CS 2312 : Lecture #1 : Logic and Proofs Review

Tuesday, August 20, 2024 9:52 PM

What is a proposition?

It is a statement that is either true or false.

Examples:

1) $5+5=10$ ✓
2) This class is apart of GW's CS education. ✓
3) What day is it? ✗
4) For all whole numbers, n^2+n+41 is a prime number. ✓

Let's construct some proposition's and relearn some notation

p: n is a prime number larger than 2

q: n is odd

$\tilde{p}$ (not p): true when p is false and vice-versa

n is not a prime number larger than 2

$p \wedge q$ (p and q) : true when both p and q are true, false otherwise

n is a prime number larger than 2 AND n is odd

$p \vee q$ (p or q): true when at least one of p or q is true, false otherwise

n is a prime number larger than 2 OR n is odd

$p \oplus q$: (p exclusive-or q) : true when exactly one of p and q is true, and false otherwise

n is a prime number larger than 2 exclusive-OR n is odd

$p \to q$: (p implies q) : false when p is true and q is false and true otherwise

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

n is a prime number larger than 2 $\to$ n is odd True ✓

$q \to p$ (q implies p / converse of $p \to q$)

n is odd $\to$ n is a prime number larger than 2

$n=9$ False ✗

$\sim p \to \sim q$ (not p implies not q / inverse of $p \to q$)

n is not a prime number larger than 2 $\to$ n is even

$n=1$ False ✗

$\sim q \to \sim p$ (not q implies not p / contrapositive of $p \to q$)

n is even $\to$ n is not a prime number larger than 2

True ✓

$p \leftrightarrow q$ (p iff q [if and only if], $p \to q$ and $q \to p$)

False

Truth Table:

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	F	F	F	T	T	F	T	F
T	T	F	T	T	F	T	T	T
F	T	T	F	T	T	T	F	F
F	F	T	F	F	F	T	T	T

Logical Equivalence:

Show that $p \to q$ is the same as $\sim p \vee q$

which is the same as $\sim q \to \sim p$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$	$\sim q$	$\sim q \to \sim p$
T	T	T	F	T	F	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	F	T	T	T	T	T

Quantified Statements:

A predicate is a proposition whose value depends on the value of a variable.

Example: n is a prime number

$P(n)$:

P - "is a prime number"

Universal Quantification:

$\forall$ = For all

$\forall x \in D$, $P(x)$ is true when for all x in the domain, $P(x)$ is true, and is false when there is some x in the domain for which $P(x)$ is false

Example: $\forall n \in \mathbb{N}$, n^2+n+41 is a prime number

$41^2+41+41$

Existential Qualifier

$\exists$ = "there exists"

$\exists x \in D$, $P(x)$ is true when there exists some x in the domain where $P(x)$ is true, and is false when for all x in the domain, $P(x)$ is false.

Example: $\exists x \in \mathbb{N}$, $x^2 < x$ ✗ False

$\mathbb{N} = (0, 1, \ldots\ldots \infty)$

$\mathbb{Z}^+ = (1, 2, \ldots\ldots \infty)$ $+ \{0\} = \mathbb{Z}$

$\mathbb{Z}^- = (-1, -2, \ldots\ldots -\infty)$

Proofs:

Axioms / Theorems / Definitions:

An integer x is even iff $\exists$ some k where $x = 2k$

An integer x is odd iff $\exists$ some k where $x = 2k+1$

An integer x is prime iff $\forall (r,s)$ where $x = r \cdot s$, $r=1$ or $s=1$. Otherwise x is composite, in other words if $\exists$ some (r,s) where $x = r \cdot s$, and both r and s are not 1, x is composite.

A real number, x, is rational iff $\exists$ some pair (r,s) where $x = \frac{r}{s}$ and $s \neq 0$

$\lfloor x \rfloor$ - the floor of x

if $\lfloor x \rfloor = n$ then $n \leq x < n+1$

$\lceil x \rceil$ - the ceiling of x

if $\lceil x \rceil = n$ then $n-1 < x \leq n$

① Direct Proofs: $\forall (x,y)$ pairs where x and $y \in \mathbb{Z}$

1) If x and y are 2 integers and $x+y$ is even, then $x-y$ is even

$x+y = 2k$ where k is an integer

$x-y = 2k - 2y$

$x-y = 2(k-y)$

$x+y-2y = 2k-2y$

$x-y = 2k-2y$

2) For all integers n, if n is odd, then n^2+n+1 is odd

3) If x is an integer and $x>1$, x^3+1 is composite (not prime)