Homework 2

1.

(a) Let $f(n) = 3^{n+1}$ and $g(n) = 3^n$. Examine

$$\begin{aligned}\lim_{n\to\infty} \frac{f(n)}{g(n)} &= \lim_{n\to\infty} \frac{3^{n+1}}{3^n} \\ &= \lim_{n\to\infty} \frac{3\cdot 3^n}{3^n} \\ &= \lim_{n\to\infty} 3 \\ &= 3\end{aligned}$$

Since 3 is a constant, then $f(n) \in \Theta(g(n))$

(b) Let $f(n) = 3^{3n}$ and $g(n) = 3^n$. Examine

$$\begin{aligned}\lim_{n\to\infty} \frac{f(n)}{g(n)} &= \lim_{n\to\infty} \frac{3^{3n}}{3^n} \\ &= \lim_{n\to\infty} \frac{(3^n)^3}{3^n} \\ &= \lim_{n\to\infty} 3^{2n} \\ &= \infty\end{aligned}$$

So $f(n) \in \Omega(g(n))$.

2.

The list of functions ranked from smallest order of growth to largest is below.

$$1000, \ln(\ln n), \sqrt{\ln(n)}, \{\log_5 n, \lg n\}, (\lg n)^2, \left(\sqrt{2}\right)^{lgn},$$
$$\{n, 1000n+3, 2^{\lg n}\}, \{n\cdot \lg n, \ln(n!)\}, \{n^2, 4^{\lg n}\},$$
$$n^3, \left(\frac{3}{2}\right)^n, 2^n, n2^n, e^n, n!, (n+1)!$$

3.

Let $T(n) = T(n/4) + B$, with initial condition $T(1) = A$, and let $n = 4^k$. We proceed with backwards subsititution.

$$\begin{aligned} T(n) &= T(4^k) \\ &= T(4^{k-1}) + B \\ &= T(4^{k-2}) + B + B \\ &\;\;\vdots \\ &= T(1) + \underbrace{B + B + \cdots + B}_{k \text{ times}} \end{aligned}$$

Thus, $T(n) = T(1) + Bk = A + Bk$. Since $n = 4^k$, then $k = \log_4 n$ so $T(n) = A + B \log_4 n$.