First Draft: Circumnavigation and obstacle avoidance guidance for UAV monitoring a ground target using Vector Fields

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Circumnavigation of a moving ground target using a UAV requires guidance to maintain a standoff distance. Fixed wing aircraft are unable to directly follow over a ground target and must turn to maintain flight. Vector Field (VF) guidance was investigated as a circumnavigation method that would converge, circulate, and track a moving target using a constant radius circle with the target as a center. Parameters of the VF were explored to understand performance and capabilities of the field. In addition, obstacles may be present and can be represented by a VF that pushes away to achieve avoidance. The VF guidance circumnavigation and obstacle avoidance was simulated using a UAV Dubins vehicle that followed a target with an unknown path.

I. Nomenclature

A	=	amplitude of oscillation
a	=	cylinder diameter
C_p	=	pressure coefficient
Cx	=	force coefficient in the x direction
Cy	=	force coefficient in the y direction
c	=	chord
dt	=	time step
Fx	=	X component of the resultant pressure force acting on the vehicle
Fy	=	Y component of the resultant pressure force acting on the vehicle
f, g	=	generic functions
h	=	height
i	=	time index during navigation
j	=	waypoint index
K	=	trailing-edge (TE) nondimensional angular deflection rate

II. Introduction

Circumnavigation of a moving target using an aerial vehicle requires a guidance system to control the surveillance aircraft's heading. There are several methods to guide a vehicle to maintain a radius around a target, but the two most prominent are direct guidance laws and Vector Fields (VF). Further complication of guidance requirements comes from a moving target where the path is not known. Typically, the target and vehicle position, and velocity are necessary for the guidance system. Additional features of a guidance system may include obstacle or areas of avoidance. The VF guidance generation was investigated to determine tracking performance of an aerial vehicle to circumnavigate (circle) a target that is moving and avoid designated areas.

III. Literature Review

Guidance for a UAV that is moving faster than another target, assumed in this case to be a ground vehicle, may be achieved using several different methods. Three recurring types of guidance seen for UAVs include Potential Fields, classical optimal laws, and Vector Fields (VF). Various methods are used to generate Potential Fields, where obstacles

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are characterized by areas of high potential and the area of lowest potential contains the goal or end point [?]. The UAV is guided from an initial position in the field, following a path of descending potential, until it reaches the area of lowest potential [?]. While potential field methods may be utilized for some guidance scenarios, they have inherent limitations [?] and are not a feasible method for circumnavigation of a target. Classical control laws were developed [? ?], are typically non-linear, and require direct control over a vehicle's heading. Vector Field methods and control laws found in literature assume constant UAV velocity [?], which is controlled using a separate system. Classical control laws were found to be complicated in terms of required calculations, and are unsuitable in this case due to their inability to circumnavigate a target. VF methods were investigated further due to their ability to provide circumnavigation of a moving and non-moving target. The VFs come in several different forms where the initial and final concept of generating the field are fundamentally different. The two primary methods considered in the generation of VFs for guidance are Lyapunov [?] and Gonçalves [?] or Gradient Vector Fields (GVF), which generate fields using completely different fundamental mechanisms. Lyapunov and GVF both appear to generate similar VFs for basic shapes (ex, circle), and are undefined at the origin (x=0,y=0). The GVF has additional features that allow for time-varying scenarios, where the surfaces move with respect to time, and the ability to support more advanced surfaces than Lyapunov.

IV. The Lyapunov Vector Field

Lyapunov VFs can be generated given a Lyapunov function as defined in Frew [?]. The exact method of transforming a Lyapunov function into a VF equation can be found [? ? ?]. Circular VF guidance, or circumnavigation, using a Lyapunov equation can be achieved from Equation 1 where v is the desired UAV velocity, r is the UAV distance from the center of the field, r_d is the desired circle radius, and x and y are the UAV positions. The Lyapunov VF can be built from a variety of continuous shapes and may include a constant wind field by summing an additional vector.

$$\overrightarrow{V}_{Lyapunov} = \frac{v}{r^2 + r_d^2} \begin{bmatrix} -x(r^2 - r_d^2) - y(2rr_d^2) \\ -y(r^2 - r_d^2) - x(2rr_d^2) \end{bmatrix} \tag{1}$$

V. The Gonçalves Vector Field

The GVF is fundamentally an interaction of two surfaces that produces a convergence and circulation path where the surfaces intersect [? ? ? ?]. The functions to express the surfaces ($\alpha_i : \mathbb{R}^n \rightarrow \mathbb{R} | i = 1, ..., n-1$) of $(n-1)$ dimensions must be positive definite function that are always differentiable. Consider a space with the following dimensions:

$$\mathbf{q} = \begin{bmatrix} x_1, x_2, ..., x_n \end{bmatrix} \tag{2}$$

where the vector field from [?] can then be expressed as:

$$\overrightarrow{V} = G \sum_{i=1}^{n-1} \alpha_i \nabla_{\mathbf{q}} \alpha_i + H \wedge_{i=1}^{n-1} \nabla_{\mathbf{q}} \alpha_i + L M^{-1} a \tag{3}$$

or in component form

$$\overrightarrow{V} = G\overrightarrow{V}_{conv} + H\overrightarrow{V}_{circ} + L\overrightarrow{V}_{tv} \tag{4}$$

where each of the three vector components can be individually expressed by their contribution. The convergence as

$$\overrightarrow{V}_{conv} = \sum_{i=1}^{n-1} \alpha_i \nabla_{\mathbf{q}} \alpha_i \tag{5}$$

and the circulation term as

$$\overrightarrow{V}_{circ} = \wedge_{i=1}^{n-1} \nabla_{\mathbf{q}} \alpha_i \tag{6}$$

which for orthogonal relationship of α_1 to α_2 reduces to

$$\overrightarrow{V}_{circ} = (\nabla\alpha_1 \times \nabla\alpha_2) \tag{7}$$

The time varying component of the VF can be calculated as

$$\overrightarrow{V}_{tv} = M^{-1} a \tag{8}$$

where

$$\overrightarrow{M} = \begin{bmatrix} \nabla\alpha_1^T \\ \nabla\alpha_2^T \\ (\nabla\alpha_1 \times \nabla\alpha_2)^T \end{bmatrix} \tag{9}$$

and

$$\overrightarrow{a} = \begin{bmatrix} \frac{\partial\alpha_1}{\partial t} & \frac{\partial\alpha_2}{\partial t} & 0 \end{bmatrix}^T \tag{10}$$

Note that the a parameter in Equation 10 is the only part of the entire VF calculation that includes change with respect to time, which includes only the moving target.

The original VF specification from Gonçalves states that the three vector field components are to be added together (Equation 4). A slight modification of straight sum would be to first normalize each components for equal weighting and then apply the G, H, and L weights. This should have the effect of equal weighting and allow the modification weights (G, H, and L) to have a more dramatic effect.

$$\overrightarrow{V} = G\|\overrightarrow{V}_{conv}\| + H\|\overrightarrow{V}_{circ}\| + L\|\overrightarrow{V}_{tv}\| \tag{11}$$

VI. Dubins Vehicle

Dubins vehicle modeling was used for simulation of a fixed wing aerial vehicle. The vehicle's velocity (V_{uav}) was assumed to be fixed with a variable heading angle (θ). The vehicle's velocity (Equation 12) and position (Equation 13) are updated at every time step. The heading angle is limited by Equation 14 and directed by the angle of the vector field.

$$\overrightarrow{V}_{uav}(k) = V_{uav} \begin{bmatrix} cos(\theta(k)) \\ sin(\theta(k)) \end{bmatrix} \tag{12}$$

$$\overrightarrow{X}(k) = \overrightarrow{V}dt + \overrightarrow{X}(k-1) \tag{13}$$

$$\dot{\theta} \leq 20deg/sec \tag{14}$$

VII. Circular Field Example

Application of the GVF for circumnavigation of a moving target will require a cylinder with moving center (α_1, Equation 15) and flat plane surface (α_2, Equation 16). The equations for convergence (17), circulation (18), and time varying (19) were used to generate the Figures 1 to **??** along with the final vector field for (G=H=L=1). Note that the plot has normalized vectors for viewing purposes so all vectors can be seen. The vector fields shown build a picture of how a vehicle can be dictated to head a specific direction to converge and then circulate around a target that may be moving.

$$\alpha_1 = (x - x_c)^2 + (y - y_c)^2 - r^2 \tag{15}$$

$$\alpha_2 = z \tag{16}$$

$$\overrightarrow{V}_{conv} = \left(\frac{\alpha_1}{r^2} \begin{bmatrix} 2(x - x_c) \\ 2(y - y_c) \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \tag{17}$$

$$\overrightarrow{V}_{circ} = \begin{bmatrix} 2(y - y_c) \\ -2(x - x_c) \\ 0 \end{bmatrix} \tag{18}$$

$$\overrightarrow{V}_{tv} = \frac{-2v_x(x - x_c) - 2v_y(y - y_c)}{(2(x - x_c))^2 + (2(y - y_c))^2} \begin{bmatrix} 2(x - x_c) \\ 2(y - y_c) \\ 0 \end{bmatrix} \tag{19}$$

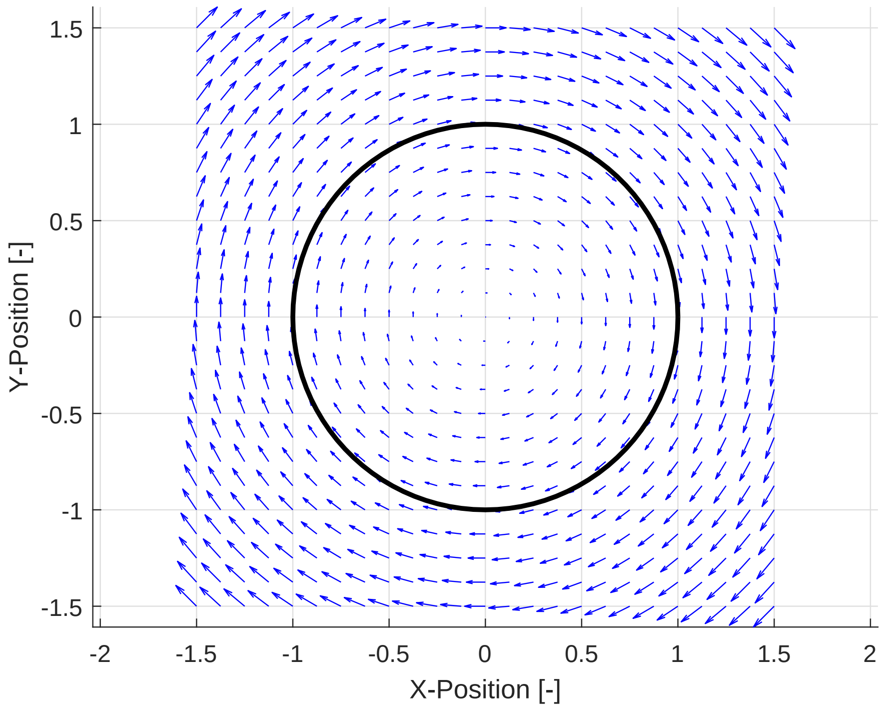

Fig. 1 GVF Circulation Field around Circle

VIII. Conclusion

A conclusion section is not required, though it is preferred. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions. *Note that the conclusion section is the last section of the paper that should be numbered. The appendix (if present), acknowledgment, and references should be listed without numbers.*

Appendix

An Appendix, if needed, should appear before the acknowledgments.

Acknowledgments

An Acknowledgments section, if used, **immediately precedes** the References. Sponsorship and financial support acknowledgments should be included here. The preferred spelling of the word "acknowledgment" in American English is without the "e" after the "g." Avoid expressions such as "One of us (S.B.A.) would like to thank. . . " Instead, write "F. A. Author thanks. . . " Sponsor and financial support acknowledgments are also to be listed in the "acknowledgments" section.

References