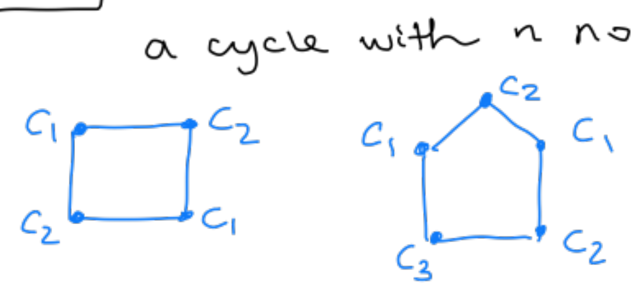
Announcements

- HW9 due Thursday
- Midterm #2 on 11/21
  - graphs through 11/14
  - double-sided handwritten sheet of paper
- No labs week of midterm (11/18, 11/20, 11/22)

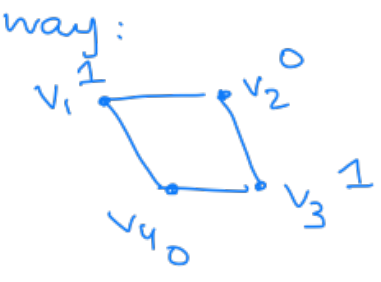
$\boxed{5d}$ Find the minimum value of k such that a cycle with n nodes is k-colorable.



- Let the cycle be $v_1, v_2, v_3, \dots, v_n, v_1$

Suppose n is even.
Color the graph in the following way:

$f(v_i) = 1$ for odd i
$f(v_i) = 0$ for even i



No adjacent nodes have the same parity.
(v_n is even, so it's only adjacent to odd nodes, v_{n-1} & v_1)

so no adjacent nodes are colored with the same color

$\Rightarrow k \leq 2$

Second part is showing that k cannot be < 2.
A graph w/ at least one edge needs at least 2 colors (from #3).

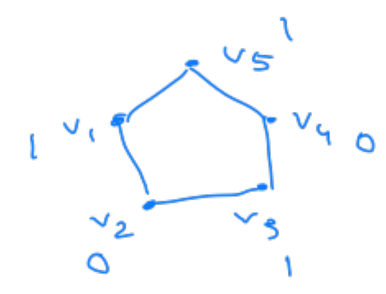
Because our cycle has an edge, $k \geq 2$.

$\boxed{\Rightarrow k = 2}$

- Suppose n is odd.

Try the same coloring above.

$f(v_i) = 1$ for odd i
$f(v_i) = 0$ for even i



Because n is odd & v_n is adjacent to v_1,
both v_1 and v_n will be colored "1"
so 2 colors is not sufficient $(k > 2)$

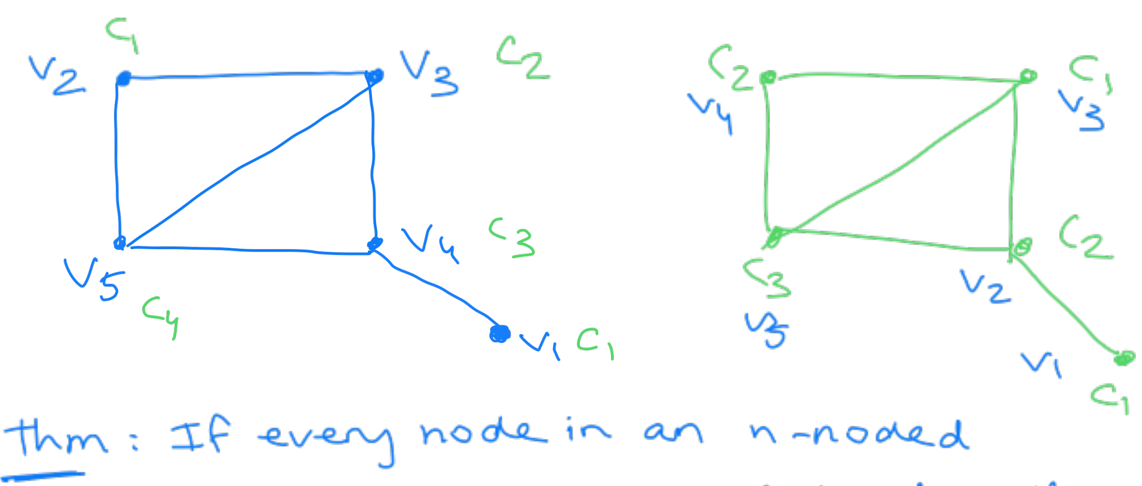
Instead, use the above approach for $v_1, v_2 \dots, v_{n-1}$
then color v_n color 2.

In this case, there are no adjacent nodes w/ the same color. $(k \leq 3)$

$\boxed{\Rightarrow k = 3}$

<u>Basic Coloring Algorithm</u> for graph $G = (V, E)$

1. Order the nodes from $v_1, v_2, \dots, v_n$
2. Order the colors from $c_1, c_2, c_3, \dots$
3. for $i = 1, 2, \dots, n$
   Assign the lowest legal color to v_i
   $\downarrow$
   no adj. nodes have same color



<u>Thm</u>: If every node in an n-noded graph G has degree $\leq d$, then the Basic Algorithm uses <u>at most</u> $d+1$ colors for G (no matter the ordering of nodes)

<u>Proof by induction</u>

<u>Induction hypothesis</u>: $P(n)$: Assume that if every node in an n-noded graph G has degree $\leq d$, then the Basic Algorithm uses at most $d+1$ colors for G.

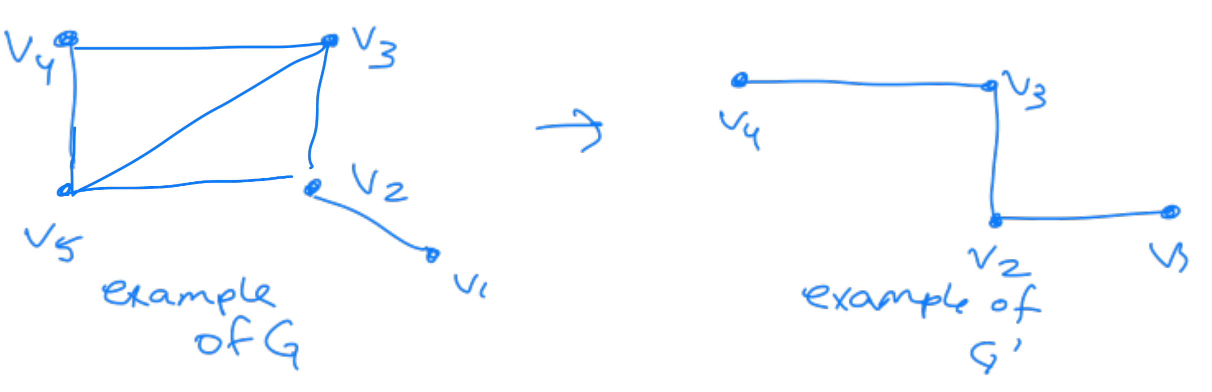
<u>Base case</u>: $n = 1 \Rightarrow 0$ edges $\rightarrow d = 0$, $\rightarrow 1$ color $= d+1$ ✓

<u>Inductive step</u>

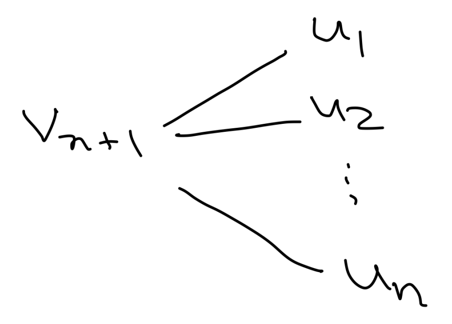
Assume $P(n)$ is true.
Let $G = (V, E)$ be any $n+1$-node graph. Let d be the max degree in G.
WTS that we can color it w/ maximum $d+1$ colors

Order the nodes $\underbrace{v_1, v_2, v_3, \dots, v_n}, v_{n+1}$

Remove v_{n+1} from G. This creates a new graph $G' = (V', E')$



G' has max degree $\leq d$ and it has n nodes, so we can use $P(n)$



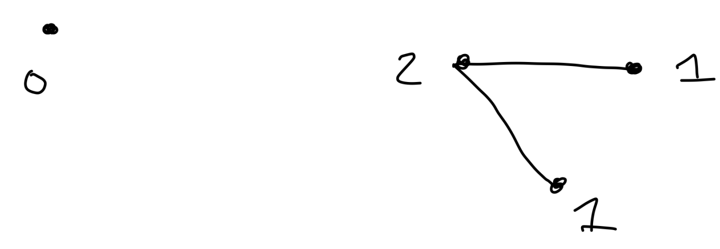
$\leq d$ neighbors

v_{n+1} has $\leq d$ neighbors b/c it has max degree d.

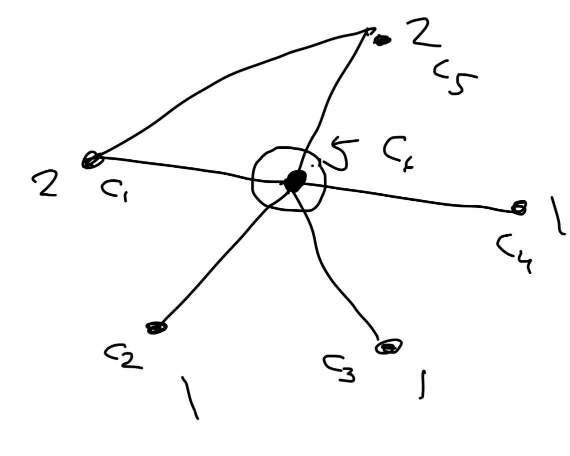
$\Rightarrow \exists$ at least 1 color out of my $d+1$ colors that's not used by any neighbor of v.

Give v_{n+1} that color.

$\Rightarrow$ Basic Alg uses $\leq d+1$ colors on G.
$\Rightarrow P(n+1)$



max degree $= 2$



<u>max degree = 5</u>