Lecture #10: Running Time / Code Snippets

Sunday, September 22, 2024 2:39 PM

Annoucements:

1. Homework #4 due Today
2. Homework #5 released today, Big-oh / Code-Snippets
3. Midterm #1: October 8th (be ready by October 3rd)
4. My office Hours after class

1. Prove that $7n-2$ is not $\Omega(n^{10})$

Assume f.p.o.c that $7n-2$ is $\Omega(n^{10})$

$7n-2 \geq c \cdot n^{10} \quad \forall n \geq n_0$

$7n \geq c \cdot n^{10}$

$7 \geq c \cdot n^9$ ↑

$c \cdot n^9 \leq 7$

$n^9 \leq 7/c$

$n \geq \sqrt[9]{7/c}$

2. Prove that $n^{1+0.0001}$ is not $O(n)$

Assume f.p.o.c that $n^{1+0.0001} = O(n)$

$n^{1+0.0001} \leq c \cdot (n) \quad \forall n \geq n_0$

$n^{0.0001} \leq c$

when $n > c^{(10^4)}$ the left hand side will be larger than Right-hand size

Analyzing Runtime of Code Snippets

Provide a tight band on the running time of the following code: (theta)

1.
```
for (i=0, i<n, i++)
  for (j=0, j<i, j=j+10)
    print ("I love CS2312")
```
← 1

Tabular Method

Iteration	i
0	0
1	1
2	2
⋮	
x	x

Iteration	j
0	0
1	10
2	20
3	30
⋮	
y	10y

Expression for Runtime

$$\text{Runtime} = \sum_{x=0}^{?} \sum_{y=0}^{?} 1$$

$j < i$

$10y < i$

$y < i/10$

$$\sum_{x=0}^{?} \sum_{y=0}^{y=i/10} 1$$

$$\sum_{y=0}^{i/10} 1 = i/10$$

$$\sum_{x=0}^{?} i/10$$

$i < n$

$x < n$

$$\sum_{x=0}^{?} x/10$$

$$\sum_{x=0}^{n} x/10$$

$$\frac{1}{10} \sum_{x=0}^{n} x = 0+1+2+3 \ldots + n = \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{20} = \Theta(n^2)$$

Prove $\frac{n(n+1)}{20} = \Theta(n^2)$

① Prove that $\frac{n(n+1)}{20} = O(n^2)$

Prove that $\frac{n(n+1)}{20} \leq c(n^2)$ for some c and all $n \geq n_0$

$\frac{n^2}{20} + \frac{n}{20} \leq c(n^2)$ → $1/10$ or $2/20$, $n_0 = 1$

$\frac{n^2}{20} + \frac{n}{20} \leq \frac{2n^2}{20} = \frac{n^2}{10}$

$\frac{n^2}{20} + \frac{n}{20} \leq \frac{n^2}{20} + \frac{n^2}{20} \leq \frac{2n^2}{20}$

Prove that $\frac{n(n+1)}{20} \geq c \cdot (n^2) \quad = \Omega(n^2)$

$c = \frac{1}{20} \quad n_0 = 1$

$\frac{n^2}{20} + \frac{n}{20} \geq n^2/20$ ✓

$= \Theta(n^2)$

2.
```
i = n
while (i ≥ 10) do
  i = i/3
  for j = 1 to n do
    print ("My favorite class is Discrete Math")
```

```
for (i=n; i ≥ 10; i=i/3)
  for (j=0; j<n, j++)
```

Iteration	i
0	n
1	n/3
2	n/9
⋮	
x	$n/3^x$

Iteration	j
0	0
1	1
2	2
⋮	
y	y

$$\sum_{x=0}^{?} \sum_{y=0}^{?} 1$$

$j < n$

$y < n$

$$\sum_{y=0}^{n} 1 = n$$

$$\sum_{x=0}^{?} n$$

$i < 10$

$\frac{n}{3^x} < 10$

$3^x \cdot 10 > n$

$3^x > n/10$

$x < \log_3(n/10)$

$$\sum_{x=0}^{\log_3(n/10)} n$$

$$n \sum_{x=0}^{\log_3(n/10)} 1$$

$= n \cdot \log_3(n/10)$

$= \Theta(n \log_3(n))$

3.
```
for i=0 to n do
  for j=n down to 0 do
    for k=1 to j-1 do
      print ("Do we want a Midterm Review Session?)
```
→ $i = 0$ to $n/4$ → $n/2$; $j = n$ to $3n/4$ → $n/2$; $k = n/4$ to $3n/4$

Superset / Subset Method
↳ O ↳ Ω

for $i=0$ to n ← $O(n)$

$j = n$ to 0 ← $O(n)$

$k = 1$ to n ← $O(n)$

$= O(n^3)$

for ($i=0$ to $n/4$) $= \Omega(n/4)$

for ($j=n$ to $3n/4$) $= \Omega(n/4)$

for ($k = n/4$ to $3n/4$) $\Omega(n/2)$ 1 to j

$= \Omega(n/4) \cdot \Omega(n/4) \cdot \Omega(n/2) = \Omega(n^3/32)$

$= \Omega(n^3)$

$\Theta(n^3)$

4.
```
for i=1 to n do
  for j=1 to i·i do
    for k=1 to j do
      print ("Short Exam with lots of easy problems or
             Long Exam with a few hard problem")
```

$\lg(n)$ ↓ $\quad O(n\lg(n))$

5.
```
for (i=1, i≤n, i=2·i) do
  for (j=1 to i) do
    print ("I can find the runtime of any code snippet");
```
1 to n

Iteration	i
0	1
1	2
2	4
3	8
⋮	
x	2^x

$i > n$

$2^x > n$

$x < \log_2(n)$

Iteration	j
0	0
1	1
2	2
⋮	
y	y

$$\sum_{x=0}^{?} \sum_{y=0}^{?} 1$$

$j < i$

$y < i$

$$\sum_{y=0}^{i} 1 = i$$

$$\sum_{x=0}^{?} i$$

$$\sum_{x=0}^{\log_2(n)} 2^x = 2^0 + 2^1 + \ldots + 2^{\log_2(n)}$$

$= 2^{\lg(n)+1} - 1$

$= 2^1 \cdot 2^{\lg(n)} - 1$

$= 2n - 1$

$\Theta(n)$

for ($i = n/128$; $i \leq n$; $i = 2 \cdot i$) $\Omega(\cancel{\lg(n)})$ $\Omega(1)$

for ($j = 1$ to $n/128$) ← $\Omega(n/128)$

$= \Omega(n\lg(n))$

Properties of Asymptotic Growth Functions $\quad = \quad n$

$[0, n] \to [0, n/8]$

$[n, 0] \to [n, 7n/8]$

$[1, j-1]$

$[1, 7n/8] \to [n/8, 3n/4]$