CS2312: Lecture #2: Proofs

Sunday, August 25, 2024 11:35 AM

Announcements:

1) Join Piazza: piazza.com/gwu/fall2024/csci2312
2) Course Website: gw-cs2312.github.io
3) Homework #1 Released Today, due before class next Tuesday
4) Lab this week!
5) My Office Hours: After-Class from 6PM → No more questions starting next Thursday!

2) $\forall n \in \mathbb{Z}$, if n is odd, then n^2+n+1 is odd

$n = 2k+1$ where $k \in \mathbb{Z}$

$(2k+1)^2 + (2k+1) + 1$

$4k^2+4k+1 + 2k+1 + 1$

$4k^2+6k+3 \rightarrow 2(q)+1$ where $q \in \mathbb{Z}$

$2(\underbrace{2k^2+3k+1}_{q \in \mathbb{Z}}) + 1$ Odd ✓

3) Let x be an integer. If $x>1$, then x^3+1 is composite (not prime)

$x^3+1 = r \cdot s$

$(x+1)(x^2-x+1)$

$\neq 1$

> 1

$x+1 > 1$ ✓

$x^2-x+1 > 1$

$x^2-x > 0$

$x^2 > x$

$\boxed{\begin{array}{l} x>1 \\ x^2>x \\ x^2-x>0 \\ x^2-x+1>1 \end{array}}$

4) $\forall x \in \mathbb{R}$ and $\forall m \in \mathbb{Z}$: $\lfloor x+m \rfloor = \lfloor x \rfloor + m$

$x = \underline{y} + \underline{\varepsilon}$

$y \in \mathbb{Z}$

$\varepsilon \in [0, \dots 1] \in \mathbb{R}$

$\lfloor y+\varepsilon+m \rfloor = y+m$

W.T.S $y = \lfloor x \rfloor$

$y = x - \varepsilon$

$y = \lfloor x \rfloor$

5) $x \in \mathbb{Z}$, $y \in \mathbb{Z}$, where $x+y$ is even, then x and y are both odd or both even

if x is odd and y is even, then $x+y$ is odd

If x is even and y is odd, then $x+y$ is odd

$2k$ $2k+1$

$4k+1$

$2(2k)+1$

6) If $3n+2$ is odd, then n is odd $(n \in \mathbb{Z})$

If n is even then $3n+2$ is even

$n = 2k$ $6k+2$

$2(3k+1)$ ✓

7) $\forall a, b \in \mathbb{R}$, if $a \cdot b$ is irrational, then either a or b, or both must be irrational

8) The product of 2 odd numbers is an odd number

$x = 2k+1$ $y = 2j+1$

$(2k+1) \cdot (2j+1) = 4kj + 2k + 2j + 1$

$2(2kj+k+j) + 1$

$\hookrightarrow q \in \mathbb{Z}$

$\boxed{2q+1}$

9) If x^2 is even, then x is even

$p \rightarrow q = \sim q \rightarrow \sim p$

If x is odd then x^2 is odd ✓

10. $\sqrt{2}$ is irrational

A rational number is any number that can be written as $\frac{p}{q}$ where $(p, q) \in \mathbb{Z}$ and have no common factors and $q \neq 0$

Assume for the purpose of contradiction that $\sqrt{2}$ is rational:

$\sqrt{2} = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$ and p and q have no common factors

$p = \sqrt{2} \cdot q$

$p^2 = 2q^2$ p^2 is even, p is even

$p = 2k$

$4k^2 = 2q^2$

$q^2 = 2k^2$

Unique Factorization Theorem: Fundamental Theorem of Arithmetic

For $\forall x \in \mathbb{Z}$, x can expressed as a multiple of prime numbers.

$x = \underbrace{p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \dots}$

Assume f.p.o.c that $\sqrt{2}$ is rational

$\sqrt{2} = \frac{p}{q}$

$p = \sqrt{2} \cdot q$

$p^2 = 2q^2$

$2^2 \cdot 5$ $2 \cdot 5$

$S(20) = 3$ $\underbrace{p^2 = 20 \quad q^2 = 10}$ $S(10) = 2$

Let $S(x)$ be the sum of the number of times each prime number shows up in the prime factorization of x

$S(20)$ $2^2 \cdot 5$ $= 3$

$S(p^2)$ is even $S(q^2)$ is even

$S(4) = 2$

$S(16) = 4$

Contradiction, since $p^2 = 2q^2$

11. The sum of 2 irrational numbers is irrational

$x = \sqrt{2}$ $y = -\sqrt{2}$ $= 0$

12. If x and y are irrational numbers, then x^y is irrational.

13) n is even $\leftrightarrow$ $7n+4$ is even

14) There are an infinite number of prime numbers

Assume there's a finite number of prime numbers

$n = a_1 \cdot a_2 \cdot a_3 \cdots a_x + 1$

$2 \quad 3 \quad 5 \quad 7 \dots$