1 Theoretical Analysis

The theoretical analysis of all the sorting algorithms were covered in class. For reference, we have the table of the relevant sorting algorithms and their efficiency for the best, worst, and average cases.

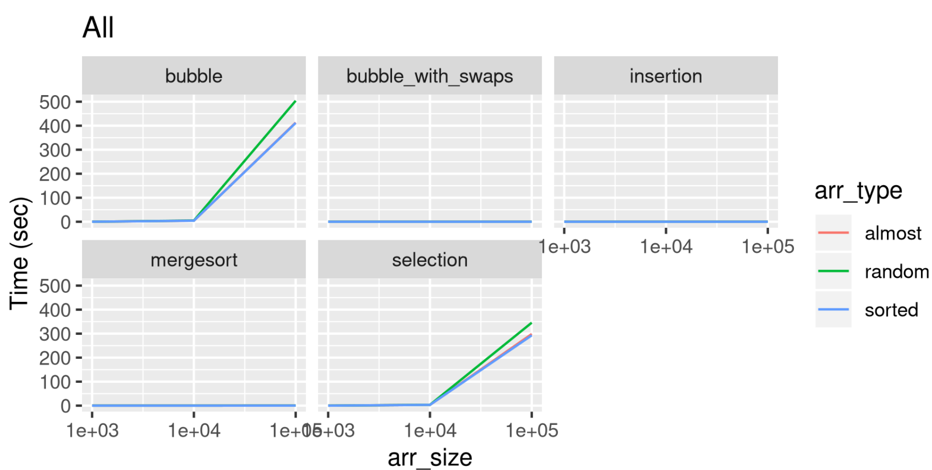
Algorithm	Best	Worst	Average
Selection	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Insertion	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n^2)$
Bubble	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Bubble (with swaps)	$\Theta(1)$	$\Theta(n^2)$	$\Theta(n^2)$
Quick	$\Theta(n \log n)$	$\Theta(n^2)$	$\Theta(n \log n)$
Merge	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$

2 Empirical Analysis

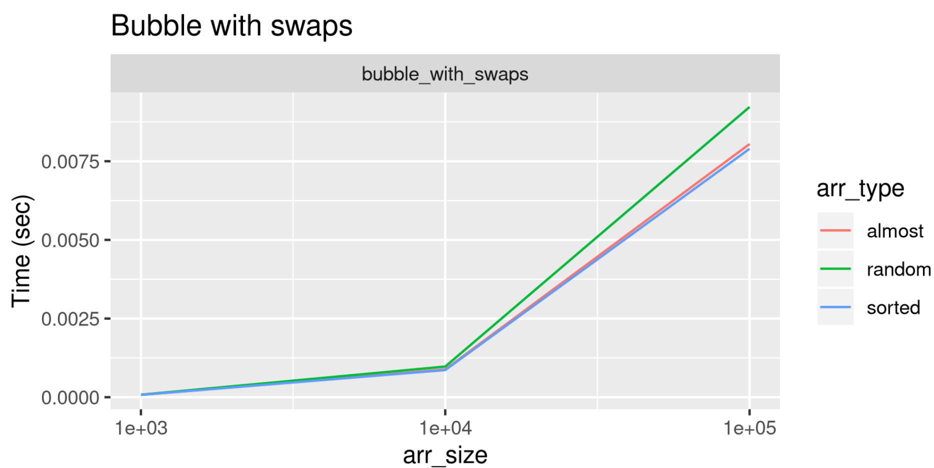
The following section covers the empirical analysis of the above sorting algorithms. Due to memory issues (for more details see), quicksort gets its own section. All the algorithms were implemented in Python, and the source code can be found on GitHub. The tables with the raw data for the analysis can be found in section BLANK.

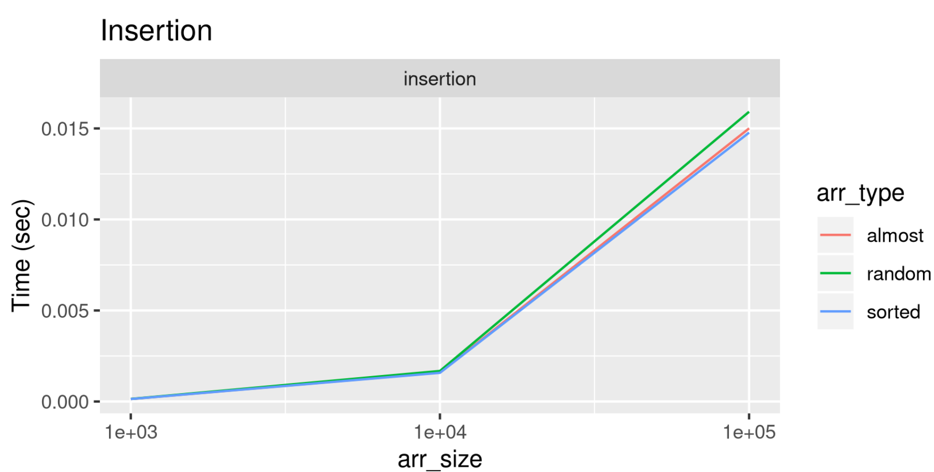
2.1 Graphs and Discussion

We have the graphs from all the algorithms below.

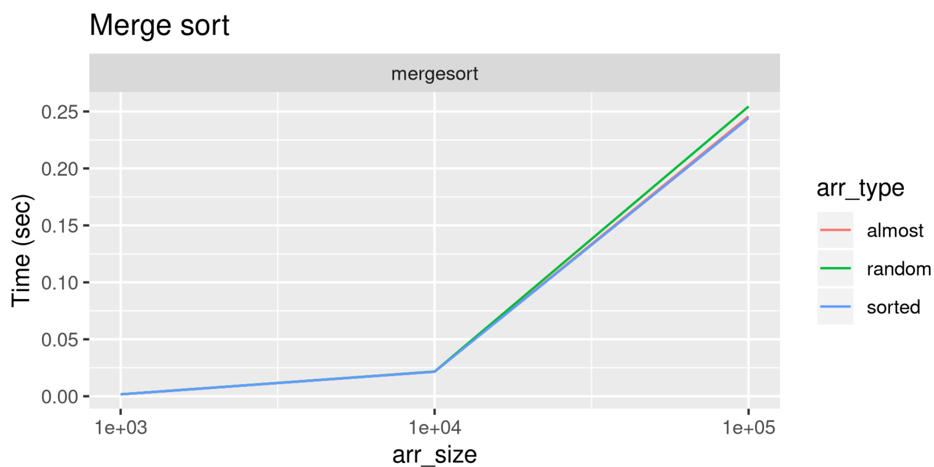


From the graphs we can see that bubble and selection sort grow much quicker than the other sorting algorithms. So much so, that the scale of the vertical axis makes it to where the other algorithms are basically a flat line. To see the growth more clearly of the other sorting algorithms, we have separate graphs.





It is hard to tell with only three data points, but we can see that all the sorting algorithms have similar shapes, albeit with bubble and selection having much steeper curves. This makes sense for the most part since most of the algorithms (except for mergesort), have efficiency $\Theta(n^2)$. From a purely theoretical view, we would expect to see that mergesort has a less steep



curve than the other algorithms since it is non-adaptive and has efficiency $\Theta(n \log n)$. However, the curve is the same and, in fact, the absolute time is much higher than either the improved bubble sort or the insertion sort. On the other hand, our theoretical analysis is supported by the data for mergesort being non-adaptive because the curves for all the array types (sorted, almost sorted, and random) are essentially the same. This does not hold for all the algorithms that are adaptive.

In tables 1 and 2, we can see that the improved bubble sort performs the best for almost sorted and random arrays respectively. Insertion sort is a close second. This hold true for sorted lists as well, which confirms our theoretical analysis with the improved bubble sort having best case efficiency $\Theta(1)$. Of course, the question of which sorting algorithm performs best on an already sorted list is a bit moot.

Selection and regular bubble sort perform much worse than the others. This shows that even if two algorithms have the same order of growth, this does not necessarily mean they are as fast as one another.

Algorithm	Average time (sec)
Bubble with swaps	0.00300
Insertion	0.00558
Mergesort	0.0897
Selection	101
Bubble	139

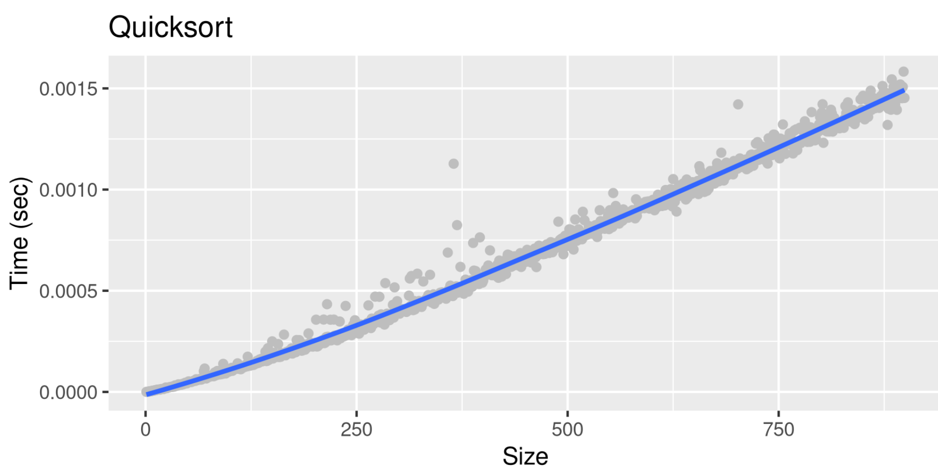
Table 1: Almost sorted

Algorithm	Average time (sec)
Bubble with swaps	0.00300
Insertion	0.00558
Mergesort	0.0897
Selection	101
Bubble	139

Table 2: Random

2.2 Quicksort

The space efficiency of quicksort is $\Theta(n)$. Therefore, in our code, when we try to apply quicksort to an already sorted array (the worst case scenario), we quickly hit Python's recursion stack limit of 1000. This is true even for random arrays bigger than around a 1000. Thus, quicksort was not included with the other algorithms. We did, however, run quicksort on arrays of random elements to confirm our theoretical analysis. The plot of the analysis can be found in figure 2.2.



From this plot, we can see that our empirical analysis supports our theoretical analysis that quicksort is of efficiency $\Theta(n \log n)$.

A Tables of Raw Data

Test