

1.

We want to multiply 2101 and 1130. We have

$$\begin{aligned}c_2 &= 21 * 11 \\c_0 &= 01 * 30 \\c_1 &= (21 + 01) * (11 + 30) - ((21 * 11) + 01 * 30) \\&= (22 * 41) - (21 * 11) - (01 * 30)\end{aligned}$$

Thus we break it down further as follows.

For $21 * 11$ we have:

$$\begin{aligned}c_2 &= 2 * 1 = 2 \\c_0 &= 1 * 1 = 1 \\c_1 &= (2 + 1) * (1 * 1) - (2 * 1 + 1 * 1) \\&= 3\end{aligned}$$

So

$$21 * 11 = 2 \cdot 10^2 + 3 \cdot 10 + 1 = 231$$

For $01 * 30$ we have:

$$\begin{aligned}c_2 &= 0 * 3 = 0 \\c_0 &= 1 * 0 = 0 \\c_1 &= (0 + 1) * (3 + 0) - (0 + 0) \\&= 3\end{aligned}$$

So

$$01 * 30 = 0 \cdot 10^2 + 3 \cdot 10 + 0 = 30$$

For $22 * 41$ we have:

$$\begin{aligned}c_2 &= 2 * 4 = 8 \\c_0 &= 2 * 1 = 2 \\c_1 &= (2 + 2) * (4 * 1) - (8 + 2) \\&= 10\end{aligned}$$

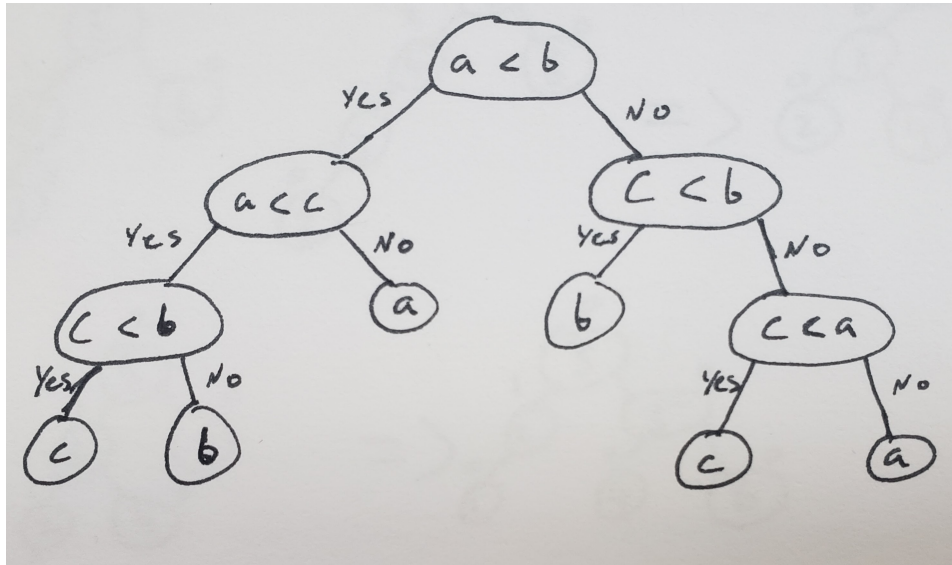
So

$$22 * 41 = 8 \cdot 10^2 + 10 \cdot 10 + 2 = 902$$

Putting it all together, we have

$$\begin{aligned}2101 * 1130 &= 231 \cdot 10^4 + (902 - 231 - 30) \cdot 10^2 + 30 \\&= 2,374,130\end{aligned}$$

2.



3.

- (a) First we sort the array with a sorting algorithm with optimal efficiency will be in the first and last positions of the array respectively. Thus, we have an efficiency as follows:

$$T(n) = T_{\text{sort}}(n) + T_{\text{select}}(n) \in \Theta(n \log n) + \Theta(1) = \Theta(n \log n)$$

- (b) First assign a variable called `min` to $A[0]$ and then loop through the array and if $A[i]$ is less than `min`, then reassign `min` to $A[i]$. Then create a variable called `max` and loop through again reassigning if an element of the array is larger than `max`. Since we had to loop through the array twice, then $T(n) = 2n$ and $T(n) \in \Theta(n)$.

4 (b).

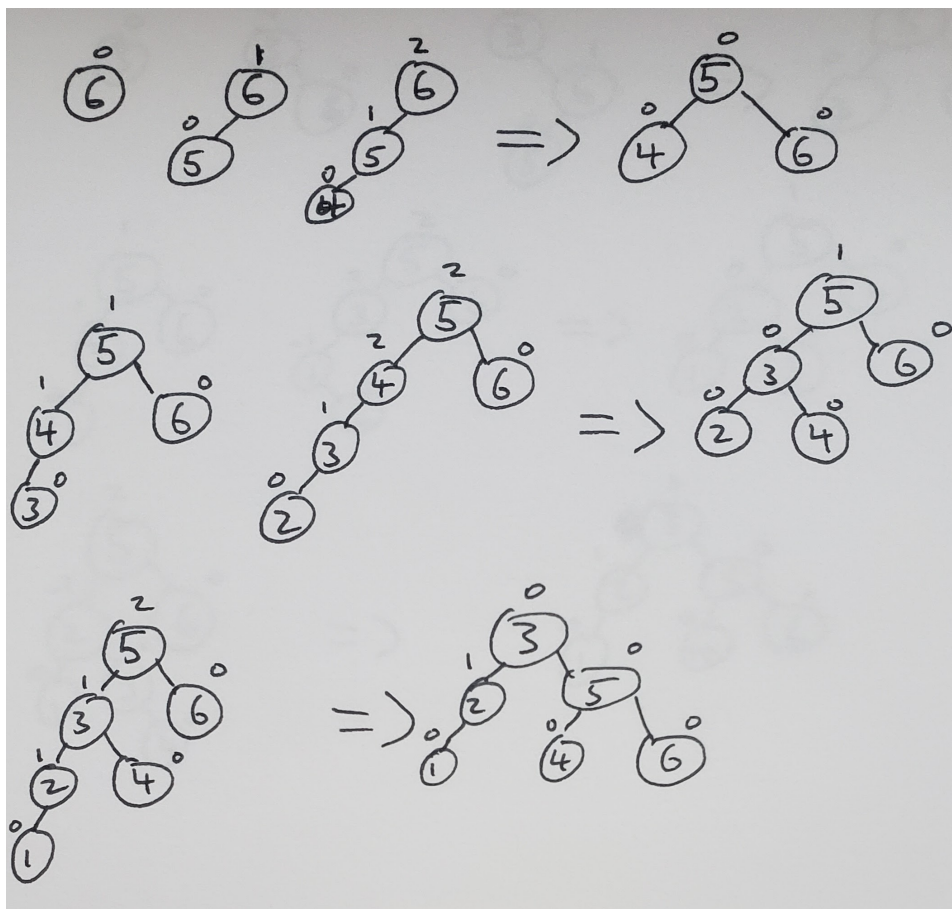


Diagram 1 (Left): Node 3 (BF 0) is the root. Node 6 (BF 0) is the right child of node 3.

Diagram 2 (Middle): Node 3 (BF -2) is the root. Node 6 (BF 1) is the right child of node 3. Node 5 (BF 0) is the left child of node 6.

Diagram 3 (Right): Node 5 (BF 0) is the root. Node 3 (BF 0) is the left child of node 5. Node 6 (BF 0) is the right child of node 5.

Diagram 4 (Left): Node 5 (BF 1) is the root. Node 3 (BF 1) is the left child of node 5. Node 6 (BF 0) is the right child of node 5. Node 1 (BF 0) is the left child of node 3.

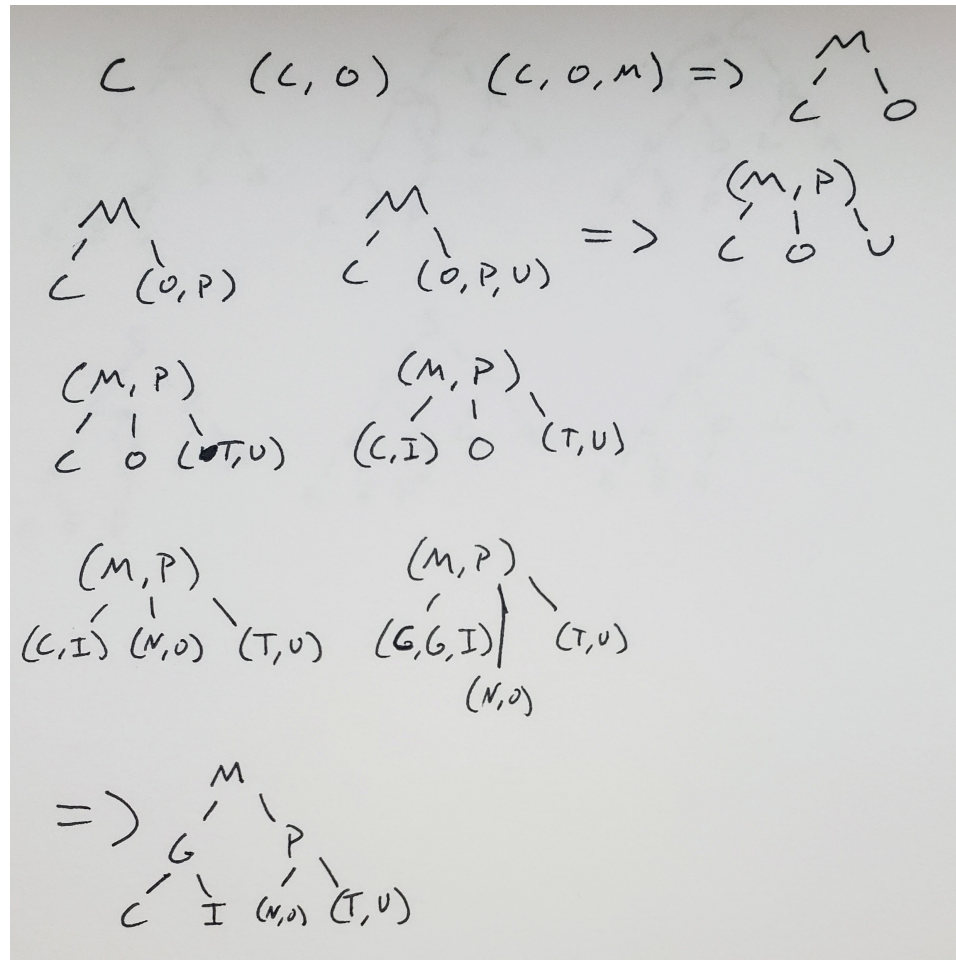
Diagram 5 (Middle): Node 5 (BF 2) is the root. Node 3 (BF 2) is the left child of node 5. Node 6 (BF 0) is the right child of node 5. Node 1 (BF -1) is the left child of node 3. Node 2 (BF 0) is the right child of node 1.

Diagram 6 (Right): Node 5 (BF 1) is the root. Node 2 (BF 0) is the left child of node 5. Node 6 (BF 0) is the right child of node 5. Node 1 (BF 0) is the left child of node 2. Node 3 (BF 0) is the right child of node 2.

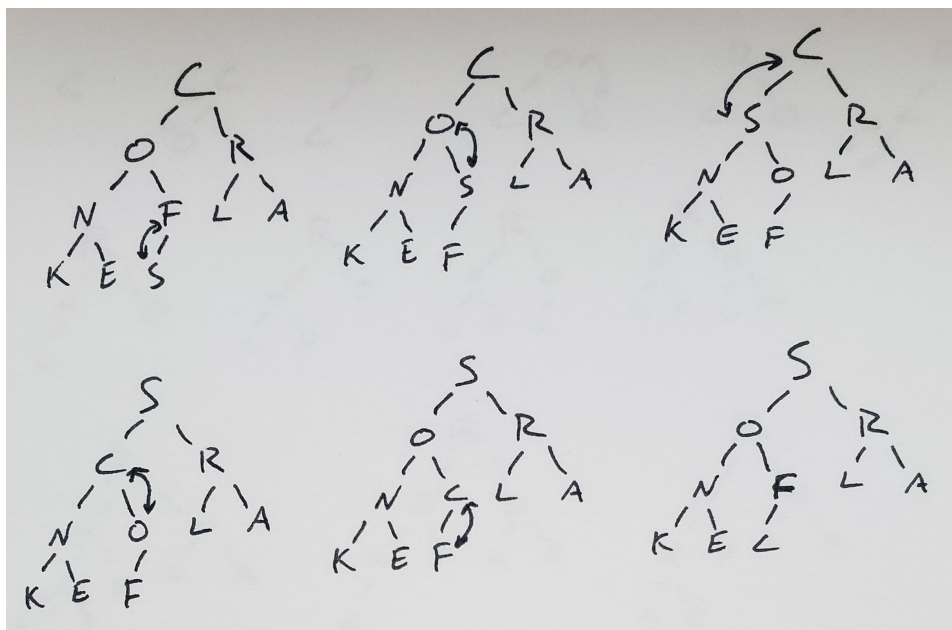
Diagram 7 (Left): Node 5 (BF 2) is the root. Node 2 (BF -1) is the left child of node 5. Node 6 (BF 0) is the right child of node 5. Node 1 (BF 0) is the left child of node 2. Node 3 (BF -1) is the right child of node 2. Node 4 (BF 0) is the left child of node 3.

Diagram 8 (Right): Node 3 (BF 0) is the root. Node 2 (BF 1) is the left child of node 3. Node 5 (BF 0) is the right child of node 3. Node 1 (BF 0) is the left child of node 2. Node 4 (BF 0) is the left child of node 5. Node 6 (BF 0) is the right child of node 5.

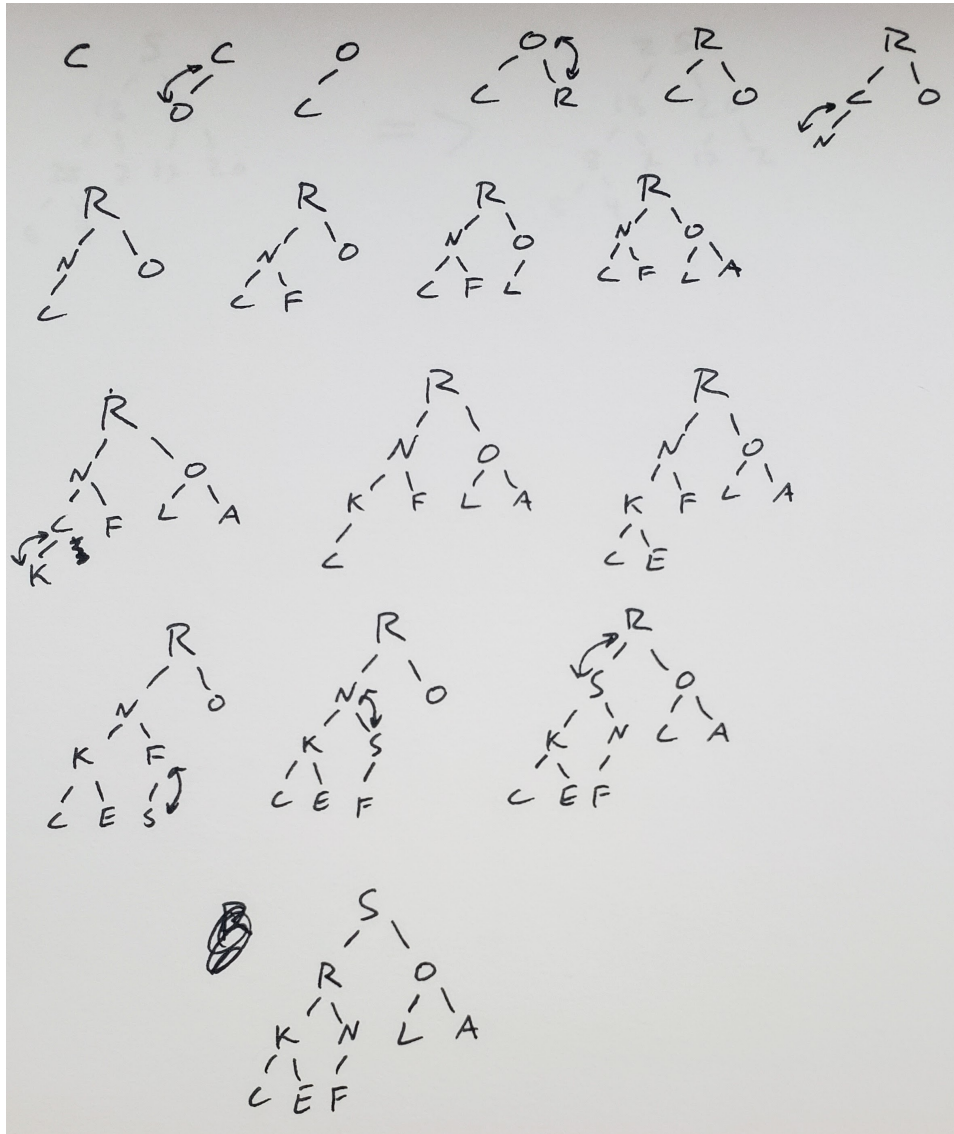
5.



6(a).

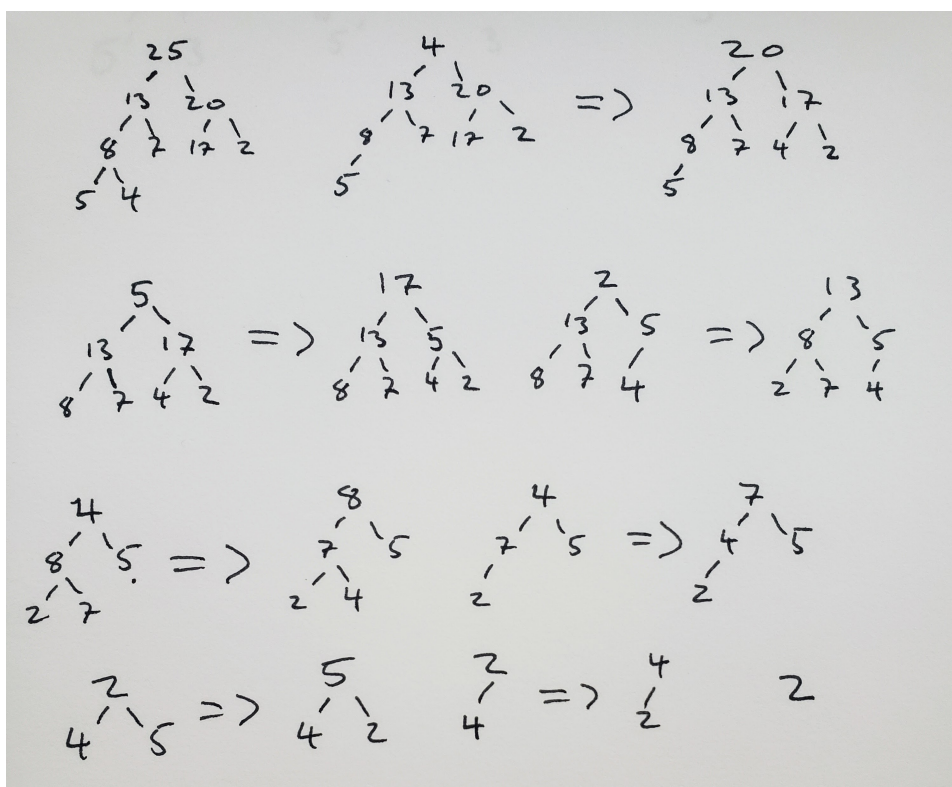
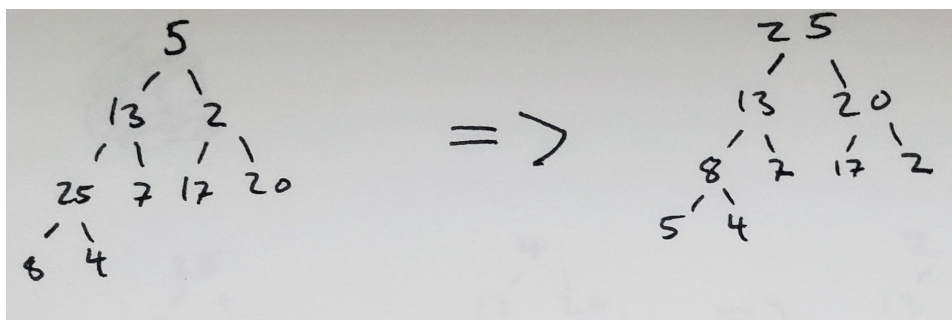


6(b).



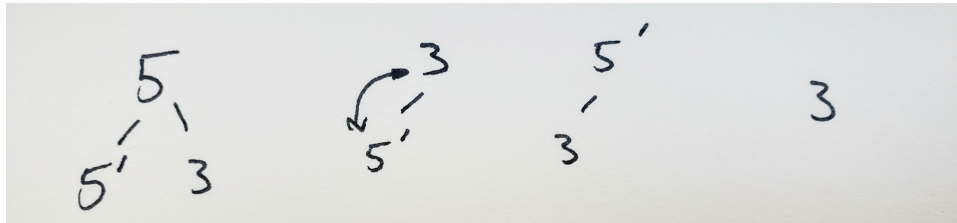
7.

First, we create the max heap, and then we proceed with the deleting procedure.



8.

A counterexample is if we want to sort the array 5, 5', 3, where we denote the second 5 with 5'.



Then the sorted array becomes 3, 5, 5'. So the two fives are swapped.

$R \xrightarrow{E} E \begin{array}{c} R \\ | \\ E \end{array} \xrightarrow{A} \begin{array}{c} R \\ / \quad \backslash \\ E \quad A \end{array} \xrightarrow{L} \begin{array}{c} R \\ / \quad \backslash \\ E \quad A \\ \swarrow \quad \searrow \\ L \quad \end{array}$

$\begin{array}{c} R \\ / \quad \backslash \\ L \quad A \\ | \\ E \end{array} \xrightarrow{I} \begin{array}{c} R \\ / \quad \backslash \\ L \quad A \\ / \quad \backslash \\ E \quad I \end{array} \xrightarrow{(*)} \begin{array}{c} I \\ / \quad \backslash \\ L \quad A \\ | \\ E \end{array} \begin{array}{c} L \\ / \quad \backslash \\ I \quad A \\ | \\ E \end{array}$

$\begin{array}{c} L \\ / \quad \backslash \\ I \quad A \\ / \quad \backslash \\ E \quad T \\ | \\ T \end{array} \xrightarrow{T} \begin{array}{c} T \\ / \quad \backslash \\ L \quad A \\ / \quad \backslash \\ E \quad I \end{array} \xrightarrow{(*)} \begin{array}{c} I \\ / \quad \backslash \\ L \quad A \\ | \\ E \end{array} \begin{array}{c} L \\ / \quad \backslash \\ I \quad A \\ | \\ E \end{array}$

$\begin{array}{c} E \\ / \quad \backslash \\ I \quad A \\ | \\ I \end{array} \xrightarrow{(*)} \begin{array}{c} I \\ / \quad \backslash \\ E \quad A \\ | \\ E \end{array} \xrightarrow{Y} \begin{array}{c} I \\ / \quad \backslash \\ E \quad A \\ | \\ Y \end{array} \begin{array}{c} Y \\ / \quad \backslash \\ I \quad A \\ | \\ E \end{array}$

$\begin{array}{c} Y \\ / \quad \backslash \\ I \quad A \\ / \quad \backslash \\ E \quad S \\ | \\ S \end{array} \xrightarrow{S} \begin{array}{c} Y \\ / \quad \backslash \\ I \quad A \\ / \quad \backslash \\ L \quad I \end{array} \xrightarrow{H} \begin{array}{c} Y \\ / \quad \backslash \\ S \quad A \\ / \quad \backslash \\ E \quad I \quad H \end{array} \begin{array}{c} Y \\ / \quad \backslash \\ S \quad H \\ / \quad \backslash \\ E \quad I \quad A \end{array}$

$\begin{array}{c} A \\ / \quad \backslash \\ S \quad H \\ / \quad \backslash \\ E \quad I \end{array} \xrightarrow{(*)} \begin{array}{c} S \\ / \quad \backslash \\ I \quad H \\ / \quad \backslash \\ E \quad A \end{array} \xrightarrow{O} \begin{array}{c} S \\ / \quad \backslash \\ I \quad O \\ / \quad \backslash \\ E \quad A \quad H \end{array} \xrightarrow{W} \begin{array}{c} W \\ / \quad \backslash \\ I \quad S \\ / \quad \backslash \\ E \quad A \quad H \quad O \end{array}$

$\begin{array}{c} S \\ / \quad \backslash \\ I \quad O \\ / \quad \backslash \\ E \quad A \quad H \end{array} \xrightarrow{(*)} \begin{array}{c} O \\ / \quad \backslash \\ I \quad H \\ | \\ E \quad A \end{array}$

10. The frequency and distribution arrays are as follows.

Array values	a	b	c	d
Frequency	2	3	2	1
Distribution	2	5	7	8

Then we build the sorted array.

	$D[0 \dots 3]$	Sorted array							
$A[7] = b$	2	5	7	8				b	
$A[6] = a$	2	4	7	8	a				
$A[5] = a$	1	4	7	8					
$A[4] = b$	0	4	7	8			b		
$A[3] = c$	0	3	7	8					c
$A[2] = d$	0	3	6	8					d
$A[1] = c$	0	3	6	7				c	
$A[0] = b$	0	3	5	7		b			

So the sorted array is a, a, b, b, b, c, c, d.