General Formulas

$$L = K - U \tag{1}$$

Simple Double Pendulum

Lagrangian

$$L = \frac{1}{2}(m_1 + m_2)l_2^2\dot{\theta}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1\dot{\theta}_2\cos(\theta_1 + \theta_2) \tag{2}$$

$$+(m_1 + m_2)gl_1\cos(\theta_1) + m_2 l_2 g\cos(\theta_2) \tag{3}$$

Euler Lagrange

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} = 0 \tag{4}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2} = 0 \tag{5}$$

Equations of Motion

$$\ddot{\theta}_1 = [-m_2 l_1 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2)\cos(\theta_1 - \theta_2) + gm_2\sin(\theta_2)\cos(\theta_1 - \theta_2) \tag{6}$$

$$-m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta 2) - (m_1 + m_2)g\sin(\theta_1)] \tag{7}$$

$$/(l_1(m_1 + m_2) - m_2 l_1\cos^2(\theta_1 - \theta_2)) \tag{8}$$

$$\ddot{\theta}_2 = [m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)\cos(\theta_1 - \theta_2) + g\sin(\theta_1)\cos(\theta_1 - \theta_2)(m_1 + m_2) + \tag{9}$$

$$l_1\dot{\theta}_1^2\sin(\theta_1 - \theta_2)(m_1 + m_2) - g\sin(\theta_2)(m_1 + m_2)] \tag{10}$$

$$/[l_2(m_1 + m_2) - m_2 l_2\cos^2(\theta_1 - \theta_2)] \tag{11}$$

Damped Double Pendulum

Lagrangian

$$L = \frac{1}{2}(m_1 + m_2)l_2^2\dot{\theta}^2 + \frac{1}{2}m_1 l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2) \tag{12}$$

$$+(m_1 + m_2)gl_1\cos(\theta_1) + m_2 l_2 g\cos(\theta_2) \tag{13}$$

Euler Lagrange

$$\frac{d}{dt}\frac{\partial L}{\dot{\theta}_i} = \frac{\partial L}{\partial \theta_i} - \frac{\partial D}{\partial \dot{\theta}_i} \tag{14}$$

Equations of Motion

$$\ddot{\theta}_1 = \frac{(-(m_1 + m_2)gl_1\sin(\theta_1) - m_2 l_1 l_2\dot{\theta}_2^2\sin(\theta_1 - \theta_2) + D_2 * (\dot{\theta}_2 - \dot{\theta}_1) - D_1\dot{\theta}_1)}{l_1^2(-m_2 - m_1 + m_2(\cos(\theta_1 - \theta_2))^2)} \tag{15}$$

$$+\frac{(\cos(\theta_1 - \theta_2) * (-D_2\dot{\theta}_2 + m_2 l_1\dot{\theta}_1^2 l_2\sin(\theta_1 - \theta_2) - m_2 g l_2\sin(\theta_2) + D_2\dot{\theta}_1))}{l_1 l_2(-m_2 - m_1 + m_2(\cos(\theta_1 - \theta_2))^2)} \tag{16}$$

$$\ddot{\theta}_2 = \frac{\cos(\theta_1 - \theta_2)(-(m_1 + m_2)gl_1\sin(\theta_1) - m_2 l_1 l_2\dot{\theta}_2^2\sin(\theta_1 - \theta_2) + D_2(\dot{\theta}_2 - \dot{\theta}_1) - D_1\dot{\theta}_1)}{l_1 l_2(-m_2 - m_1 + m_2\cos^2(\theta_1 - \theta_2)} \tag{17}$$

$$-\frac{(m_2 + m_1) * (-D_2\dot{\theta}_2 + m_2 l_1\dot{\theta}_1^2 l_2\sin(\theta_1 - \theta_2) - m_2 g l_2 sin(\theta_2) + D_2 * \dot{\theta}_1)}{m_2 l_2^2(-m_2 - m_1 + m_2\cos^2(\theta_1 - \theta_2))} \tag{18}$$

$$Parameters : m_1 = 1, m_2 = 1, l_1 = 1, l_2 = 1, g = 9.81 \tag{19}$$