CSCI 1311 Lab 1

2025-01-27

Welcome!

Let's get to know each other. Share:

- your name,
- an interesting fact about you,
- and something you look forward to this semester.

Recap

1. A proposition is ?
2. $\neg p$ is true when p is ?
3. $p \wedge q$ is true when ?
4. $p \vee q$ is true when ?
5. $p \oplus q$ is true when ?
6. $p \implies q$ is false when ?
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9. The **contrapositive** of $p \implies q$ is ?
10. A proposition is logically equivalent to its ?
11. A proposition's converse is logically equivalent to the proposition's ?
12. $p \longleftrightarrow q$ is true when ?
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14. $\forall$, the universal quantifier, means ?
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1. A proposition is a statement that is either true or false.
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15. $\exists$, the existential qualifier, means ?

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15. $\exists$, the existential qualifier, means "there exists" (e.g. $\exists x \in \mathbb{Z}, x$ is even).

Recap: notation

- $\in$ = "in"
- $\mathbb{N}$ = the set of natural numbers = $\{0, 1, 2, \ldots, \infty\}$
- $\mathbb{Z}^+$ = the set of positive integers = $\{1, 2, 3, \ldots, \infty\}$
- $\mathbb{Z}^-$ = the set of negative integers = $\{-1, -2, -3, \ldots, -\infty\}$
- $\mathbb{Z}$ = the set of integers = $\{-\infty, \ldots, -1, 0, 1, \ldots, \infty\}$

Recap

1. An integer x is even if $\exists\, k \in \mathbb{Z}$ where $x =$?
2. An integer x is odd if $\exists\, k \in \mathbb{Z}$ where $x =$?
3. An integer x is prime if ?
4. An integer x is composite if ?
5. A real number $r \in \mathbb{R}$ is rational if ?
6. $\lfloor x \rfloor$ is the floor of x. $\lfloor x \rfloor = n \implies$?
7. $\lceil x \rceil$ is the ceiling of x. $\lceil x \rceil = n \implies$?

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1. An integer x is even if $\exists\, k \in \mathbb{Z}$ where $x = 2k$
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5. A real number $r \in \mathbb{R}$ is rational if $\exists (x, y), r = \frac{x}{y}, y \neq 0$
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7. $\lceil x \rceil$ is the ceiling of x. $\lceil x \rceil = n \implies n - 1 < x \leq n$

Proven in class

- $\forall x, y \in \mathbb{Z}, x + y$ is even $\implies x - y$ is even
- $\forall n \in \mathbb{Z}, n$ is odd $\implies (n^2 + n + 1)$ is odd
- $\forall x \in \mathbb{Z}, x > 1 \implies (x^3 + 1)$ is composite
- $\forall x \in \mathbb{R}, m \in \mathbb{Z}, \lfloor x + m \rfloor = \lfloor x \rfloor + m$
- $\forall x, y \in \mathbb{Z}, (x + y$ even $) \implies x$ and y are both odd or both even
- $\forall n \in \mathbb{Z}, 3n + 2$ is odd $\implies n$ is odd
- $\forall a, b \in \mathbb{R}, a \cdot b$ is irrational $\implies$ either a, b, or both must be irrational
- $\forall x, y \in \mathbb{Z}, x$ is odd and y is odd $\implies x \cdot y$ is odd
- $\forall x \in \mathbb{Z}, x^2$ is even $\implies x$ is even
- $\sqrt{2}$ is irrational

Recap: proof by contradiction

Assume for the purpose of contradiction that $P(x)$ is **true.**

We reason that $P(x) \implies Q(x)$.

However, we know that $Q(x)$ is **false.** It is impossible for $Q(x)$ to be true and false at the same time, so we have a contradiction.

Therefore, $P(x)$ is **false.**

The Unique Factorization Theorem: Fundamental Theorem of Arithmetic

$\forall x \in \mathbb{Z}$, x can be expressed as a product of prime numbers:

$x = p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots \cdot p_n^{e_n}$

1. Show that for any two integers m and n, $m^2 + n^2$ has the same parity as $m + n$.
2. Prove that $\sqrt{6}$ is irrational.

Reminders

- Homework
- Office hours

CSCI1311: Discrete Structures I

Solutions Lab 1

January 27, 2025

Problem 1: Show that for any two integers m and n, $m^2 + n^2$ has the same parity as m + n.
Solution:
Consider the following two cases which cover all possibilities.
Case 1: m + n is odd
We can write m + n = 2k + 1, for some k ∈ Z.
Then, we have,

$$\begin{aligned} m^2 + n^2 &= (m + n)^2 - 2mn \\ &=(2k + 1)^2 - 2mn \\ &=4k^2 + 4k + 1 - 2mn \\ &=2(2k^2 + 2k - mn) + 1 \text{ which is odd, as } 2k^2 + 2k - mn \in Z. \end{aligned}$$

Case 2: m + n is even
We can write m + n = 2k, for some k ∈ Z.
Similarly, we have,

$$\begin{aligned} m^2 + n^2 &= (m + n)^2 - 2mn \\ &=(2k)^2 - 2mn \\ &=4k^2 - 2mn \\ &=2(2k^2 - mn) \text{which is even, as } 2k^2 - mn \in Z. \end{aligned}$$

We have shown that m + n has the same parity as $m^2 + n^2$ and we are done.

Problem 2: Prove that $\sqrt{6}$ is irrational.
Solution:
We will use a proof by contradiction.
Assume for the sake of contradiction that $\sqrt{6}$ is rational.
By the definition of a rational number, write $\sqrt{6}$ as $\frac{a}{b}$ where a and b have no common divisors other than 1 (we call them relatively prime natural numbers) and b≠0.
This means that $6 = \frac{a^2}{b^2}$
$6b^2 = a^2$, If $6 \mid a^2$, then $2 \mid a^2$ which implies that a must be even (recall the lemma from slide 11 from Lecture 3.5).

Because a is even, let a = 2c for some integer c.

$6b^2 = a^2$

$2.3.b^2 = (2c)^2$

$2.3.b^2 = 2.2.c^2$

$3.b^2 = 2.c^2$ If 2 | (3. b^2), then 2 | b^2 which implies that b must be even (see Lemma above). So, clearly, a and b are both even.

However, this presents a contradiction: a and b must be relatively prime natural numbers, and thus cannot both be divisible by the same factor, 2.

Therefore, we can say that $\sqrt{6}$ is irrational.

Alternate Solution:

We will use a proof by contradiction. Assume for the sake of contradiction that $\sqrt{6}$ is rational. By the definition of a rational number, we write $\sqrt{6}$ as $\frac{a}{b}$ where a and b have no common divisors other than 1 (we call them relatively prime natural numbers) and b≠0.

This means that 6 = $\frac{a^2}{b^2}$

$6b^2 = a^2$

From here, we case the possible parities of a and b. Note that a and b cannot both be even or else they will share a common factor of 2.

Case 1: a and b are both odd, let a = 2k + 1 and let b = 2l + 1

$6b^2 = a^2$

$6(2l + 1)^2 = (2k + 1)^2$

$6(4l^2 + 4l + 1) = 4k^2 + 4k + 1$

$2(12l^2 + 12l + 3) = 2(2k^2 + 2k) + 1$

The LHS is even, and the RHS is odd; thus, the two sides cannot be equal. This is a contradiction.

Case 2: a is even and b is odd, let a = 2k and let b = 2l + 1

$6b^2 = a^2$

$6(2l + 1)^2 = (2k)^2$

$6(4l^2 + 4l + 1) = 4k^2$

$3(4l^2 + 4l + 1) = 2k^2$

$12l^2 + 12l + 2 + 1 = 2k^2$

$2(6l^2 + 6l + 1) + 1 = 2(k^2)$

The LHS is odd, and the RHS is even; thus, the two sides cannot be equal. This is a contradiction.

Case 3: a is odd and b is even, let a = 2k + 1 and let b = 2l

$6b^2 = a^2$

$6(2l)^2 = (2k + 1)^2$

$6(4l^2) = 4k^2 + 4k + 1$

$2(12l^2) = 2(2k^2 + 2k) + 1$

The LHS is even, and the RHS is odd; thus, the two sides cannot be equal. This is a contradiction. These cases cover all possibilities, and we reach a contradiction in every case. Therefore, we can say that $\sqrt{6}$ is irrational.