CSCI 1311 Lab 1

2025-01-27

Welcome!

Let's get to know each other. Share:

- your name,
- an interesting fact about you,
- and something you look forward to this semester.

Recap

1. A proposition is ?
2. $\neg p$ is true when p is ?
3. $p \wedge q$ is true when ?
4. $p \vee q$ is true when ?
5. $p \oplus q$ is true when ?
6. $p \implies q$ is false when ?
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14. $\forall$, the universal quantifier, means "for all" (e.g. $\forall x \in \mathbb{Z}, 2x$ is even).
15. $\exists$, the existential qualifier, means ?

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15. $\exists$, the existential qualifier, means "there exists" (e.g. $\exists x \in \mathbb{Z}, x$ is even).

Recap: notation

- $\in$ = "in"
- $\mathbb{N}$ = the set of natural numbers = $\{0, 1, 2, \ldots, \infty\}$
- $\mathbb{Z}^+$ = the set of positive integers = $\{1, 2, 3, \ldots, \infty\}$
- $\mathbb{Z}^-$ = the set of negative integers = $\{-1, -2, -3, \ldots, -\infty\}$
- $\mathbb{Z}$ = the set of integers = $\{-\infty, \ldots, -1, 0, 1, \ldots, \infty\}$

Recap

1. An integer x is even if $\exists\, k \in \mathbb{Z}$ where $x =$?
2. An integer x is odd if $\exists\, k \in \mathbb{Z}$ where $x =$?
3. An integer x is prime if ?
4. An integer x is composite if ?
5. A real number $r \in \mathbb{R}$ is rational if ?
6. $\lfloor x \rfloor$ is the floor of x. $\lfloor x \rfloor = n \implies$?
7. $\lceil x \rceil$ is the ceiling of x. $\lceil x \rceil = n \implies$?

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1. An integer x is even if $\exists\, k \in \mathbb{Z}$ where $x = 2k$
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1. An integer x is even if $\exists\, k \in \mathbb{Z}$ where $x = 2k$
2. An integer x is odd if $\exists\, k \in \mathbb{Z}$ where $x = 2k + 1$
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5. A real number $r \in \mathbb{R}$ is rational if $\exists (x, y), r = \frac{x}{y}, y \neq 0$
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Proven in class

- $\forall x, y \in \mathbb{Z}, x + y$ is even $\implies x - y$ is even
- $\forall n \in \mathbb{Z}, n$ is odd $\implies (n^2 + n + 1)$ is odd
- $\forall x \in \mathbb{Z}, x > 1 \implies (x^3 + 1)$ is composite
- $\forall x \in \mathbb{R}, m \in \mathbb{Z}, \lfloor x + m \rfloor = \lfloor x \rfloor + m$
- $\forall x, y \in \mathbb{Z}, (x + y$ even $) \implies x$ and y are both odd or both even
- $\forall n \in \mathbb{Z}, 3n + 2$ is odd $\implies n$ is odd
- $\forall a, b \in \mathbb{R}, a \cdot b$ is irrational $\implies$ either a, b, or both must be irrational
- $\forall x, y \in \mathbb{Z}, x$ is odd and y is odd $\implies x \cdot y$ is odd
- $\forall x \in \mathbb{Z}, x^2$ is even $\implies x$ is even
- $\sqrt{2}$ is irrational

Recap: proof by contradiction

Assume for the purpose of contradiction that $P(x)$ is **true.**

We reason that $P(x) \implies Q(x)$.

However, we know that $Q(x)$ is **false.** It is impossible for $Q(x)$ to be true and false at the same time, so we have a contradiction.

Therefore, $P(x)$ is **false.**

The Unique Factorization Theorem: Fundamental Theorem of Arithmetic

$\forall x \in \mathbb{Z}$, x can be expressed as a product of prime numbers:

$x = p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots \cdot p_n^{e_n}$

1. Show that for any two integers m and n, $m^2 + n^2$ has the same parity as $m + n$.
2. Prove that $\sqrt{6}$ is irrational.

Reminders

- Homework
- Office hours