Theorem If G colors a knot and every subgroup of G is normal, then G is cyclic.

Proof. Let x be a label of a strand. If $G = \langle x \rangle$, then we are done. So assume $\langle x \rangle \subsetneq G$. Let y label a strand and let $y \in G \setminus \langle x \rangle$. Then x and y are conjugate. Thus, $y = gxg^{-1}$ for some $g \in G$ and therefore $y \in g\langle x \rangle g^{-1}$. Since every subgroup of G is normal, then $g\langle x \rangle g^{-1} = \langle x \rangle$. A contradiction, showing $y \notin G \setminus \langle x \rangle$, implying that $y \in \langle x \rangle$.

Now assume $y \in G$ does not label a strand. Let L be the set of labels of the knot, so $\langle L \rangle = G$. Since y does not label a strand then $y \in G \setminus L$. But, $L \subseteq \langle x \rangle$, so $G \subset \langle x \rangle$. A contradiction, showing that $G \cong \langle x \rangle$.