CSci 2312: Discrete Structures II: Graph Coloring

A graph is k-*colorable* if each node can be assigned one of k colors so that no two adjacent nodes have the same color.

1. Show that a graph that is n-colorable is also n'-colorable for all $n' \geq n$.
2. Show that if a graph cannot be colored with n colors, it cannot be colored with n' colors, for all $n' < n$.
3. Show that a graph with at least one edge needs at least 2 colors.
4. Show that the following graphs are n-colorable (as always, proofs are required):
   (a) Any graph with n nodes.
   (b) Any sub-graph of a graph which is n-colorable.
5. In each of the following cases, find the minimum value of k such that the graph is k-colorable. (As always, proofs are required):
   (a) The empty graph with n nodes.
   (b) The star with n nodes, $n \geq 2$. (A *star* is a graph with a single node (the *hub*) adjacent to all others; all nodes that are not the hub are adjacent only to the hub and no other node.)
   (c) The clique with n nodes. (A *clique* is another name for the complete graph.)
   (d) A *cycle* with n nodes. (A *cycle* is another name for what the text book refers to as a simple circuit: a trail which ends and begins at the same node and has no other repeated node.)
6. Let n be the minimum number of colors required to color a graph G. Let G' be a proper subgraph of G (that is, $G' \neq G$). Show that n', the minimum number of colors required to color G', is such that $n' \leq n$. Give an example of G and G' where $n' = n$ and one where $n' < n$.
7. Show that a graph with maximum degree at most r is $(r + 1)$-colorable.
8. A *bipartite* graph is another name for a 2-colorable graph. Show that a graph is bipartite if and only if it contains no odd cycle.