Gravitational waves from compact binary coalescence: physics, parameters, and degeneracies

PyCBC Inference Workshop - Portsmouth

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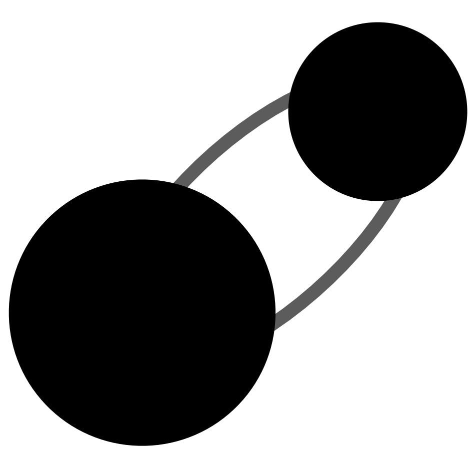
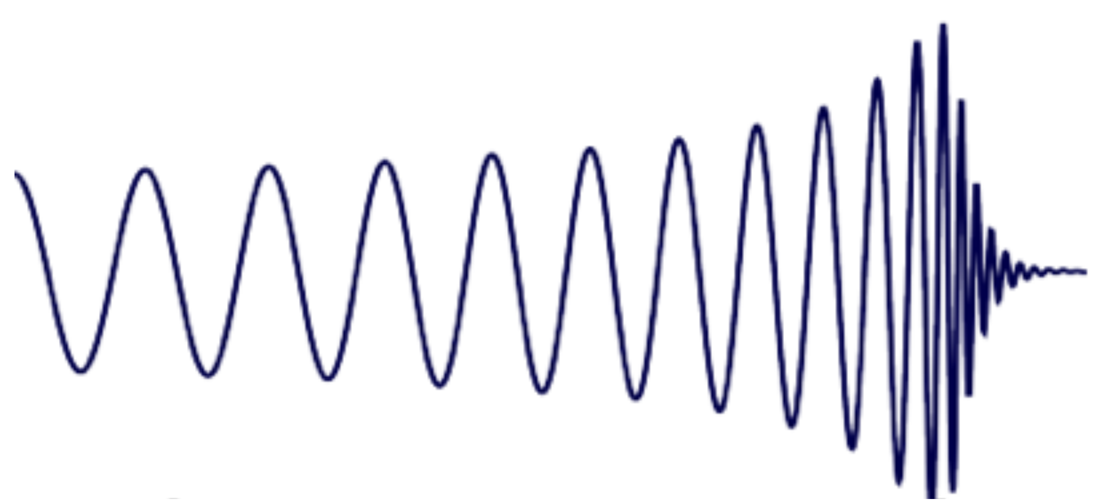
14th May 2019

Motivation

- We are curious and want to know about how our Universe works!
- With GWs we can potentially study a huge range of physics and astrophysics.
- From fundamental principles to stellar evolution and cosmology





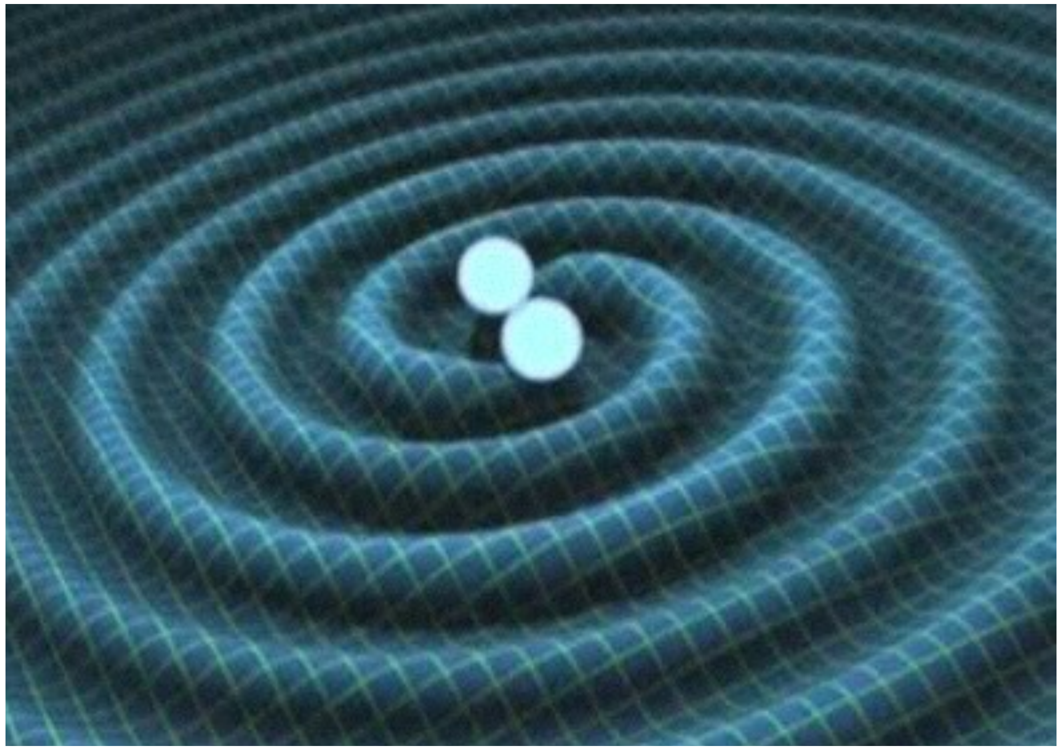
Credit: Bronwen and Lasky.

Astrophysical Sources of Gravitational Waves

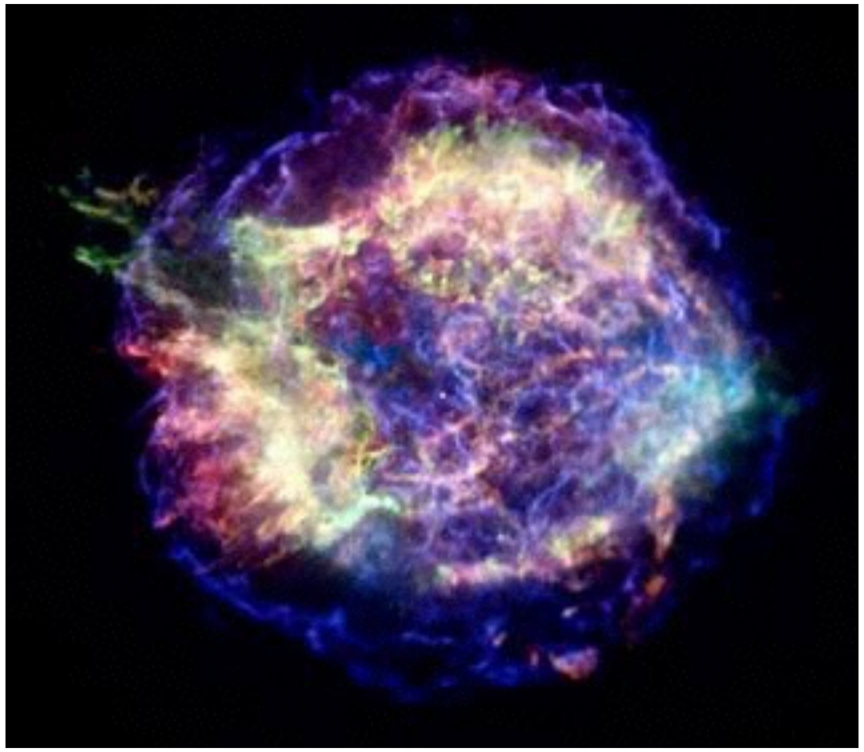
Wobbling Neutron Stars

"Continous"

"Stochastic" Background

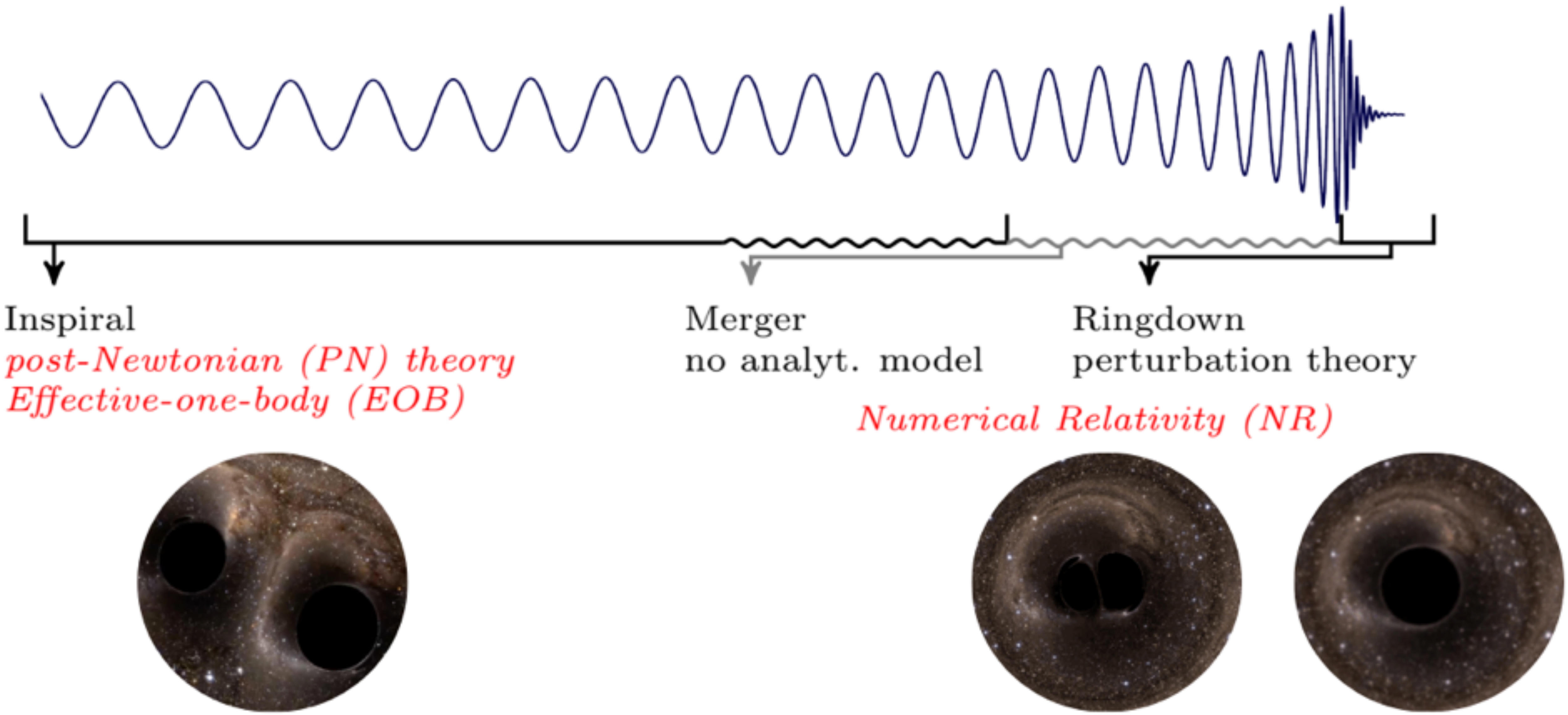


Compact Binaries



Supernovae "Burst"

Compact Binary Coalescence (CBC)

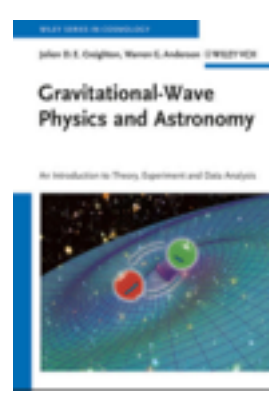


Credit: Ohme, SXS Collaboration

Outline

- Physics
- Parameters
- Degeneracies
- Waveform Models

Gravitational-Wave Physics and Astronomy: An Introduction to Theory, Experiment and Data Analysis
Book by Jolien D. E. Creighton and Warren G. Anderson



Linearised Gravity

- Gravitational waves (GWs) can be seen by perturbing flat spacetime (to linear order) $$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

- Linearised Einstein equations for the perturbation is a wave equation*

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta} \longrightarrow -\Box \bar{h}_{\alpha\beta} = \frac{16\pi G}{c^4} T_{\alpha\beta}$$

- In TT gauge and z-direction $$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

* neglecting discussion on gauge

Physics: Gravitational Wave Generation - simplified approach

- GWs result from the acceleration of masses.
- We can compute the GW signal from the orbital motion (assuming weak fields and slow motion)

Metric Perturbation Tensor

2nd Time Derivative Quadrupole Tensor

$$\bar{h}^{ij}(t, \boldsymbol{x}) \simeq \frac{2G}{c^4 r} \ddot{I}^{ij}(t - r/c)$$

Distance To Source

Retarded Time

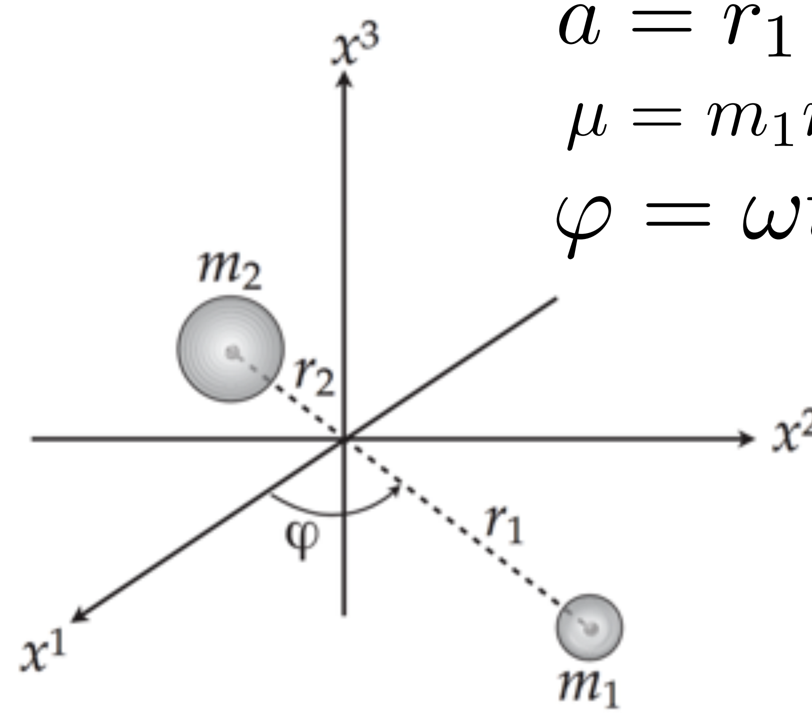
Physics: Gravitational Wave Generation - Binary Point Particles

- Let's calculate the *conservative* GW from orbiting point particles

$$I^{ij}(t) = \int x^i x^j \tau^{00}(t - r/c, \boldsymbol{x})\, d^3\boldsymbol{x}$$

Mass Density

$$a = r_1 + r_2$$
$$\mu = m_1 m_2 / M$$
$$\varphi = \omega t$$



$$x(t) = a\cos(\varphi)$$
$$y(t) = a\sin(\varphi)$$
$$z(t) = 0$$

- Assume circulate, planar motion

- Non-vanishing components of Quadrupole Tensor

$$I_{12} = I_{21}$$

$$I_{11} = \mu a^2 \frac{1+\cos(2\varphi)}{2} \quad I_{22} = \mu a^2 \frac{1-\cos(2\varphi)}{2} \quad I_{12} = \mu a^2 \frac{\sin(2\varphi)}{2}$$

Physics: Gravitational Wave Generation - Binary Point Particles

- 2nd time derivative of Quadrupole Tensor

$$\ddot{I}_{11} = -2\mu a^2 \omega^2 \cos(2\varphi)$$
$$\ddot{I}_{22} = 2\mu a^2 \omega^2 \cos(2\varphi)$$

$$\ddot{I}_{12} = \ddot{I}_{21} = -2\mu a^2 \omega^2 \sin(2\varphi)$$

- Final metric perturbation expression

$$h_{ij}^{\mathrm{TT}} = -\frac{4G\mu a^2\omega^2}{c^4 r}\begin{bmatrix} \cos 2\varphi & \sin 2\varphi & 0 \\ \sin 2\varphi & -\cos 2\varphi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Physics: Gravitational Wave Generation - Binary Point Particles

- GW Polarisations are thus:

$$\varphi = \omega t$$

$$h_+ = -\frac{4G\mu a^2 \omega^2}{c^4 r} \cos 2\varphi$$

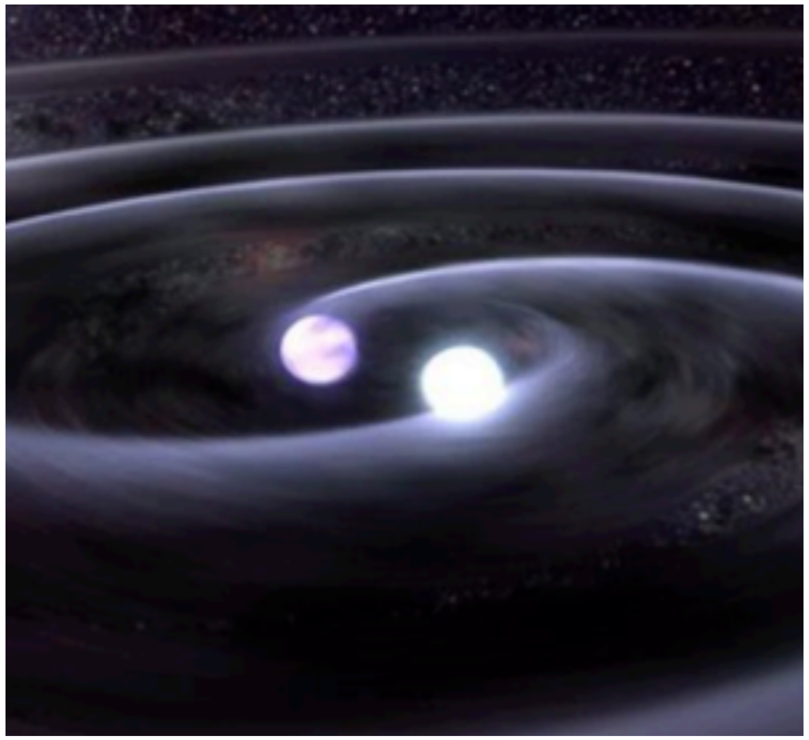
$$h_\times = -\frac{4G\mu a^2 \omega^2}{c^4 r} \sin 2\varphi$$

GW frequency is *twice* the orbital frequency (due to symmetry)

$$f_{\mathrm{GW}} = 2f_{orb} = \omega/\pi$$

Typical Amplitude: $\sim 10^{-21}$

GW Sources



Compact Binaries

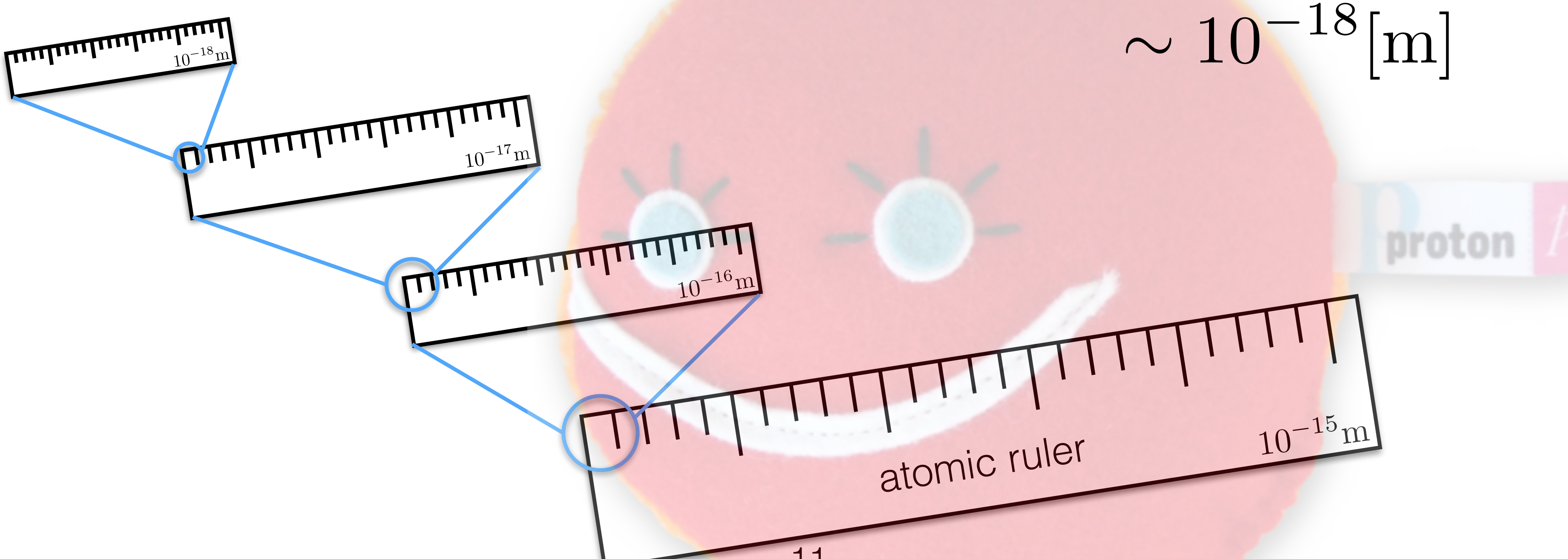
Typical strength of GW from compact binary mergers

$$h \sim 10^{-21}$$

Produces a change in length of a 4km arm of

$$\Delta L = 10^{-21} \times 4000[\mathrm{m}]$$

$$\sim 10^{-18}[\mathrm{m}]$$



Physics: Gravitational Wave Generation - Binary Point Particles

- From circular motion we can relate the separation and orbital angular frequency

$$v = a\omega$$

- Combine with Kepler's third law $GM = \omega^2 a^3$

- Relationship between orbital velocity and the GW frequency

$$v = (\pi G M f_{\mathrm{GW}})^{1/3}$$

Physics: Gravitational Wave Generation - Binary Point Particles

- Where is the inspiral?
- Energy is lost via GW emission
- Non-conservative effects, GW back reaction

GW Luminosity [Energy/Time]

3rd Time Derivative Quadrupole Tensor

$$L_{\mathrm{GW}} = \frac{1}{5}\frac{G}{c^5}\left\langle \dddot{I}_{11}^2 + \dddot{I}_{22}^2 + 2\dddot{I}_{12}^2 \right\rangle = \frac{32}{5}\frac{c^5}{G}\eta^2\left(\frac{v}{c}\right)^{10}$$

Suppressed by high power on velocity

where $\eta = \mu/M$ is the *symmetric mass ratio*

Physics: Gravitational Wave Generation - Binary Point Particles

- To calculate how the binary evolves under GW radiation reaction we equate the rate of change in (Newtonian) orbital energy to GW luminosity

$$E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{G m_1 m_2}{a} \quad = -\frac{1}{2} \mu v^2$$

- Energy balance equation $L_{\mathrm{GW}} = -dE/dt$

- Chain rule to get velocity w.r.t. time $\dfrac{dE}{dt} = \dfrac{dE}{dv}\dfrac{dv}{dt}$

$$\frac{d(v/c)}{dt} = \frac{32\eta}{5} \frac{c^3}{GM} \left(\frac{v}{c}\right)^9$$

Physics: Gravitational Wave Generation - Binary Point Particles

- From this we can compute the GW frequency evolution with time

$$\frac{df_{\rm GW}}{dt} = \frac{f_{\rm GW}}{dv}\frac{dv}{dt}$$

$$\frac{df_{\rm GW}}{dt} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f_{\rm GW}^{11/3}$$

"Chirp Mass" $\mathcal{M}_c = \eta^{3/5} M = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

- Frequency increases with time (chirping)

- Only depends on special combination of the masses

Physics: Estimating The Distance

- Putting it all together the GW polarisations take the form

Apparent "loudness" Intrinsic "loudness"

Amplitude Distance Phase

$$h \propto A(\mathcal{M}_c, r) \cos[\varphi(\mathcal{M}_c)]$$

- By measuring the chirp mass from the phase we can estimate the distance from the amplitude

- GWs are hence "self-calibrating" signals

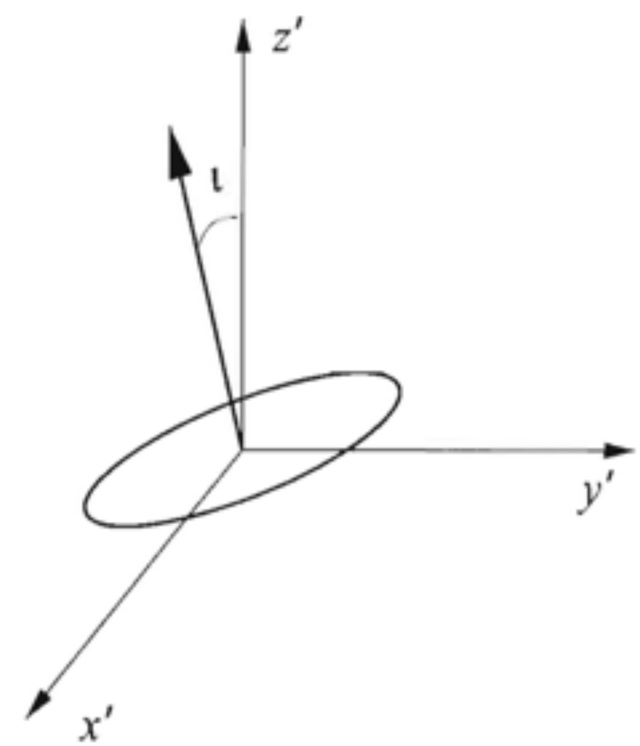
  Schutz Nature(1986)

Physics: Angular Behaviour

- GW polarisation including viewing angle

$$h_+ = (1 + \cos^2(\iota))A(t)\cos[\varphi(t)]$$
$$h_\times = \cos(\iota)A(t)\sin[\varphi(t)]$$

- Iota is the inclination angle
- Maximum amplitude for “face-on/away”
  - Circularly Polarised
- Cross polarisation is zero for “edge-on”
  - Linearly Polarised



Physics: Order of magnitude luminosity

- Gravitational Wave luminosity peaks around

$$L_{\mathrm{GW}} \propto G/c^5 \sim 3.6 \times 10^{52}\,\mathrm{W}$$

- Near merger binaries can this, albeit for a fraction of a second

- Compared to the Universe it is > 100 times more luminous

$$L_{\odot} \sim 10^{26}\,\mathrm{W}$$

$$L_{\mathrm{galaxy}} \sim 10^{36-38}\,\mathrm{W}$$

$$L_{\mathrm{Universe}} \sim 10^{46-50}\,\mathrm{W}$$

Going beyond Newtonian Order

- Matched filter based analyses require accurate models of the GW signal

- Accuracy requires us to go beyond Newtonian order

- Post-Newtonian (PN) theory is an expansion of the GR equations of motion in terms of the velocity v/c

- Count orders beyond Newtonian: nPN order: $(v/c)^{n/2}$

- For details see [Blanchet LLR]

Going beyond Newtonian Order

PN Kepler's 3rd Law $$\omega^2 = \frac{GM}{a^3}\left[1 + \frac{GM}{c^2 a}(\eta - 3) + ...\right]$$

Leads to PN orbital energy and GW Luminosity

$$E(v) = -\frac{1}{2}\eta v^2\left[1 - \left(\frac{3}{4} + \frac{1}{12}\eta\right)v^2 + ...\right]$$

$$L_{\mathrm{GW}} = \frac{32}{5}\frac{c^5}{G}\eta^2\left(\frac{v}{c}\right)^{10}\left[1 - \left(\frac{1247}{336} + \frac{35}{12}\eta\right)v^2 + ...\right]$$

Newtonian 1PN

CBC Parameters

- 15D - For non-eccentric, binary black holes

Component Masses

Distance

Sky Location

Polarisation Angle

$$m_1, m_2, \vec{S}_1, \vec{S}_2, D, \iota, \alpha, \delta, \psi, \phi_0, t_0$$

Component Spin Angular Momentum

Inclination

Reference Phase

Coalescence Time

CBC Parameters

- 15D - For non-eccentric, binary black holes

$$m_1, m_2, \vec{S}_1, \vec{S}_2, D, \iota, \alpha, \delta, \psi, \phi_0, t_0$$

- +2 for eccentricity e, f_{e}

- +1 (at least) for each Neutron Stars

$$\Lambda_{\mathrm{NS}} \longleftarrow$$ Neutron Star Tidal Deformability

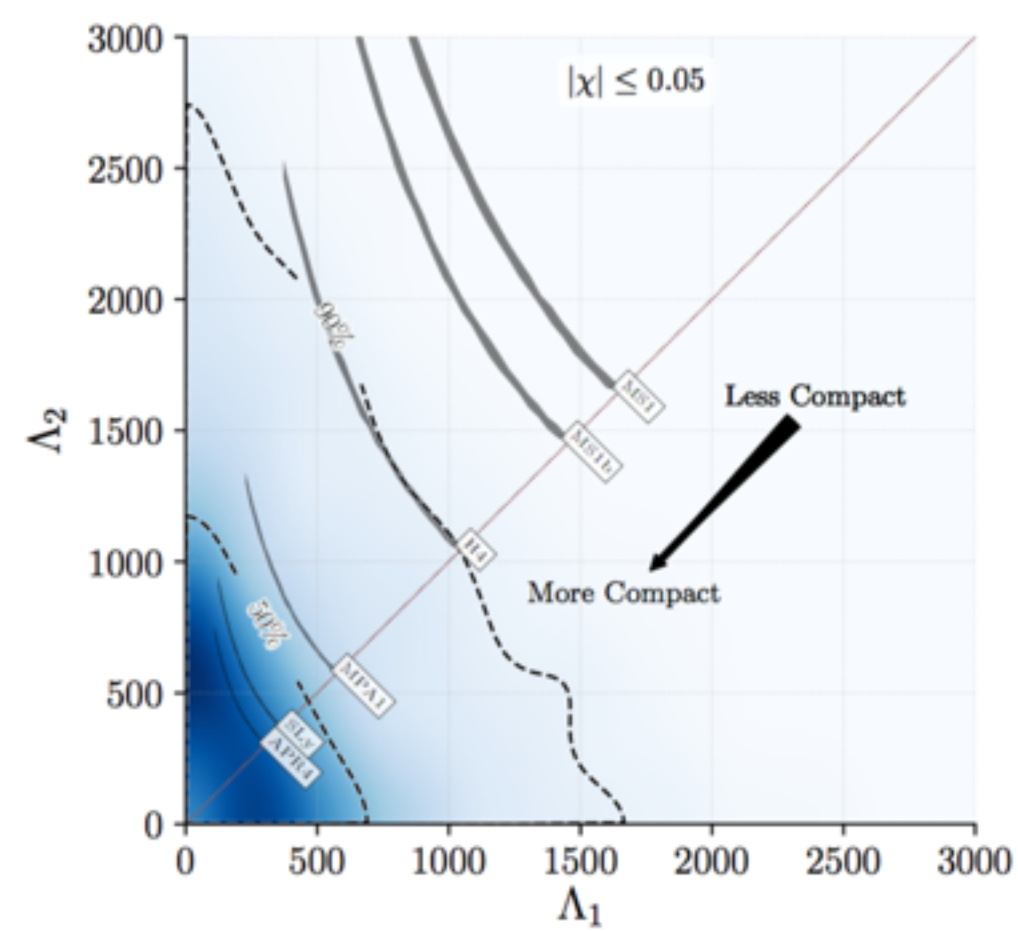
$$\Lambda_{\mathrm{BH}} = 0$$

- Tidal Effect start 5PN order

- Tidal deformability encodes the Equation of state $P = P(\rho)$

- Determines the properties of NS

GW170817 Results



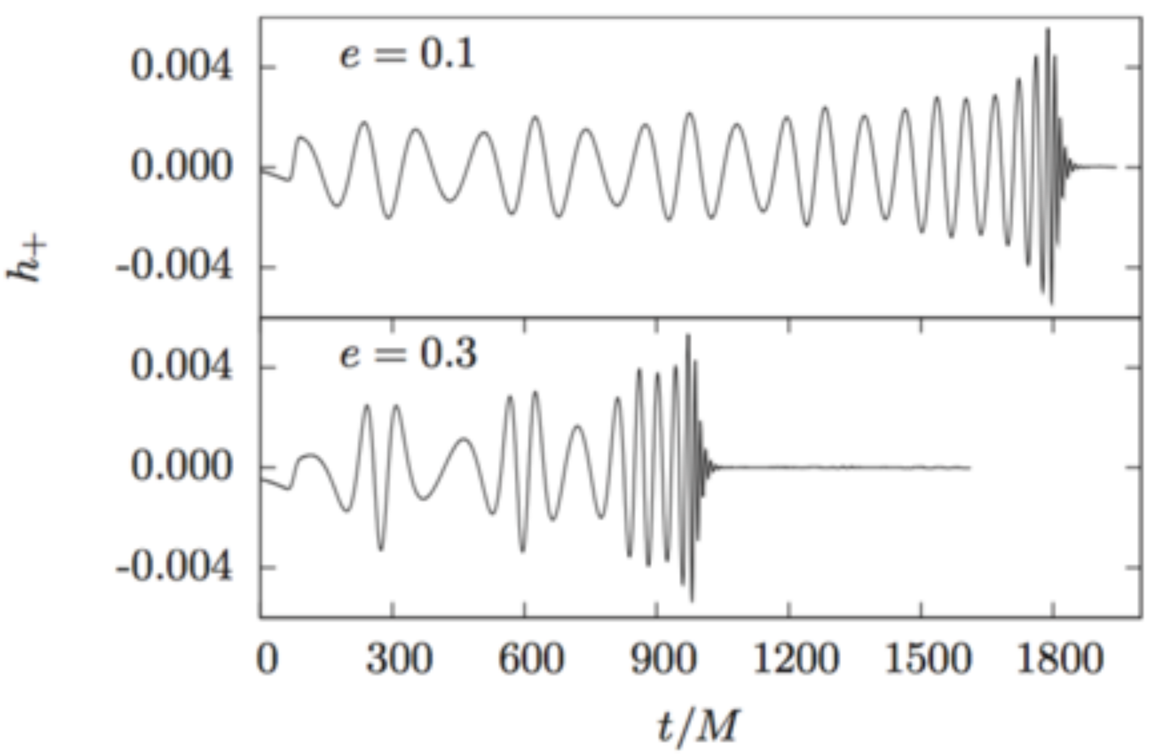
CBC Parameters

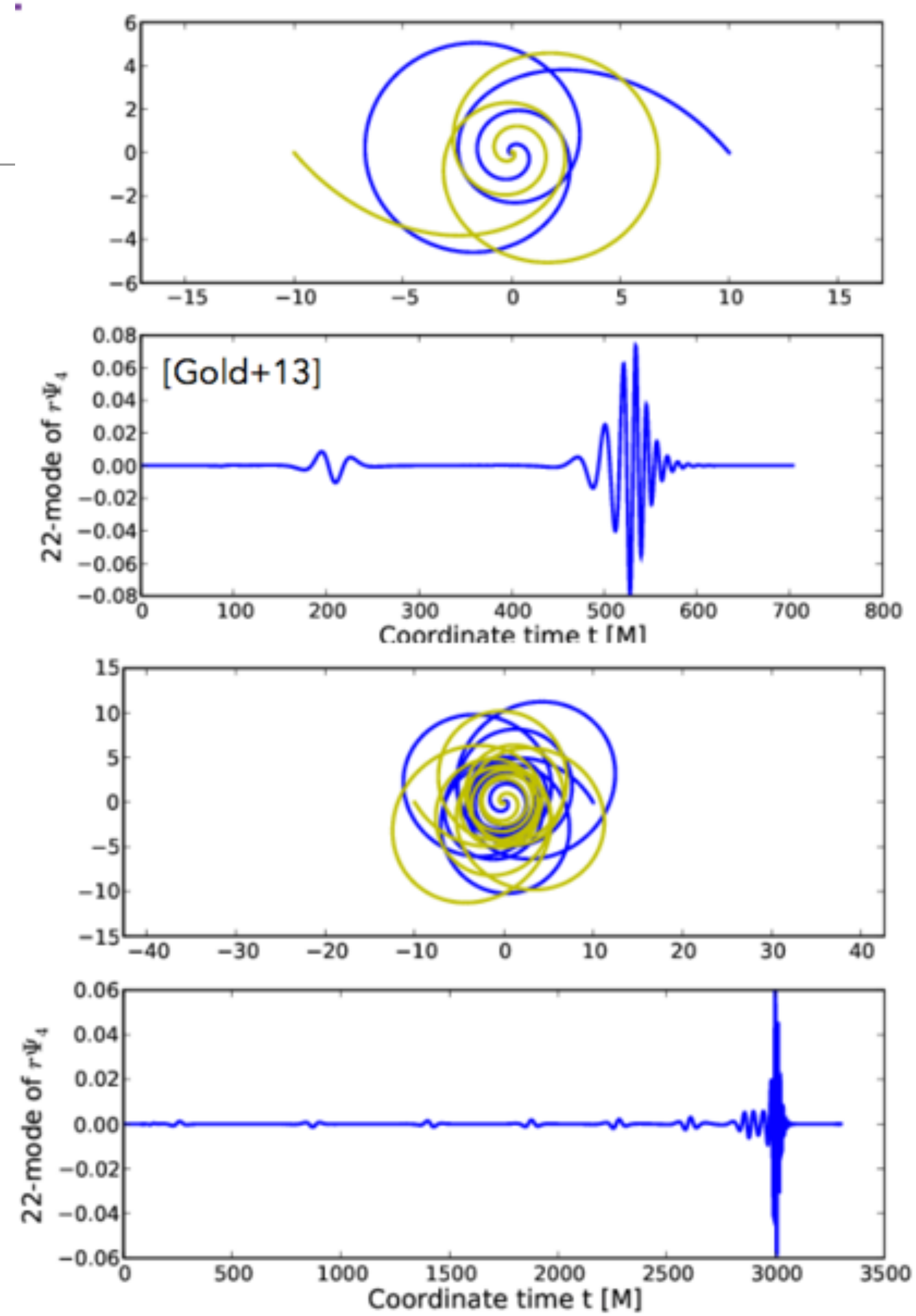
- Evolution of binaries via GW have a very rich phenomenology
- Which mainly depends on the intrinsic parameters

Intrinsic	Extrinsic
$m_1, m_2, \vec{S}_1, \vec{S}_2,$ $e, f_\mathrm{e}\ \Lambda_1, \Lambda_2$	$D, \iota, \alpha, \delta, \psi, \phi_0, t_0$
Changes dynamics	Changes how we observe the same physical system

Eccentricity

- Eccentricity - GWs tend to circularise the orbit [Peters&Matthews+63]
- Burst like features during periapsis
- We expect LIGO-Virgo BBHs to have circularised



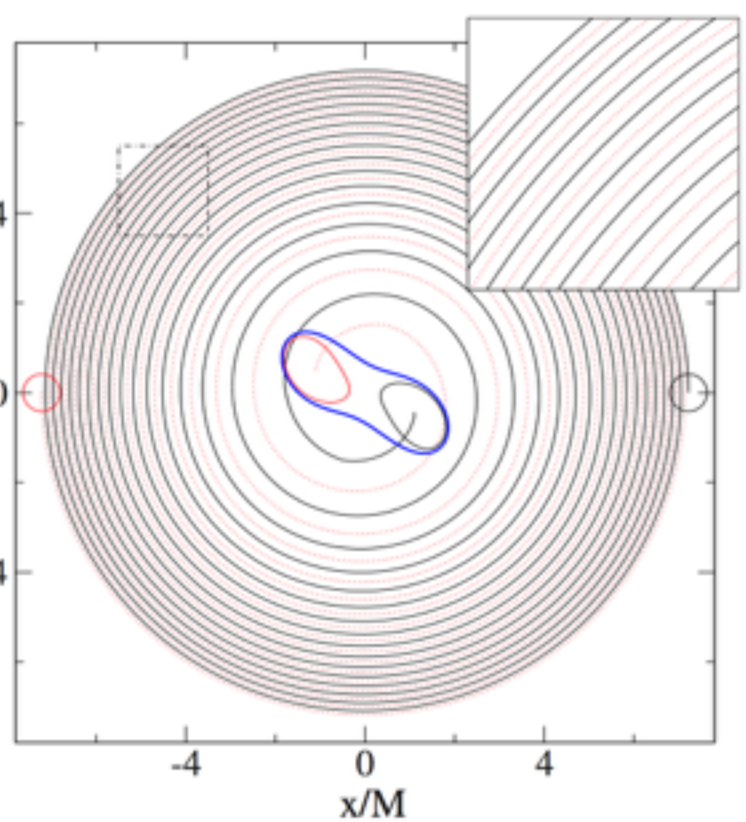


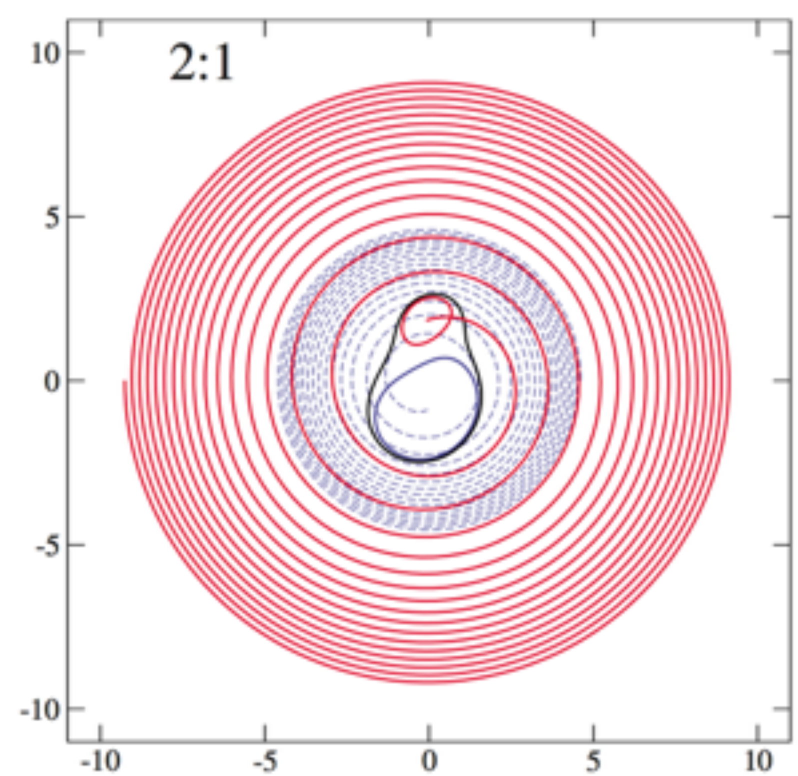
Mass-Ratio

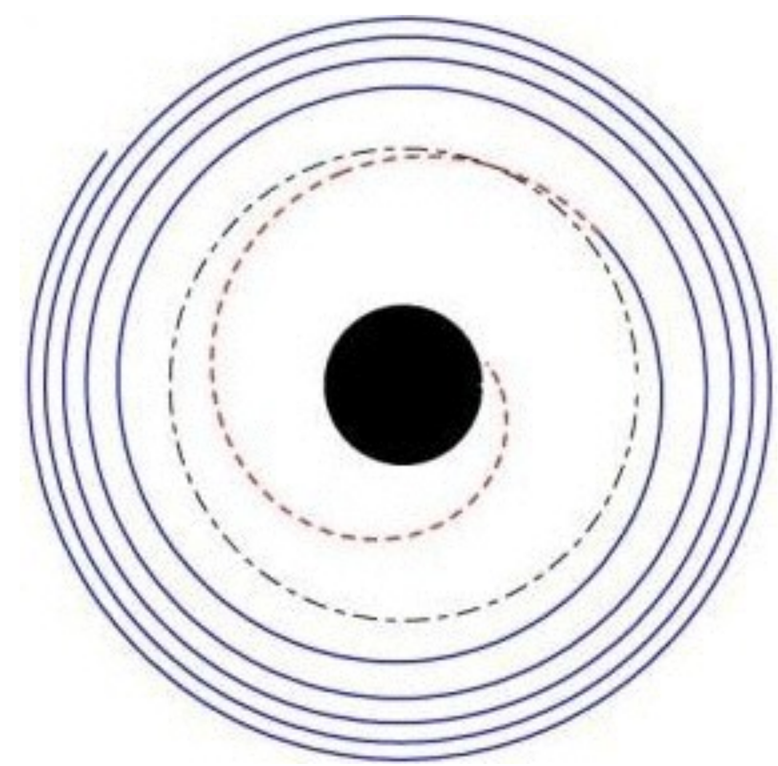
$$q = m_1/m_2 \quad (m_1 \geqslant m_2)$$

- For Binary Black Holes the total mass sets the time scale
- Mass-Ratio sets the dynamics

Increasing Mass-Ratio







Higher Modes

$$h_+ - i h_\times = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\iota, \phi) h_{\ell m}$$

- Typical to decompose into spherical harmonics*
- Leading order: Quadrupole

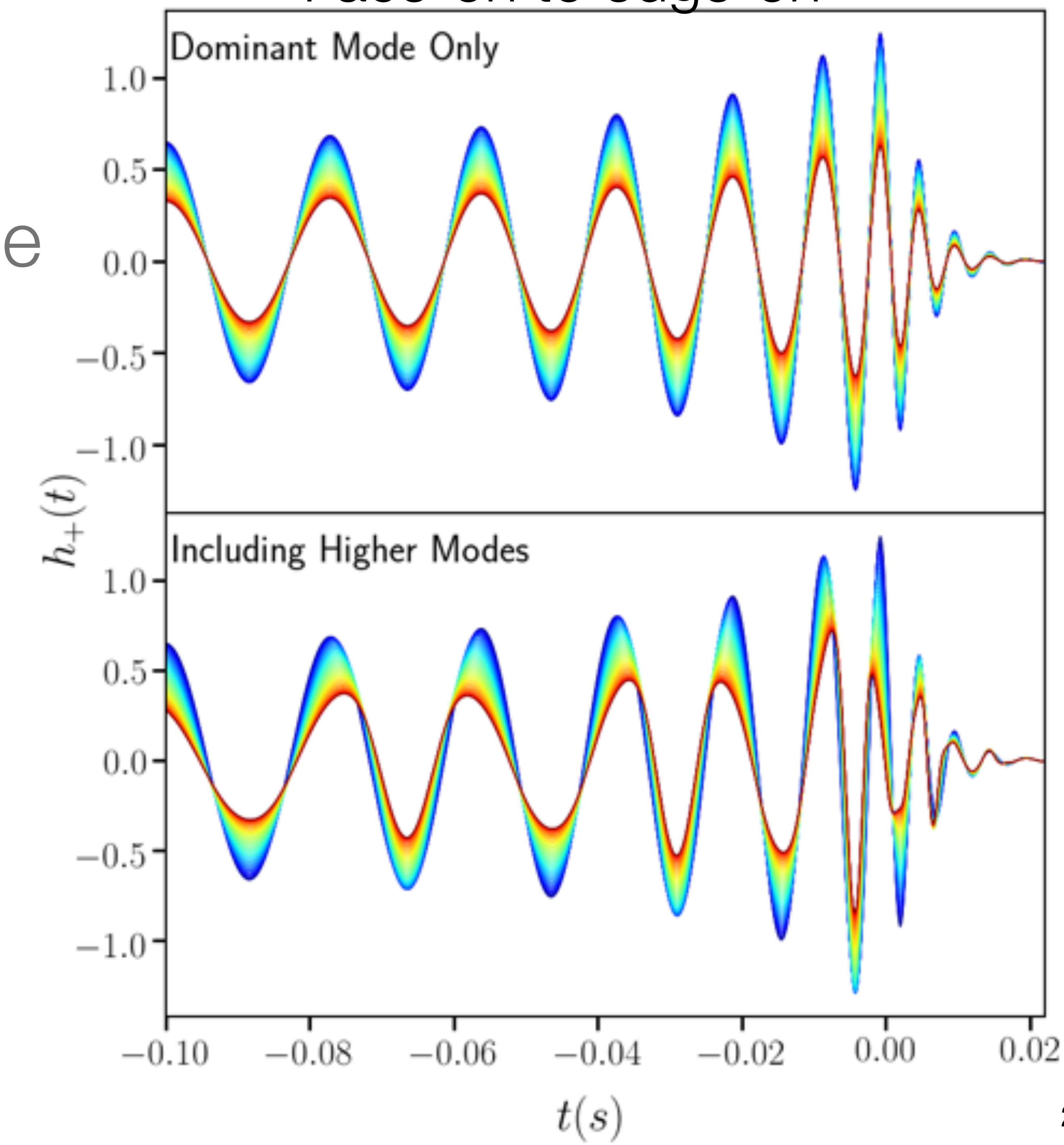
$$\ell = |m| = 2$$

- All others "Higher Modes"

$$\ell \neq |m| \neq 2$$

- Important for high mass-ratio, inclination and precession

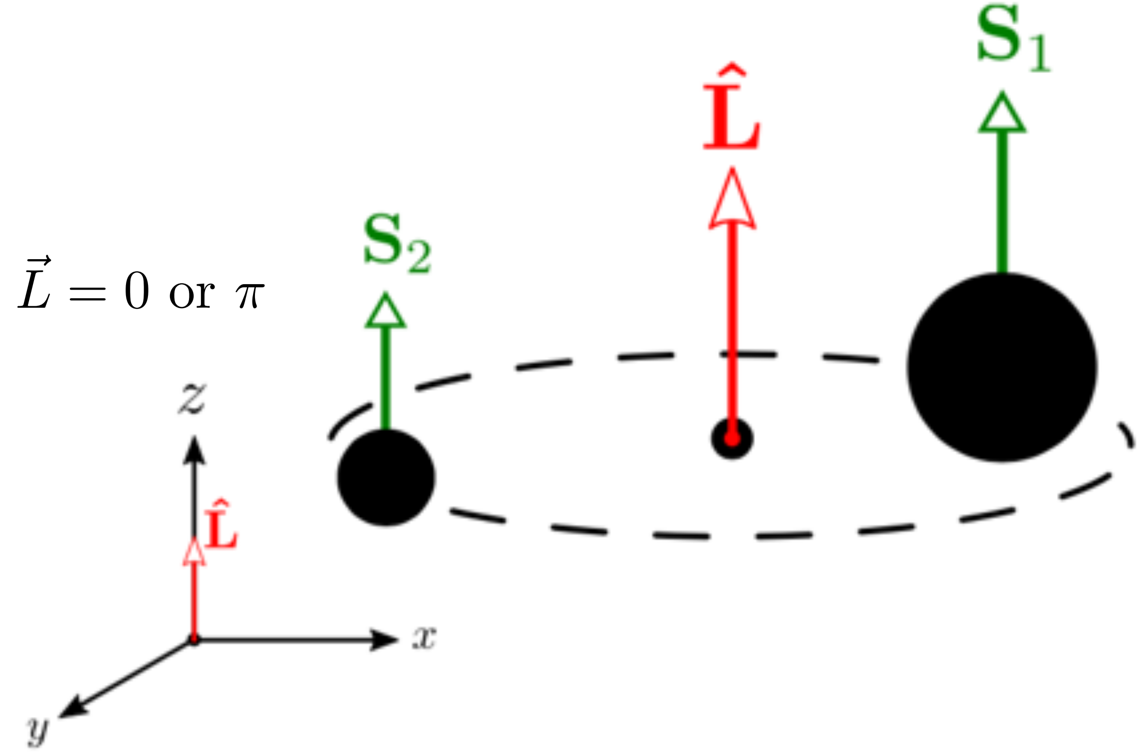
Blue to Red
Face-on to edge-on

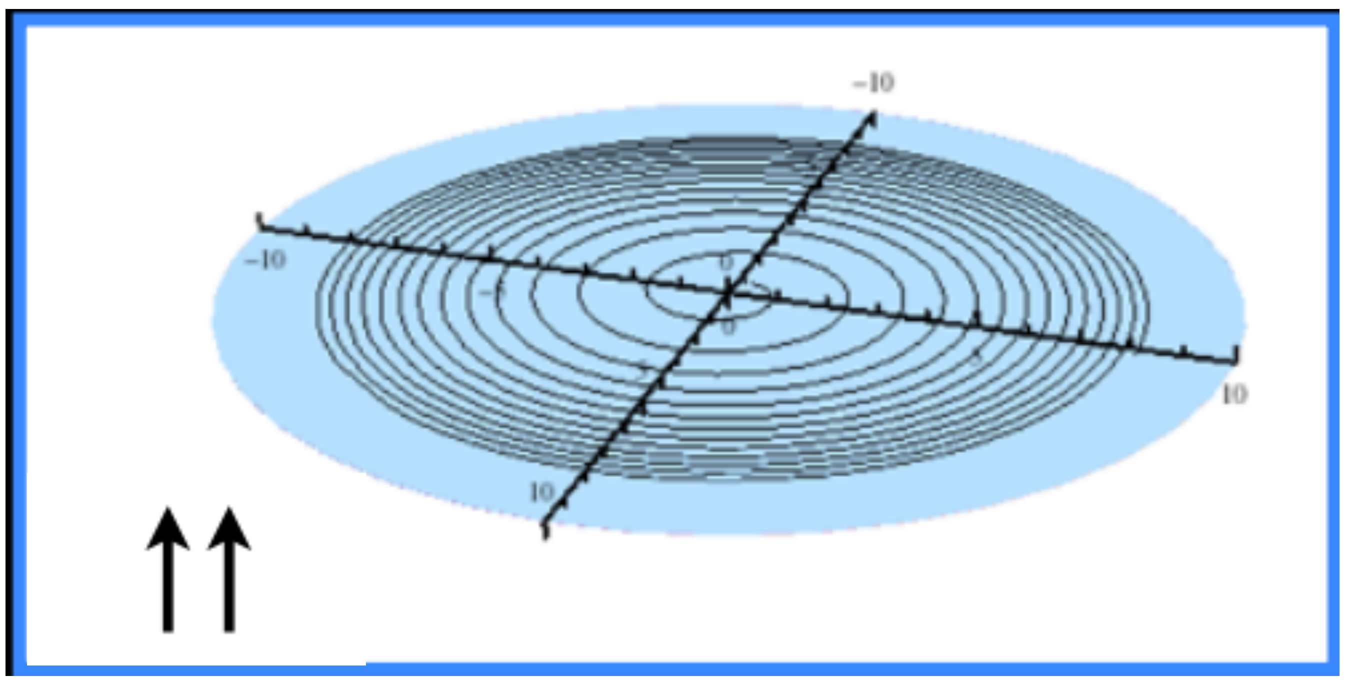


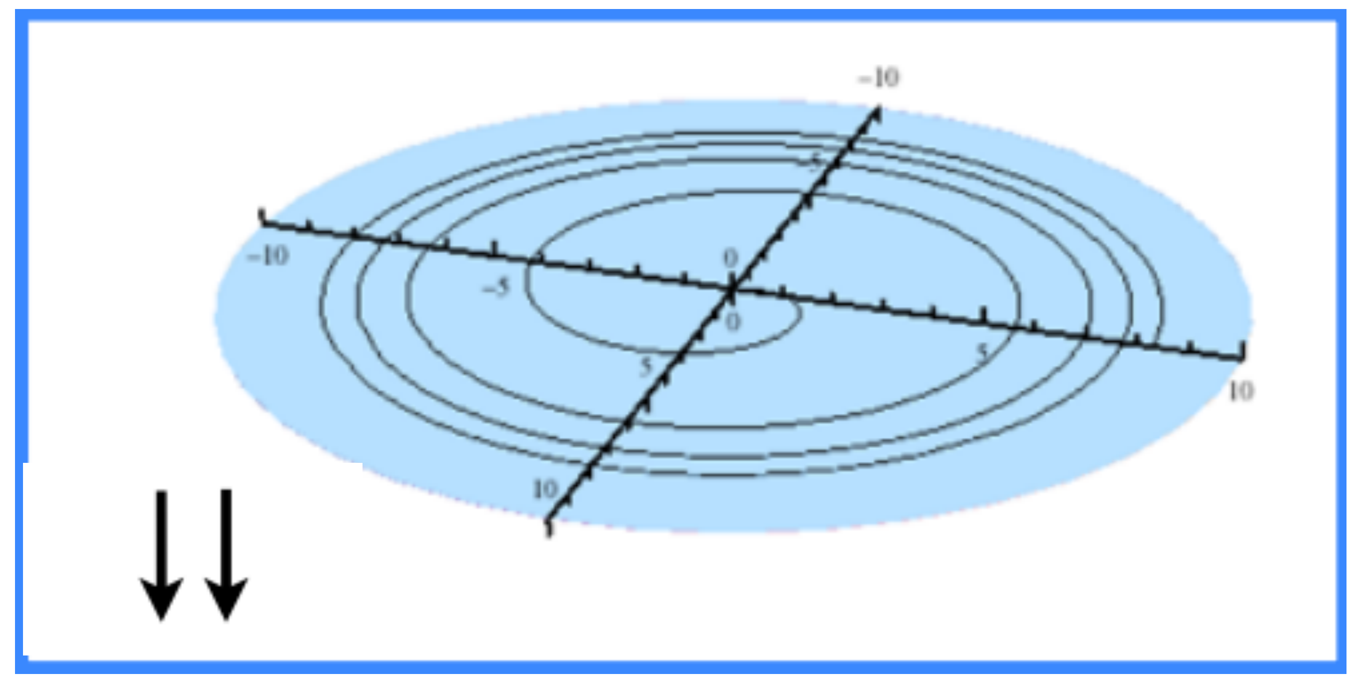
*Spin Weight = -2

Spin - Aligned

$\vec{S}_i \cdot \vec{L} = 0 \text{ or } \pi$







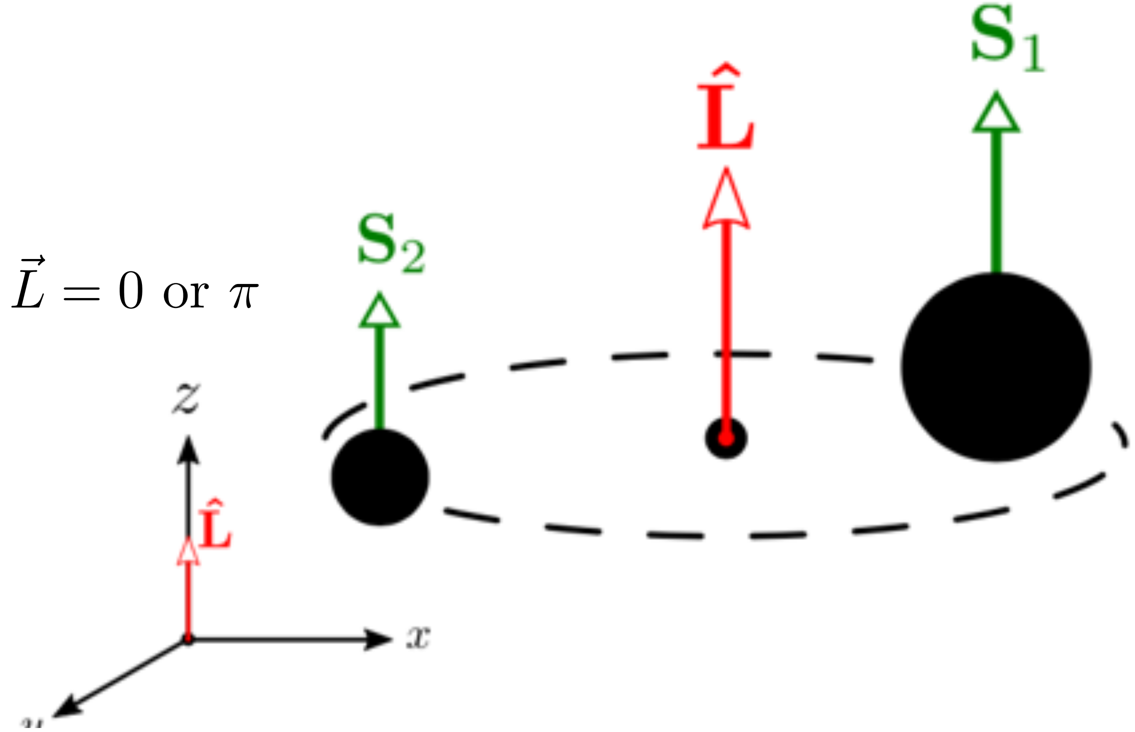
Orbital “Hang-up” Effect

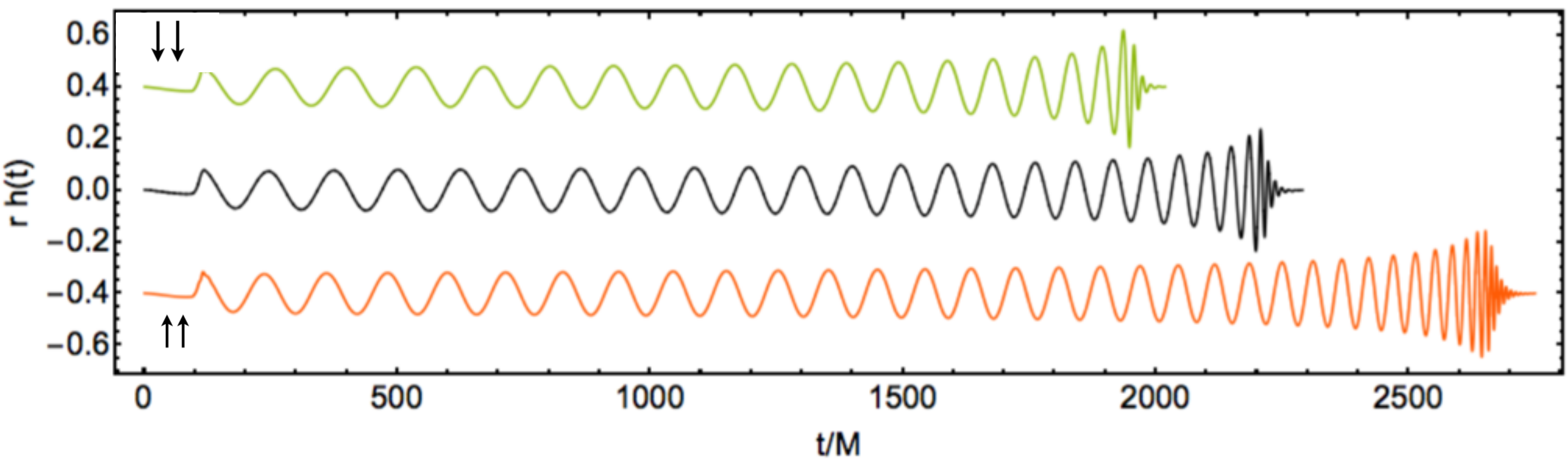
BHs with aligned spins can orbit each other for longer

Image Credit: Mark Hannam

Spin - Aligned

$\vec{S}_i \cdot \vec{L} = 0 \text{ or } \pi$

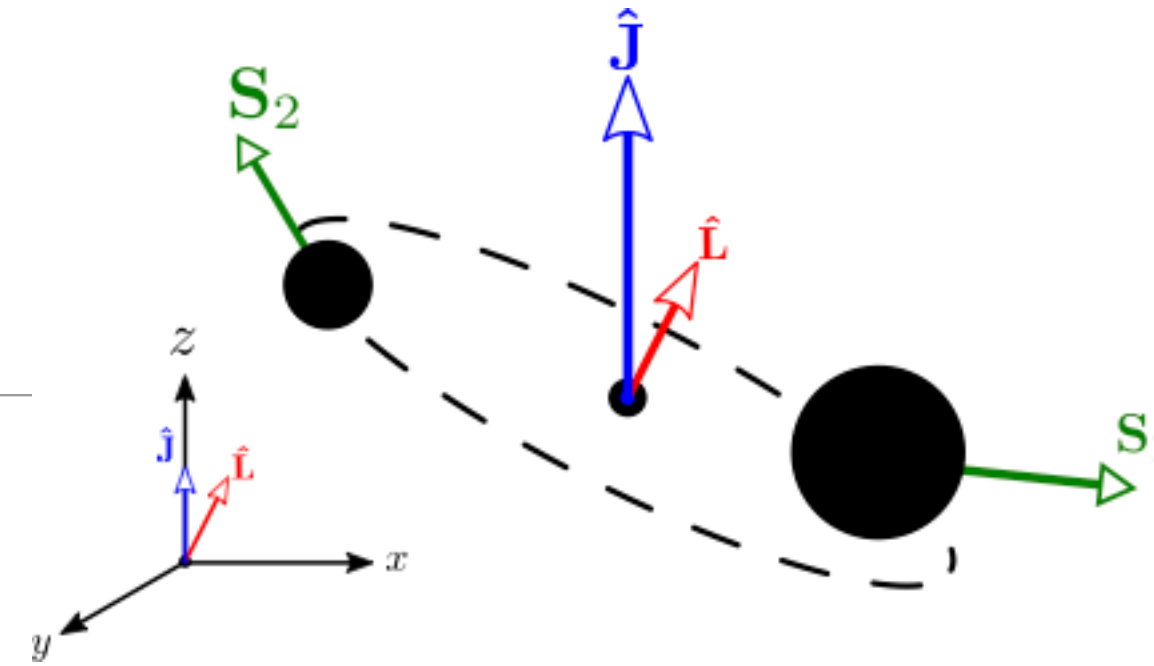


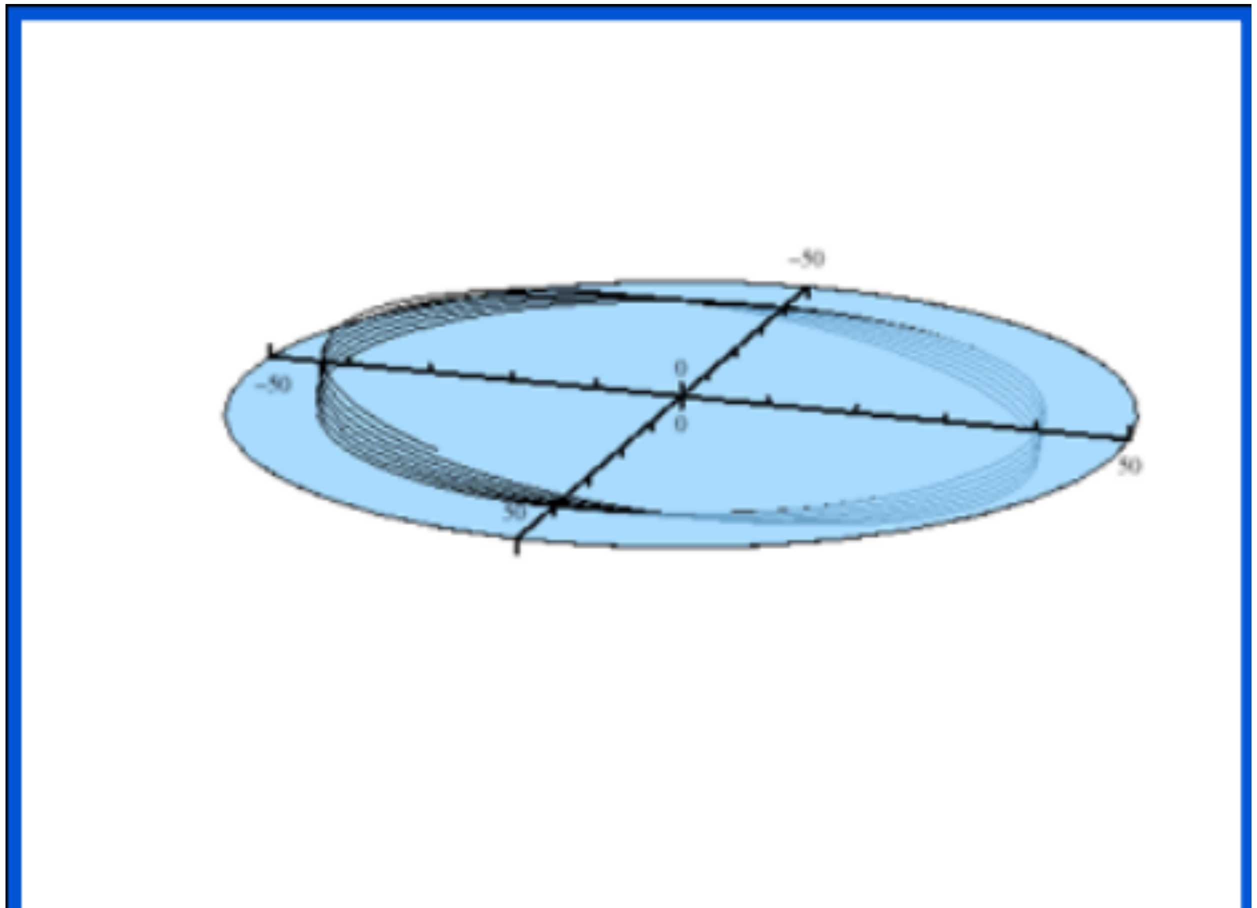


Visible in the waveform

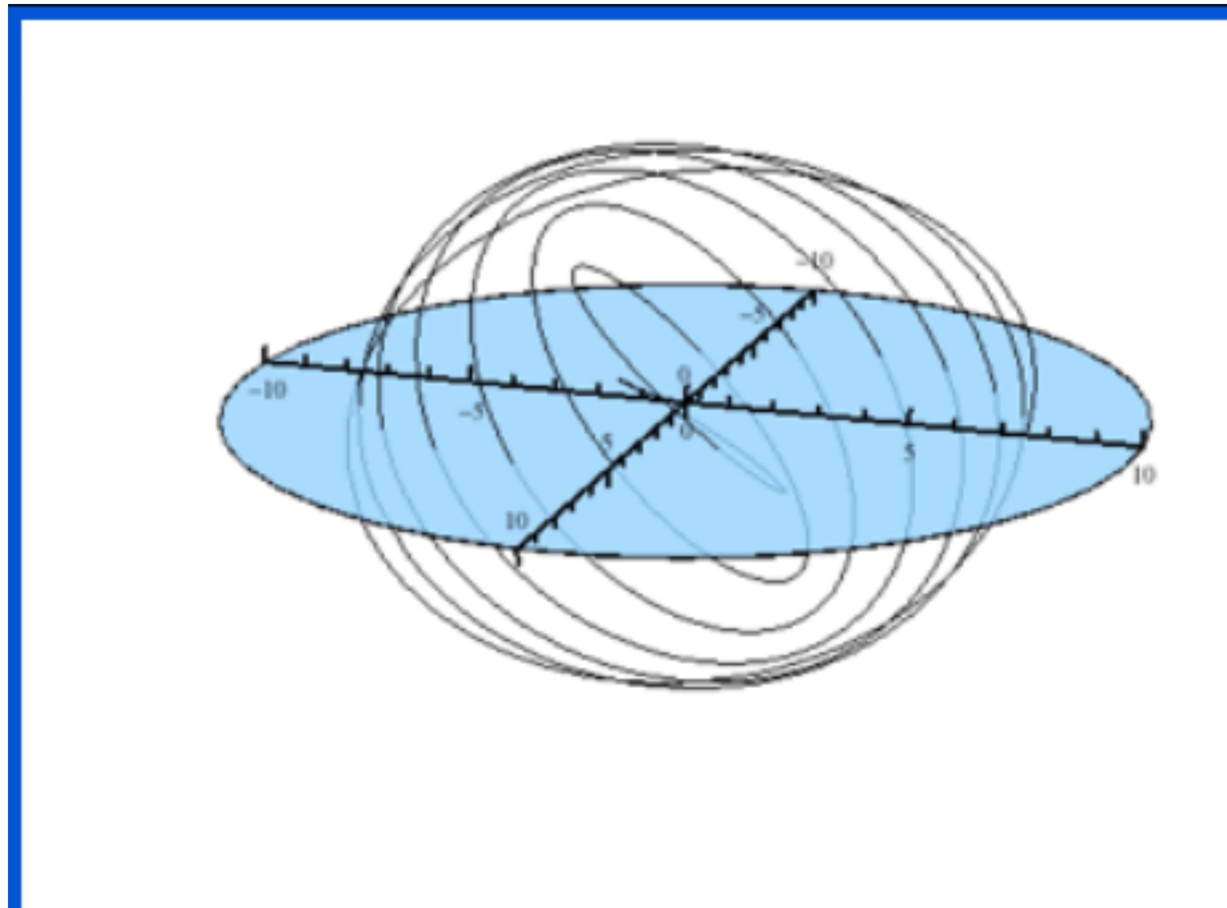
Orbiting closer implies orbiting faster. Emit GWs at higher frequencies

Spin Precession





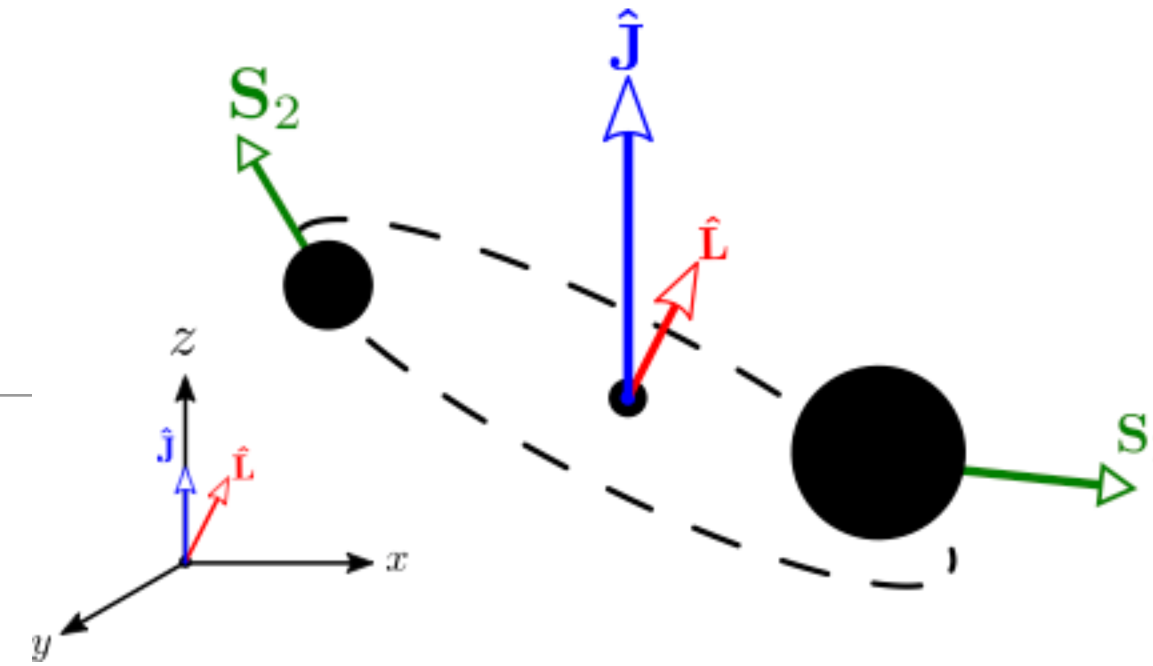
During Inspiral
Slowly tilting orbital plane

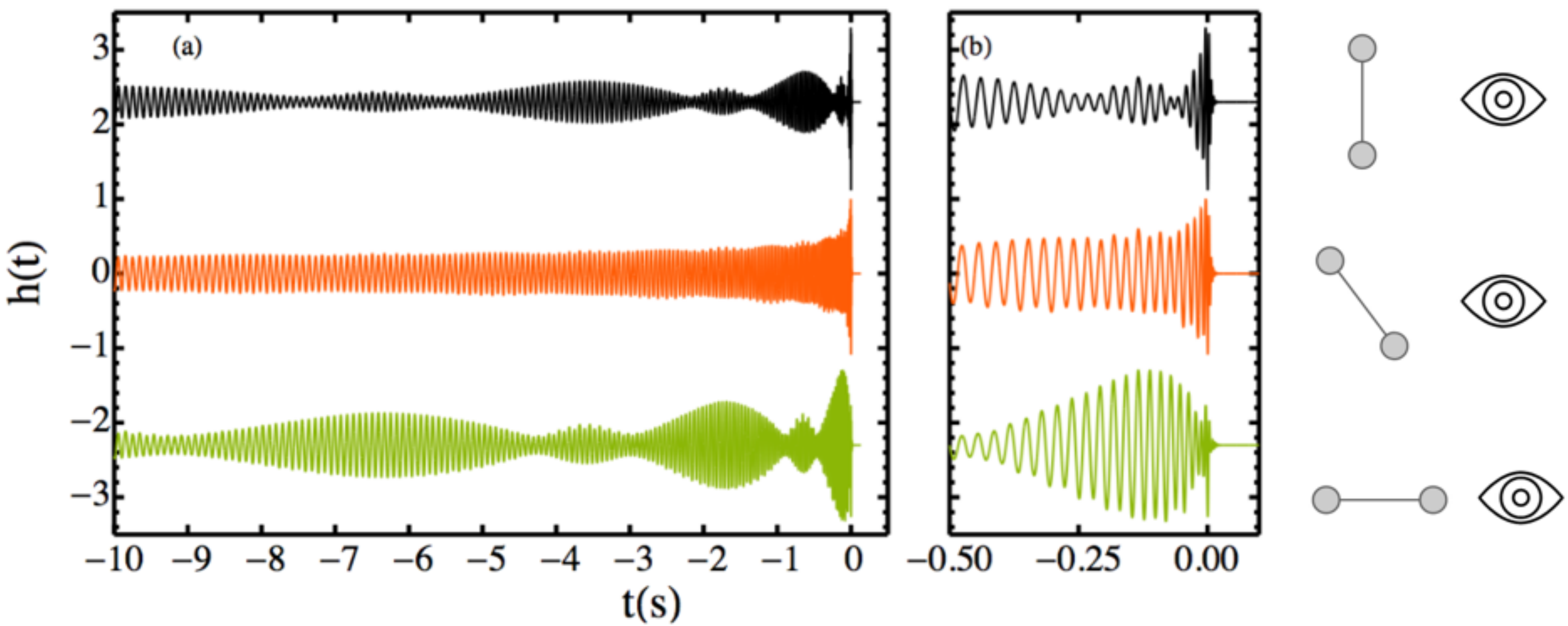


Near Merger
Rapidly changing orbital plane

Image Credit: Mark Hannam

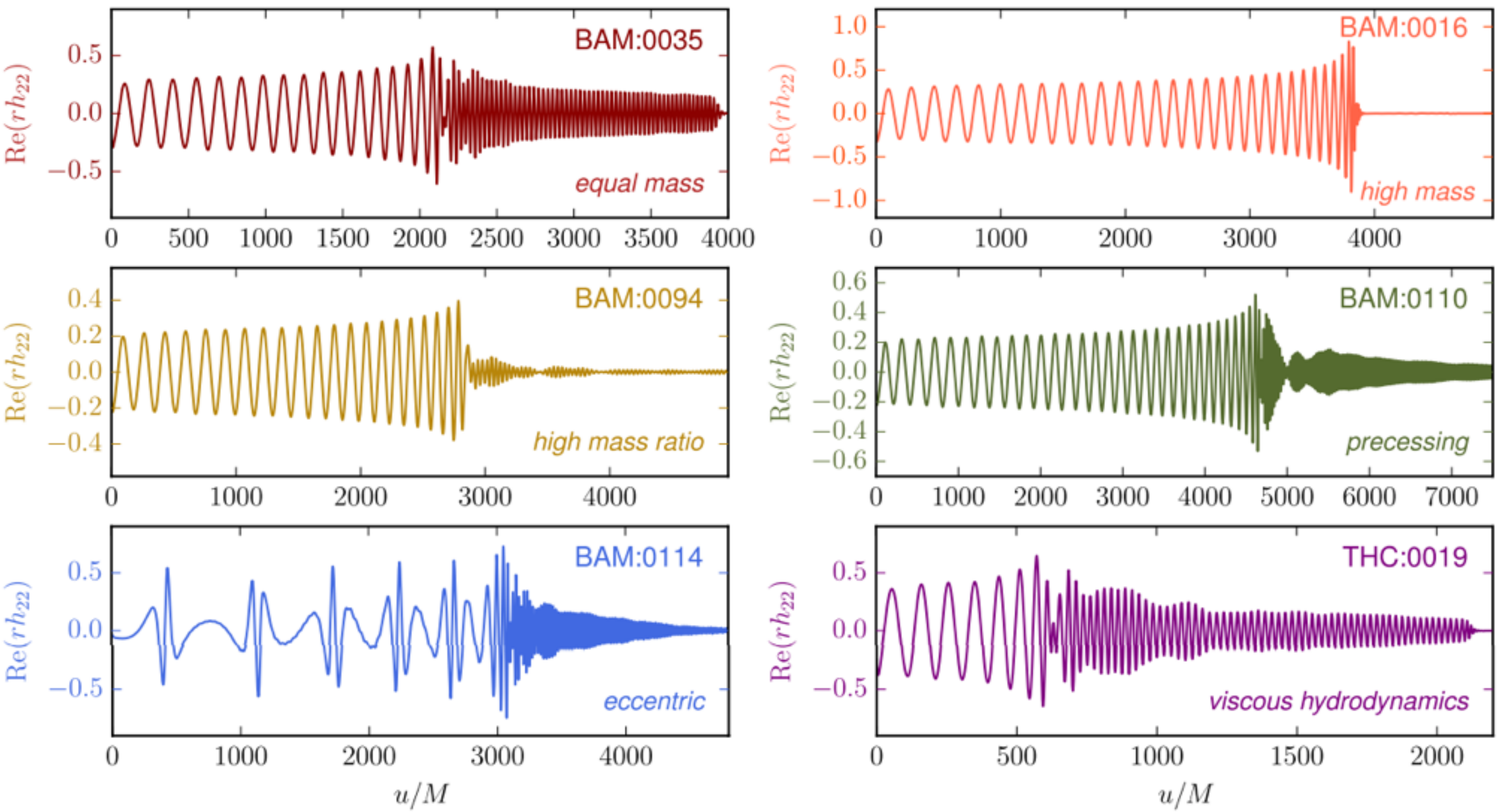
Spin Precession





Viewed at different inclination angles the waveform can look very different

Binary Neutron Stars

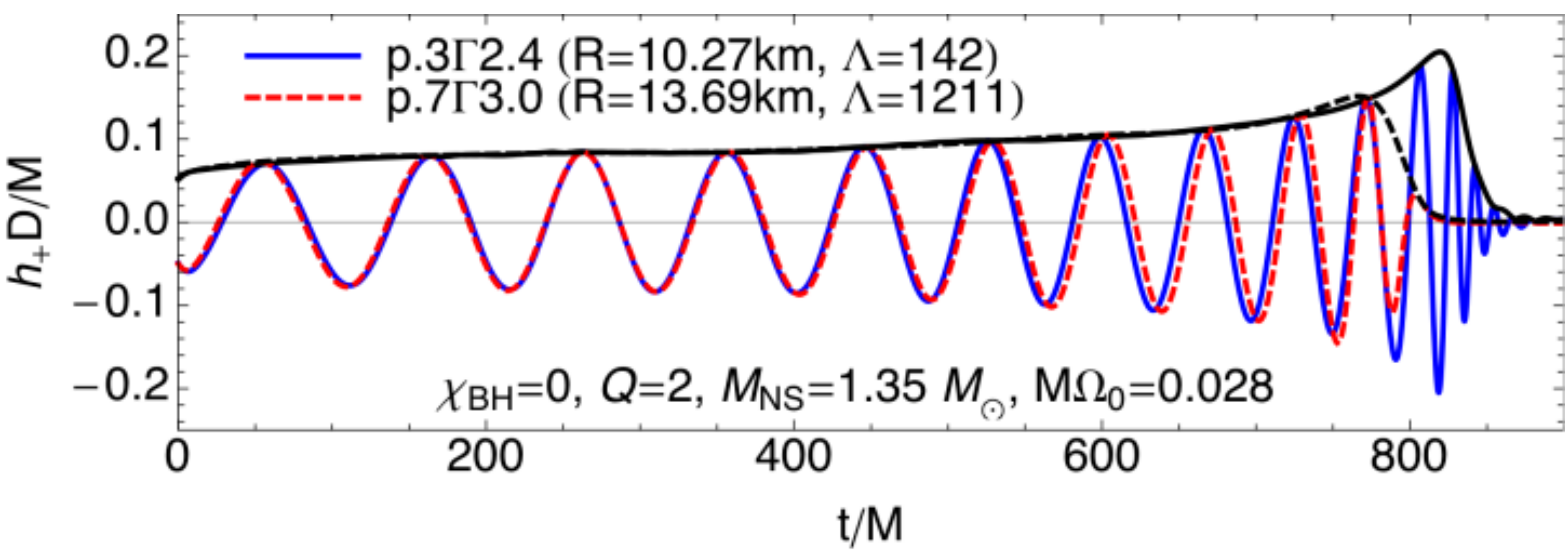


Dietrich et al

Mixed Binary: Neutron Star - Black Hole

- Depending on parameters e.g. mass-ratio, tidal deformability the Neutron star could be *tidally disrupted.*

- In such cases the GW inspiral emission terminates early



Degeneracies

- Along certain directions in parameter space (changing combinations of parameters appropriately) the resulting dynamics and waveform look very similar

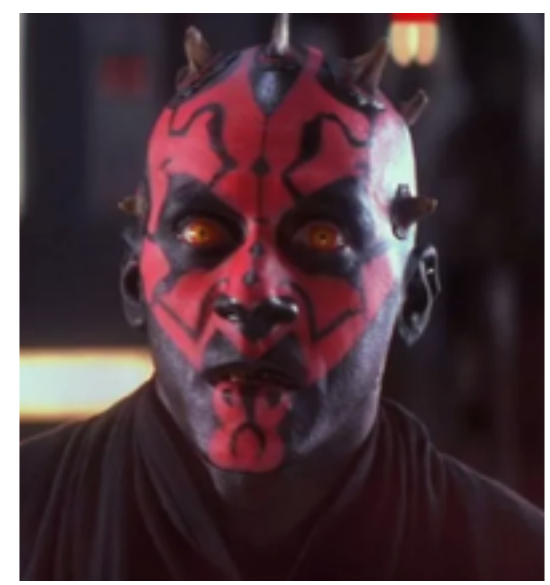
- Double edged laser sword

Degeneracies

- Along certain directions in parameter space (changing combinations of parameters appropriately) the resulting dynamics and waveform look very similar

- Double edged laser sword

Effective Dimensionality Reduced, More efficient Sampling, …



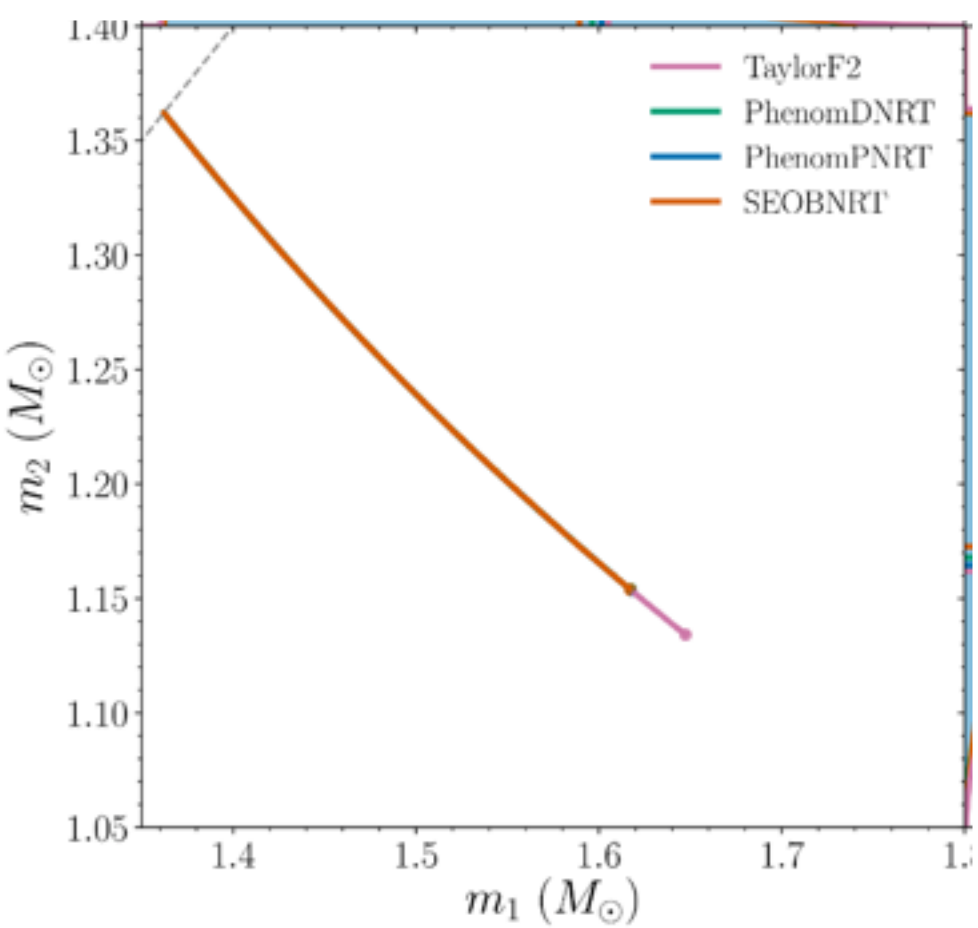
Ability to do physics reduced :(

Degeneracies - Chirp Mass

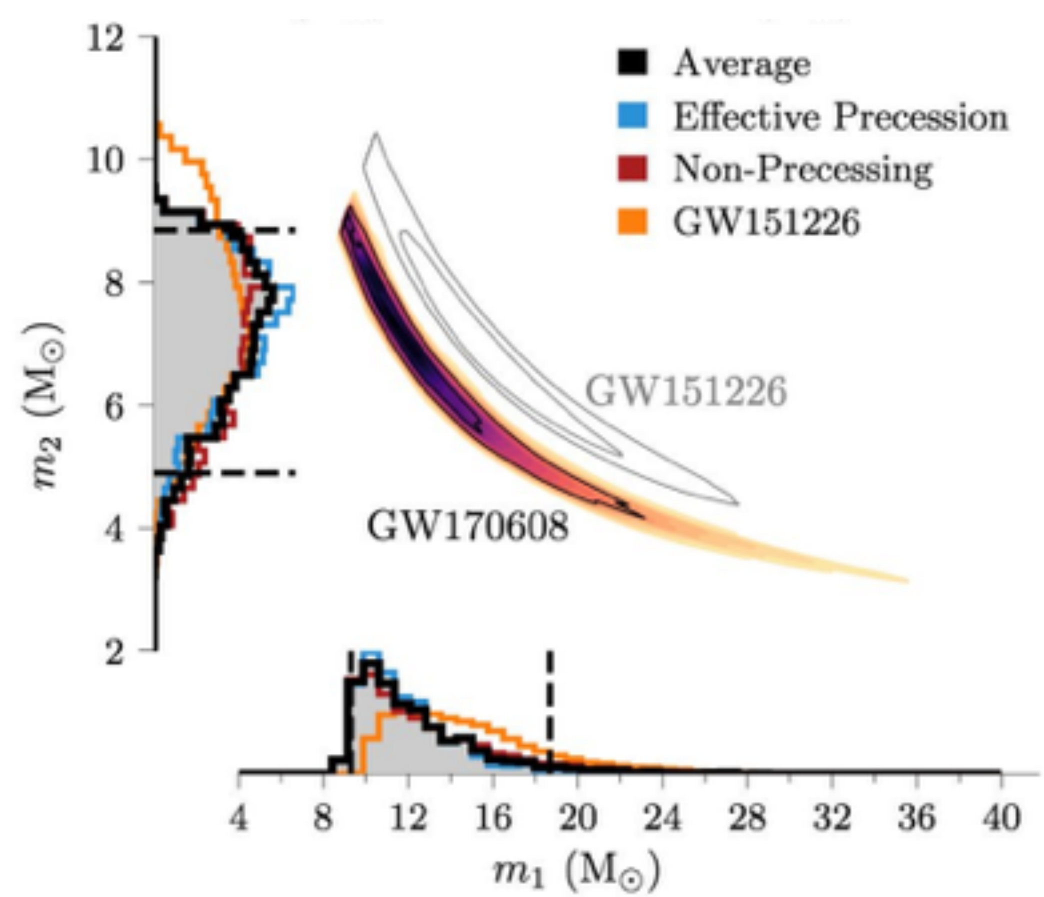
Chirp Mass $$\mathcal{M}_c = \eta^{3/5} M = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Leading order parameter

Degeneracy stronger for lower mass binaries



Binary Neutron Star



Binary Black Hole

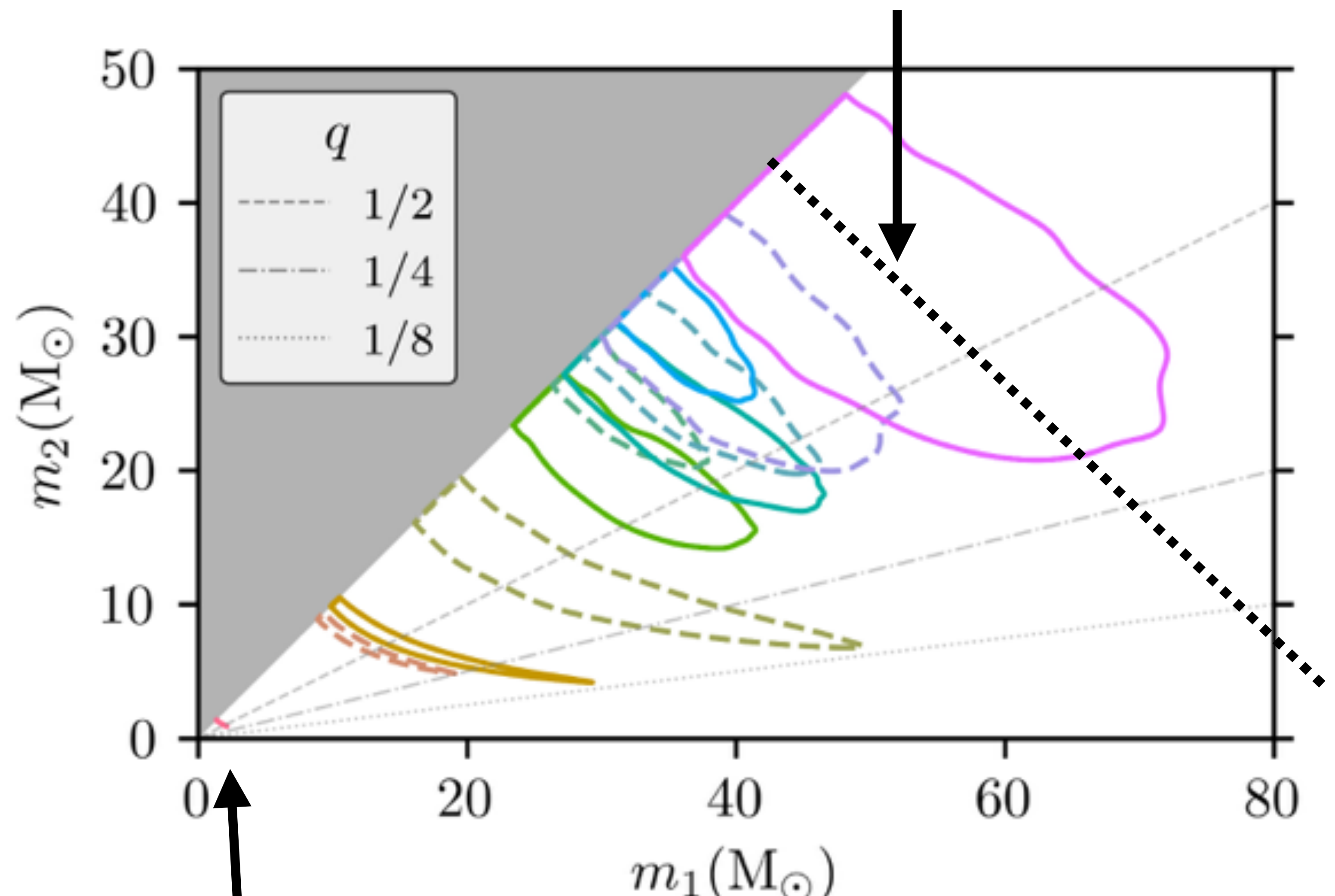
Degeneracies - Chirp Mass

As the mass increases the total mass become the relevant parameter
Parameter close to final BH

- Limits our ability to measure the individual masses and the mass-ratio

- Measuring individual masses is vital to informing Supernova physics - mass gaps

Const Total Mass



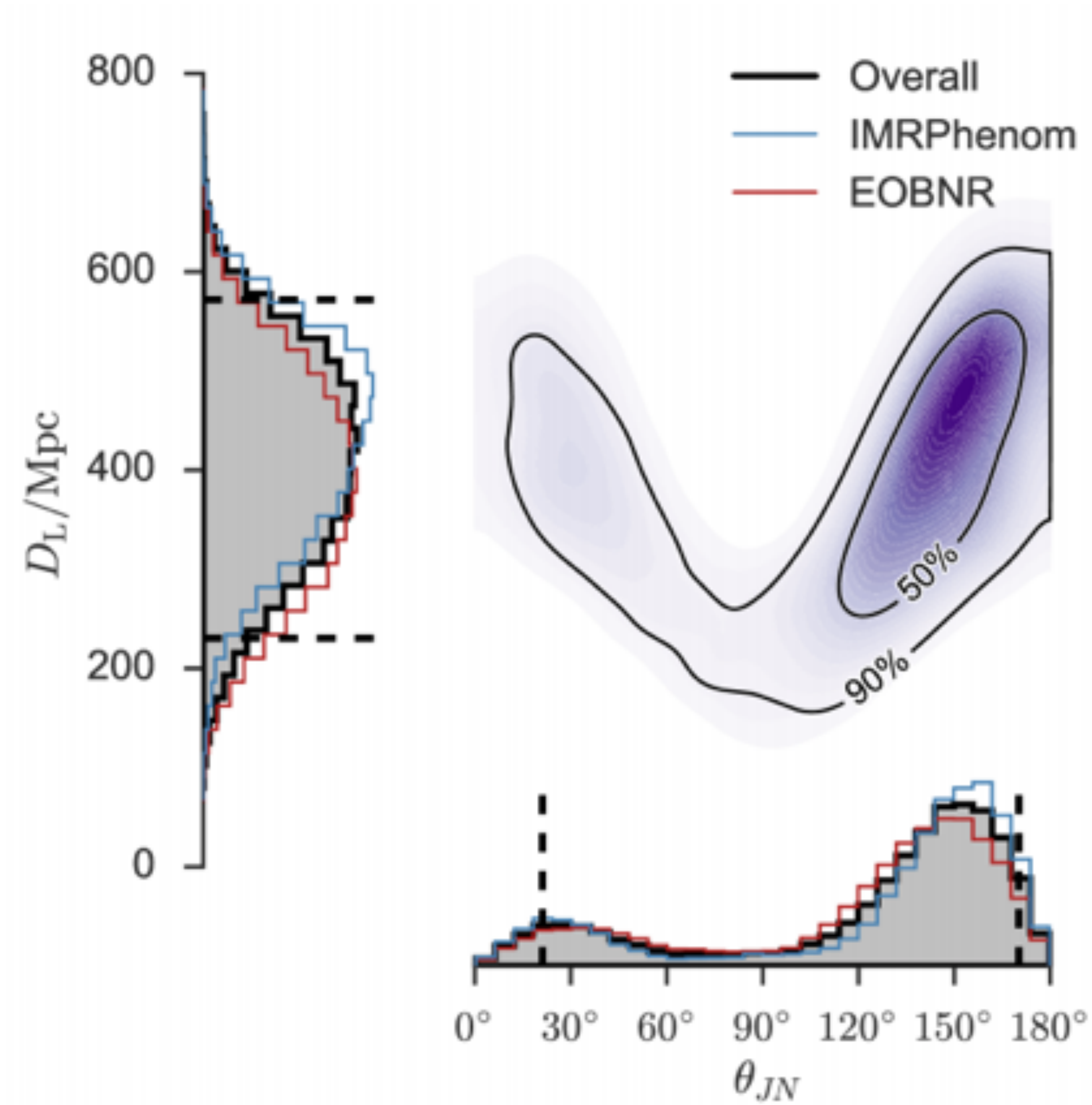
Binary Neutron Star

Degeneracies - Distance/Inclination

- When including only the quadrupole term (as is often done) then the changing the inclination angle only changes the amplitude, not the phasing.

- Change in amplitude is identical to a change in distance

- Including higher modes can help break this degeneracy



Degeneracies - Mass-ratio/Spin

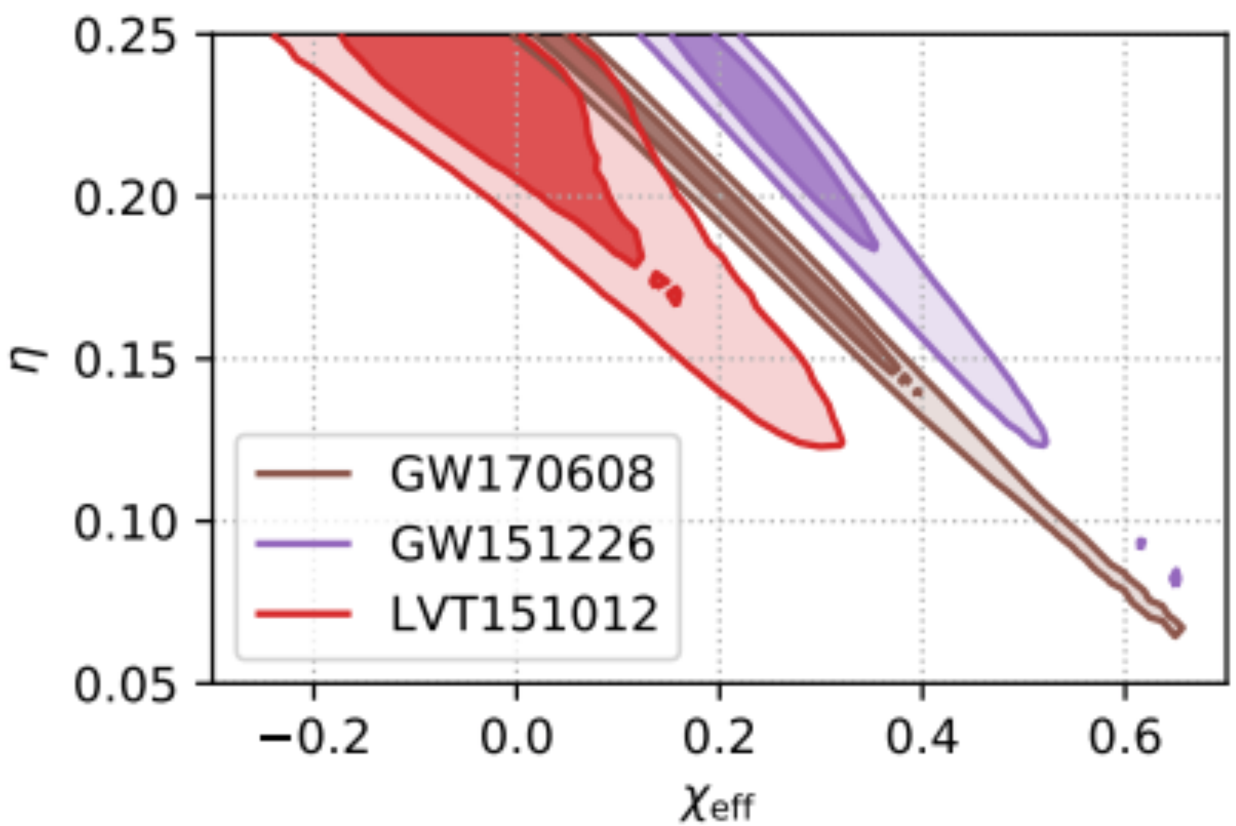
- In order to see this degeneracy easily let's return to Post-Newtonian theory

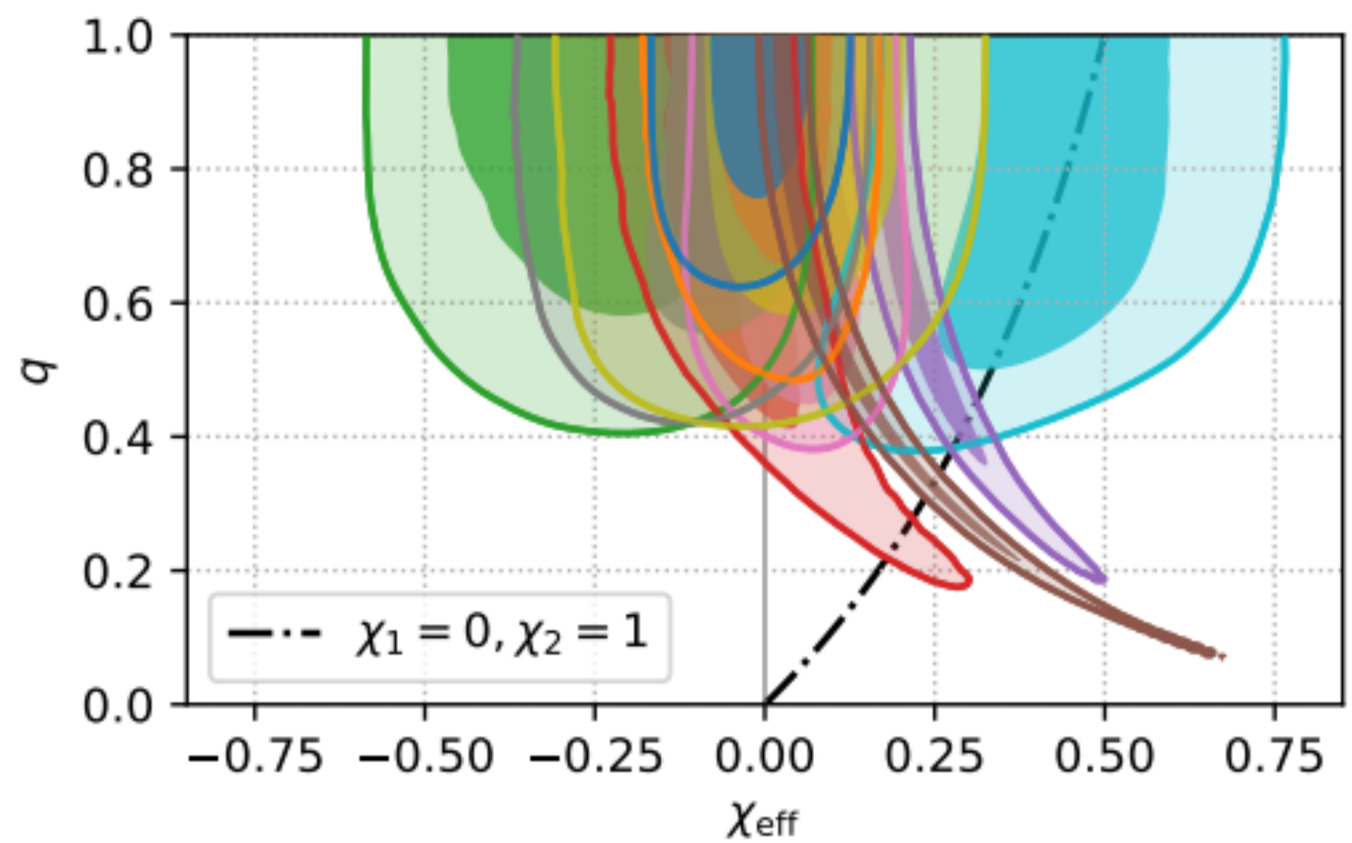
- At 1.5PN order a particular combination of mass-ratio and spin appears in the same term.

$$E(v) = -\frac{M\eta v^2}{2}\left\{1 + v^2\left(-\frac{3}{4} - \frac{\eta}{12}\right) + v^3\left[\frac{8\,\delta\chi_a}{3} + \left(\frac{8}{3} - \frac{4\eta}{3}\right)\chi_s\right]\right.$$

- Over a certain region of parameter space there terms can compensate for each other

Degeneracies - Mass-ratio/Spin





Ref: 1806.10610

	Type	Domain	Spin	HM	Ecc	Tidal	Notes
TaylorF2	Inspiral	FD	Non-Prec	No	In Progress	Yes	Fast PN Inspiral
SpinTaylorT(1,2,5)	Inspiral	TD	Prec	In progress	No	Yes	PN Precession
SEOBNRv4(_opt)	IMR	TD	Non-Prec	No	No	No	Fast version use ROM
SEOBNRv4HM	IMR	TD	Non-Prec	Yes	No	No	HM extension of above
SEOBNRv3(_opt)	IMR	TD	Prec	No	No	No	Precessing version of SEOBNRv2 (Improvement underway)
SEOBNRv4T	Inspiral-Merger (BNS)	TD	Non-Prec	No	No	Yes Faster version see SEOBNRv4T_surrogate	Tidal version of SEOBNRv4
SEOBNRv4_ROM	IMR	FD	Non-Prec	No	No	No	Fast FD version of SEOBNRv4
IMRPhenomD	IMR	FD	Non-Prec	No	No	No See IMRPhenomD_NRTidal	
IMRPhenomHM	IMR	FD	Non-Prec	Yes	No	No	HM extension of above
IMRPhenomPv2	IMR	FD	Effective Prec	No	No	No See IMRPhenomPv2_NRTidal	Precessing Version of PhenomD
IMRPhenomPv2_NRTidal	Inspiral-Merger (BNS)	FD	Effective Prec	No	No	Yes	Tidal version of PhenomPv2
NRSur7dq2	IMR	TD	Prec	Yes	No	No	Very accurate, restricted range
NRHybSur3dq8	IMR	TD	Non-Prec	Yes	No	No	Very accurate, restricted range
Lackey_Tidal_2013_SEOBNRv2_ROM	IMR NSBH	FD	Non-Prec	No	No	Yes	restricted range: $2 < q < 5$

Waveform Models: Not an exhaustive list…

Things Not Discussed

- Other degeneracies
  - Sky Location
  - Phase and Polarisation
- Numerical Relativity
- Waveform Modelling
- Black Hole Perturbation Theory (for the ringdown)