

1. Prove that $9n-2 = \Theta(n)$

$$f(n) = 9n-2 \quad g(n) = n$$

$$f(n) = \Theta(g(n))$$

$$1) f(n) = O(g(n)) \quad 2) f(n) = \Omega(g(n))$$

$$\text{W.T.S that } 9n-2 = O(n)$$

$$\text{W.T.S that } 9n-2 \leq c \cdot n \text{ for all } n \geq n_0$$

$$c = 9$$

$$n_0 = 0$$

$$\text{W.T.S } 9n-2 \leq 9n \text{ when } n > 0$$

$$9n \leq 9n$$

$$9n-2 \leq 9n$$

$$\text{W.T.S that } 9n-2 \geq c \cdot n \quad \forall n \geq n_0$$

$$c = 1$$

$$n_0 = 1$$

$$\text{W.T.S that } 9n-2 \geq n \quad \forall n \geq 1$$

$$\text{W.T.S that } 9n \geq 2 \quad n \geq 1$$

$$\text{W.T.S that } n \geq 1/4 \quad n \geq 1/4$$

$$9n-2 \geq n \quad \checkmark$$

2. Prove that $20n^3 + 50n \lg(n) + 20 = O(n^3)$

$$\text{W.T.S: } 20n^3 + 50n \lg(n) + 20 \leq c \cdot n^3 \quad \forall n \geq n_0$$

$$c = 90$$

$$n_0 = 1$$

$$\text{W.T.S that } 20n^3 + 50n \lg(n) + 20 \leq 90 \cdot n^3$$

$$20n^3 \leq 20n^3 \quad \forall n \geq 1$$

$$50n \lg(n) \leq 50n^3 \quad \forall n \geq 1$$

$$20 \leq 20n^3 \quad \forall n \geq 1$$

$$20n^3 + 50n^3 + 20n^3 \leq 90n^3 \quad \checkmark$$

3. Prove that $2^{50} = O(1)$

$$2^{50} \leq c \cdot 1$$

$$c = 2^{50} \quad \checkmark$$

4. Prove that $10 \lfloor n \rfloor = O(n)$

$$\text{W.T.S that } 10 \lfloor n \rfloor \leq c \cdot n$$

$$c = 10$$

$$n_0 = 1$$

$$\text{W.T.S that } 10 \lfloor n \rfloor \leq 10 \cdot n \quad \checkmark$$

5. Prove that $n^2/4 - 100n = \Theta(n^2)$

$$\text{Big-O: W.T.S that } n^2/4 - 100n = O(n^2)$$

$$\text{W.T.S } n^2/4 - 100n \leq c \cdot n^2 \quad \forall n \geq n_0$$

$$c = 101$$

$$n_0 = 1$$

$$\text{W.T.S that } n^2/4 - 100n \leq 101n^2$$

$$n^2/4 - 100n \leq n^2/4 \leq n^2 \leq 101n^2 \quad \checkmark$$

$$\text{W.T.S that } n^2/4 - 100n \geq c \cdot n^2 \quad \forall n \geq n_0$$

$$\text{W.T.S } n^2/4 - c(n^2) \geq 100n$$

$$\text{W.T.S } \frac{n^2}{4} - 4c \frac{n^2}{4} \geq 100n$$

$$\text{W.T.S } n^2 - 4c(n^2) \geq 400n$$

$$\text{W.T.S } n^2(1-4c) \geq 400n$$

$$\text{W.T.S } (1-4c) \geq \frac{400}{n}$$

$$\text{W.T.S } (1/4 - c) \geq \frac{100}{n}$$

$$\text{W.T.S } (1/4 - c)n \geq 100$$

$$c = 1/6$$

$$(1/6)n \geq 100$$

$$n_0 = 600$$

$$\lim_{n \rightarrow \infty} \frac{n^2/4 - 100n}{n^2} = \frac{n/4 - 100}{n} = 1/4$$

Prove that $\lg(n) = O(n)$

BC:

I.H: $\lg(k) = O(k)$ for some $k \geq 1$

TS: W.T.S that $\lg(k+1) = O(k+1)$

$$\lg(k+1) \leq \lg(2k)$$

$$\downarrow$$

$$\lg(2) + \lg(k)$$

$$\boxed{1 + k}$$

7. Prove that $5n^{100} = O(2^n)$

8. Prove that $7n-2$ is NOT $\Omega(n^{10})$

9. Prove that $n^{1+0.001}$ is NOT $O(n)$