Homework Problem for ADMM Project

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Homework assignment is designed to understand the basics of the ADMM method.

1. The alternating direction method of multipliers (ADMM) is an algorithm used to solve optimization problems of the form.

$$\min_{x,y} \ \text{f(x) + g(y)} \ \text{s.t.} Ax + By = c$$

   where $f(x) : \mathbb{R}^d \to \mathbb{R}$ and $g(x) : \mathbb{R}^p \to \mathbb{R}$.
   What is the Lagrangian function of the ADMM problem?

   Solution:

$$L(x, y, z) = f(x) + g(y) - \langle z, Ax + By - c \rangle$$

2. The Spider-ADMM modifies the Lagrangian by adding an additional penalty term (also known as the augmented lagrangian function) and linearizing f(x) (but we'll ignore this in the homework set).

$$L_\rho(x, y, z) = L(x, y, z) + \frac{\rho}{2} \|Ax + By - c\|^2$$

   To find the optimal x and y, a popular method is to utilize gradient descent to iteratively minimize the augmented lagranian function with respect to x, y, then z. Given x_t, y_t, and z_t and learning rate λ, what would be the update to find $x_{t+1}, y_{t+1}, z_{t+1}$?

   Solution

$$\begin{aligned}
x_{t+1} &= x_t - \lambda \nabla_x (L(x, y_t, z_t) + \frac{\rho}{2} \|Ax + By_t - c\|^2) \\
y_{t+1} &= y_t - \lambda \nabla_y (L(x_{t+1}, y, z_t) + \frac{\rho}{2} \|Ax_{t+1} + By - c\|^2) \\
z_{t+1} &= z_t - \lambda (Ax_{t+1} + By_{t+1} - c)
\end{aligned}$$