Homework 1

1. Section 1.1 #6

(a) We find $\gcd(31415, 14142)$ by Euclid's Algorithm.

$$\begin{aligned}
\gcd(31415, 14142) &= \gcd(14142, 3131)\\
&= \gcd(3131, 1618)\\
&= \gcd(1618, 1513)\\
&= \gcd(1513, 105)\\
&= \gcd(105, 43)\\
&= \gcd(43, 19)\\
&= \gcd(19, 5)\\
&= \gcd(5, 4)\\
&= \gcd(4, 1)\\
&= \gcd(1, 0)\\
&= 1
\end{aligned}$$

(b) We see that the number of iterations for Euclid's algorithm is 11. The number of iterations for the method of consecutive integers is 14142, or approximently 1286 times faster.

Writing a quick Python program to run one million simulations reveals that, on average, Euclid's algortihm takes about 11 iterations and the consecutive integers method takes about 39960 iterations, or about 3633 times faster.

2. Section 1.1 #8

Let $m, n \in \mathbb{Z}$ with $m < n$. Applying Euclid's algorithm, we have $\gcd(m, n) = \gcd(n, m \bmod n)$. But since $m < n$, then $m \bmod n = m$, so $\gcd(n, m \bmod n) = \gcd(n, m)$. In other words, the two inputs are swapped.

The inputs swapping can only happen on the first iteration and if $m < n$. For any integers a, b, $a > b \bmod a$, since $b \bmod a$ is the remainder of b when devided by a and the remainder is smaller than the quotient, by the Division Algorithm.

3. Section 1.2 #3

Only (a) is an algorithm (provided we know how to compute a square root). The problems with (b) and (c) is that they are ambiguous, i.e we are not given how to compute A or h_a respectively.

4. Section 1.2 #5

(a) Let n be a positive integer and let $r = n - 2a$, where a is a positive integer. Then prepend r to the binary representation of n. Then assign n to a and repeat this process until $n = 0$.

(b)

5. Section 1.3 #4

(a) Let the two islands and the two banks of the river be the vertices and the bridges be the edges of a graph. Now we reformulate the question in graph theoretic terms: does this graph have an Euler cycle?

(b)

6. Section 1.3 #8

(a) We first translate this map into a graph with each region being a vertex and two vertices are connected if they share an edge on the map. Doing so, we have the following graph:

(b) We can color the graph with four colors like so:

7. Section 2.1 #1

(a) Computing the sum of n numbers

  (i) n

  (ii) Addition of two numbers

  (iii) No

(b) Computing $n!$

  (i) n

  (ii) Multiplication of two integers

(iii) No

(c) Finding the largest element in a list of n numbers

(i) n

(ii) Comparison of two objects

(iii) No

8. Section 2.1 #2(a)

The basic operation is addition of two numbers. Let T be the total number of operations and let N be the total number of elements in the two matrices. Since the number of elements of an $n \times n$ matrix is n^2, then $N = 2n^2$. And since we add two matrices straight across, $T = n^2$. Therefore, $T = N/2$.

9. Section 2.1 #4(a)

In the best case, we would get lucky and select a matching pair in the first two selections. So the smallest number of gloves selected is 2.

In the worst case, we would select every glove of a particular handedness, so we would have 11 gloves. In the next selection, we would be guaranteed a match. So the largest number of gloves selected is 12.

Bonus. Section 1.2 #2

The order of traversals that will take 17 minutes is as follows:

People sent	Time taken	Total time
1, 2	2 min	2 min
2	2 min	4 min
3, 4	10 min	14 min
1	1 min	15 min
1, 2	2 min	17 min