THE GEORGE WASHINGTON UNIVERSITY
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# CS 2312: Lab 05

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# Lab 05

- Show how TAs would think about and solve past HW problems
- Midterm Q&A

# Composite Proof

- Show that $2^{35}(2^{33}-1)+1$ is composite
- All composites have factors other than 1 and themselves
- Therefore factoring should complete our proof

# Composite Proof

$$2^{35}(2^{33}-1)+1$$

$$2^{68}-2^{35}+1$$

$$(2^{34}-1)^2$$

$2^{34}-1$ is a factor $>1$

# Divisibility Proof

- Show that $a|b \Leftrightarrow da|db$
- Converting to divisibility definition will let us use algebra
- Definitions are always useful to keep in mind!

$$a|b \Leftrightarrow \exists q \in \mathbb{Z} \text{ s.t. } b = qa$$

# Divisibility Proof

$$a|b \Leftrightarrow \exists q \in \mathbb{Z} \text{ s.t. } b = qa$$

$$\Leftrightarrow \exists q \in \mathbb{Z} \text{ s.t. } db = qda$$

$$\Leftrightarrow da|db$$

Backwards direction is the same since d != 0

# Strong Induction Proof

- Show that $\forall n \in \mathbb{Z} \geq 6,\ \exists a, b \in \mathbb{Z}^+ \text{ s.t. } n = 3a + 4b$
- We can always increment a to prove for n = k+3
- We need three base cases!

# Base Cases + Induction Hypothesis

$$6 = 3(2) + 4(0) = 3a + 4b \checkmark$$

$$7 = 3(1) + 4(1) = 3a + 4b \checkmark$$

$$8 = 3(0) + 4(2) = 3a + 4b \checkmark$$

We can assume that the statement holds for n = k, n = k-1, n = k-2

# Induction Step

Assuming true for $n = k - 2$, prove statement for $n = k + 1$:

$$k - 2 = 3a + 4b$$

$$\Rightarrow k + 1 = 3(a + 1) + 4b$$

$$\Rightarrow k + 1 = 3a' + 4b'$$

$$let\ a' = a + 1,\ \ b' = b \in \mathbb{Z}^+$$

# Big-O Proof

- Show that $2n^3 + 5n^2 + 4n + 3 \in O(n^5)$
- Since big-O is a loose upper bound, we can make useful overestimates and prove piece by piece
- Let b = 1, B = 2 + 5 + 4 + 3 = 14

# Big-O Proof

$$2n^3 \leq 2n^3 \Rightarrow 2n^3 \leq 2n^5$$

$$5n^2 \leq 5n^2 \Rightarrow 5n^2 \leq 5n^5$$

$$4n \leq 4n \Rightarrow 4n \leq 4n^5$$

$$3 \leq 3 \Rightarrow 3 \leq 3n^5$$

# Big-O Proof

$$2n^3 + 5n^2 + 4n + 3 \leq 14n^5$$

$$0 \leq |2n^3 + 5n^2 + 4n + 3| \leq Bn^5$$

$$\Rightarrow f(n) \in O(n^5)$$

# Q&A Time!

- Midterm on 10/8
- Are there any questions you have or topics you would like to review?