6.11.2023

MODULE 4

Network Analysis

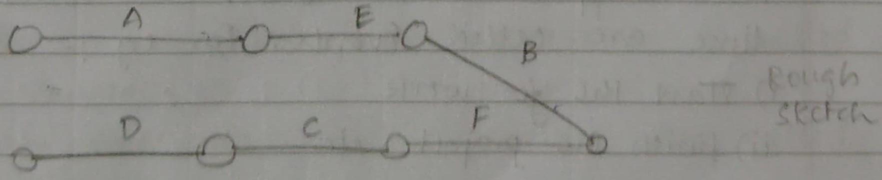
A project is a composed of a no. of jobs and activities or tasks that are related to each other. And all of these should be completed in order to finish the project.
There are 2 prominant methods to do this
1) ~~Criterial~~ Critical path method (CPM)
2) ~~Pret~~ PERT (Project Evaluation Review Technique)

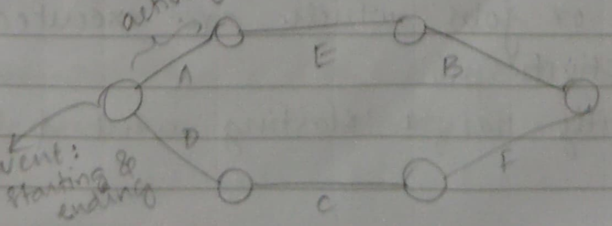
Q: Assume that a statue is to be erected in a village square on a stone base which is to be built ~~in~~ on a cement concrete foundation. The statue is to be made on another place, ~~ma~~ move to the base and install. The various operations of the entire project are below ~~in~~ a random order.

A - make statue,
B - lift the statue into place,
C - construct concrete foundation,
D - compact & level the site
E - move statue to village square,
F - construct stone base.

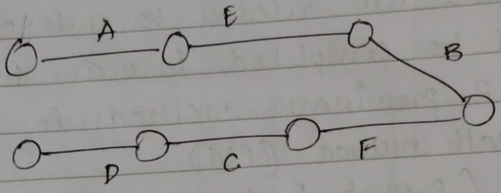
Construct network ~~dat~~ diagram of the project.

A>

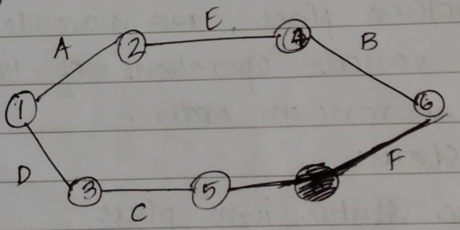




In this project, we can construct the base & make statue simultaneously at 2 different places. Accordingly we can rough sketch of the networking diagram.



In every network, there will be a starting event & finishing event, so the correct final network design of the above ~~B~~ project can be respectively as



O → begining or ending of ~~such~~ some activity

—— → activity

To represent activities we will be using line segments or symbols like A, B, C ... or pair of nos. in the form (1, 2) or (3, 4).

Some happens which occur at some instant of time are called Events. For eg :-

i) start the ~~jt~~ work

ii) finish the project etc...

The tasks or jobs which are executed are called Activities.

Each activity has a starting event & ending event.

If the starting event is 1 & end event = 2, the activity can be represented as

① ——— ②

or

1 to 2

or using some notations / symbols

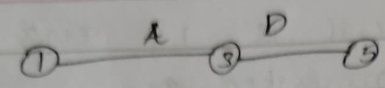
$\underline{\quad A \quad}$

A network is a combination of several activities & events. Two events are said to be ~~adjacent~~ adjacent events if there is an ~~event~~ activity connecting them.
2 activities are said to be adjacent if there is an event common to them.
Chain is ~~a~~ sequences of adjacent activities.
The last event showing the end of the project is called Sink.
The 1st event showing the begining of the project is called Source.
The tail event is the one which marks the begining of the ~~event~~ activity & head event ~~mak~~ marks the end of an ~~event~~ activity.

~~Sour~~ Successor events and predecessor events:
The events that follow an event is called Successor events.
The events that occur before an event are called Predecessor events.

~~Successor activity~~: An activity 'A' is said to be predecessor of an activity 'D' if D follows A.

eg:



Here, 'A' is said to be the immediate predecessor ~~predecessor~~ of D. & 'D' is said to be immediate successor of A.

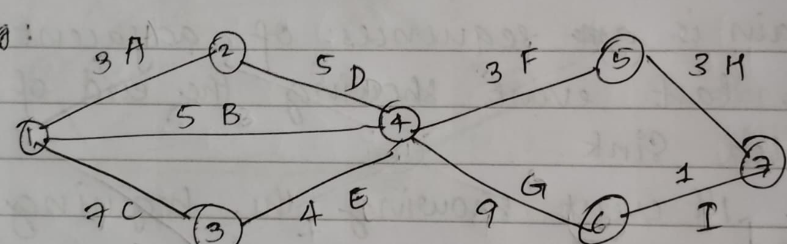
→ Earliest Occurence time of an event :- (Earliest event time) EOT

EOT of an event is the earliest possible time at which the event can occur.

* $EOT(j) = \max_{i} \{ EOT(i) + duration(i,j) \}$

i is ~~varying~~ varying

eg:



$EOT(1) = 0$

$EOT(2) = \max \{ EOT(1) + \text{duration of } (1,2) \}$

$= 0 + 3 = 3$ //

$EOT(3) = \max \{ EOT(1) + duration(1,3) \}$

$= 0 + 7 = 7$ //

$EOT(4) = \max \{ EOT(1) + duration(1,4),$
$EOT(2) + duration(2,4),$
$EOT(3) + duration(3,4) \}$

$= \max \{ 0+5, 3+5, 7+4 \}$

$= \max \{5, 8, 11\}$

$= 11$

//

$EOT(5) = 11+3 = 14$

$EOT(6) = 11+9 = 20$

$$EOT(7) = \max \begin{Bmatrix} EOT(5) + d(5,7) \\ EOT(6) + d(6,7) \end{Bmatrix}$$

$$= \max\{14+3,\ 20+1\}$$

$$= 21$$

An activity is said to be critical activity if a delay in the execution of ~~an~~ that activity delays the whole project.

Critical Path :-

Critical path in a project is that sequence of activities which determine the total project time. It is composed of a sequence of critical activities.

→ Latest occurrence time of an event :- (LOT) Latest event time.

The LOT of an event is the latest allowed time by which the event ~~to the~~ must occur to keep the project on schedule.

The LOT of the event 'i' is calculated by the formula

$$* \ LOT(i) = \min_{j}\{LOT(j) - \text{duration}(i,j)\}$$

$LOT(7) = 21$

$LOT(6) = \min(LOT(7) - \text{duration}(6,7))$

$= \min(21-1)$

$= 20$

$LOT(5) = \min\{LOT(7) - \text{duration}(5,7)\}$

$= \min\{21-3\}$

$= 18$

$$LOT(4) = \min \begin{Bmatrix} LOT(5) - \text{duration}(4,5), \\ LOT(6) - \text{duration}(4,6) \end{Bmatrix}$$

$= \min\{18 - 3,\ 20 - 9\}$

$= \min\{15,\ 11\}$

$= 11$

$LOT(2) = \min\{LOT(4) - \text{duration}(2,4)\}$

$= \min\{11 - 5\}$

$= 6$

$LOT(3) = \min\{LOT(4) - \text{duration}(3,4)\}$

$= \min\{11 - 4\}$

$= 7$

$LOT(1) = \min\{LOT(2) - \text{duration}(1,2),\ LOT(3) - \text{duration}(1,3)\}$

$= \min\{6 - 3,\ 7 - 7\}$

$= \min\{3, 0\}$

$= 0$

~~Start~~

→ Start time & finish time of an Activities :-

EST(i,j) Earliest start time of an activity (i, j) : It is the earliest time by which the activity can start.

It is exactly equal to EOT(i).

ie., $EST(i,j) = EOT(i)$

Earliest finish time of an activity :- EFT(i,j)

It is the earliest time by which the activity can be finished.

$EFT(i,j) = EOT(i) + \text{duration}(i,j)$

Latest finish time of an Activity :- LFT (i, j)

It is the latest time by which the activity can be finish without delaying the completion of the project.

$$LFT(i, j) = LOT(j)$$

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Latest start time of an activity :-

It is the latest time by an activity can be started without delaying the completion of the project.

$$LST(i, j) = LFT(i, j) - \text{duration of } (i, j)$$

- CPM (Critical Path Method) :-

It is a network technique which consists of

i) planing the sequence of activities to be performed in the network.

Scheduling the time & resource to various

Controlling the ~~pre~~ performance so that they are not deviating from the plan.

ii) This method is applicable for those ~~th~~ types of ~~for~~ repetative project or for those ~~fair~~ projects for which fairly accurate time estimate for completion of each activity is known before hand. & for which the the cost estimates can be made with fair degree of accuracy.

- Important steps in CPM :-

i) List all the activities & draw a network ~~dr~~ diagram.

ii) compute EOT & LOT of events.

iii) calculate EST, ~~in a~~ EFT, LST, LFT for each event.

iv) Compute the float of each activity.

v) Identify the critical path.
vi) Compute the total project time by summing the duration of critical activities.
~~vii)~~

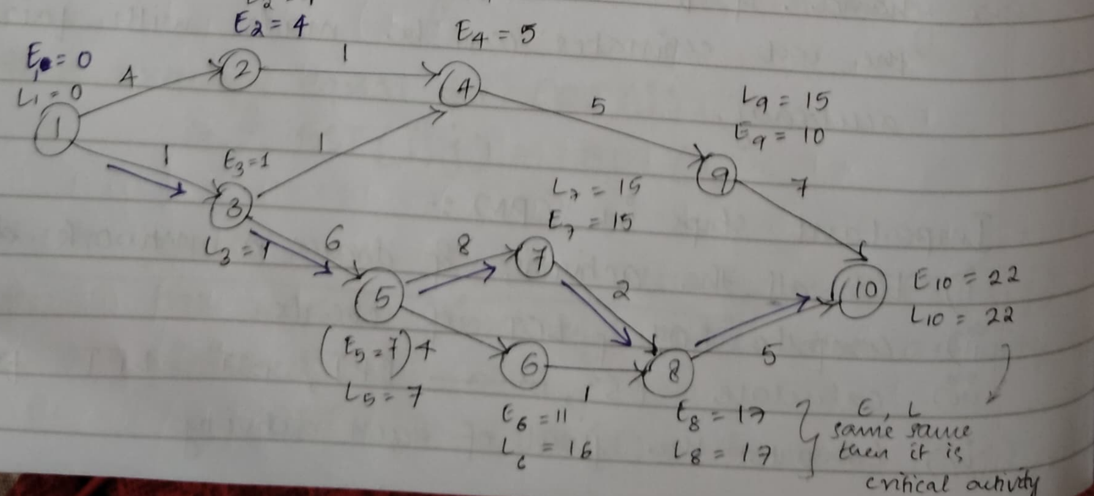
- Method of identifying critical activity :-
i) Corresponding to each activity (i, j) we compute EOT(i), LOT(i), EOT(j), LOT(j), then if this activity (i,j) will be critical if EOT(i) = LOT(i) & EOT(j) = LOT(j).

Q. A project schedule has the following characteristics

Activities	Time
1 - 2	4
1 - 3	1
2 - 4	1
3 - 4	1
3 - 5	6
4 - 9	5
5 - 6	4
5 - 7	8
6 - 8	1
7 - 8	2
8 - 10	5
9 - 10	7

~~PART A :-~~
a) Construct network diagram.
b) Compute EOT & LOT of each event.
c) Comput EST, LST, EFT, LFT of each activities.
d) Find the critical path & project time.

A→



(forward pass)

$EOT(1) = 0$

$$EOT(j) = \max_i \{ EOT(i) + duration(i,j) \}$$

$EOT(2) = EOT(1) + 4 = 4$

$EOT(3) = EOT(1) + 1 = 1$

$$EOT(4) = \max \begin{cases} EOT(2) + duration(2,4) \\ EOT(3) + duration(3,4) \end{cases}$$

$= \max \{ 4+1, 1+1 \}$

$= \max (5, 2)$

$= 5$

$EOT(5) = EOT(3) + dur(3,5)$

$= 1 + 6 = 7$

$EOT(6) = EOT(5) + dur(5,6)$

$= 7 + 4 = 11$

$EOT(7) = EOT(5) + dur(5,7)$

$= 7 + 8$

$= 15$

$$EOT(8) = \max \begin{cases} EOT(7) + dur(7,8) \\ EOT(6) + dur(6,8) \end{cases}$$

$= \max \{ 15 + 2, 11 + 1 \}$

$= \max \{ 17, 12 \}$

$= 17$

$EOT(9) = EOT(4) + dur(4,9)$

$= 5 + 5 = 10$

$$EOT(10) = \max \begin{cases} EOT(9) + dur(9,10) \\ EOT(8) + dur(8,10) \end{cases}$$

$= \max \{ 10 + 7, 17 + 5 \}$

$= \max \{ 17, 22 \}$

$= 22$

~~t10 =~~

(backward pass)

$L_{10} = 22$

$$LOT(i) = \min_{j} \left[LOT(j) - duration(i, j) \right]$$

$L_9 = \cancel{\min\{} L_{10} - dur(9,10)$
$= 22 - 7 = 15$

$L_8 = L_{10} - dur(8,10)$
$= 22 - 5 = 17$

$L_7 = L_8 - dur(7,8)$
$= 17 - 2$
$= 15$

$L_6 = L_8 - dur(6,8)$
$= 17 - 1$
$= 16$

$L_5 = \min \begin{cases} L_6 - dur(5,6), \\ L_7 - dur(5,7) \end{cases}$

$= \min \begin{cases} 16 - 4, \\ 15 - 8 \end{cases}$

$= \min\{12, 7\}$
$= 7$

$L_4 = L_9 - dur(4,9)$
$= 15 - 5 = 10$

$L_3 = \min \begin{cases} L_4 - dur(3,4), \\ L_5 - dur(3,5) \end{cases}$

$= \min \begin{cases} 10 - 1, \\ 7 - 6 \end{cases}$

$= \min(9,1) = 1$

$L_2 = L_4 - dur(2,4)$
$= 10 - 1$
$= 9$

$L_1 = \cancel{0} \min \begin{cases} L_2 - dur(1,2) \\ L_3 - dur(1,3) \end{cases}$

$= \min \begin{cases} 9 - 4, \\ 1 - 1 \end{cases}$

$= \min\{5, 0\} = 0$

$\rightarrow$

Critical activities :-

E_1, L_1

E_{10}, L_{10}

E_7, L_7

E_8, L_8

E_{10}, L_{10}

The ~~activit~~ critical path is :-

$$1 \xrightarrow{1} 3 \xrightarrow{6} 5 \xrightarrow{8} 7 \xrightarrow{2} 8 \xrightarrow{5} 10$$

Project time $= 1+6+8+2+5 = 22$

$EST(i,j) = EOT(i)$, $EFT(\overset{tail}{i},\overset{head}{j}) = EOT(i) + \text{duration}(i,j)$

$LST(i,j) = LFT(i,j) - \text{duration}(i,j)$, ~~LFT(i,j)~~ $= LOT(j) - \text{duration}(i,j)$

Activity	time	Earliest time EST	Earliest time EFT	Latest time LST	Latest time LFT
1 - 2	4	0	4	5 ~~0~~	9
1 - 3	1	0	1	0	1
2 - 4	1	4	5	9	10
3 - 4	1	1	2	9	10
3 - 5	6	1	7	1	7
4 - 9	5	5	10	10	15
5 - 6	4	7	11	12	16
5 - 7	8	7	15	7	15
6 - 8	1	11	12	16	17
7 - 8	2	15	17	15	17
8 - 10	5	17	22	17	22
9 - 10	7	10	17	15	22

$LFT(i,j) = LOT(j)$

→ Slack time and float time :-

Slack denotes the flexibility range within which an event can happen. slack of an event ~~ES~~ is ~~denote~~ defined by the difference b/w the ~~last EOT &~~ LOT & EOT.

$$\text{Slack}(i) = LOT(i) - EOT(i)$$
$$= L_i - E_i$$

~~This term is associated with the activit~~
The float is associated with the activity time.
Float denotes the range within which the activity starts or finishes without affecting the completion of the project.
There are 4 types of floats :-
i) Total float
ii) Free float
iii) Independent float
iv) Interfering float

- Total float is defined by the ~~de~~ difference b/w LFT & EFT of an activity. or it is defined by the difference b/w LST & EST of the activity.

$$\text{Total float}(i,j) = LST(i,j) - EST(i,j)$$
OR
$$= LFT(i,j) - EFT(i,j)$$

* If the activity is critical, total float = 0
In previous qn,

$$\text{total float}(1,3) = LST(1,3) - EST(1,3)$$
$$= 0 - 0 = 0$$

$$\text{total float}(3,5) = LST(3,5) - EST(3,5)$$
$$= 1 - 1 = 0$$

- Free float :-

Free float is that portion of the total float that can be used by an activity without delaying succeeding activities.

Free float = EST (successor event) – EFT (current event)

- Independent float is defined as the excess of min available time over the required activity duration. i.e. It is the amount of time in which an activity could be delayed if preceeding activity is finished at their lastest & following activities start at their earliest.

It is given by,

Independent float = EST (succeeding activity) – LFT (preceding activity) – duration (activity)$_{(i,j)}$

- Uses of float :-
- Floats are used to solve resource leveling & resource allocation problems.
- Floats give some flexibility in rescheduling some activities so as to smoothen the level of resources or ~~activity~~ allocate the limited resource as best as possible.

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→ Project Evaluation & Review Technique (PERT) :-

PERT is a ~~manufact~~ management technique in which we try to use logical disciplines in planning & controlling projects. In this method we use network diagram consisting of events. The successive events are joined by arrows. The main ~~it~~ objective of PERT analysis is to find out which a job ~~to~~ could be finished in a given date. This system is prefered for those projects or ~~pr~~ operations which are of ~~pro~~ probabilistic in nature i.e, not repetetive in which the precise time determination of various activities is not ~~predictable~~ practicable.

PERT is useful technique in project planning & control. It gives the planner a perfect idea about the sequences of activities and their times. It is useful in minimizing delay & ~~interaction~~ interruptions ~~of~~ of ~~in~~ various activities. It helps in co-ordinating the various parts of the over all job & seeing that every ~~preceedor~~ predecessor activity is finished in time for the following activity to continue. It shows a way how ~~to~~ a project can be finished earlier than the original schedule. The main assumption in PERT is that the activity duration is independent ie., the time required for one activity has nothing to do with another activity.

- Time Estimates in PERT analyse :-

  The PERT calculations are probabilistic. There are 3 ~~named~~ main time estimates :-

  i) Optimisatic time estimate (t_o)

  ii) ~~Persim~~ ~~Per~~ Pessimistic time estimate (t_p)

  iii) most likely time estimate (t_m)

- Optimistic time of a ~~activity~~ activity is the ~~ts~~ shortest possible time in which an activity can be completed under ideal condition (t_o).

- Pessimistic time is the max. time that ~~would~~ would be required to finish the activity if ~~everything~~ everything went wrong & abnormal situation reveal.

- Most likely time is the time ~~in~~ which the activity take most frequently. This time estimate ~~to~~ lie b/w optimistic time & pessimistic time.

→ Steps invoked in PERT analysis :-

i) Identify the events & activities & draw a suitable n/w diagram.

ii) Events are numbered in asc. order from Left to right.

iii) Obtain the various time estimates for each activities such as t_o, t_m, t_p.

iv) Compute the ~~except~~ ~~except~~ expected time for each activity using the formula :-

$$\boxed{t_e = \frac{t_o + 4t_m + t_p}{6}}$$

v) Using the expected activity ~~estimate~~ time compute EOT & LOT.

~~EOT~~

vi) Compute the float for each activity. The activities with 0 float are critical activity.

vii) Find the total expected time by adding the time estimate for various critical activities.

viii) Find the variance time estimate of all activities using the formula :-

$$\boxed{var\,(i,j) = \left(\frac{t_o - t_p}{6}\right)^2}$$

ix) Find the ~~of~~ probability of finishing the project on some fixed target by using values from ~~time~~ table of normal distribution.

The value, ~~$z = \frac{x - \bar{x}}{SD}$~~

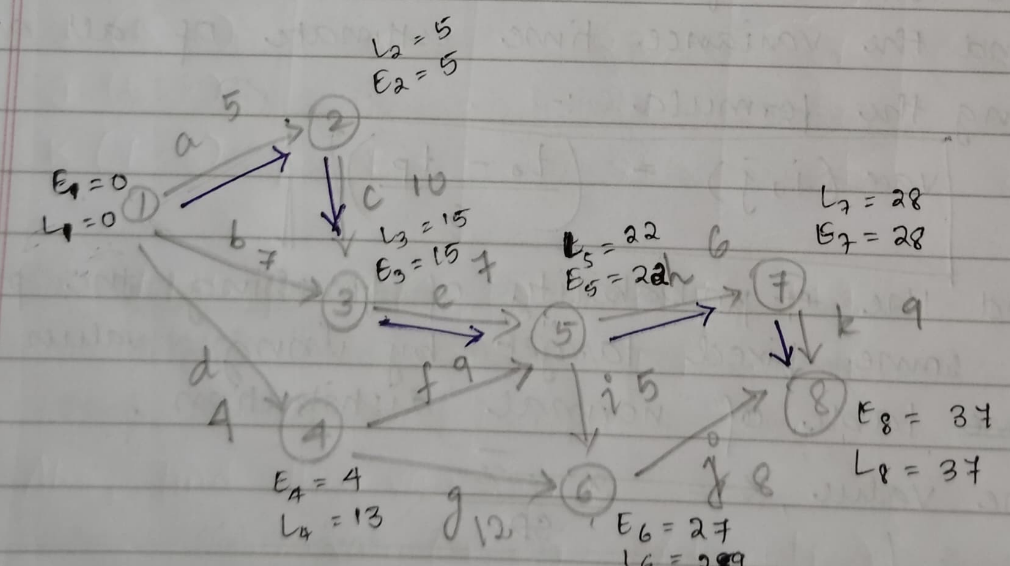
$$\boxed{z = \frac{\text{due Date} - \text{expected date of completion}}{\sqrt{\text{sum of variance of critical activities of the critical path}}}}$$

Q: The following table gives the different time estimates such as t_o, t_m, t_p ~~to~~ for each ~~to~~ activity of a project.

task	$\underset{a}{1-2}$	$\underset{b}{1-3}$	$\underset{c}{2-3}$	$\underset{d}{1-4}$	$\underset{e}{3-5}$	$\underset{f}{4-5}$	$\underset{g}{4-6}$	$\underset{h}{5-7}$	$\underset{i}{5-6}$	$\underset{j}{6-8}$	$\underset{k}{7-8}$
t_o	4	5	8	2	4	7	8	4	3	5	6
t_p	6	9	12	6	10	15	16	8	7	11	12
t_m	5	7	10	4	7	8	12	6	5	8	9

i) Compute expected time for each activity
ii) Compute earliest event time & LOT.
iii) Compute EST, EFT, LST, LFT values ~~for~~ of all activities
iv) Identify the critical path & project time.

A→



~~E_2~~ $EOT(2) = \max\{EOT(1) + dur(1,2)\}$
$= 0 + 5$
$= 5$

$EOT(3) = \max \begin{cases} EOT(1) + dur(1,3), \\ EOT(2) + dur(2,3) \end{cases}$

$= \max \begin{cases} 0+7, \\ 5+10 \end{cases}$

$= \max(7, 15)$

$= 15$

$EOT(4) = \max \{ 0+4$

$= 4$

$EOT(5) = \max \begin{cases} 4+9, \\ 15+7 \end{cases}$

$= \max(13, 22)$

$= 22$

$EOT(6) = \max \begin{cases} 4+12, \\ 22+5 \end{cases}$

$= 27$

$EOT(7) = 22+6 = 28$

$EOT(8) = \max \begin{cases} 28+9, \\ 27+8 \end{cases}$

$= \max(37, 35)$

$= 37$

$L_5 = \min \begin{cases} 28-6, \\ 29-5 \end{cases}$

$= \min \{22, 24\}$

$= 22$

$L_4 = \min \begin{cases} 22-9, \\ 29-12 \end{cases}$

$= \min \{13, 17\}$

$= 13$

$L_3 = 22-7 = 15$

$L_2 = 15-10 = 5$

$L_1 = \min \begin{cases} 5-5, \\ 15-7, \\ 13-4 \end{cases}$

$= 0$

$L_8 = 37$

$L_7 = \min \{ LOT(8) - \text{duration}(7, 8)$

$= 37 - 9$

$= 28$

$L_6 = 37 - 8$

$= 29$

The critical path is

$$① \xrightarrow{5} ② \xrightarrow{10} ③ \xrightarrow{7} ⑤ \xrightarrow{6} ⑦ \xrightarrow{9} ⑧$$

~~Projected~~ project time $= 5 + 10 + 7 + 6 + 9$

$= 37$ //

activity	time	earliest time		latest time		Total float	variance
		EST	EFT	LST	LFT		
A (1-2)	5	0	5	0	5	0	1/9 ✓
B (1-3)	7	0	7	8	15	8	4/9
C (2-3)	10	5	15	5	15	0	4/9 ✓
D (1-4)	4	0	4	9	13	9	1 ~~[illegible]~~ 4/9
E (3-5)	7	15	22	15	22	0	1 ~~6/9~~ ✓
F (4-5)	9	4	13	13	22	9	16/9
G (4-6)	12	4	16	17	29	13	16/9
H (5-7)	6	22	28	22	28	0	4/9 ✓
I (5-6)	5	22	27	24	29	2	4/9
J (6-8)	8	27	35	29	37	2	1
K (7-8)	9	28	37	28	37	0	1 ✓

Total float $(i, j) = LFT - EFT$

$= LST - EST$

Also, find the variance :

$$var(i, j) = \left(\frac{t_o - t_p}{6}\right)^2$$

Variance of critical activities $\Rightarrow \frac{1}{9} + \frac{4}{9} + 1 + \frac{4}{9} + 1$

$\Rightarrow \frac{9}{9} + 1 + 1 = 3$ //

Std deviation $\Rightarrow \sqrt{3}$

v) what is the probability that project could be finished ~~in~~ within 33 days ?

vi) what is ... needed to ...

A> v) $P(x \leq$ ...

$\Rightarrow P$ ...

vi) $P(x$ ...

classmate
Date
Page

vi) what is the probability that atleast 40 days are needed to ~~complete~~ finish the project ?

A) v) $P(\overset{\text{no. of days}}{X} \le 33)$

$$\Rightarrow P\left(\frac{X-37}{\sqrt{3}} \le \frac{33-37}{\sqrt{3}}\right) =$$

$$= P\left(Z \le -\frac{4}{\sqrt{3}}\right)$$

$$= P(Z \le -2.309)$$

$$= P(Z \ge 2.309)$$

$$= 0.5 - P(0 \le Z < 2.309)$$

$$= 0.5 - 0.4896$$

$$= 0.0104$$

$$Z = \frac{X - \bar{X}}{\sigma}$$

variance
1/9 ✓
4/9
4/9 ✓
1 ~~1/16~~ 4/9
1 ~~16/9~~ ✓
16/9
16/9
4/9 ✓
4/9
1
1 ✓

vi) $P(X \ge 40) = \cancel{P\left(X - 37\right.}\ P\left(\frac{X-37}{\sqrt{3}} \ge \frac{40-37}{\sqrt{3}}\right)$

$$= 0.5 - P(0 < Z < 1.73)$$

$$=$$