/21] \, S_2 , \ [1 \ 4/21 \ 5/21] \, S_1 \}$

$= \{ 5/21 , \ 4/21 \}$

All the net evaluations are $\geq 0 \Rightarrow$

Current solution is optimal $\Rightarrow x_1 = \frac{2}{7}$

$x_2 = \frac{9}{7}$

$z = \frac{13}{7}$

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Qn. Using RSM Max $z = x_1 + 2x_2$ subject to,

$$x_1 + x_2 \leq 3$$
$$x_1 + 2x_2 \leq 5$$
$$3x_1 + x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

Standard form,

$$z - x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3 = 0$$
$$x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 3$$
$$x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 5$$
$$3x_1 + x_2 + 0s_1 + 0s_2 + s_3 = 6$$

Matrix form

$$\begin{bmatrix} 1 & -1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 5 \\ 6 \end{bmatrix}$$

IBFS

$n - m = 5 - 3 = 2$ $\quad x_1, x_2 = 0 = \text{NBV}$

$s_1 = 3, \; s_2 = 5, \; s_3 = 6$

BV	B^{-1}				X_B	X_k	Ratio X_B/X_k	x_1	x_2
	B_0	B_1	B_2	B_3					
z	1	0	0	0	0	-2		-1	-2
S_1	0	1	0	0	3	1	3	1	1
← S_2	0	0	1	0	5	$\boxed{2}$	$\boxed{5/2}$	1	2
S_3	0	0	0	1	6	1	6	3	1

Net evaluation $= \{[1\ 0\ 0\ 0]\,x_1,\ [1\ 0\ 0\ 0]\,x_2\}$

$= \{-1,\ -2\}$

$$X_k = B^{-1} x_2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Iteration II

Modify B^{-1}

B_1	B_2	B_3	X_B	X_k	
0	0	0	0	-2	$R_1 \to R_1 + 2R_3$
1	0	0	3	1	$R_2 \to R_2 - R_3$
0	1	0	5	$\boxed{2}$	$R_3 \to R_3/2$
0	0	1	6	1	$R_4 \to R_4 - R_3$

Improved table

B_1	B_2	B_3	X_B	X_{k+1}
0	1	0	5	0
1	$-1/2$	0	$1/2$	0
0	$1/2$	0	$5/2$	1
0	$-1/2$	1	$7/2$	0

BV	B^{-1}				X_B	X_k
	B_0	B_1	B_2	B_3		
z	1	0	1	0	5	
s_1	0	1	$-1/2$	0	$1/2$	
x_2	0	0	$1/2$	0	$5/2$	
s_3	0	0	$-1/2$	1	$7/2$	

x_1	s_2
-1	0
1	0
1	1
3	0

Net evaluation $= \{[1\ 0\ 1\ 0]\, x_1,\ [1\ 0\ 1\ 0]\, s_2\}$

$= \{0, 1\}$

All net evaluations are $\geq 0 \Rightarrow$

$x_1 = 0, \quad x_2 = 5/2, \quad \text{Max } z = 5$

Qn. Using RSM, Max $z = x_1 + x_2$ subject to,

$x_1 + \frac{x_2}{2} \leq 2,$

$3x_1 + 2x_2 \leq 12$

$x_1, x_2 \geq 0,$

Standard form,

$z - x_1 - x_2 + 0s_1 + 0s_2 = 0$

$x_1 + \frac{x_2}{2} + s_1 + 0s_2 = 2$

$3x_1 + 2x_2 + 0s_1 + s_2 = 12$

Matrix

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1/2 & 1 & 0 \\ 0 & 3 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 12 \end{bmatrix}$$

IBFS

$n - m = 4 - 2 = 2$

$x_1 = x_2 = 0$.

$s_1 = 2$

$s_2 = 12$

BV	B^{-1}			X_B	X_k	Ratio
	B_0	B_1	B_2			
z	1	0	0	0	-1	
s_1	0	1	0	2	$\boxed{1}$	$\boxed{2}$
s_2	0	0	1	12	3	4

x_1	x_2
-1	-1
1	1/2
3	2

Net eval $= \{[1\ 0\ 0]\,x_1,\ [1\ 0\ 0]\,x_2\}$

$= \{-1, -1\}$

$X_k = B^{-1} x_1$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

Iteration II

B_1	B_2	X_B
0	0	0
1	0	2
0	1	12

X_k:
-1 $\quad R_1 \to R_1 + R_2$
$\boxed{1}$
3 $\quad R_3 \to R_3 - 3R_2$

Modified table

B_1	B_2	X_B	X_k
1	0	2	0
1	0	2	1
-3	1	6	0

~~Next table~~

BV	B^{-1}			X_B	X_k	Ratio
	B_0	B_1	B_2			
z	1	1	0	2	$-1/2$	
← x_1	0	1	0	2	$\boxed{1/2}$	4
s_2	0	-3	1	6	$1/2$	12

s_1	x_2
0	-1
1	$1/2$
0	2

Net eval $= \{[1\ 1\ 0]\, s_1,\ [1\ 1\ 0]\, x_2\}$

$= \{1, \underset{\uparrow}{-1/2}\}$

$-1 + \frac{1}{2}$

$-\frac{3}{2} + 2$

$X_k = B^{-1} x_2$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1/2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

Iteration (II).

Modified table

B_1	B_2	X_B	X_k	
1	0	2	$-1/2$	$R_1 + \frac{1}{2}R_2$
1	0	2	$\boxed{1/2}$	$2R_2$
-3	1	6	$1/2$	$R_3 - \frac{1}{2}R_2$

B_1	B_2	X_B	X_k
2	0	4	0
2	0	4	1
-4	1	4	0

BV	B^{-1}			X_B	X_k
	B_0	B_1	B_2		
Z	1	2	0	4	
x_2	0	2	0	4	
S_2	0	-4	1	4	

S_1	x_1
0	0
1	1
0	0

Net eval $= \{[1\ 2\ 0]S_1, [1\ 2\ 0]x_1\}$

$= \{2, 2\}$

All net evaluations are $\geq 0 \Rightarrow$

$x_1 = 0$, $x_2 = 4$, Max $Z = 4$