Project 1

Theoretical Analysis

We have two algorithms to find the nth Fibonacci number, F_n, one recursive and one iterative. We proceed with the theoretical analysis of these two algorithms, and find the order of growth of each.

Recursive algorithm

We have the following recursive definition for F_n:

$$F(n) = F(n-1) + F(n-2)$$

To find the general form for $F(n)$, we solve a linear homogenous recurrence relation, and have

$$F(n) = \frac{1}{\sqrt{5}}(\varphi)^n - \frac{1}{\sqrt{5}}(\hat{\varphi})^n$$

with

$$\varphi = \left(\frac{1+\sqrt{5}}{2}\right) \text{ and } \hat{\varphi} = \left(\frac{1-\sqrt{5}}{2}\right)$$

To find the order of growth of $F(n)$, we need to calculate the limit as n approaches infinity. Since $|\hat{\varphi}| \approx 0.62 < 1$, then

$$\lim_{n\to\infty} \frac{1}{\sqrt{5}}(\hat{\varphi})^n = 0.$$

And since, $|\varphi| \approx 1.62 > 1$, then

$$\lim_{n\to\infty} \frac{1}{\sqrt{5}}(\varphi)^n = \infty.$$

So,

$$\lim_{n\to\infty} F(n) = \frac{1}{\sqrt{5}} \lim_{n\to\infty} (\varphi)^n + (\hat{\varphi})^n = \frac{1}{\sqrt{5}} \lim_{n\to\infty} (\varphi)^n = \infty.$$

Thus, $F(n) \in \Theta(\varphi^n)$ or $F(n) \in \mathcal{O}(2^n)$

Iterative algorithm

Empirical Analysis