~~We know that:~~

(3) a) To show that sum of posterior probabilities of classes = 1 under assumption of logistic regression.

Soln: We know: $Pr(G=k/X=x) = \dfrac{e^{\beta_{k0}+\beta_k^T x}}{1+\sum_{l=1}^{K-1} e^{(\beta_{l0}+\beta_l^T x)}}$

We also know: $\log \dfrac{Pr(G=k/X=x)}{P(G=K/X=x)} = \beta_{k0}+\beta_k^T x$

$\therefore \dfrac{Pr(G=k/X=x)}{Pr(G=K/X=x)} = e^{\beta_{k0}+\beta_k^T x}$

$\Rightarrow Pr(G=K/X=x) = \dfrac{1}{1+\sum_{l=1}^{K-1} e^{\beta_{l0}+\beta_l^T x}}$

Now:- $\sum_{k=1}^{k=K-1} Pr(G=k/X=x) = \dfrac{\sum e^{\beta_{k0}+\beta_k^T x}}{1+\sum_{l=1}^{K-1} e^{\beta_{l0}+\beta_l^T x}}$

Now sum of posterior probabilities of all classes

$= \sum_{k=1}^{k=K-1} Pr(G=k/X=x) + Pr(G=K/X=x)$

$= \dfrac{\sum_{k=1}^{K-1} e^{\beta_{k0}+\beta_k^T x}}{1+\sum_{l=1}^{K-1} e^{\beta_{l0}+\beta_l^T x}} + \dfrac{1}{1+\sum_{l=1}^{K-1} e^{\beta_{l0}+\beta_l^T x}}$

$= \dfrac{1+\sum_{k=1}^{K-1} e^{\beta_{k0}+\beta_k^T x}}{1+\sum_{l=1}^{K-1} e^{\beta_{l0}+\beta_l^T x}} = 1$

Hence, Proved.

(b) $p(X) = \dfrac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}} \rightarrow$ Given.

To show :- $\dfrac{p(X)}{1-p(X)} = e^{\beta_0+\beta_1 X}$

Soln: We know: $p(X) = \dfrac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}$

$$= 1-p(X) = 1-\frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}} = \frac{1}{1+e^{\beta_0+\beta_1 X}}$$

$$\therefore \frac{p(X)}{1-p(X)} = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}} \times \frac{1+e^{\beta_0+\beta_1 X}}{1}$$

$$= e^{\beta_0+\beta_1 X}$$

Hence, Proved.