BEGINNING ALGEBRA MADE USEFUL

CHAPTER 4

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Chapter 4
Introduction to Quadratic Functions

In Chapters 2 and 3, we studied several aspects of linear functions, including slope, *x*- and *y*-intercepts, solving equations, and modeling. We considered patterns in tables, patterns illustrated with tiles, appearances of graphs, etc. We also compared linear patterns to non-linear patterns.

Noticing that not all patterns and functions are linear, many questions arise:

- To what family of functions might a non-linear function belong?
- Are any non-linear functions related to linear functions? If so, how are they related?
- Can we use what we learned about linear functions to analyze other families of functions?

In Chapter 4, we begin to consider a function family that is not linear. We begin in context and with patterns. We look for connections that link the new function family to linear functions. Then we use some of what we've explored to make sense of the new function family.

4.1 Another Type of Function?

As with linear functions, non-linear functions can arise in context. In this lesson, we will analyze the numbers of performers based on group number in two books by Kathi Appelt, Bats on Parade and Bat Jamboree. Though written for children, the underlying mathematics is much more advanced. The simplicity of the context helps us makes sense of the more complex mathematics.

As you complete the Batty Functions activity, look for several non-linear patterns that arise in the stories.

Activity: Batty Functions

1. Listen to the story, *Bats on Parade*, by Kathi Appelt. (Find online a YouTube video of someone reading *Bats on Parade*.) Read the scenarios found in or adapted from the story. Scenario A describes the characters in the story as they appear in the book. In Scenario B, you will also count the flag bearer as shown in the story. In Scenario C, the story is adapted to include more flag bearers.

   Scenario A: The first group has 1 marcher in 1 row. The second group has 2 marchers in each of 2 rows. The third group has 3 marchers in each of 3 rows, and so on.
   Scenario B: Each group that joins the parade also has a single flag bearer. Include the flag bearer in the number of marchers in each group.
   Scenario C: Each group that joins the parade has as many flag bearers as columns in the group. For example, the third group of bats marches in 3 rows and 3 columns and has 3 flag bearers.

   a. In the table on p. 146, draw a tile pattern to match each Scenario A, B, and C.

Scenario	Step 1	Step 2	Step 3	Step 4	Step 5
A					
Number of Tiles					
B					
Number of Tiles					
C					
Number of Tiles					
D					
Number of Tiles					

b. Write the numbers of tiles for each of Scenarios A, B, and C in the table on p. 147. The step number above becomes the group number in the table on p. 147.

Group Number	Scenario A Number of Marchers in the Group	Scenario B Number of Marchers in the Group	Scenario C Number of Marchers in the Group	Scenario D Number of Performers in the Group	Scenario E Number of Performers So Far
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
x					

2. For each of Scenarios A, B, and C:
   a. What patterns do you see in the data?
   b. Are any of the patterns linear? How do you know?
   c. How are the columns of data related to each other?

3. a. Graph each set of data with Group Number as the independent variable and the number of marchers for the scenario as the dependent variable.
   b. How are the graphs related to each other?

4. a. For each data set, find an equation giving the number of marchers in group x. Write the equations in the last row of the table above.
   b. Are any of the equations linear? Should they be? Explain.

5. Listen to another story, *Bat Jamboree*, also by Kathi Appelt. (Google *Bat Jamboree* to find this story online in a YouTube video.) Complete the tables on pp. 146 and 147 for Scenarios D and E.

   Scenario D: Each group has the same number of performers as the group number.
   Scenario E: The number of performers who have appeared so far is the number of new performers and all performers who appeared before the new group. For example, when Group 3 performs, the number of performers who have appeared so far plus the new group are 1 + 2 + 3 = 6 performers.

6. a. Compare the data for Scenario E to the data for Scenarios A, B, and C. What do you notice?
   b. Is the data for any of Scenarios A, B, or C related to Scenario E? How?
   c. Graph the data with Group Number as the independent variable and Scenario E as the dependent variable.
   d. How is the graph related to the graphs in problem 3a?
   e. Determine an equation to model the data for Scenario E as it relates to group number.
7. Share your work on *Batty Functions*. Resolve differences and ask questions before continuing.

Adapted from Mathematical Explorations: Batty Functions: Exploring Quadratic Functions through Children's Literature by Charlene Beckmann and Jessica Roy, *Mathematics Teaching in the Middle School*, Vol. 13, Issue 1, August 2007, pp. 52–64.

Activity: Solving Functions Graphically

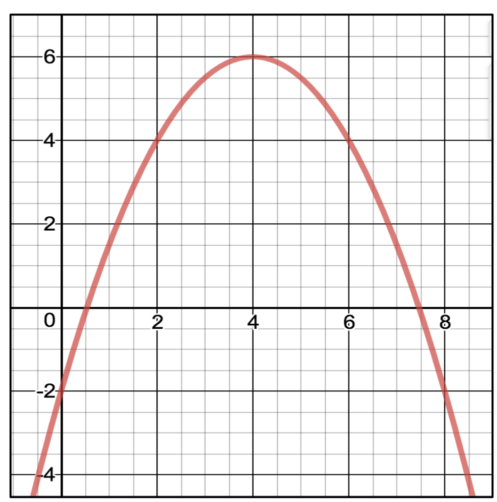
In *Batty Functions*, you found many equations, most of them non-linear. Relationships between some of the data sets helped you determine other equations. What questions might we ask about the data sets that can be answered using the equations? Brainstorm with your group. Write at least 3 questions about *Batty Functions* that can be answered using one or more of the equations. Share your questions with the class.

Questions that are frequently asked about *Batty Functions* include:

- How many bats are in group 7? Group n?
- If a group has 37 bats, what is the group number? If a group has b bats, what is the group number?
- What is a reasonable domain for each function?
- What is a reasonable range for each function?

Did you ask questions like these? Look at the graphs you drew for *Batty Functions*. How can a graph help you answer these questions? How can a table help? Choose some of the questions about *Batty Functions* that you and others wrote. Answer as many of them as you can in context. If you still have questions, write them down to ask once the *Batty Functions* questions have been answered.

Look at the graph at right. Suppose the graph represents the height, *h*, of a ball *t* seconds after it is thrown. Ask and answer similar questions as those above for this context and graph. Can you find the answer exactly in each case? Why or why not?



4.1 Homework

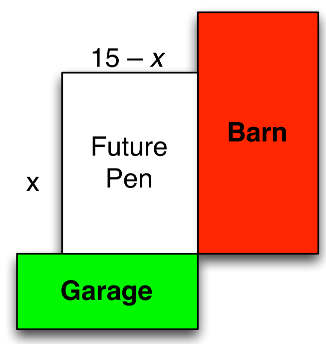
1. Some of the patterns from *Batty Functions* might be familiar.
   a. Which patterns from *Batty Functions* have you seen before?
   b. Where have they come up in the past? In this text?
   c. Can you now find equations for previous patterns from this text that you were wondering about? List the context, data, and equation.

2. Look back through patterns you have explored in this text.
   a. Find examples of non-linear patterns. List where you found the patterns.
   b. Identify which non-linear patterns are similar to one or more of the Batty Functions.

3. How many total bats appear in *Bat Jamboree*? How do you know?

4. Notice that most of the *Batty Functions* are non-linear. Look at the equations you found to represent each of the *Batty Functions*.
   a. The Batty Function described for Column B arises from the story directly. How did you determine the equation?
   b. The Batty Function described for Column C is related to the first one. How?
   c. The Batty Function described for Column D is also related to the first one. How did you determine the equation?
   d. How did you determine the equation that represents the Batty Function described for Column F?
   e. Are any of the equations above related to linear functions? How?

5. Terri wants to build a rectangular pen for her goats. A barn will form the wall on one side of the pen. A garage will form another wall next to the barn. No fencing is needed on these sides of the pen. She has 15 yards of fencing to use to make the pen. Help Terri determine the dimensions that will make the area of the pen as large as possible. (Recall: For a rectangle, Area = Length x Width.)



   a. Explain how the figure at right is related to this context.
   b. Some possible lengths to use all 15 yards of fencing are shown in the table. Complete the table for other dimensions.

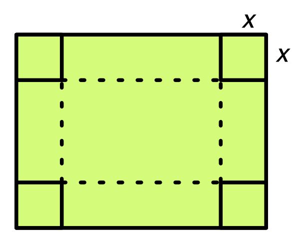
x	1	2	3	4	5				
15 – *x*	14	13	12						
Area of pen	14	26	36						

   c. What patterns do you notice in the table?
   d. Write an equation that can be used to find the area of the pen based on the side lengths of the pen.
   e. Graph the equation. For what value of x is the area largest? How do you know?
   f. Is the area function linear or non-linear function? Why do you think so?

6. Holly wants to build a rectangular pen for her horses. One side of the pen will share the wall of the stable. The other three sides of the pen will be fenced. Her stable is 100 feet on one side. She can afford 150 feet of fencing.
   a. Draw a diagram to illustrate this situation.
   b. Find at least 5 pairs of dimensions that will use all of the fencing Holly can afford. Build an organized table with the dimensions. Also find the area of the pen in each case.
   c. Determine an equation that can be used to find the area of the pen.
   d. Determine the dimensions that give the largest area for the pen.
   e. What is the largest area?
   f. Is the area function linear or non-linear function? Why do you think so?

7. a. Make an open top box by cutting out squares and folding up the corners of an 8.5 by 11 sheet of paper. See figure below.
   b. Record the dimensions and the volume of the box in the table below. (Recall: For a rectangular prism (a box), Volume = Height x Length x Width.)

Depth, x	Height	Width	Area of Base	Volume
0	8.5	11		
1				
2				
3				
4				
x				



   c. Find equations for the height of the box, the width of the box, the area of the base, and the volume of the box. Write them in the last row of the table. How do you know your equations are correct? Use the figure provided to show how you found the equations.
   d. What is the largest side length of a square that can be cut from each corner of the paper and still be able to make a box? Explain.
   e. What are the dimensions of the box with smallest volume? What is the smallest volume? Explain.
   f. What are the dimensions of the box with largest volume? What is the largest volume of a box created as directed in this problem? How do you know?
   g. Build the box with largest volume. Are you surprised by its appearance?
   h. Is the volume function linear or non-linear function? Why do you think so? Is it the same type of non-linear function as the functions in problems 5 and 6? Explain.

8. a. Complete the table to show the sums of consecutive even numbers beginning with 2.

Number of consecutive even numbers	1	2	3	4	5	6	7	8
Sum of consecutive even numbers	2	2 + 4 = 6	2 + 4 + 6 =					

b. Graph the data in the table.
c. Find an equation to fit the table.
d. Is the function linear or non-linear function? Why do you think so?
e. What is the domain of the function given the context?
f. What is the range of the function given the context?
g. What is the sum of the first 100 even numbers?
h. Will the sum of consecutive even numbers ever be 100?
i. Repeat problem 8 for the sums of consecutive odd numbers beginning with 1.
j. Have you seen either of these patterns before? Where?

4.2 Functions Arising from the Products of Two Lines

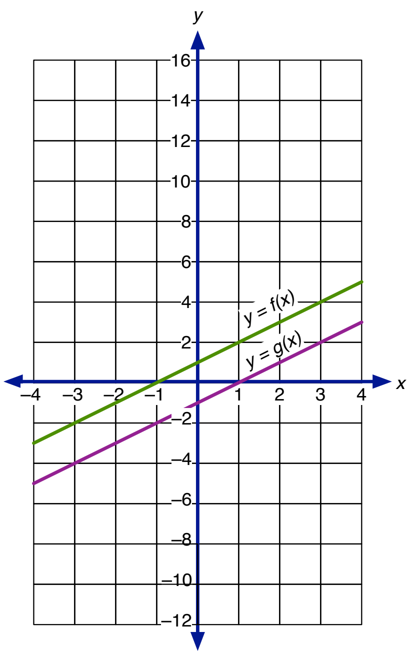
You have likely noticed that we are now investigating a new type of function. The graphs have an obvious curve to them; they are not straight lines. As you investigate products of linear functions, think about *Batty Functions* and what you remember about non-linear functions from your previous work in mathematics courses.

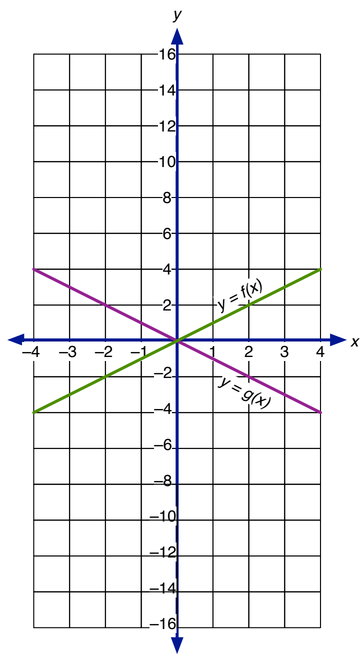
Activity: Investigating Products of Linear Functions

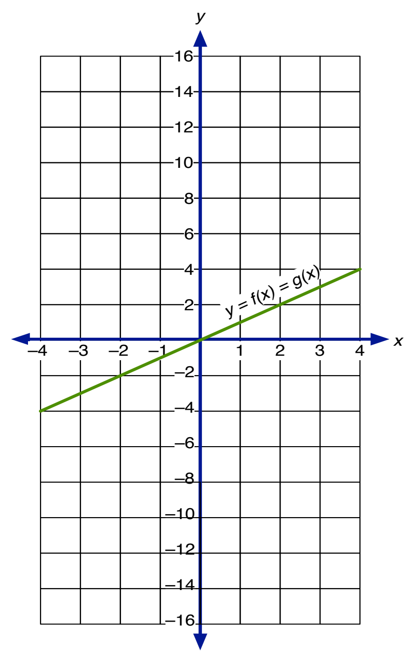
Solve the problems that follow to investigate the appearance of products of linear functions. Think about these questions as you work: To what family of functions do products of linear functions belong? What shape is the graph of this function family? (Note: For a review of function notation, please see pp. 122 and 123.)

1. Graphs of lines are provided in the figures below. Complete the following for each pair of graphs:
   a. For each value of x in the table, determine the values of $f(x)$, $g(x)$, and the product $h(x) = f(x) \bullet g(x)$.
   b. Plot the points $(x, h(x))$ on the graph.

c. Connect the points to show the graph of $y = h(x)$.







x	***f*(*x*)**	***g*(*x*)**	***h*(*x*) = *f*(*x*) • *g*(*x*)**
–4			
–3			
–2			
–1			
0			
1			
2			
3			

x	***f*(*x*)**	***g*(*x*)**	***h*(*x*) = *f*(*x*) • *g*(*x*)**
–4			
–3			
–2			
–1			
0			
1			
2			
3			

x	***f*(*x*)**	***g*(*x*)**	***h*(*x*) = *f*(*x*) • *g*(*x*)**
–4			
–3			
–2			
–1			
0			
1			
2			
3			

2. a. What observations can you make about the graph of $y = h(x)$ based on the graphs of $y = f(x)$ and $y = g(x)$? Explain why your observations make sense.
   b. What is the shape of the product graph? Why is the product graph shaped this way?
   c. What type of function arises from multiplying two linear functions together?

3. a. In order for a product of two numbers to be zero, what do you know about one or both of the numbers you are multiplying together?
   b. A function arising from the product of two lines has an x-intercept at $x = 3$. The equation is written as a product of two linear functions. What do you know about one of the linear functions? How does this information relate to problem 3a?

4. Through investigating the product of two lines, we are able to find an equation of a function from the equations of the lines that are multiplied together to form it.
   a. Find an equation for each of the products of the pairs of lines in problem 1.
   b. How are the x-intercepts of each line related to the x-intercepts of the product graph?
   c. How are the x-intercepts of each function related to its equation?

Activity: Multiplying Lines Electronically

Now that you have graphed the product of two lines by hand, explore the product of two lines using Desmos as you complete the student page, *Multiplying Lines with Desmos*. Do you get the same results? What more do you notice?

Multiplying Lines with Desmos

1. Set up Desmos to play with products of linear functions, as follows:
   a. Type $y = x - p$ into the first input line. Click on the p button to create a slider for p. Start the slider at $p = 0$.
   b. Type $y = x - q$ into the second input line. Click on the q button to create a slider for q. Start the slider at $q = 1$.
   c. Type $y = (x - p)(x - q)$ into the third input line.
   d. Type $y = x^2$ into the fourth input line.

2. Play with the p slider.
   a. How does the graph of the product of two lines change as you change p?
   b. Why does the change in p cause the product graph to appear as it does?
   c. What happens to the product graph when p is less than 0? Why does that make sense?
   d. From your previous mathematical experiences, what is the graph of the product called?

3. Play with the q slider until you can explain what is happening to the product graph and why it is happening. We call the graph of the product of two lines a parabola. A parabola is the graph of the quadratic function.

4. Edit the first input line to be $y = a(x - p)$. Create a slider for a. Edit the third input line to be $y = a(x - p)(x - q)$. Play with the a slider.
   a. For what values of a does the parabola appear to be wider than $y = x^2$? Why do you think a has this effect on the parabola?
   b. For problem 4a, is the parabola wider than $y = x^2$? Why or why not? Argue from the equations, not the appearance of the graphs.
   c. For what values of a does the parabola appear to be steeper than $y = x^2$?
   d. Explain what a does to the parabola.

5. Turn off $y = x^2$ (click on the colored button to the left of the equation). Reset the a-slider so that $a = 1$. Play with the sliders for p and q. Compare the lines to the parabola. What do the lines and the parabola always have in common no matter how you change p and q?

6. Continue to play with all three sliders, a, p, and q.
   a. Try each of these three possibilities:
      i. Both lines have positive slopes.
      ii. Both lines have negative slopes. (What additional change do you need to make?)
      iii. One line has a positive slope and the other line has a negative slope.
   b. How can you tell from the lines when a parabola created from the product of the lines will open upward? Reason from the equations of the lines and the parabola.
   c. How can you tell from the lines when a parabola created from the product of the lines will open downward? Reason from the equations of the lines and the parabola.
   d. How can you tell from the lines where the parabola will intersect the x-axis?
   e. How can you tell from the lines where the parabola will intersect the y-axis?
   f. What is a line of symmetry?
   g. How can you find the line of symmetry for a function that is the product of two lines? Why does this make sense?
   h. Each parabola has a highest or lowest point, called the vertex. What information about the vertex can you determine from the lines? Explain.

Activity: Polygraph for Parabolas

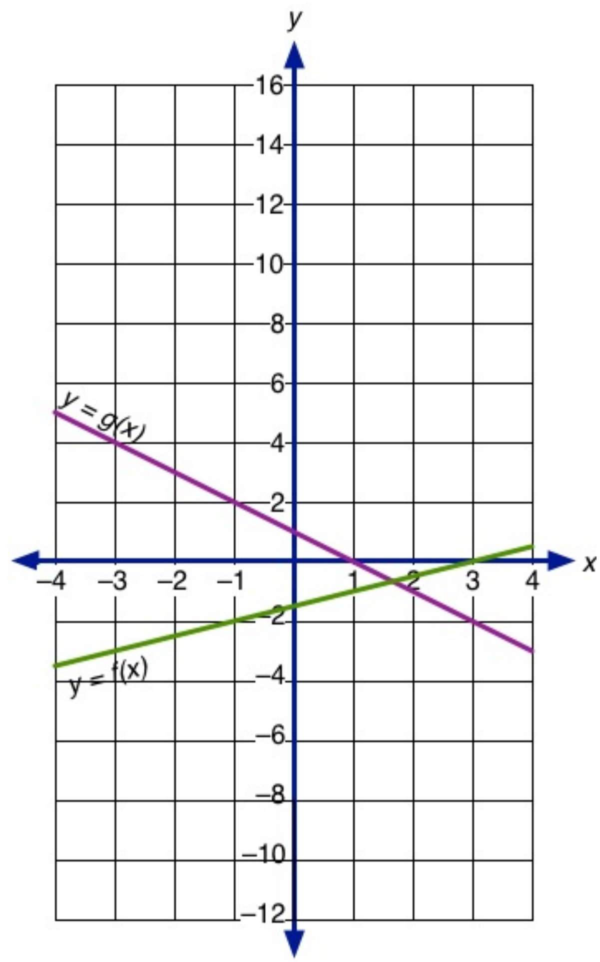
Now that we have learned something about how quadratic functions arise in context and their connection to linear functions, let's spend some time describing them. You teacher will provide a Class Code for the Desmos activity, *Polygraph: Parabolas* in the Quadratic Bundle on Teacher Desmos. You will be randomly paired with another classmate. Polygraph is a variation of the commercial game, *Guess Who?*. Play 2 rounds of the game or as directed by your teacher.

When time is called, your teacher will provide you another Class Code, this time for the Desmos activity, *Polygraph: Parabolas, Part 2*. Complete the activity, taking notes on vocabulary that will help you describe and discuss quadratic functions and their graphs more efficiently.

4.2 Homework

1. Study the graphs at right. Answer problems 1a through 1f for the lines provided.
   a. Complete the table. Do not estimate any of the coordinates.

x	$f(x)$	$g(x)$	$h(x) = f(x) \bullet g(x)$
–3			
–2			
–1			
0			
1			
2			
3			



   b. What is the x-intercept? Write the point as an ordered pair, (x, y).

      for $y = f(x)$ __________________
      for $y = g(x)$ __________________
      for $y = h(x)$ __________________

   c. How are the x-intercepts for $y = h(x)$ related to the x-intercepts of $y = f(x)$ and $y = g(x)$? Why does that make sense?
   d. From your table, draw an accurate graph of $y = h(x)$. Connect the points in a smooth curve. State everything you can about why the graph looks as it does.
   e. The vertex of a parabola is its highest or lowest point. Find the vertex for the function $y = h(x)$. How do you know you're correct?

f. Write an equation for each function. Do not simplify the equation for $y = h(x)$.

$y = f(x) =$ ____________________

$y = g(x) =$ ____________________

$y = h(x) =$ ____________________

2. The table below includes several quadratic functions, each provided as a product of two linear factors. Most of the equations are in the form, $y = a(x - p)(x - q)$, the factored form of a quadratic function. Graph each function.
   a. Find the x-intercepts, y-intercept, and vertex for the quadratic graph (parabola). Record each point in the table as an ordered pair.
   b. Record the direction the parabola opens, Up or Down.

	Factored Form, $y = a(x - p)(x - q)$	x-intercepts		y-intercept	Vertex, (h, k)	Opens (Up or Down)
A	$y = (x)(x + 1)$					
B	$y = 2(x)(x - 1)$					
C	$y = -0.5(x)(x + 2)$					
D	$y = (-1)(x + 3)(x - 5)$					
E	$y = (-1)(x - 2)(x + 2)$					
F	$y = (-x + 2)(-2x - 4)$					

   c. The final equation is not yet in factored form though it shows two factors. Write the equation in factored form. What information about the quadratic function is more evident in factored form than in the form provided in the table?

3. a. How can you find the x-intercepts from the factored form of a quadratic function?
   b. How can you tell from the factored form of a quadratic function whether or not the graph of the product will open upward?
   c. How is the y-intercept of the quadratic function related to the linear factors?
   d. How can you find the vertex from the factored form of a quadratic function?

4. Each quadratic function is given in standard form, $y = ax^2 + bx + c$. For each function:
   a. Graph the quadratic function.
   b. Complete all but the last column of the table. Record points as ordered pairs.

	Standard Form $y = ax^2 + bx + c$	x-intercepts		y-intercept	Vertex, (h, k)	Opens (Up or Down)	Factored Form $y = a(x - p)(x - q)$
A	$y = x^2 + 5x + 6$						
B	$y = -x^2 - 5x - 6$						
C	$y = x^2 - x - 2$						

D	$y = 2x^2 - 4x - 6$						
E	$y = -0.5x^2 + 3x - 4$						
F	$y = x^2 - 1$						

c. Suppose one x-intercept of a parabola is at point (4, 0). Write the equation of a line that has the same x-intercept. Explain how you know your equation is correct.

d. Determine a linear equation that shares one of the x-intercepts. Write the equation and graph the line with the parabola. Repeat for the other x-intercept. If a line does not share an x-intercept with the parabola, think more about the equation of the line, fix it, and re-graph.

e. Graph the product of the lines you found in problem 4d. How does the product compare with the graph of the quadratic function given in standard form? If the graphs are not the same, adjust this equation so the graphs coincide. Compare the equation in factored form with the equation in standard form. What do these equations have in common?

f. Two of the equations in the first table could have been factored differently than they were. Study both examples to see the possibilities:

$$y = 2(x)(x - 1) = (2x)(x - 1) = (x)(2x - 2)$$
$$y = -0.5(x)(x + 2) = (-0.5x)(x + 2) = (x)(-0.5x - 1)$$

In each example, the first version is considered to be the factored form of the equation. For each equation in this table, determine the factored form and write it in the table. Describe how to find the factored form of a quadratic function.

5. a. Some students say that the vertex of a parabola coincides with the intersection of the two lines that are multiplied together to find the quadratic function. Is this a true statement? Why or why not?

   b. Some students remember from previous mathematics classes that the line of symmetry for a quadratic function can be found using the expression, $\frac{-b}{2a}$. Where do a and b come from? If we don't have an equation for the quadratic function, is this method useful?

   c. Are there any quadratic functions that cannot be factored? If not, why not? If so, give three examples that cannot be factored and explain why they are not factorable.

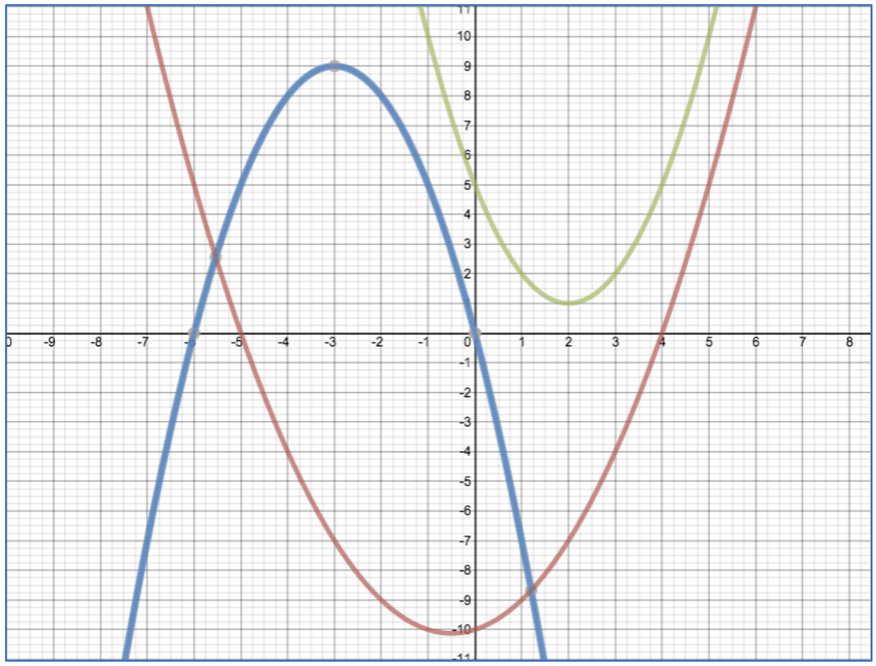
6. a. How many quadratic functions can share the points (1, 0) and (–3, 0)? How do you know? Give examples of equations that contain these two points.

   b. How many points must you know in order to fit them with a quadratic function? Why do you think so?

   c. Suppose three points all lie on the same line. Is it reasonable to exactly fit the points with a quadratic function? Why or why not?

7. Find the factored form of the equation, $y = -3x^2 + 12x + 15$. Find out everything you can about this function (x-intercepts, y-intercept, vertex, etc.)

8. Study the parabolas in the figure at right.
   a. If possible, find an equation for each graph. Explain how you know each equation is correct.
   b. If you cannot find an equation for a graph, explain why it is not possible with your current knowledge of quadratic functions.



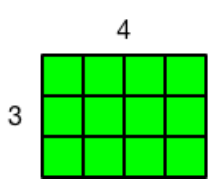
4.3 Multiplying Linear Factors and Factoring Quadratic Expressions

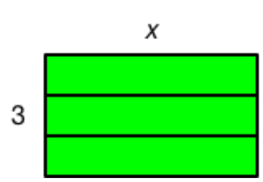
This lesson is designed to help learners refresh their conceptual understanding of multiplying terms and simplifying the result. This lesson helps you build on your understanding of area and multiplication of numbers to simplify expressions such as $x(x + 3)$, $5x(x - 1)$, $(x + 5)(x + 3)$, $(x - 2)(x + 2)$, or $(2x + 1)(x - 4)$. If you are proficient at multiplying these expressions together, this section is optional. If you find this work challenging, revisit *How to Learn Math for Students*, Lesson 5, video 2: *Drawing and Representing*. Multiplying binomials is modeled from 2:20–3:35 minutes in this video.

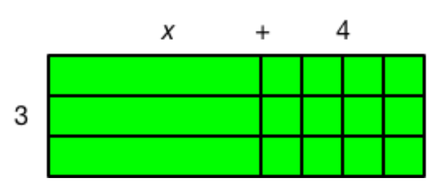
Activity: Conceptual Underpinnings of Multiplying Binomials

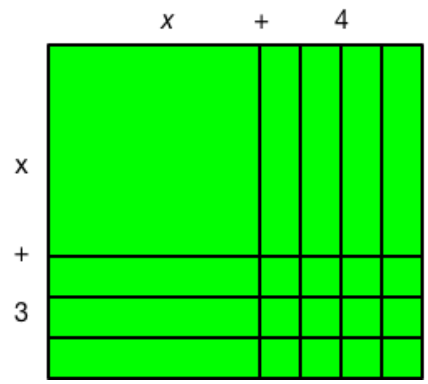
Determine the area of the first rectangle. It might help to think about the number of tiles you need to cover the surface of a coffee table with the dimensions shown.

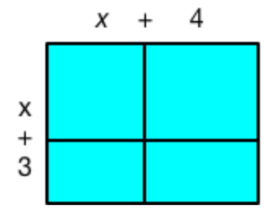
Study the remaining examples. Determine the area of the other rectangles whose side lengths are shown. The first four examples are shown using an algebra tile model for area. The final example uses a pseudo-area model that is often referred to as 'the box method.' It represents the same product as the algebra tile model to its left.











Note that the algebra tile model preserves the scale of the side lengths and areas. The box method does not necessarily represent scale accurately though some effort is made to keep the dimensions of regions with the same dimensions equal in length. In this case, the dimensions representing x for both horizontal and vertical sides are equal. Which of these models do you prefer, if any?

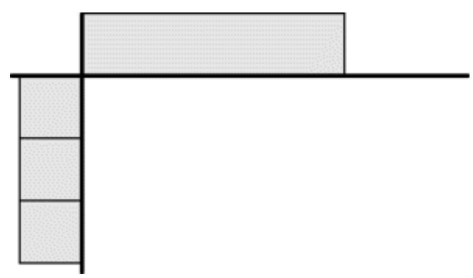
Complete the student page, Using Algebra Tiles to Multiply Linear Factors and Factor Quadratic Expressions, using the model you prefer (make minor adaptations as needed for the box method). Be ready to ask questions over any problems you find challenging.

Use both graph paper and algebra tiles to complete the student page, Using Algebra Tiles to Multiply Linear Expressions and Factor Quadratic Expressions. If you do not have algebra tiles available, use an algebra tile app online or on an electronic tablet. The apps, Multiplication Activities, on this website can help you complete this student page: http://media.mivu.org/mvu_pd/a4a/homework/index.html. Alternatively, you can use a hand drawn area model to help you find the product of two linear expressions.

Using Algebra Tiles to Multiply Linear Factors and Factor Quadratic Expressions

Materials:
- one set of algebra tiles and one algebra tile frame for each pair of students
- one sheet of graph or grid paper for each pair of students

1. On a piece of graph paper, draw a rectangle with dimensions 6 units by 4 units. What is the area of the rectangle? How does the graph paper help you determine the area?
2. Draw a rectangle to show the area of a rectangle with one dimension of 3 units and the other dimension of 4 + 5 units.
   a. Show the area the rectangle as the sum of the area of two regions.
   b. Show the area as a single result.
   c. What property is illustrated by the example?
3. You can use algebra tiles to visualize the product of two expressions containing variables.
   a. One of the algebra tiles has two different side lengths. The short side length is 1 unit; the longer side length is x. Trace the tile. Label its side lengths.
   b. What is the area of the tile in problem 3a?
   c. Use the tile in problem 3a to determine the side lengths of the other two sizes of algebra tiles. Trace each tile. Label each side length.
   d. For each tile in problem 3c, find the area and write the number in the center of each tile you drew.
4. a. To find the product of x and 4, what algebra tile pieces are needed to represent x and 4?
   b. What is the area of a rectangle whose dimensions are x and 4? Show this area with algebra tiles and label the dimensions of the sides. Draw a picture.
5. a. The algebra tile frame at right shows the dimensions of x and 3 on the outside of the frame (indicated with heavy black lines). Notice that one factor is placed along the upper edge of the frame; the other factor is placed along the left edge of the frame. Fill in the rectangle so the dimensions of the pieces you use match with side lengths of the pieces along the outside of the frame.



   b. Find the sum of the areas of the pieces forming the interior of the rectangle to determine the product $x \bullet 3$.
6. a. Suppose you want to find the product of x and $x + 2$. What algebra tile pieces are needed to represent x and $x + 2$?
   b. Use your work in problem 5a to place the algebra tiles along the edge of the frame to find the product of x and $x + 2$. Use algebra tiles to fill in a rectangle so the dimensions of the pieces you use match with the dimensions of the pieces along the outside of the frame. Draw a picture of your work with tiles.
   c. Find the sum of the areas of the pieces forming the interior of the rectangle to determine the product of x and $x + 2$.

d. Check that your product for x and $x + 2$ makes sense as follows:
   - Substitute $x = 4$ into x and $x + 2$. Find the product of these two values.
   - Substitute $x = 4$ into the product you found in problem 6c.

   Do your results agree? If not, revisit how you constructed your rectangle in problem 6c and check that your revised product works.

7. a. Outline a rectangle with dimensions $x + 3$ and $x + 4$. Fill in the rectangle with algebra tile pieces to find the product $(x + 3)(x + 4)$.
   b. Check your result by choosing a value for $x \neq 0$ and substituting the value into $x + 3$, $x + 4$, the product of these two binomials, and the algebraic expression you obtained for $(x + 3)(x + 4)$.
   c. Use algebra tiles to illustrate the product of $(2x + 1)(3x + 2)$. Write the product.
8. Algebra tiles can also be used to work backwards from a product to find, if possible, two factors that generate that product.
   a. Collect the pieces for $x^2 + 6x + 8$.
   b. Arrange the pieces to form a rectangle. The factors are the dimensions of the rectangle showing the area $x^2 + 6x + 8$. What are the factors?
   c. Use the process you used in problem 8b to find the factors of $2x^2 + 5x + 2$.
   d. How do the algebra tile pieces help you think about factoring?

Adapted from Beckmann, C.E., Thompson, D.R., and Rubenstein, R.N., *Teaching and Learning High School Mathematics*, Hoboken, NJ: John Wiley & Sons, Inc., 2010, pp. 258–9.

With the other members of your group, create a few more problems in the form $ax^2 + bx + c$ that other groups can arrange into rectangles to find the factors. Pass the expressions to other groups to see if they can work backwards from the product its factors.

4.3 Homework

1. a. In Lesson 1.6, you worked with algebra tiles to combine like terms. Revisit problems 2 and 3 to remind you of this work.
   b. For each expression:
      - Draw algebra tile representations for each term.
      - Combine like terms using the algebra tile representations. Show and explain your work.
      - Write a simplified expression for the combined algebra tile representations.
        i. $(x^2 + 3x + 2) + (2x^2 + x - 3)$
        ii. $(x^2 - 3x + 2) + (2x^2 + x - 3)$
        iii. $(2x^2 + x + 3) - (x^2 + 3x + 2)$
        iv. $(2x^2 + x - 3) - (x^2 + 3x + 2)$
2. There are free apps online to help you make sense of multiplying factors and factoring quadratic expressions using algebra tiles. Use the app on this website, http://media.mivu.org/mvu_pd/a4a/homework/index.html. Read the introductory pages carefully. These pages tell you how to use the apps. Navigate through the introduction using the arrow in the upper right corner. Watch the videos and play with the apps for

Multiplication Activities. Look for *Multiplication* and *Factoring Trinomials*. Solve at least 2 problems at each difficulty level, until you conceptually understand the link between the algebra tiles and the expressions they represent.

3. Use what you learned in Lesson 4.3 and Lesson 4.3 Homework to solve the problems on the student page, *Multiplying and Factoring with Algebra Tiles*. Be ready to ask questions in class. The student page can be found after this homework problem set. If you choose to use the box method, think carefully about which terms arise in which parts of the box to help you solve problems 2, 3, and 4.

4. a. Find the standard form of each quadratic function in the table in problem 2 of the Lesson 4.2 Homework. Record the equations in the table.
   b. Verify that each standard form equation is equivalent to the factored form given in the table. Describe your verification process.

Multiplying and Factoring with Algebra Tiles

Materials: One set of algebra tiles and one algebra tile frame for each pair of students

$(x + 1)(x + 2)$	$(2x)(x + 2)$
$(2x + 1)(x + 2)$	$(x + 1)(x - 2)$
$(x - 1)(x - 2)$	$(2x + 1)(x - 2)$

1. For each expression in the table above:
   a. In the space provided, draw a picture of an area model to show the product of the two terms.
   b. Multiply the factors together to get a sum of terms.
   c. Replace x with 7 in both original and final expressions. Check to see that you get the same result in both cases.

2. Use algebra tiles to work backwards from a product to find two factors that generate that product.
   a. Collect the pieces for $x^2 + 7x + 12$.
   b. Arrange the pieces to form a rectangle. The dimensions of the rectangle are the factors of $x^2 + 7x + 12$. What are the factors?
   c. Repeat problem 2b using the pieces for $x^2 + 8x + 12$ and then for $x^2 + 13x + 12$.
   d. Compare your work for all three expressions. What do you notice?
   e. How do the algebra tile pieces help you think about factoring?

3. Use algebra tiles to factor these expressions.
   a. $2x^2 + 5x + 3$ b. $2x^2 + 7x + 3$
   c. How does your work change from one to the next? Discuss the arrangements of tiles. Which tiles should you position first?

4. Use what you learned in problems 2 and 3 to factor the following expressions:
   a. $x^2 + 4x + 4$ b. $x^2 - 2x - 3$ c. $x^2 - 4$

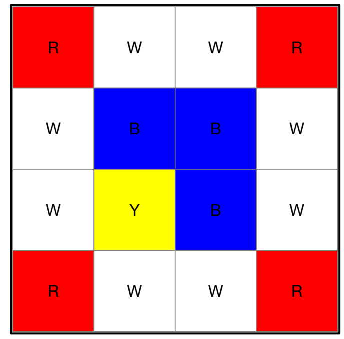
4.4 Rates of Change of Quadratic Functions

Activity: Quadratic Quilts and Rates of Change for $y = ax^2 + bx + c$

Because the area of bedding is measured in square inches, it might not be surprising that quilts provide a context in which to study quadratic functions. We use the context of making quilts to also study rate of change for quadratic functions. Recall that the rate of change of a linear function was always constant. We will notice that quadratic functions change differently. Explore the student page, *Quadratic Quilts*, to explore quadratic rate of change.

Exploring Quadratic Quilts

1. Esther designs and makes quilts. The completed square at right is called a quilt block. She makes different sizes of square quilts using the same pattern and colors for each block in each quilt. To ensure that each quilt block matches, she purchases all of the fabric she needs at the same time.



   a. The quilt block at right has dimensions 1×1. Determine the number of same-size small square pieces of each color Esther needs for the quilt block.

   b. How many small square pieces of each color does she need for each of the different projects listed? Complete the table.

Quilt Dimensions in Blocks	Project Type	x	Number of Quilt Blocks	Number of Yellow (Y) Squares	Number of Blue (B) Squares	Number of Red (R) Squares	Number of White (W) Squares
1×1	Potholder	1	1	1	3	4	8
2×2	Nightstand cover	2	4		12		
3×3	Doll blanket	3	9				
4×4	Baby blanket	4					
5×5	Lap blanket	5					
6×6	Tablecloth	6					
7×7	Twin bed cover	7					

   c. Consider one color at a time. What is the relationship between the quilt's dimension, x, and the number of individual squares needed to make the quilt?

   d. Determine an equation for each relationship in problem 1c.

2. a. Complete the table at right. Let $f(x) = x^2$.

   b. Describe how $f(x) = x^2$ is changing as x increases.

   c. Describe any patterns or symmetry you see in the table.

   d. Without graphing the function, state what you know about the graph from your table and responses to problems 2b and 2c.

   e. Sketch the graph without plotting points.

x	$y = f(x)$	$g(x) = f(x) - f(x-1)$	$g(x) - g(x-1)$
–3			
–2			
–1			
0			
1			
2			
3			
4			

3. a. Complete a table like the one at right for each equation you found in problem 1d.
   b. Complete a table like the one at right for $f(x) = ax^2 + bx + c$.
   c. What patterns do you see in the first differences, $f(x) - f(x-1)$?
   d. What patterns do you see in the second differences, $g(x) - g(x-1)$?
   e. What do these patterns suggest about the rates of change of all quadratic functions?

x	$y = f(x)$	$g(x) = f(x) - f(x-1)$	$g(x) - g(x-1)$
–3			
–2			
–1			
0			
1			
2			
3			
4			

4. a. Graph the parent function, $f(x) = x^2$.
   b. What are the coordinates of the y-intercept? x-intercept?
   c. Describe the shape of the graph.
   d. Why does the graph look as it does? Compare the table values to the graph.
   e. What new information is provided by the graph that is not evident in the table?
   f. What information is provided in the table that is not evident in the graph?

5. The vertex of a quadratic function is the point at which it changes direction.

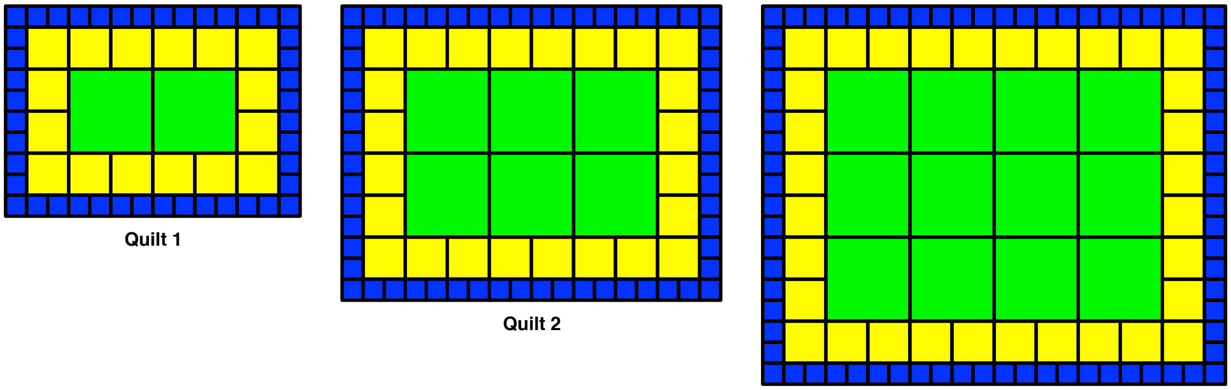
a. What are the coordinates of the vertex of $f(x) = x^2$?
   b. Graph each equation from problem 1d. How are the graphs related to $f(x) = x^2$?
   c. For each equation in problem 1d, what is the vertex for each graph?

6. a. Two other important points on the graph of $f(x) = x^2$ are (1, 1) and (−1, 1). Graph each equation from problem 1d for x in [−5, 5]. What points correspond with (1, 1) and (−1, 1) for these quadratic function family members? Why do you think so?
   b. For functions in the quadratic function family, why are the points that correspond to (0, 0), (1, 1), and (–1, 1) on $f(x) = x^2$ important?
   c. How might these points help you determine the equation of the function represented by the table or graph? Explain.

Adapted from Beckmann, C.E., Thompson, D.R., and Rubenstein, R.N., Teaching and Learning High School Mathematics, Hoboken, NJ: John Wiley & Sons, Inc., 2010, pp. 252–4.

4.4 Homework

1. Tiffany makes quilts for different purposes. All of the quilts are made using the same three sizes of squares: 8 inch, 4 inch, and 2 inch. Study each quilt. The first 3 sizes of quilts are shown. Tiffany makes larger sizes but always in the same pattern as these.
   a. Determine the number of each size square needed to make the quilts. Fill in the table on p. 166.

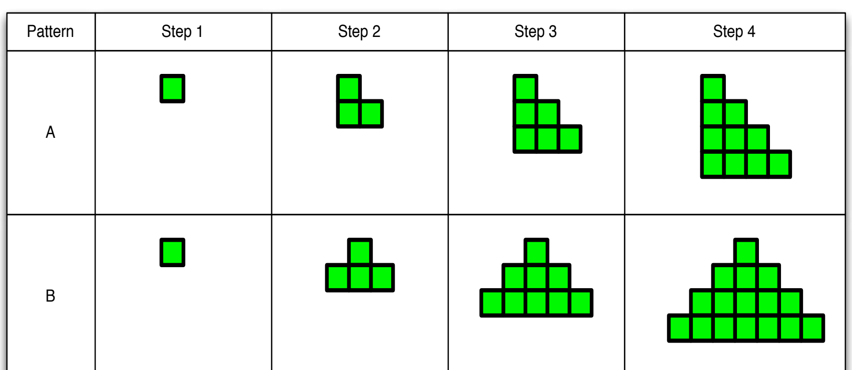


Quilt 1 Quilt 2 Quilt 3

b. Extend the quilt pattern to other sizes of quilts. Complete the table.

Quilt Number	Number of 8-inch Squares	Number of 4-inch Squares	Number of 2-inch Squares
1			
2			
3			
4			
5			
6			
Equation			
Function Family			
Explain your function family choice.			

2. Study Patterns A and B in the figure at right. Visualize the patterns being built with blocks.



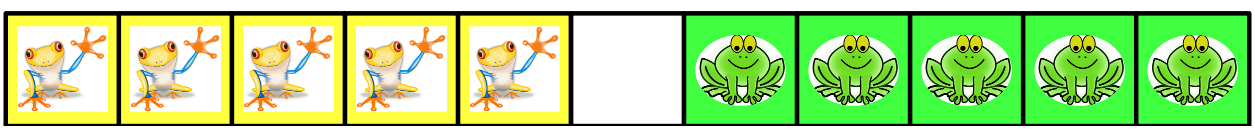
To get Pattern C (not shown), double all but the center column of Pattern B. To build Pattern C with blocks, build 4 sets of stair-steps around a central tower so that the stair-steps are sticking out at right angles to each other.

a. Record how many blocks you need to build each step of each pattern.

Step Number	1	2	3	4	5	x
Pattern A: Number of Squares						
Pattern B: Number of Squares						
Pattern C: Number of Squares						

b. Plot the data using different colors for each graph. Label the graphs A, B, and C. Indicate the scales on the axes.
c. For each set of data, determine if the data represents a linear function, quadratic function, or some other type of function. Explain your choice. (Do not depend solely on the graph. Graphs can be deceiving.)
d. You have seen the data for Patterns A and B before. Find equations to fit both sets of data. Verify that each equation fits at least 3 points of the corresponding data set.
e. For Pattern C, determine the value of a in the equation y = ax2 + bx + c. Explain how you know you are right.
f. Plot the graph of the equation y = ax2 using the value of a you found in problem 2e.
g. Does the graph fit the data? If not, is the graph too high or too low?
h. Try to adjust your equation to fit the data. Play with values of b and values of c until your graph seems to fit the data. Record the equation.
i. Try at least 3 data points for Pattern C to see if the equation you found in problem 2h works. Adjust the equation further if needed.

3. In Leapfrogs, two different colors of objects are arranged in the colored spaces to represent two types of frogs on a log.



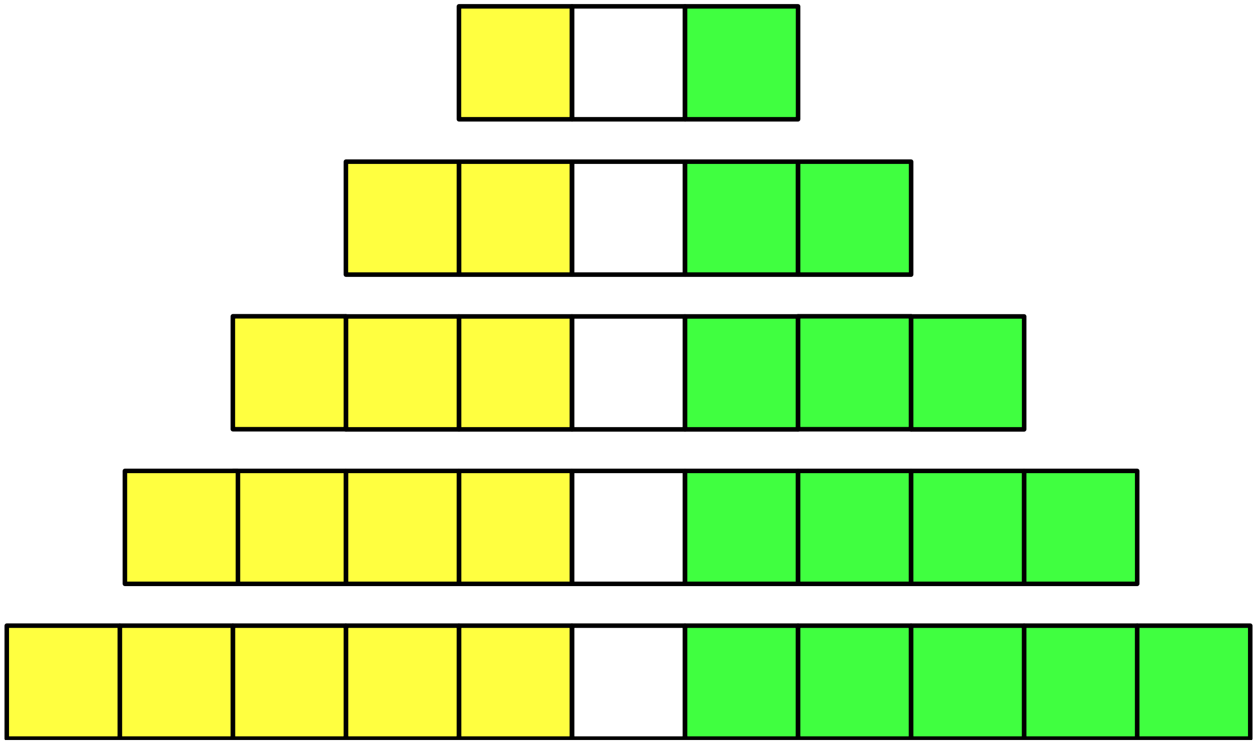
Your job is to interchange the frogs on one end of the log with the frogs on the other end of the log. There are two restrictions:

- A frog can move into an adjacent empty space.
- A frog can jump over a frog of the other color.

You need 5 each of two different types of small objects (for example, pennies and dimes, cubes of 2 colors, paper clips and coins) and the Leapfrogs gameboard below. Objects on the yellow spaces are referred to as Yellow Frogs. Objects on the green spaces are referred to as Green Frogs. Using the same number of frogs on each end of the board, find the least number of moves needed to interchange the colors of frogs as the number of frogs increases.

Adapted from: Mason, John, Leone Burton, and Kaye Stacey. *Thinking Mathematically* (Revised Edition). Harlowe, UK: Prentice Hall, 1985, p. 57.

Leapfrogs Gameboards



a. Record the **least number of moves** needed to interchange the two colors of frogs.

Number of Yellow Frogs	1	2	3	4	5	F
Least Number of Moves Needed to Interchange Frogs						

b. Plot the data. To what function family do the data belong? Why do you think so? (Your analysis must include more than looking at the graph.)

c. Find an equation that fits the data. What is the least number of moves, *M*, needed to interchange *F* frogs on each end of the log? Use the process in problem 2e through 2j.

4. a. What is the equation to find the area of a circle given its radius?
   b. How is the diameter of a circle related to its radius?
   c. Write the equation for the area of a circle in terms of its diameter.
   d. Complete Columns C, E, and G.

A	B	C	D	E	F	G
Pizza Size	Diameter, D (in inches)	Inches^2 of pizza	Base price, B, for Cheese Pizza	Price of cheese pizza per inch^2	Single Topping Price, T	Topping Price per inch^2
Small	10		7		1.00	
Medium	12		9		1.25	
Large	14		11		1.50	
Extra Large	16		13		1.75	
Equation	D	A(D) =	B(D) =		T(D) =	

e. The cheese pizza prices for a local pizza parlor are shown in Column D. Is the pizza parlor’s method for pricing pizzas reasonable? Why or why not? Discuss area versus cheese pizza price in your response.
f. Suggest a better equation to use to set the price for each size cheese pizza so the customer is paying the same amount for each square inch of cheese pizza regardless of the pizza’s diameter.
g. Use your equation from problem 4f to find prices for each cheese pizza in the table.
h. Consider the topping price per square inch in Column G. Is the pizza parlor’s method for pricing toppings reasonable? Discuss area versus per topping price in your response.
i. Suggest an equation to set the topping price for each size pizza so the customer is paying the same amount per topping for each square inch of pizza regardless of the pizza’s diameter.
j. Use your equation from problem 4i to find prices for added toppings.
k. To what function family does each equation in Columns C, G, and E belong? How do you know?

5. Revisit the Tile Patterns in Lesson 2.9 and Lesson 2.9 Homework.
   a. Determine which tile patterns are quadratic.
   b. Explain how you can tell from the arrangement of tiles.
   c. Explain how you can tell from the table.
   d. Find an equation for each quadratic pattern. Use the process in problems 2e–2i, if needed.

6. As with linear functions, you can use an electronic graphing tool to fit regression equations to the data. (See Lesson 3.4 for a reminder.)
   a. Revisit each of the problems above. Fit a quadratic regression equation to each data set.
   b. For each data set, compare the regression equation with the equation you found in other ways. How close was your equation to the regression equation? Do you need to adjust your thinking or were your equations correct?
   c. A student page describing how to find the regression equation using a Texas Instruments (TI) graphing calculator follows. If you need a reminder of how to use Desmos to find a regression equation, click on the ? icon (Help) in the upper right corner of the Desmos screen, click on Regressions, then follow the directions provided.

Entering, Graphing, and Plotting Data
Finding Regression Equations

Entering the Data

Use your graphing calculator to enter the data:

- Press STAT then choose 1: Edit…
- Enter the values for the independent variable (domain) into L1.
- Enter the values for the dependent variable (range) into L2.

Graphing the Data

- Press STAT PLOT (2ND Y=) then choose 1: Plot1… Press ENTER
- Settings:
  - On,
  - Scatter plot (first graph type)
  - Xlist: L1
  - Ylist: L2
  - Mark: +
- Press GRAPH

Setting the Viewing Window

- Press WINDOW
- Use the table to determine
- Xmin, Xmax, Xscl
- Ymin, Ymax, Yscl

OR

- Press ZOOM then scroll down to 9: ZoomStat, Press ENTER

Fitting a Function to the Data

- Press STAT
- Move the cursor to the right to highlight CALC
- Scroll down to the function type you want to use, for example, 5: QuadReg, then Press ENTER (to fit a quadratic function)
- Indicate the lists of the data separated by commas, for example, L_1, L_2.
- Choose where you want to place the function equation, for example, Y_1. Press VARS, move the cursor to the right to highlight Y-VARS, Press ENTER to choose 1: Function, scroll to highlight the Y-variable you want, then press ENTER again.
- The screen should read: 5: QuadReg L_1, L_2, Y_1. Press ENTER (to fit a quadratic function)
- The regression equation will be in Y_1 or whatever Y-variable you chose.
- Press GRAPH to plot the regression equation with the data.

4.5 Function Families

In Chapters 2 and 3, we saw that linear functions have very predictable behaviors depending on the values of m and b in the equation, y = mx + b. All linear functions are members of the linear function family. Each function family has a parent function, the member of the function family in its most basic form. The parent of the linear function family is the function y = x or in function notation, f(x) = x. In this form, we see m = 1 and b = 0. In Lesson 4.5, we examine other families of functions.

Activity: Transformation of Functions

In this activity, we examine the absolute value function family whose parent function is f(x) = |x| and the quadratic function family whose parent function is f(x) = x2. To investigate these function families, we need three different parameters, a, h, and k. By changing each parameter, we will see how changes in a, h, and k affect the graphs of the parent functions.

To begin, we create a large coordinate plane on the floor with both x- and y-axes labeled from –10 to 10. Volunteers use string to mark the x- and y-axes and sticky notes to label the axes. (Use 3 x 3 inch sticky notes or larger to label the axes). Make the grid large enough so that volunteers can stand next to each other on the x-axis. A tiled floor is very helpful!

We also need to prepare a set of hang tags that volunteers can wear. Tags can be made from light color cardstock. Punch a hole in the center of a short side, thread string, yarn, or ribbon through the hole so the tag can hang round a volunteer's neck. Label each tag with values of x between –10 and 10. It is helpful if numbers are not all the same distance apart and if the positive numbers chosen are not all the same as the absolute value of the negative numbers chosen. A possible set of numbered tags might include: –10, –7, –4, –1, 0, 2, 5, 8, and 10.

Let's begin:

1. Each of 9 volunteers wears a tag with a number on it. The number on the tag represents your assigned value of *x.*
2. Stand on the *x*-axis at the point, (*x*, 0), where *x* is the number on your tag.
3. Move to the ordered pair (*x*, |*x*|). Tape a colored cord to the floor to mark the graph you're standing on. You have just created the graph of *y* = |*x*|. Discuss the appearance of the graph.
4. At your teacher's request, transform *y* = |*x*| in various ways. Consider the equation,

$$y = a \bullet \left| x - h \right| + k$$

   a. Repeat directions 2 and 3. The point to which you move is the ordered pair indicated in the table, as guided by your teacher. Use a different color of string for each graph.
   b. What are the values of a, h, and k for each graph? Write them in the table.
   c. Explain WHY the new graph is located where it is. Use the equation and the graph in your explanation. Compare each new graph to y = |x| and other related graphs.
   d. In the final column, write the equation of the function defined by each ordered pair.

Move to:	a	h	k	Equation
$(x, \|x\|)$				
$(x, \|x\| + 1)$				
$(x, \|x\| - 2)$				
$(x, -\|x\|)$				
$(x, 0.5\bullet\|x\|)$				
$(x, \|x-2\|)$				
$(x, \|x+1\|)$				
$(x, 0.5\bullet\|x-2\|)$				
$(x, 0.5\bullet\|x\| - 2)$				

Return the room to its original condition. Use Desmos to explore transformations of the parent quadratic function, $y = x^2$.

5. Continue the investigation using Desmos.
   a. Enter $y = a(x - h)^2 + k$ into the entry line of Desmos. This is another important form of a quadratic function.
   b. Set up sliders for a, h, *and* k.
   c. Play with k first. What does k do? Why does the graph change as it does? Explain from the equation.
   d. Play with a next. What does a do? Why does the graph change as it does? Explain from the equation.
   e. Play with h. What does h do? Why does the graph change as it does? Explain from the equation.

6. a. What is significant about h and k? What is significant about a?
   b. This form of the quadratic equation, $y = a(x - h)^2 + k$, is called the Vertex Form. Why do you think that is?

7. Your teacher will provide you a Class Code to complete the Desmos activity, *Card Sort: Parabolas* in the Teacher Desmos Quadratic Bundle. Complete the activity. The fourth slide is challenging because it does not include scales for any of the graphs. Describe how you knew how to sort each of the equations to fit the graphs provided. Provide illustrations in your description.

8. Quadratic functions can be written in the following forms*:

   Standard Form, $y = ax^2 + bx + c$

   Vertex Form, $y = a(x - h)^2 + k$

   Factored Form, $y = a(x - p)(x - q)$ (*Some quadratic functions cannot be written in this form. Do you know why?)

   a. What information about a quadratic function is most evident when it is given in:

      i. Standard Form?

      ii. Vertex Form?

      iii. Factored Form?

   b. What information about a quadratic function is the same for each form of the equation?

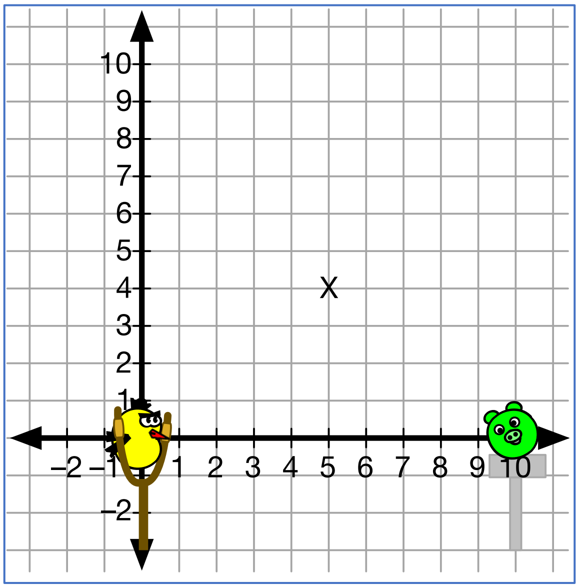
Activity: Angry Birds and Quadratic Functions

The video game, *Angry Birds*, has become so popular that now there are toys and movies created with the popular animals defending their homes from the visiting pigs. Why do we care? Because every time you launch an angry bird, the bird takes a parabolic flight path! To find the equation for the path of an angry bird, revisit your work with Transformations of Functions. Complete the student page, *Angry Birds and Quadratic Functions*, to think more deeply about transforming functions.

Angry Birds and Quadratic Functions

1. The graph below shows an Angry Bird waiting to be launched to hit a pig. The bird must go from the origin (0, 0), through the point (5, 4), and hit the pig located at (10, 0).

   What equation will allow the Angry Bird to take the right path? Write an equation. Show the work you do to find the equation in the space to the right of the picture.



2. Find both coordinates of the vertex of the quadratic function in problem 1. Write the vertex as an ordered pair. How do you know your solution is correct?
3. Suppose the Angry Bird is launched from (0, 2), the pig is sitting on a pedestal at (10, 2), and the bird's path must go through (5, 6). Adjust your equation in problem 1 for this new scenario. Explain how you know you are correct.
4. Find both coordinates of the vertex of the equation in problem 3. How do you know your solution is correct?
5. Suppose the Angry Bird was launched from (−1, 0). What equation would you need to use to hit a pig at (9, 0) and go through the point, (h, 4), where h is the x-coordinate of the vertex? Show your work.

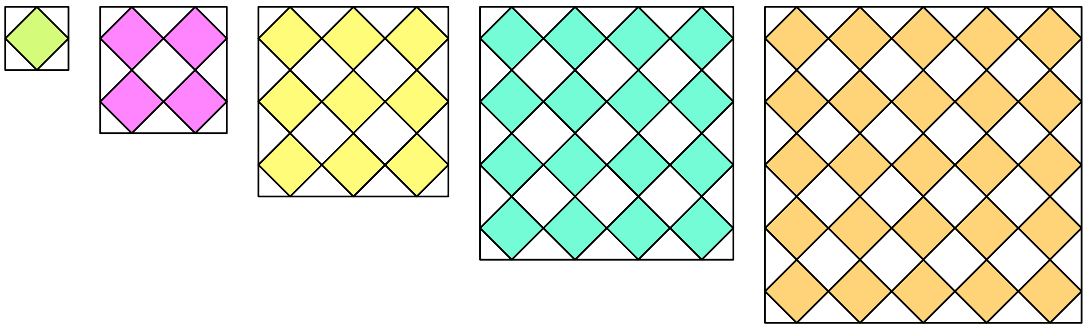
4.5 Homework

1. Your teacher will provide you a Class Code to complete the Desmos activity, *Match My Parabola* in the Teacher Desmos Quadratic Bundle. Describe how you were able to find equations for the points and graphs provided. What do you know about quadratic functions that helped you complete this activity?
2. Your teacher will provide you a Class Code to complete the Desmos activity, *Marbleslides: Parabolas* in the Teacher Desmos Quadratic Bundle. Complete the activity. This fun activity gives you a chance to test your ability to transform functions and earn stars.

3. Complete the student page, *Tiling Tables*, to extend your newly acquired skills with transformations of functions.

Tiling Tables

1. Kelly makes square tables and then tiles the tops. To cover the tabletops, she uses quarter tiles for each corner, half tiles along the sides, and full square tiles to fill in the rest. The first 5 sizes in her line of tabletops are shown below. Both side lengths of the first tabletop are 10 inches.



   a. Complete the table for the first 6 tabletops.

A	B	C	D	E
Table number, *n*	**Dimension of one side, *D***	**Number of quarter tiles, Q**	**Number of half tiles, *H***	**Number of full tiles, *F***
1	10	4	0	1
2			4	5
3				
4				
5				
6				
n				

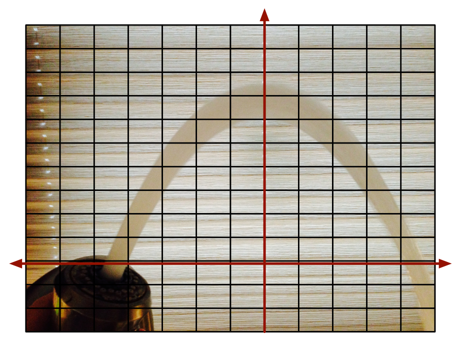
   b. Look for patterns in the table. Describe at least 3 patterns you see in the data.

2. Let the table number, *n*, be the independent variable. Plot the data in the columns indicated. Analyze the data in the table. Determine to what function family the data belongs. Explain how you know.
   a. Columns A and B:
   b. Columns A and C:
   c. Columns A and D:
   d. Columns A and E:

3. Find equations for each set of data in problem 2. Enter the equations into the last row of the table.

4. a. Write any quadratic equations in standard form.
   b. Is it possible to factor the quadratic equation? Why or why not?
   c. Subtract the constant from the standard form of the equation. Write this new equation.
   d. Compare the graph of the equation in problem 4a to the graph of the equation in problem 4c. How do these graphs compare with each other? How do you know?
   e. Factor the equation you found in problem 4c.
   f. Find the vertex of the equation in 4c from the factored form of the equation (in problem 4e).
   g. How does this vertex relate to the vertex of the equation in problem 4a? Find the vertex of the equation in problem 4a.
   h. Write the vertex form of the quadratic equation in problem 4a.
   i. Show that the equations in problems 4a and 4h are equivalent.

4. Consider the picture of water flowing from a spout at right. A coordinate grid has been imposed on the picture so that the *y*-axis goes through the vertex and the *x*-axis is the line on the blind just above the spout. Outline a curve that coincides with either the lower edge of the water flow or the upper edge of the water flow. Refer to your chosen curve as you answer the following questions. The grid increments by 1 unit for both *x* and *y* scales.



   a. To what function family does this curve belong? How do you know?
   b. Fit an equation to the graph you chose for the scale given. Determine the values of the parameters, a, h, and k in the equation, $y = a \bullet f(x - h) + k$. Explain how you know each parameter is correct.
   c. What is a sensible domain for the function that models this flow of water? Explain.
   d. What is a sensible range for the function that models this flow of water? Explain.
   e. How would the picture change if the water pressure increased? What parameters would change? How would each change? Why do you think so?
   f. If the water pressure decreased, how would the picture change? What parameters would change? How would each change? Why do you think so?
   g. Suppose $a = 2$, $h = 0$, and $k = 3$ for the equation, $y = a \bullet f(x - h) + k$. Describe the location of the spout and the water pressure as compared with the water pressure being used in the picture. Explain how you know you are correct.

5. a. Revisit Lesson 4.2 Homework problems 2 and 4. Find the vertex form for each of the quadratic equations in both tables. Record them next to the tables in problems 2 and 4.
   b. How can you verify that each form represents the same quadratic function?
   c. For the forms of the same function you entered into the tables in problems 2 and 4, verify that each form of the equation represents the same quadratic function.

d. Revisit Lesson 4.2 problem 7. Determine equations in vertex form for all three graphs. Explain your work. Use grid points to determine equations; do not estimate coordinates of any points.

4.6 What Do You Know about Functions?

In Lesson 2.8, you reviewed the big ideas of linear functions. From your work in Lessons 4.1 through 4.5, what do you think are the big ideas of quadratic functions? Brainstorm with your group. Write each new idea on an index card or a sticky note. Arrange the cards or sticky notes in a way that shows how ideas are linked to each other. Your arrangement is a concept map. When your group is satisfied with your concept map, compare your concept map to that of other groups. Add any ideas you missed. Discuss with the class any ideas that you think are misplaced or that could be placed differently.

Activity: Linear, Quadratic, and Other Functions Card Sort

Can you distinguish linear from quadratic and other functions? The function card sort helps you think about similarities and differences among functions. See if you can find cards with functions that belong to the same function family.

There are three groups of cards. Functions on Cards 1 through 5 are represented in context and as tables. Functions on Cards 6 through 10 are represented through practical situations. Functions on Cards 11 through 15 are represented through children's literature. Cards in the same function family do not represent the same equation in this case.

1. Print out and cut the cards apart.
2. Sort each card into one of the categories: Linear, Quadratic, or Other.
3. Explain how you know a card fits the category you chose and cannot fit another category.
4. For each card in the Linear and Quadratic categories, find an equation to fit the representation.
5. a. Do all of the cards you categorized as Other fit the same function family? Explain.
   b. If you think there is more than one function family represented by the cards you sorted as Other, sort the cards into piles that seem to represent the same function family. Describe each category. Explain why you sorted the cards as you did.
   c. Find equations for as many of the cards in the Other category as possible. Explain how you know each equation is correct.

Write the number of calories c as a function of the diameter d of a pepperoni pizza.

Diameter	Calories
8"	955
10"	1493
12"	2150
14"	2926
16"	3821

1

Write the number of calories c as a function of the number of chai lattes, n, you consume.

Number of Beverages	Calories
1	194
2	388
3	582
4	776
5	970

2

Write the number of cookies c as a function of the number of children, n, sharing them.

Number of Children	Number of Cookies
1	12
2	6
3	4
4	3
6	2

3

Write the number of cases of soda needed, c, as a function of the number of rows, n, in a stack.

Number of Rows	Number of Cases
1	2
2	5
3	9
4	14
5	20

4

Write the rebound height of a ball, h, as a function of the number of times, n, the ball has bounced.

Number of Bounces	Ball Height
0	12
1	6
2	3
3	1.5
4	0.75

5

A book club has a one-time membership fee of $25. Members receive 20% off each purchase. Write the price, P, you will pay overall as a function of the amount, A, of your purchase before discount.

6

Your grandmother lives 200 miles away. Write the amount of time, t, it will take you to get to her house as a function of the speed, s, you drive.

7

You have $100 in your pocket. Gas costs $2.49 per gallon. Write the amount you have left as a function of the number of gallons of gas you buy.

8

On Day 0, you email a video to five of your friends. On Day 1, each of those friends forwards the video to 5 friends. On Day 2, each person who received the video on Day 1 does the same. Write the number of persons who have received the video as a function of the number of days since the video was posted.

9

The diameter of a vegetable can is the same as its height. Write the visible surface area of the can's label as a function of its diameter, d.

10

In *One Hundred Hungry Ants* by Elinor J. Pinczes (1993), 100 ants are trying to get to a picnic. To travel faster they arrange themselves in lines, with each line having the same number of ants. Write the number of ants, A, per line as a function of the number of lines, L.

11

In *Bats on Parade* by Kathi Appelt (1999), the nth group of bats marches in a group of $n \times n$ bats plus a flag bearer. Write the number of animals A in a group as a function of the group number, n.

12

In *One Grain of Rice* by Demi (1997) a peasant girl, Rani, is rewarded for her honesty. She asks for 1 grain of rice on day 1, 2 grains on day 2, 4 grains on day 3, 8 grains on day 4, and so on. Write the number of grains of rice, G, that Rani receives on day d as a function of the day number.

13

In *The Twelve Circus Rings* by Seymour Chwast (1996), the first circus ring has 1 performer, the second ring has 2 + 1 performers, the third has 3 + 2 + 1 performers, and so on. Write the number of performers, p, as a function of the ring number n.

14

In *Math Curse* by Jon Scieszka and Lane Smith (1995), you are asked to estimate how many m&m's you need to measure the Mississippi River. Write the number of m&m's needed to measure a river, m, as a function of its length in miles, L. (Note: An m&m is about 1 cm in diameter.)

15

4.6 Homework

1. Complete the student page, *Growing Tile Patterns* (pp. 181 and 182). How can you tell from a tile pattern that it is growing linearly? How can you tell from a tile pattern that it is growing quadratically?

2. The Saffir-Simpson Hurricane Scale rates a hurricane's intensity using wind speed and storm surge, which is the abnormal rise in sea level accompanying a hurricane or other intense storm. The scale also estimates the potential damage and flooding expected along the coast from a hurricane landfall.
(Resource: http://www.cnn.com/2007/US/07/06/hurricane.scale/, retrieved April 23, 2020.)

   Let the category number be the independent variable. Let the highest wind speed in the category be the dependent variable.

Category	Highest wind speed in miles per hour
1	95
2	110
3	130
4	155

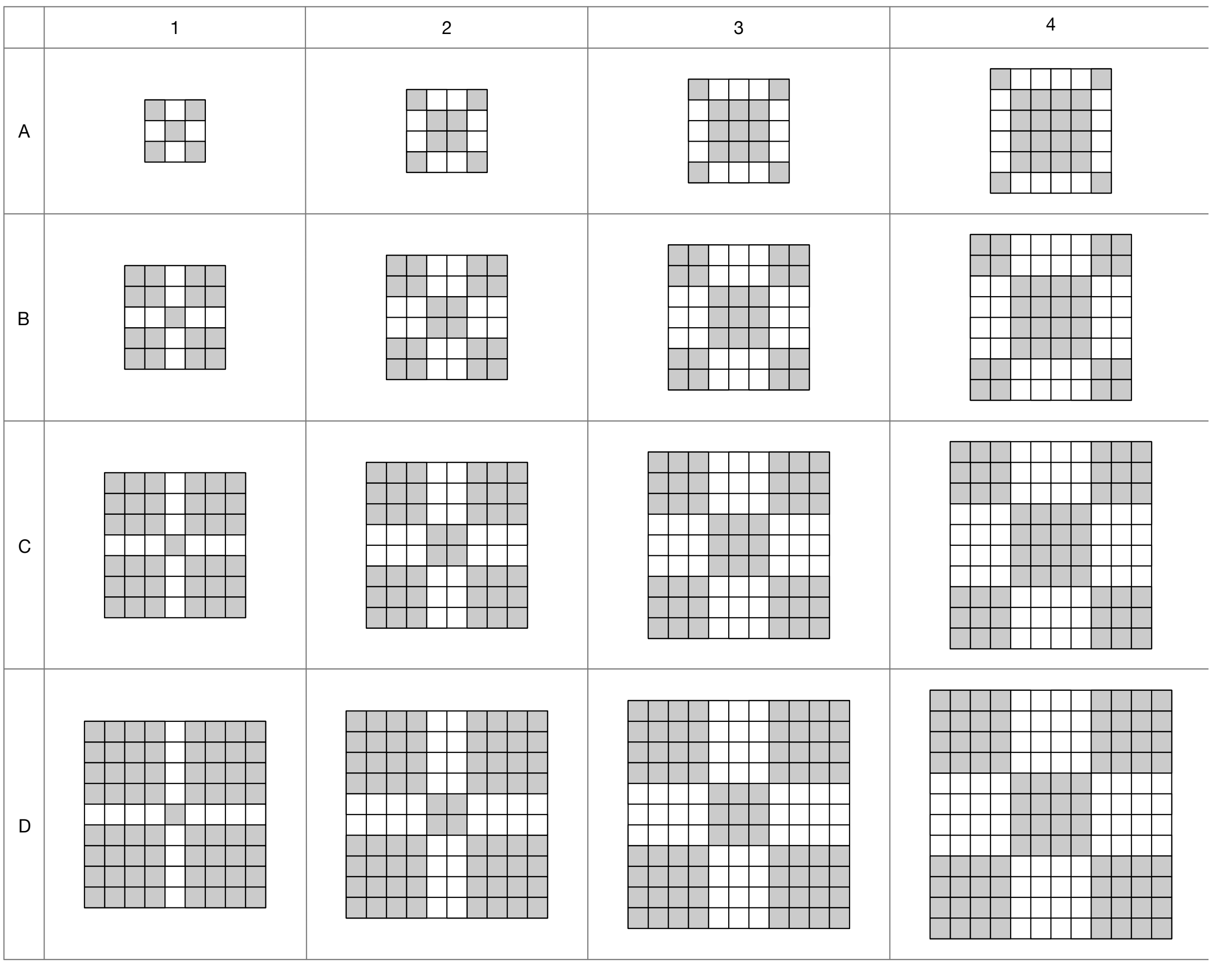
   a. To what function family does this data belong? Why do you think so?
   b. How can you tell from the table?
   c. Graph the data. How can you tell from the graph?
   d. Determine an equation to fit the data. Explain your process.
   e. No highest wind speed is given for a Category 5 hurricane. If the data continued to follow the pattern in the table, what would be the highest wind speed for a Category 5 hurricane?
   f. Hurricanes have been getting stronger over the past several years prompting the National Hurricane Center to propose a Category 6. If the data continued to follow the pattern in the table, what would be the highest wind speed for a Category 6 hurricane?
   g. The highest recorded wind speed was a gust of 253 miles per hour during Tropical Cyclone Olivia on April 10, 1996. Into what hurricane category would this wind speed fall? Explain.

3. Complete the student page, *Forms of Quadratic Functions Card Sort* (p. 183). Be ready to discuss your solutions with your group. Write down questions you still have.

Growing Tile Patterns

There are 10 growing tile patterns to investigate in the figure below.

- Patterns A through D are in Rows A through D respectively, with Step 1 in Column 1 and Step 4 in Column 4.
- Patterns 1 through 4 are in Columns 1 through 4 respectively, with Step 1 in Row A and Step 4 in Row D.
- Diagonal 1 begins in the upper left corner at Step 1and moves diagonally to the lower right corner at Step 4.
- Diagonal 2 begins in the upper right corner at Step 1and moves diagonally to the lower left corner at Step 4.



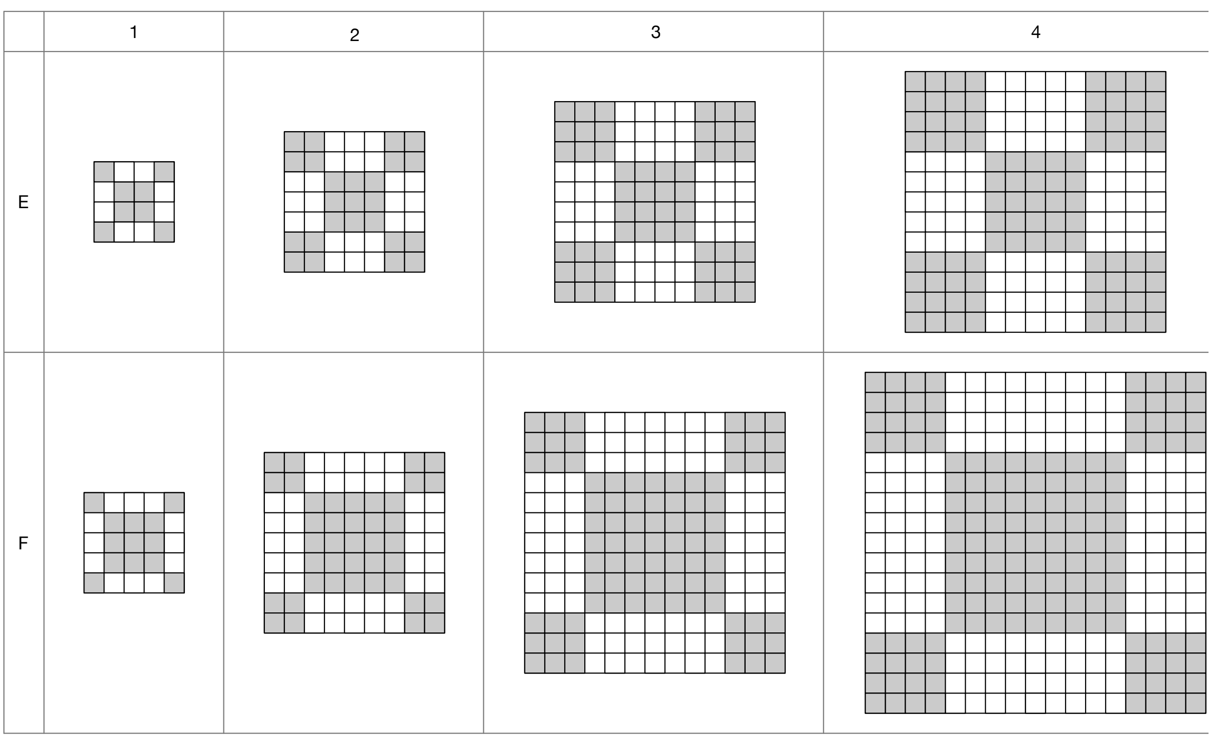
Choose one tile pattern to investigate. Solve each problem for the pattern you choose:

1. Study the pattern.
   a. Describe how the number of white tiles is changing as the pattern progresses from Step 1 to Step 4.
   b. Describe how the number of shaded tiles is changing as the pattern progresses from Step 1 to Step 4.
   c. Draw Step 5. Explain how you know it follows the same pattern.
   d. Is Step 5 possible for all 10 patterns? Why or why not?

2. Complete the table showing the number of white, shaded, and total tiles for each step.

Step Number	Number of Light Colored Tiles	Number of Dark Colored Tiles	Total Number of Tiles
1			
2			
3			
4			
5			
x			

3. Find patterns in the table. How are the numbers changing in each column?
4. Find equations to fit white, shaded, and total tiles, respectively, assuming the pattern continues predictably. Enter the equations in the last row of the table.
5. How are the equations you found related to how each pattern of tiles is growing?
6. How are the equations related to each other?
7. Suppose you have 200 white tiles and 200 shaded tiles.
   a. What is the largest step number of your chosen tile pattern you can create? How do you know?
   b. If the tiles are 1-inch squares, how large will the project be?
8. Patterns E and F are shown below. Solve the problems above for one of these patterns.



Forms of Quadratic Functions Card Sort

There are three forms of a quadratic function. Each one is important in its own way. Each one makes one or more aspects of a quadratic function evident. The forms are:

Standard: $y = ax^2 + bx + c$, Vertex: $y = a(x - h)^2 + k$, Factored: $y = a(x - p)(x - q)$

1. A card sort and recording sheet are included below. Cut the cards apart.
   a. Sort the cards to find pairs of equations that represent the same quadratic function. Write the equations in the table below.
   b. Verify algebraically that the cards you have matched name the same function. Show your work! (Check the graphs, too!)
2. a. Find the vertex and x-intercepts for each card set. List them as ordered pairs in the table.
   b. One equation form is missing from each set. Determine the missing form and write the missing equation (replace letters with appropriate numbers) in the table below. Verify algebraically that the missing equation matches the other equations in the set.
3. What features of the graph of a quadratic function are evident from:
   a. The standard form of the equation?
   b. The vertex form of the equation?
   c. The factored form of the equation?
4. a. Find all three forms of a quadratic function with x-intercepts at $x = 2$ and $x = -1.5$.
   b. Is there only one function that satisfies the conditions in problem 4a? How do you know?
   c. Find an equation for a quadratic function that satisfies the conditions in problem 4a and contains the point (3, 6). How many such quadratic functions are there?
5. a. A quadratic function has vertex (2, 4) and an x-intercept at $x = 0$. Where is the other x-intercept? How do you know?
   b. Find an equation for a quadratic function that fits the conditions in problem 5a. Show that your equation works.

Card Set	Standard Form	Vertex Form	Factored Form	Vertex	x-intercepts
1					
2					
3					
4					

Cards to Sort:

$y = (x + 1)(x - 3)$	$y = 2(x + 1)(x - 1)$	$y = (x + 0.5)^2 - 0.25$	$y = 2(x - 0)^2 - 2$
$y = x^2 + x$	$y = -0.5(x)(x - 4)$	$y = (x - 1)^2 - 4$	$y = -0.5x^2 + 2x$