Asymptotics

CS 2312, Discrete Structures II

Solution: $\sqrt{n} \in O(n)$

- Need to show that:

There exist b, B > 0 such that:

$n \geq b \Rightarrow \sqrt{n} \leq Bn$

- B=1 and b=1. Note that both are positive.
- Begin with the hypothesis:

$n \geq b \Rightarrow n \geq 1 \Rightarrow n^2 \geq n \Rightarrow n \geq \sqrt{n} \Rightarrow \sqrt{n} \leq Bn$

Could also do:

$n \geq b \Rightarrow n \geq 1 \Rightarrow \sqrt{n} \geq 1 \Rightarrow \sqrt{n}\,\sqrt{n} \geq \sqrt{n} \Rightarrow n \geq \sqrt{n}$
$\Rightarrow \sqrt{n} \leq Bn$

Definition of Big-Oh

$$f(n) \in O(g(n)) \Leftrightarrow \exists B, b > 0 \; such \; that \; 0 \leq | \, f(n) \, | \leq Bg(n) \; \forall \, n \;\; \geq b$$

- Students tend to think that f $\in$ O(g) means that g is larger than f
  - But this neglects the role played by b and B
- So what does it really mean?

A misunderstanding

- F is O(g)
- Is g > F? Not always

Examples: F, g to show that

F is O(g) => g >= F

is false

g(n) = n

F(n) = n+3

What the definition means

$$f(n) \in O(g(n)) \Leftrightarrow \exists B, b > 0 \; such \; that \; 0 \leq | f(n) | \leq Bg(n) \; \forall \, n \; \geq b$$

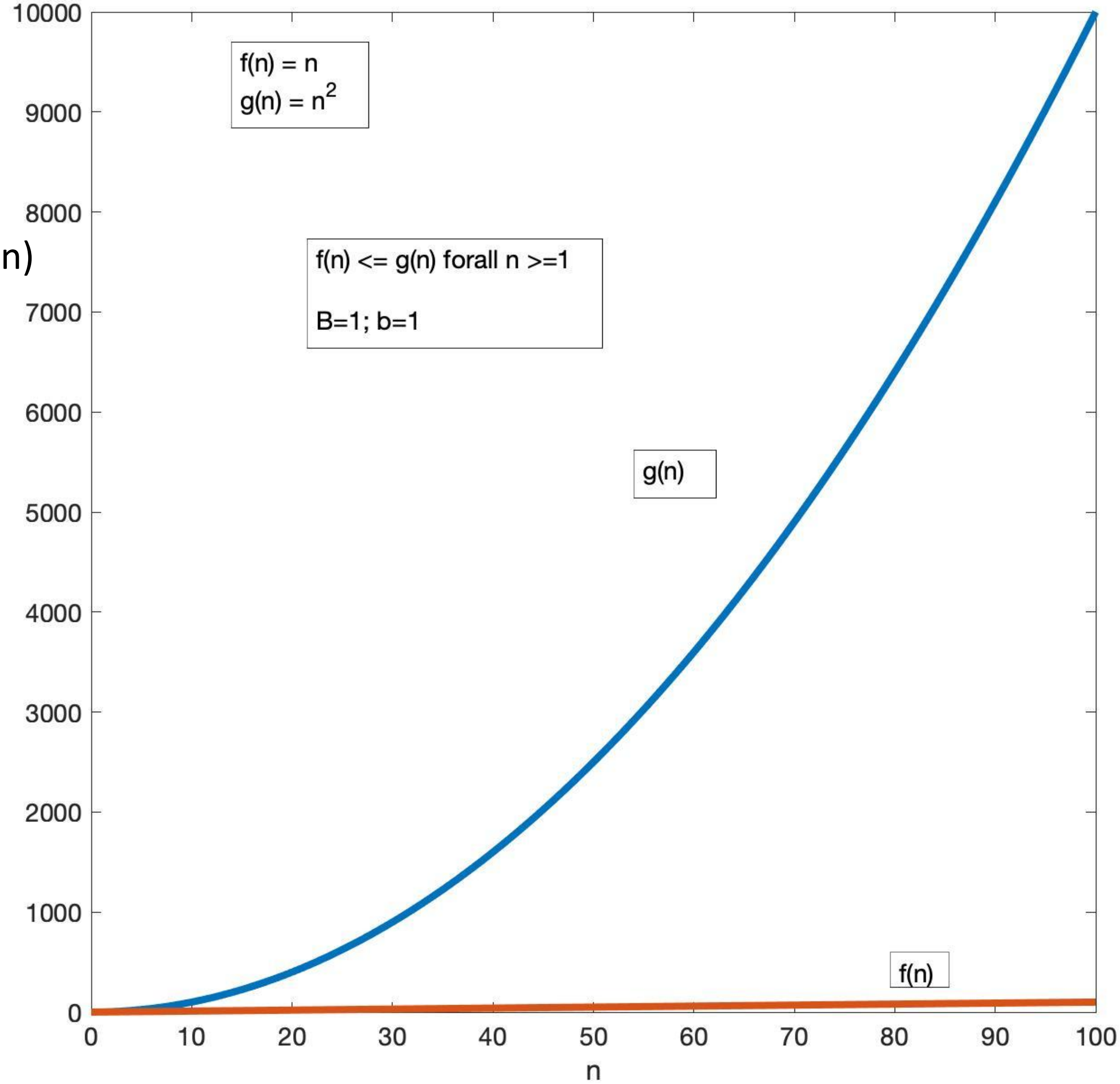
- In the long run (forever after a value *b*)
  - Does not matter what happens when $n < b$

- *f* is bounded above by a multiple *B* of *g*
  - So *g* itself can be very small, because you can choose *B* to be very large

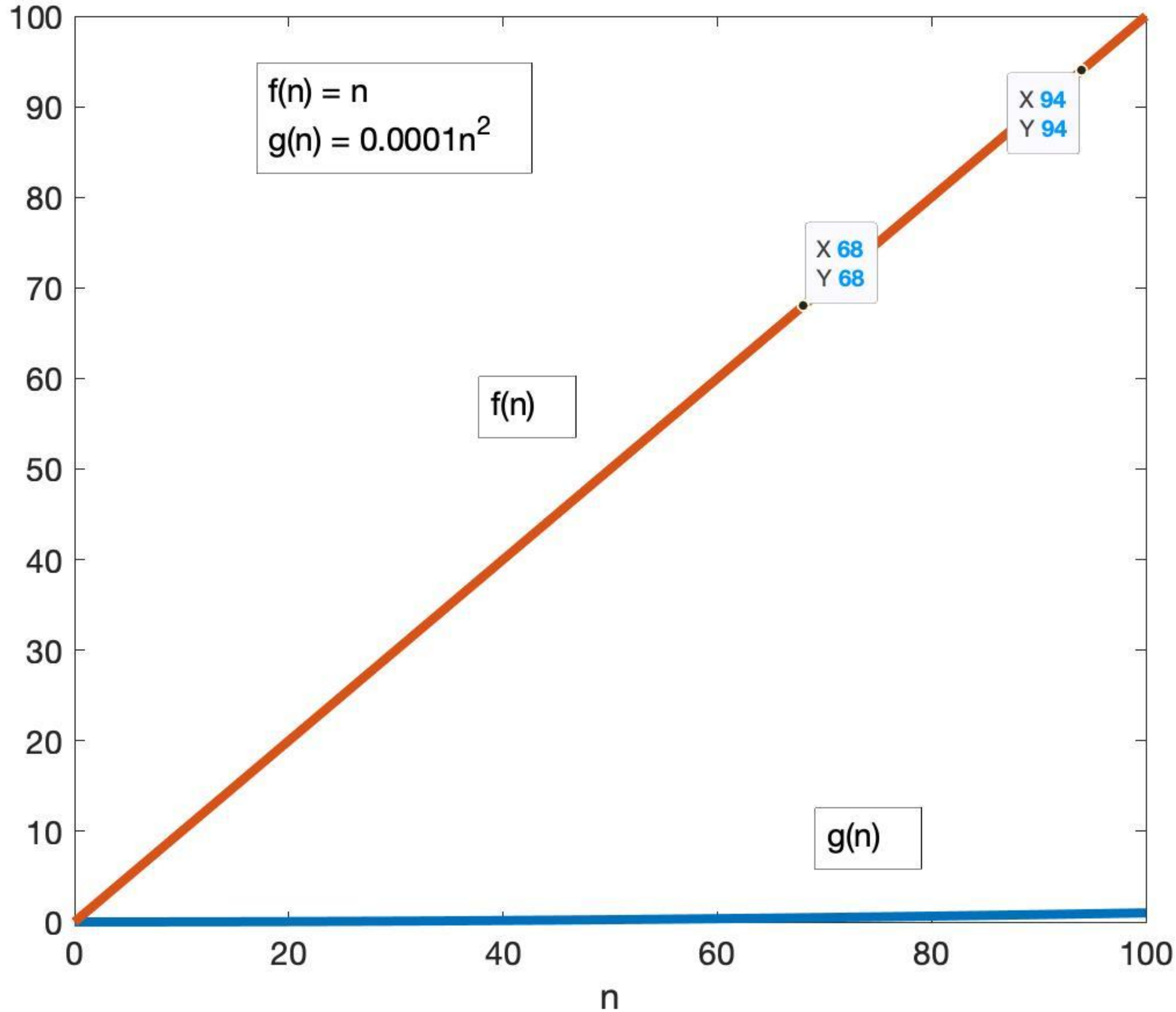
This feels intuitive

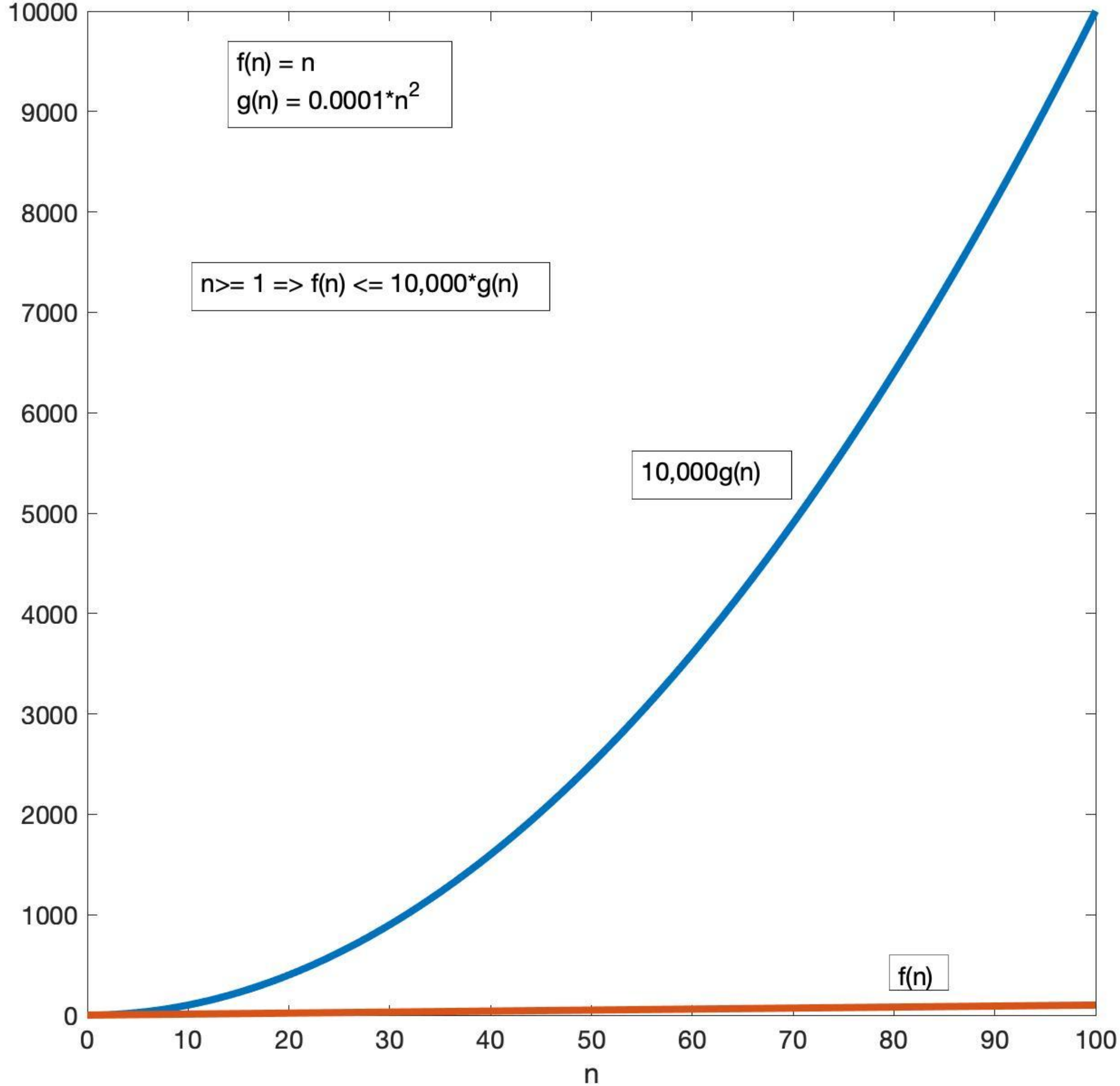
g(n) quickly outgrows f(n)

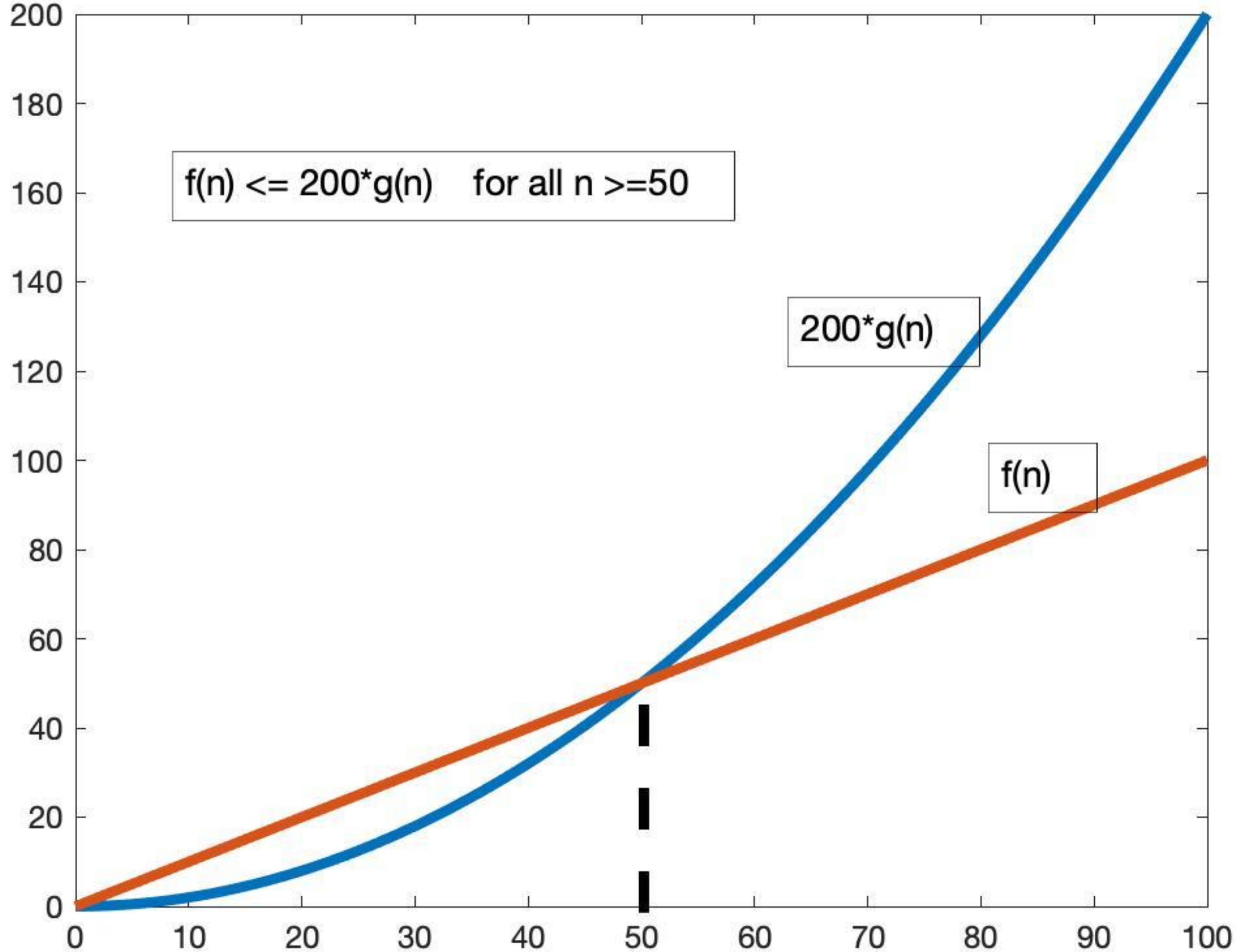
And f ∈ O(g)



But here too,
$f \in O(g)$




f(n) = n
g(n) = 0.0001*n^2
n>= 1 => f(n) <= 10,000*g(n)
10,000g(n)
f(n)
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0
10
20
30
40
50
60
70
80
90
100
n



f(n) <= 200*g(n)   for all n >=50
200*g(n)
f(n)
200
180
160
140
120
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80
90
100