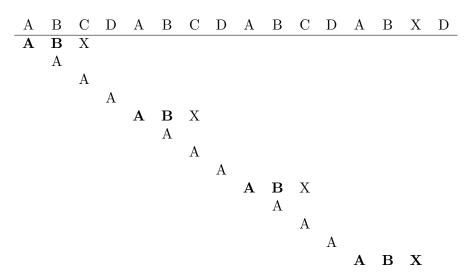
Comp Sci 3130 Gabriel Wallace

Homework 3

1.

We search for the substring "ABX" in the string "ABCDABCDABCDABXD" Successful matches are in bold and unsuccessful are in regular text.



We see that there are 9 successful and 12 unsuccessful matches.

2. Section 3.2 #8 (a)

```
ALGORITHM CountSubstrings(T[0 \dots n])
count \leftarrow 0
for i \leftarrow 0 to n-1 do
if T[i] = `A` then
for j \leftarrow i to n do
if T[j] = `B` then count \leftarrow count + 1
end if
end for
end for
```

In the best case, we have a string with no A's in it, and we never enter the second for loop. Thus, we would only iterate through the string once, so

$$C_B(n) \in \Theta(n)$$

Comp Sci 3130 Gabriel Wallace

In the worst case, we would have a string of just A's, so we would enter the second loop every single time. Thus, we have

$$C_W(n) = \sum_{i=0}^{n-1} \sum_{j=i}^n 1$$

$$= \sum_{i=0}^{n-1} (n-i+1)$$

$$= (n+1) + (n+0) + \dots + 1$$

$$= \frac{(n+2)(n+1)}{2}$$

So,

$$C_W(n) \in \Theta(n^2)$$

3. Section 3.4 #8

If we have an array of n elements, then we can generate a permutation of the array and then check if that permutation is ordered. WE will always have to make n-1 comparisons, and at worst, we'll have to check n! permutations. Thus, we'll have to make at most (n-1)n! comparisons. So the efficiency of the worst case is

$$C_W(n) \in O((n+1)!)$$

4. Section 4.1 #7

5. Section 4.1 #11 (a)

The array with the largest number of inversion are reverse sorted arrays, since every A[i] would be larger than every subsequent element. Thus, they would have

$$\sum_{i=0}^{n-1} (n-1-i) = (n-1) + (n-2) + \dots + 2 + 1 = \frac{n(n-1)}{2}$$

number of inversions.

The smallest number would be sorted arrays and they would have 0 inversions.

- **6.** Section 4.4 #8 (a, b, c, d)
 - (a) Decrease by a constant factor

(b)
$$C(n) = C\left(\frac{n}{3}\right) + 2$$
 with $C(1) = 1$ and $n = 3^k$

(c) $C(n) = C(3^{k})$ $= C(3^{k-2}) + C(3^{k-1}) + 2 + 2$ \vdots $= C(3^{k-k}) + \underbrace{2 + 2 + \dots + 2}_{k \text{ times}}$ = 1 + 2k

Since $n = 3^k$, then $k = \log_3 n$. So $C(n) = 2 \log_3 n + 1$.

- (d) The efficiency for binary search is $2\log_2 n + 1$. Since the base of the log is smaller for binary than ternary search, then ternary is more efficient. However, both algorithms are of the same efficiency class.
- **7.** Section 4.5 #2

We find the median, and since the size of the array is 7 then

$$k = \lceil 7/2 \rceil = 4$$

We proceed with the quickselect algorithm.

0	1	2	3	4	5	6
\overline{s}	i					
9	12	5	17	20	30	8
	s	i				
9	5	12	17	20	30	8
		s				i
9	5	8	17	20	30	12
8	5	9	17	20	30	12

Comp Sci 3130 Gabriel Wallace

So s = 2 < 3 = k - 1, and we partition again with the right subarray.

0	1	2	3	4	5	6
			s	i		
			17	20	30	12
				s		i
			17	12	30	20
			12	17	30	20

So s=4>3=k-1, and we partition again with the left subarray. However, the left subarray is a singleton, so no swaps occur. Thus, s=3=k-1 and 12 is the median.