Suspension Commissioning Handbook*†

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Contents

*Upstream url: `https://github.com/gw-vis/vis-commissioning-tex`

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1 Introduction

1.1 About this document

This is a document with suspension commissioning tasks described in detail, as in mathematical/theoretical/code/procedure details. We sincerely hope that this document can serve as a handbook/guideline for suspension commissioning, and as an educational document for people, who want/need to participate in suspension commission activities, to know more about the activities.

This document is dynamic, and it should be, as we get input and suggestions from people, and as technology evolves. With that said, if you have an opinion on particular tasks or methods described in this document or suspension commissioning in general, feel free to submit an issue here. Alternatively, you can contact the maintainer (Terrence for now) via any communication channels (emails, slack, whatever). We will update the document accordingly.

As we all know, people have taken different approaches for suspensions commissioning in the past. And, there were not enough communications between subgroups/people on how suspensions should be configured. With this repository, we hope to create an open environment for people to exchange ideas regarding suspension commissioning. And, we wish the document to be filled with ideas and methods that we all agree on, so all of us will commit to follow this document when participating commissioning activities.

In this document, we will mainly review some of the techniques and methods involved in suspension commissioning tasks. This document is organized as follows, the follow two subsections will be discussing the role of suspension commissioning. In particular, we will be talking about what are the expectations of the hardware aspect of the suspensions as we are given the hardware to work with. We will also be discussing what should interferometer commissioners expect from suspension commissioning. In section 2, we will be discussing the goals that we should work towards to achieve. We will be briefly discussing the background behind these goals and requirements. These are mainly taken from [19] and we strongly recommend readers to read it for detailed explanation. In section [ref sec. 3], we will be discussing some of the tasks involved in suspension commissioning. In particular, we will discussing software tasks, such as sensor diagonalization, and control. We will also be discussing, in detail, some of the techniques that can be used to tackle these problems. At last, section [ref sec. 4] is dedicated to the discussion of performance measurement, which is a series of tests to check if we have met the suspension commissioning requirements.

1.2 Suspension commissioning as a stepping stone

Suspension commissioning is about a series of tasks that get us from hardware installation/upgrade to interferometer commissioning. In other words, we do whatever is necessary to the suspensions into a state where interferometer commissioners can "use" the suspensions to do interferometer alignment and locking, without tweaking the internals of the suspension systems. With that said, we should have certain expectation on the initial state of the suspension (as hardware teams handover the suspensions to us), and interferometer commissioning team should expect something from us. So, this document is about defining those expectations and how do we achieve them.

1.2.1 Hardware expectations

Before we start working on anything, the suspensions have to be ready for commissioning. That is, we shouldn't be tweaking the hardware of the suspensions All tasks that needed to be done at the hardware level, is by definition, not our tasks. We should expect the suspensions to be "healthy", as in, the dynamics of the suspensions are exactly what one would expect, and the sensors and actuators should be fully functional and calibrated. At the end of the hardware installation process, there's an acceptance test, which every suspension should pass. This should, in principle, make sure that the suspensions are "ready". But, if we observe anything out of the normal, we should check the system's vitals and explain what's wrong to hardware experts.

There are many ways that hardware fault can leave you head-scratching for a long period of time. And, sometimes they can be hard to catch, especially when there's no testing pipeline in KAGRA. I learnt this from painful experience. I was asked to work on the optical lever on the signal recycling mirror (SR) suspensions. In particular, I was working on something we called "diagonalization" where we align the sensor signals to some desirable basis. I worked out the math but I can't seem to get the signals decoupled, regardless of the fact that the same method worked for other optical lever. At the end, we discovered that one sensor (QPD segment) has a higher gain than the others and that was simple a hardware fault that has haunted me and the Type-B team for months [20, 3]. Also, there's nothing that prevents the suspensions to go "unhealthy" even if it was perfectly fine. Systems can degrade and degradation can happen. For example, one of the PRM magnet came off during commissioning [7, 15]. Fortunately, commissioners were able to pin point the source of error and asked the vibration isolation system (VIS) team to check it. The lesson learnt here, is that, we shouldn't trust what was given to us, even if the suspensions were given to us by Albert Einstein himself. People make mistakes, and there's no exception. We are scientist, and we need to be skeptical[1]. So, think carefully if any abnormality is due to hardware fault. And, do consult hardware experts and demand a hardware fix when you got stuck.

Here is a non-exhaustive list of problems that I can think of[2], that can only be fixed at a hardware level, but can limit the outcome of suspension commissioning, and demand fixes:

- Suspension in contact with components near it (e.g. Touching a cable). This will usually result in a very low Q-factor in certain vibration modes.
- Sensor not within good operation range. This will pose a limit in the amount of actuation during feedback control. This can also result in wrong sensors/actuator diagonalization. And, this can lead to higher sensor noise.
- Stepper motors stuck. Static load on actuators cannot be offloaded to stepper motors and hence limiting the actuation force during feedback control.
- Suspension cannot be moved in full range (e.g. the BF stage of the SRM [1]).
- Abnormal coupling (e.g. Same signal but with phase difference as reported in [16]).
- Very different coil actuation efficiency, indicating a non-functional actuator.

Fixing these problems are not within the scope of suspension commissioning. We are suppose to work with a healthy system so we get maximum potential out of the hardware. We should ask for a fix, before proceeding. If it can't be fixed, we should discuss possible limitations and possible workarounds together.

1.2.2 What should interferometer commissioners expect?

We cannot guarantee that the outcome of suspension commission are exactly needed for interferometer commissioning. There are few reasons: 1) There are too many uncertainties. 2) Various topics regarding the suspensions are not well-studied, and are still being researched. 3) There are some performances that cannot be confirmed until we have the interferometers. And most importantly, 4) We don't exactly know what realistically interferometer commissioning needs, except we can only refer to theoretical studies like [19], which can be outdated and not really aligned with the real deal here in KAGRA. Therefore, regarding the last point, we strongly encourage interferometer commissioners to cooperate by sharing information and suggestions.

In particular, these information will be very useful:

[1] If there's anything I (Terrence) learnt from Rana, it's this.

[2] Please submit an issue if you can think of more that are worth mentioning.

- What is the realistic displacement level requirement, in RMS and peak, for the optics?
- At what optics displacement level did you observe lock-loss?
- At what seismic noise level did lock-loss occur?
- Is lock-loss correlated with the seismic noise level?
- How would you like to control the suspensions?
- What are the entry points? Do you want to control the suspensions directly from the setpoints? Or do you want to control the suspension using directly from the actuation signals?
- Why did you run into actuation saturation? Do you want the signals to be offloaded to higher stages, like the preisolator?
- What are some problematic cross-couplings (e.g. length-to-pitch), and in what condition you would like them to be decoupled?

While we cannot provide any guarantee, we can and should provide warranty. If the suspensions are failing, it is suspension commissioners' job to fix or improve it. Interferometer commissioners should **not** be tackling suspension tasks like, internal damping of the resonances, sensors decoupling, or seismic noise attenuation. And, interferometer commissioners should expect the suspensions to behave like a blackbox (as you don't need to know what's happening internally), and as some sort of servo (as it steers the optics). If the suspensions become problematic and is hindering interferometer commissioning, interferometer commissioners should work with suspension commissioners to tackle the problem. Previously, such kind of interface was clearly lacking[3] and we hope this can be improved in the future. We need good communication for KAGRA to work. So, here we should say that **suspension commissioning is not a "once done and for all" process**. Instead, it is something that should be recursively done, as we get "complaints" from interferometer commissioners during interferometer commissioning.

2 Goal

Qualitatively speaking, the goal of suspension commissioning is simply to bring the suspensions into a state where interferometer commissioners can use the suspensions, via defined entry points, to steer the mirrors and make an interferometer, which is sensitive enough to be a gravitational wave detectors. Althrough, as mentioned, there's no realistic requirements, which are derived from previous experience, we adopt theoretical requirements derived from [19]. In particular, in order for KAGRA to function as an gravitational wave detector, the suspensions have to satisfied 2 types of requirements, displacement noise requirement, residual motion requirement. In this section, we will briefly discuss the requirements, but will not go into the details, as this is not the scope of this document. Readers are recommended to refer to the external references cited.

2.1 Displacement noise requirement

Ground-based gravitational wave detectors have a detection band starting from 10 Hz. The detectable effect of gravitational waves comes in the form of strain, which loosely speaking, is the fractional change in length. To detect gravitational waves, we simply use an L-shape interferometer and measure the change in the arm lengths in two directions. We expect targeted gravitational waves to have a peak amplitude of 10^{-21}. While the arm lengths of the interferometer are in the order of 10^3 m, the change in arm lengths caused by targeted gravitational wave is expected to be in the order of $10^{-21} \times 10^3$ m $= 10^{-18}$ m, which is a very tiny amount of displacement. Effectively, this means that the test masses (TM), i.e. the end mirrors, must have a displacement level lower than that. Giving a safety factor of 10, i.e. signal-to-noise ratio of around 100, this converts a displacement noise requirement of 10^{-19} m at 10 Hz, roughly speaking. Figure. 1 shows the displacement noise requirement of the optics including the test mass (TM), beamsplitter (BS), signal recycling mirror (SRM), and power recycling mirror (PRM) above 10 Hz. As can be seen, the requirements of optics other than the TMs are much lower. The displacement noise

[3] As Nakano-san is a superman who tackles every problem on his own.

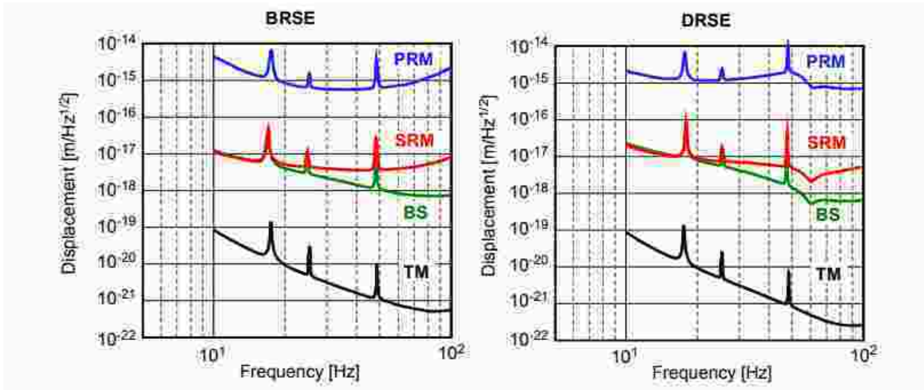

Figure 1: Displacement noise requirements of the test mass (TM) (black), beamsplitter (BS) (green), signal recycling mirror (SRM) (red), and power recycling mirror (PRM) (blue), under boardband resonant sideband extraction configuration (BRSE) (Left) and detuned resonant sideband extraction scheme (DRSE) (Right). Retrieved from [19].

level of the BS and the SRM is at 10^{-17} m, while that of the PRM is at 10^{-15} m. Table .1 summarizes the displacement noise of various optics at 10 Hz. Therefore, the partial goal of suspension commissioning is to make sure these displacement noise levels requirement are met. The suspensions of the optics are designed specifically to

	Displacement level (m/$\sqrt{\text{Hz}}$)
TM	1×10^{-19}
BS	1×10^{-17}
PRM	2×10^{-15}
PR2, PR3	1×10^{-15}
SRM	1×10^{-17}
SR2, SR3	5×10^{-18}

Table 1: Summary of longitudinal displacement noise level (at 10 Hz) of the optics including the folder mirrors of the recycling cavity (PR2, PR3, SR2, SR3). Retrieved from [19].

attenuate the seismic noise at 10 Hz to the displacement level specified in table. 1 via passive isolation. Therefore, the suspensions intrinsically satisfied these requirements already without any tweaking. However, the main concern here is not about passive isolation. Instead, it's about active isolation/feedback control, as we need to keep the control noise below these displacement level, while maintaining certain disturbance rejection capability. This brings us to the other requirement: residual motion.

2.2 Residual motion requirement

In previous section, we discussed what are the requirements for KAGRA to be sensitive enough to become a gravitational wave detector. Here, we will discuss the requirements for KAGRA to be an interferometric gravitational wave detector. In particular, we will discuss velocity, angular fluctuation, and displacement level requirement. Again, here we only briefly mention the rationale behind such requirements and readers are strongly recommended to read [19] for detailed explanations.

2.2.1 Residual velocity requirement

In order for KAGRA to become an interferometric gravitational wave detector, the main optics must be manipulated to form various optical cavities, where feedback control is used to "lock" two mirrors at relatively stable separations. The technique used is called Pound-Drever-Hall (PDH) technique [6]. To use this technique, the separation between two optics stay within the operating range where the control signal for using PDH technique becomes linear. In [19], a velocity requirement of the is derived by considering the maximum actuation power of the TM coils. Consider a case where the optics are moving towards the operating range. As the optics enter the range, the actuators applies maximum force on the optics, causing the optics to decelerate maximally. If the optics are put to halt before they exit the operating range, then lock-acquisition is achieved. This only happens when the velocity of the optics is lower than a certain threshold, which we define as a requirement. The velocity requirements the main optics are derived in [19] and is summarized in table. 2.

	Velocity requirement (RMS) (μm/s)
TM, BS, SR	0.5
PR	2

Table 2: Summary of main optics' velocity requirement. Retrieved from [19].

2.2.2 Residual angular displacement requirement

Now, I don't really understand the rationale behind these angular fluctuation requirements[4] and it seems that different sources suggests different things. I will simply report the requirements from some sources.

In [19], it was reported that the RMS angles of the mirrors should not produce beam spot fluctuation on the optics by larger than an RMS value of 1 mm. Using the beam paths as the lever arm, the angle fluctuation requirements are calculated and shown in table. 3. However, in another source regarding beam jittering at LIGO [14], it's mentioned

	Angular fluctuation RMS (μrad)
TM	0.2[19, 10] / 0.01[2, 14]
BS	4
PRM	45
PR2	20
PR3	2
SRM	25
SR2	10
SR3	1

Table 3: Angular fluctuation requirements of the optics. Retrieved from [19, 10] unless otherwise specified.

that the angular fluctuation requirement of the test masses at LIGO is 10^{-8} rad. The same value is also used to derive the wavefront sensor sensitivity requirement at KAGRA [2]. It's worth noting that both of these sources were also cited in [19]. So unless there's extra comment regarding this, let's just set the requirement for the TM to the lower one (10^{-8} rad) as a worst case scenario[5].

2.2.3 Longitudinal displacement level requirement

At last, the longitudinal displacement requirement was derived also from the maximum actuation power of the force coils at the TM stage. Doing so, we assume that only these actuators are used during lock acquisition, but this may not be the case, so here again we are assuming a worst case scenario. Table. 4 summarizes the displacement RMS value requirement of the main optics in KAGRA. This concludes the residual motion requirements for the core optics. So, the other half of the goal is to actively control the suspensions so requirements as shown in Table. 2, 3, and 4 are met, while not violating noise specification as shown in Table. 1. In the following sections, we shall discuss/review some of the methods and techniques that can be used to achieve these goals. We will also discuss some ways that we used to verify the satisfaction these requirements.

[4]Please help me to write this section if you know the correction explanation. As far as I know, the angular fluctuation requirement comes from the fact that the interferometer needs to be aligned and that the beam spots are within good range around the center of the optics. But, I don't really know how people got those numbers.

[5]There's a chance that we cannot confirm that we actually satisfied this requirement at the end of suspension commissioning as there's no sensors sensitive enough to measure such small level of fluctuation.Let's hope that our optical levers are sensitive enough

	Displacement RMS requirement (μm)
TM	0.01
BS	3.3
SRM	3.3
SR2, SR3	1.6
PRM	560
PR2, PR3	280

Table 4: Longitudinal displacement RMS requirement of the optics. Retrieved from [19].

2.3 Miscellaneous

This section mainly refers to artificial requirements that are set in [13].

2.3.1 Seismic noise

As a vibration isolation system, the main goal of the suspensions is to isolate the sole external disturbance, that is, the seismic disturbance. Therefore, the optics must satisfy the displacement noise and residual motion requirements under the influence of a background seismic noise. However, the residual motion of the optics vary with seismic noise level, which means that the performance of the suspension evaluated at a certain moment may not be the same at the other, as the seismic noise is dynamic. So, we purpose an additional constraint here. We require that the displacement noise and residual motion requirements to be satisfied under the influence of a disturbance equivalent to a background seismic noise at the 90$^{\text{th}}$ percentile level in the winter season. The choice of this seismic noise meant to be conservative as it's considered a very violent disturbance. To give some perspective of how the seismic noise level is, the seismic noise at the 90$^{\text{th}}$ percentile in KAGRA is shown in Fig. 2 [12]. As is mentioned,

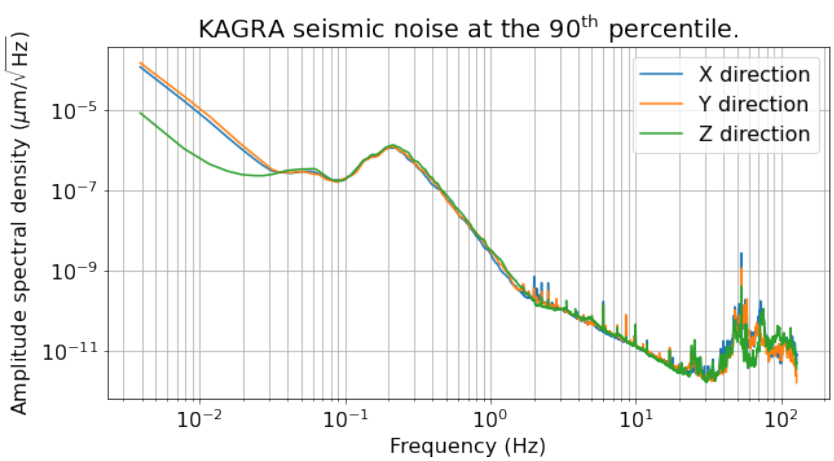

Figure 2: The amplitude spectral density of seismic noise in KAGRA at 90$^{\text{th}}$ percentile in x (blue), y (yellow), and z (green) direction. Data retrieved from [12].

this is a conservative requirement, and it can be rare to observe such huge disturbance. This means that it can be hard to test the suspension performance naturally. To this end, as we shall see in later sections [ref later sections], we purpose to inject, using the actuators, artificial time series that has the same amplitude spectral density (ASD) of the measured seismic noise.

2.3.2 Guardian

Guardian is an automation system used in advanced LIGO (aLIGO) [18]. It's a platform where operators can program, using Python, certain operations that takes the suspensions from one defined states to another via certain defined paths. The details of Guardian will not be discussed here, readers are strongly recommended to read the "living" document from aLIGO [18]. We will only briefly introduce what we're trying to achieve with Guardian and what people should expect. Here, we will relay information given in [13, 11]. Fig. 3 shows the (proposed?) state-

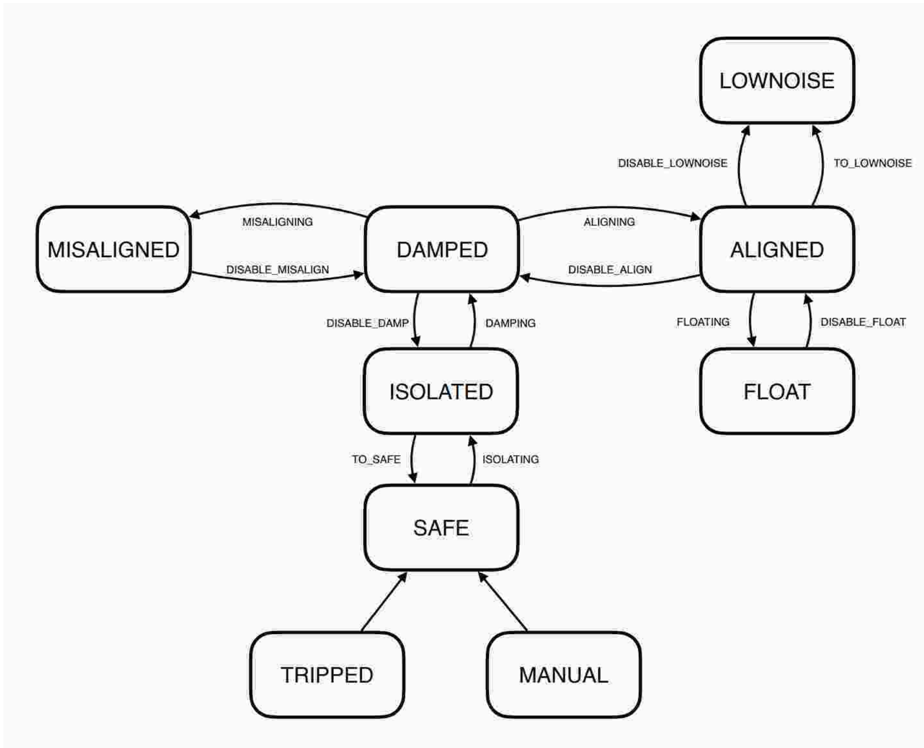

Figure 3: State-transition diagram of the (proposed?) VIS Guardian. Retrieved from [11]

transition diagram of the VIS Guardian. As can be seen, there are 9 Guardian states, namely TRIPPED, MANUAL, SAFE, ISOLATED, DAMPED, MISALIGNED, ALIGNED, FLOATED, and LOWNOISE. Here, description in [11] is lacking so we will be further elaborating the behavior of these states.

In the TRIPPED state, as is indicated by the name of the state, the suspension is tripped. This is triggered by a builtin watchdog system in the real-time model that monitors for abnormal behaviors. The suspension can "jump" (In Guardian language) into this state from every other state. This happens when the watchdog's tripped.

As for the MANUAL state, not sure what that means. But my guess is that this is a state with the master switch turned on, so actuation signals can go through for measurement and diagnostic purposes.

In the SAFE state, actuation signals cannot go to the actuators, essentially immobilizing the suspension.

In the ISOLATED state, active isolation, usually at the stage closest to the ground (likely preisolator/inverted pendulum), is engaged. The controls systems are engaged such that two things are achieved: 1) Coarsest alignment with integration action, and 2) Seismic noise suppression. The controls at the higher stage will bring lower stages into operating point (e.g.centering the optical levers), but will perturb lower stage resonances via control noise injection.

In the DAMPED state, controls at the lower stages will be engage to suppress the these resonances. At this point, the suspension should be able to satisfy the residual motion requirements as stated in Sec. 2.2.

In the ALIGNED state, the optics is steered to be coarsely aligned and is ready to be handed over to inferometer control. Distinct from the ALIGN state, In the MISALIGN state, the optics is steered away from the aligned position.

At last, at the LOWNOISE state, controls that may compromise the detector sensitivity will be shut off or replaced by a lower noise version. At this point, the suspension should satisfy the displacement noise requirements as stated in Sec. 2.1.

3 Suspension Commissioning Tasks

In this section, we will describe what are some specific tasks needed to be done in order to achieve the aforementioned goals in Sec. 2. In next section, Sec. 4, we will describe some of the mathematical details on how to complete these tasks, and how to evaluate the performance of the suspensions. In the section after the next, Sec. 5, we will review some methods that has been proposed but are not considered baseline. Here, we encourage readers to choose and implement the methods themselves. There's no best method. And, we should emphasize that it's not important to use the same methods for all suspensions, but rather, to evaluate all suspensions using the same evaluation methods. As long as we have a consistent evaluation scheme that ensures the requirements are met, that's it needs for KAGRA to work.

3.1 List of tasks

We will provide a list of tasks here in this section. Further elaboration will be provided in the next. These tasks are defined with the assumption that we have healthy hardware and we have sensors (and actuators) calibrated, and they are listed in order.

1. (If not done already) Sensor noise measurement (and modeling).
2. (If not done already) Seismic noise measurement (and modeling). Use a worst-case spectrum, e.g. 90^{th} percentile seismic noise in the winter season.
3. Control Matrices
   (a) Install initial sensing matrices and actuation matrices from first principles.
   (b) Modify sensing matrices or apply "diagonalization"/"decoupling" matrices so the readout is mapped to the desired basis (Cartesian coordinate + Euler angles).
   (c) (Optional) Modify actuation matrices or apply "diagonalization"/"decouping" matrices so the actuations are also in the desired basis.
4. Inter-calibration of sensors.
5. Further sensor noise reduction tasks, e.g. sensor fusion and sensor correction. Redo step 1.
6. Transfer function (TF) measurements and modeling. Do for
   - Diagonal actuation TFs in a stage-by-stage basis,
   - (Optional) Cross actuation TFs in a stage-by-stage basis, and
   - TFs from each displacement to optics/test mass (TM) degrees of freedom (DoF).
7. From the highest stage (closest to the ground) to lowest stage, design the control filter and do the following
   (a) Predict the closed-loop displacement level and
   (b) estimate the residual motion and displacement noise contribution to the optics using
      - the seismic noise measurement/open-loop displacement levels from step 2 or step 7f,
      - sensor noise measurement from step 1, and
      - the displacement-to-optics displacements transfer functions from step 6.
   (c) Check stability using stability critera (Nyquist plot and stability margins.) and transfer functions from step 6.
   (d) If any of the above failed, tune the control filter.

(e) Install the control filters and close the loop.

(f) Measure open-loop displacement levels of the next stage (Keep the controls at upper stage engaged.) and move on the next stage.

(g) Repeat step 7 until all local control-loops at all stages are closed.

8. Measure the residual motion using the sensors at the optics stage as an out-of-loop sensor (Do not engage the control-loops that use the optics' sensors). If this fails, find the problematic stage and redo all controllers starting from there.
   - We can measure the spectrum directly, given that the sensors at the optics are not coupled to the ground motion and that the ground motion is close to what we are targeting.
   - Alternatively, we can inject a timeseries equivalent to the target seismic noise at the highest stage to simulate the worst-case scenario.
9. (If there exists an interferometer) Measure actuation to differential arm length control signal (DARM) transfer function and measure the displacement noise. If this fails, find the problematic stage and redo all controllers starting from there.

3.2 Further elaboration

In this section we will discuss why do we need to complete the tasks as stated in Sec. 3.1 and why are they listed in that order. We will be discussing in the language of control theory. If you're not familiar with the topic, please refer to introductory textbooks such as [17] and [9].

Consider the control diagram as shown in Fig. 4. $R(s)$, $D(s)$, and $N(s)$ are the external inputs, and they are the setpoint, disturbance, and noise respectively. $R(s)$ is usually a DC setpoint purposed for coarse alignment. It has no non-zero frequency content so we'll just mention its existence here and will drop it in further discussions. $D(s)$ can be any arbitrary disturbance to that particular degree of freedom, that directly goes to the output $X(s)$ It can be thought as the output $X(s)$ when there's no actuation, and can be defined as

$$D(s) \equiv \lim_{K(s)\to 0} X(s; K(s)) \tag{1}$$

$X(s)$ is the output and can be thought as the actual displacement, not measured, of the DoF we aimed to control. This can be any of the arbitrary displacement at any stage of the suspension. $N(s)$ is the sensing noise, not exactly the sensor's intrinsic noise, but is defined as

$$N(s) \equiv \lim_{K(s)\to \infty} -X(s; K(s)) \tag{2}$$

$U(s)$ is the actuation signal. $P(s)$ is the actuation transfer function of this particular degree of freedom, and is defined as

$$P(s) \equiv \frac{X(s)}{U(s)} \tag{3}$$

At this point it because tedious to specify everything as a function of s, so will simply omit to write the s dependencies and assume all variables with capital letters are all functions of s unless otherwise specified.

The first thing we would like to note is that the suspension is not automatically configured in such a way Fig. 4 is valid. This is because the sensors on a stage are not necessarily aligned to the DoFs that we are interested in. The location of the sensors are usually well known. With a geometric matric transformation, we will be able to obtain an initial "map" or sensing matrix that maps the sensing readouts to the DoFs that we are interested in. See [5, 4, 11, 8] for references. However, this initial sensing matrix gives good calibration of the DoFs but might not be perfect so residual cross-couplings might still exist. This is done by modifying the sensing matrix / applying a second matrix along the signal path, i.e. "diagonalization". Similarly, the actuators might not be aligned to the desired DoFs and an actuation matrix is needed to align that. With the sensors and actuators aligned, Fig. 4 becomes valid for individual DoFs on a stage.

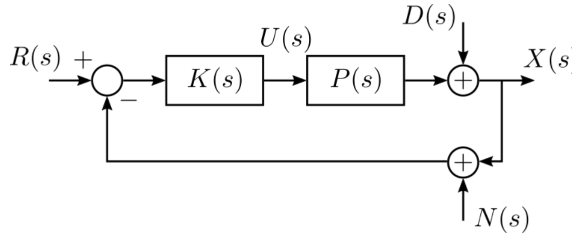

Figure 4: Typical control block diagram of a single degree of freedom. $R(s)$: reference/setpoint, $D(s)$: external disturbance, $X(s)$: displacement, $N(s)$ sensing noise, $K(s)$: controller, $P(s)$: actuation transfer function.

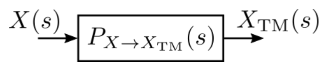

Figure 5: Displacement to optics' displacement path

Now, in Fig. 4, X represents the displacement of in an arbitrary DoF of an arbitrary stage. As shown in Fig. 5, there exists paths $P_{\mathrm{X}\rightarrow \mathrm{X}_{\mathrm{TM}}}$, such that the displacement at any stage is transferred to the optics displacement X_{TM}. In reality, since there are many DoFs, there exists many paths like Fig. 5, as shown in Fig. 6. Hence, without the optics local control, the amplitude spectral density of the optics displacement $\hat{X}_{\mathrm{TM}}(f)$ is a quadrature sum of all these contribution, i.e.

$$\hat{X}_{\mathrm{TM}}(f) = \left[\sum_i \left|P_{X_i\rightarrow X_{\mathrm{TM}}}\right|^2 \hat{X}_i(f)^2\right]^{\frac{1}{2}}, \tag{4}$$

where X_i is the displacement of the i^{th} DoF, $\hat{X}_i(f)$ is the amplitude spectral density of X_i and $P_{X_i\rightarrow X_{\mathrm{TM}}}$ the transfer function from X_i the optics displacement X_{TM}.

Now, individual contributions from different displacements in Eqn. (4) can vary by orders of magnitude, and so they are often viewed in logarithmic scale. And because of this, the Eqn. (4) can be approximated as

$$\hat{X}_{\mathrm{TM}}(f) \approx \max_i \left(\left|P_{X_i\rightarrow X_{\mathrm{TM}}}\right| \hat{X}_i(f)\right). \tag{5}$$

With this in mind, the strategy is simple. We need to make sure that the displacement of the optics can satisfy the displacement requirements and residual motion requirements stated in Sec. 2.1 and Sec. 2.2. So, for each stage and each DoF, we estimate individual $\left|P_{X_i\rightarrow X_{\mathrm{TM}}}\right| \hat{X}_i(f)$ and make sure that they satisfy the requirements instead. To estimate this, let's refer back to Fig. 4. The displacement X in the closed-loop configuration reads

$$X = \frac{1}{1+KP}\,D - \frac{KP}{1+KP}\,N, \tag{6}$$

and the amplitude spectral density reads

$$\hat{X}(f) = \left[\left|\frac{1}{1+KP}\right|^2 \hat{D}(f)^2 + \left|\frac{KP}{1+KP}\right|^2 \hat{N}(f)^2\right]^{\frac{1}{2}}. \tag{7}$$

If we know the the amplitude spectral densities $\hat{D}(f)$ and $\hat{N}(f)$, and the plant P, we can estimate the amplitude spectral density $\hat{X}(f)$ when designing the controller K. If we know $\hat{X}(f)$ and $P_{X\rightarrow X_{\mathrm{TM}}}$, then we can estimate $\left|P_{X\rightarrow X_{\mathrm{TM}}}\right| \hat{X}(f)$, i.e. the displacement level of the optics due to displacement X. Then, we can tune K such that $\left|P_{X\rightarrow X_{\mathrm{TM}}}\right| \hat{X}(f)$ satisfies the requirements. This is how we can design the controllers such such that the requirements are met.

But realistically, we cannot design the controllers in arbitrary order. This strategy requires the a top-down hierarchical control tuning scheme, i.e. tuning the controllers stage-by-stage from the stage closest to the ground. This is because the amplitude spectral density of the disturbance $\hat{D}(f)$ is not easy to estimate. In fact, the disturbance at a lower stage depends on the displacement level at a higher stage, which depends on the controller of that higher stage. This is demonstrated in Fig. 6. The only disturbance we can estimate is the disturbance at the highest stage D_1, i.e. the preisolator (IP), whose disturbance is closely related to the seismic noise X_g.

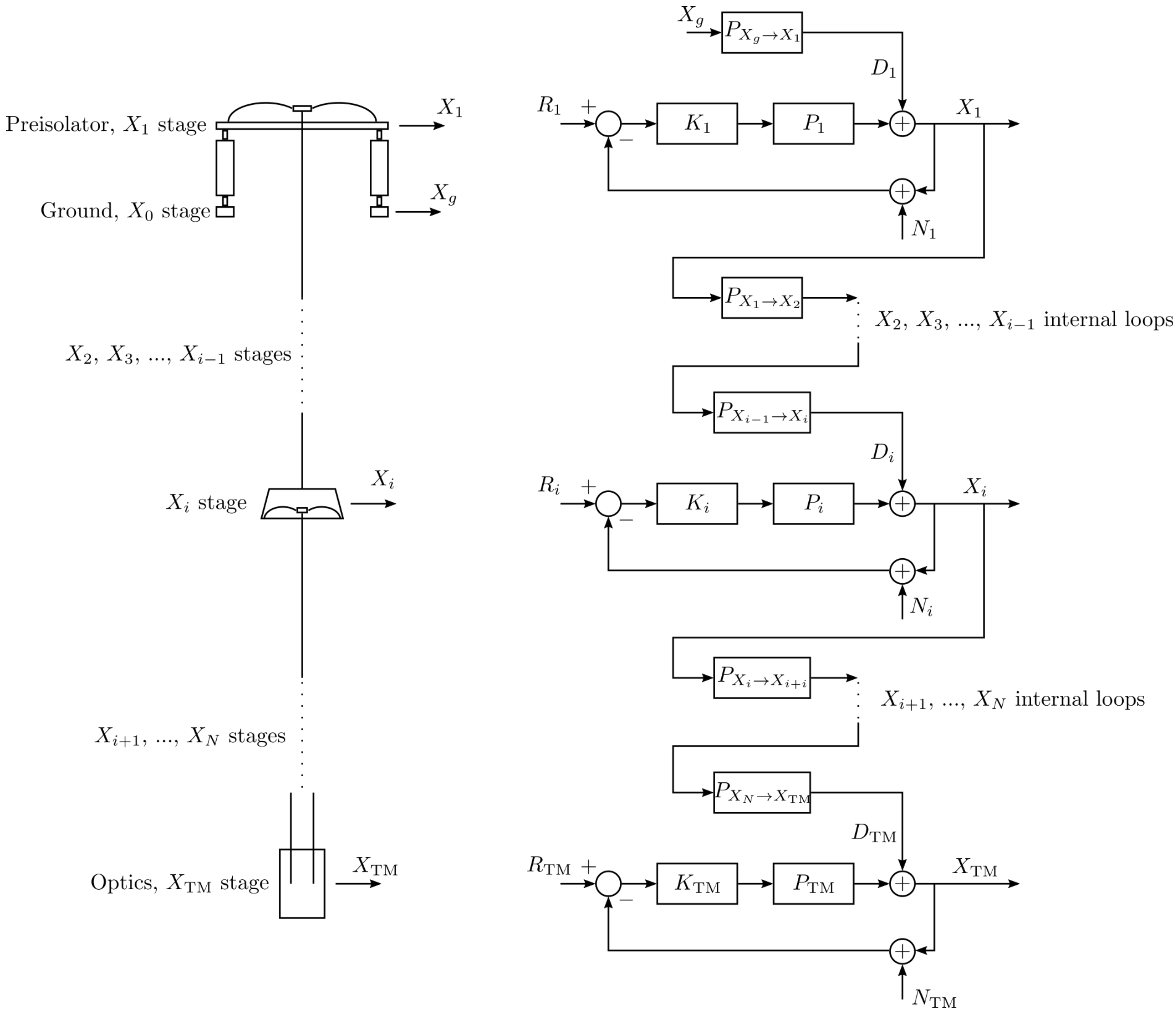

Figure 6: Control block diagram of a general multiple stage suspension with N number of stages. The disturbance of a lower stage is connected to the displacement at a higher stage. Here we use numbered subscripts to denote stages further from the ground for simplicity. In reality, the stages are named something like F1, F2, BF, IM, etc... The optics' displacement X_{TM} is an example.

Now, let's re-elaborate the strategy using Fig. 6, which shows a general hierarchical control architecture of KAGRA's suspensions. The strategy goes as follows. We specify a conservative seismic noise lever for X_g, say the seismic noise level at the 90$^{\mathrm{th}}$ percentile in winter season as shown in Fig. 2, so we know $\hat{X}_g(f)$. The ASD of disturbance is given by

$$\hat{D}_1(f) = \left|P_{X_g \to X_1}\right| \hat{X}_g(f), \tag{8}$$

and $P_{X_g \to X_1}$ can be measured or estimated directly by P_1, but with a unity gain at DC. Then, we can estimate $\hat{N}_1$, the sensing noise at the preisolator stage. After that, we measure P_1, the actuation transfer function of the preisolator stage. From here, we have all the ingredients to estimate the displacement level $\hat{X}_1$ using Eqn. (7), when designing the controller K_1. Then, we can also measure the transfer function $P_{X_1 \to X_{\mathrm{TM}}}$[6]. With this, we can

[6]It's another story on how to measure this.

calculate the optics' displacement due to X_1 by

$$\hat{X}_{\mathrm{TM}}(f) \approx \left|P_{X_1\to X_{\mathrm{TM}}}\right|\hat{X}_1(f). \tag{9}$$

We then tune K_1 such that Eqn. (9) satisfies requirements in Sec. 2.1 and Sec. 2.2. After tuning, K_1 is fixed and so is X_1. Hence, we can estimate the disturbance on the next stage via

$$\hat{D}_2(f) = \left|P_{X_1\to X_2}\right|\hat{X}_1(f). \tag{10}$$

Alternatively, we can simply estimate D_2 by measuring the open-loop spectrum of X_2 when the control of X_1 is engaged. But there're two assumptions here, 1) $D_2 \gg N_2$, so the readout is mostly D_2 and that 2) D_1 is suppressed, so X_1 doesn't change a lot when X_g changes. Then, we repeat the process for stage 2, stage 3 after that, and so on until all control loops are closed.

To verify the control performance, we need to actually measure $\hat{X}_{\mathrm{TM}}(f)$ and verify that it satisfies the requirements in Sec. 2.1 and Sec. 2.2. However, with all controls engaged, measuring the optics' displacement using the local sensors could yield a wrong result. The optics' readout $X_{\mathrm{TM,readout}}$, i.e. the bottom-left path in Fig. 6, can be written as

$$X_{\mathrm{TM,readout}} = \frac{1}{1+K_{\mathrm{TM}}P_{\mathrm{TM}}}\,D_{\mathrm{TM}}, \tag{11}$$

which can be theoretically zero when K_{TM} is high. From Eqn. (7), we know that this is not true. And in fact, the lower bound of the displacement is

$$\underline{\hat{X}}_{\mathrm{TM}}(f) \approx \min\left(\hat{D}_{\mathrm{TM}}(f),\, \hat{N}_{\mathrm{TM}}(f)\right). \tag{12}$$

This is a well known measurement error when using "in-loop" sensors for measurement. To measure the optics displacement properly, we must use sensors that is not part of the control loops, i.e. using an "out-of-loop" sensor. The candidate for this is a second optical lever, which doesn't exist, or the interferometer, which also doesn't exist at the moment. Therefore, we must disengage the controls at the optics stage before we can measure the residual motion of the optics. We assume that the displacement level of the optics will only be lower when the optics controls are engaged. Then, if the measurement above satisfies the residual motion requirements, then having the optics controls engaged will guarantee that the requirements are met as well.

4 Suspension Commissioning Baseline Methods

4.1 Performance evaluation

5 Suspension Commissioning "Advanced" Methods

5.1 Control optimization methods

5.2

References

[1] Fabian Arellano. SRM work: Bottom Filter, Payload and Top Filter limit switches. https://klog.icrr.u-tokyo.ac.jp/osl/?r=15518.

[2] Yoichi Aso, Yuta Michimura, Kentaro Somiya, Masaki Ando, Osamu Miyakawa, Takanori Sekiguchi, Daisuke Tatsumi, and Hiroaki Yamamoto. Interferometer design of the KAGRA gravitational wave detector. *Phys. Rev. D*, 88:043007, Aug 2013.

[3] Mark Barton. Comment to SR2 OPLEV diagonalization. https://klog.icrr.u-tokyo.ac.jp/osl/?r=8419.

[4] Mark Barton. SR Suspension Diagonalization Matrices. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=7663.

[5] Mark Barton and Enzo Tapia. BS Suspension Diagonalization Matrices. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=7205.

[6] Eric D. Black. An introduction to Pound–Drever–Hall laser frequency stabilization. *American Journal of Physics*, 69(1):79–87, 2001.

[7] Yoshinori Fujii. PRM inspection, one magnet had come off. https://klog.icrr.u-tokyo.ac.jp/osl/?r=8255.

[8] Tsang Terrence Tak Lun. Sensing Matrices for Optical Levers (OpLev) of the KAGRA Main Optics. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=12874.

[9] Jenő Hetthéssy László Keviczky, Ruth Bars and Csilla Bányász. *Control Engineering*.

[10] Yuta Michimura. What to do with light levers at KAGRA (Translated). https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=2403.

[11] Kouseki Miyo. All of the Vibration Isolation System in KAGRA. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=12144.

[12] Kouseki Miyo. Seismic noise of KAGRA mine. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=10436.

[13] Kouseki Miyo. Suspension Commissioning. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=12444.

[14] Guido Mueller. Beam jitter coupling in advanced LIGO. *Opt. Express*, 13(18):7118–7132, Sep 2005.

[15] Masayuki Nakano. MICH get locked for 45 min!! https://klog.icrr.u-tokyo.ac.jp/osl/?r=8182.

[16] Masayuki Nakano. PReQua. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=11861.

[17] Katsuhiko Ogata. *Modern Control Engineering*.

[18] Jameson Graef Rollins. Advanced LIGO Guardian: Overview and Coder's Introduction. https://dcc.ligo.org/LIGO-G1400016/public.

[19] T. Sekiguchi. *Study of Low Frequency Vibration Isolation System for Large Scale Gravitational Wave Detectors*. PhD thesis, Tokyo U., 2016. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=4155.

[20] Terrence Tsang. SR2 OPLEV diagonalization. https://klog.icrr.u-tokyo.ac.jp/osl/?r=8411.